Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

4-Trig-functions/4.1-Sine/72-4.1.1.3-g-tan-^p-a+b-sin-^m

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [208]. This is test number [72].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	99.04 (206)	0.96 (2)
Mathematica	96.63 (201)	3.37 (7)
Maple	85.58 (178)	14.42 (30)
Fricas	85.58 (178)	14.42 (30)
Giac	75.00 (156)	25.00 (52)
Mupad	74.04 (154)	25.96 (54)
Maxima	67.79 (141)	32.21 (67)
Sympy	2.40 (5)	97.60 (203)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

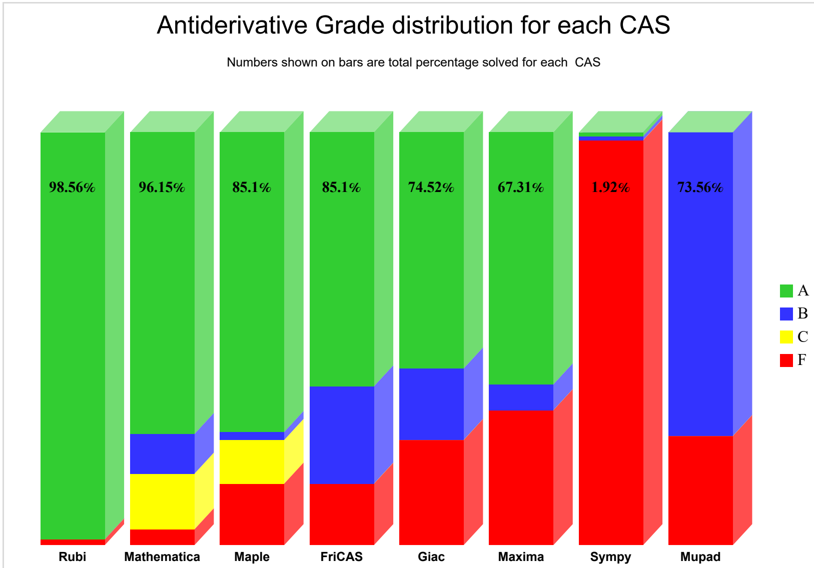
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

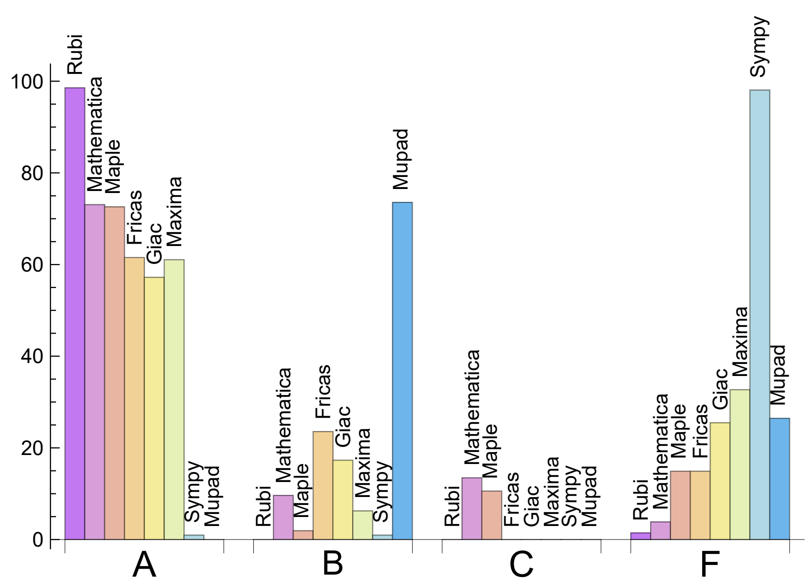
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.558	0.000	0.000	1.442
Mathematica	73.077	9.615	13.462	3.846
Maple	72.596	1.923	10.577	14.904
Fricas	61.538	23.558	0.000	14.904
Maxima	61.058	6.250	0.000	32.692
Giac	57.212	17.308	0.000	25.481
Sympy	0.962	0.962	0.000	98.077
Mupad	0.000	73.558	0.000	26.442

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	2	100.00	0.00	0.00
Mathematica	7	100.00	0.00	0.00
Fricas	30	86.67	13.33	0.00
Maple	30	100.00	0.00	0.00
Giac	52	57.69	40.38	1.92
Mupad	54	0.00	100.00	0.00
Maxima	67	67.16	10.45	22.39
Sympy	203	94.09	5.91	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.24
Fricas	0.32
Rubi	0.52
Mathematica	2.50
Giac	4.13
Maple	5.77
Mupad	7.19
Sympy	15.87

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	94.40	1.67	78.00	1.73
Maxima	125.87	1.16	95.00	1.01
Maple	132.85	1.09	116.00	1.05
Rubi	138.89	1.04	113.50	1.00
Fricas	263.16	1.81	157.50	1.46
Mathematica	264.42	1.91	115.00	1.02
Mupad	306.85	2.36	231.50	2.31
Giac	1579.29	18.60	156.00	1.42

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

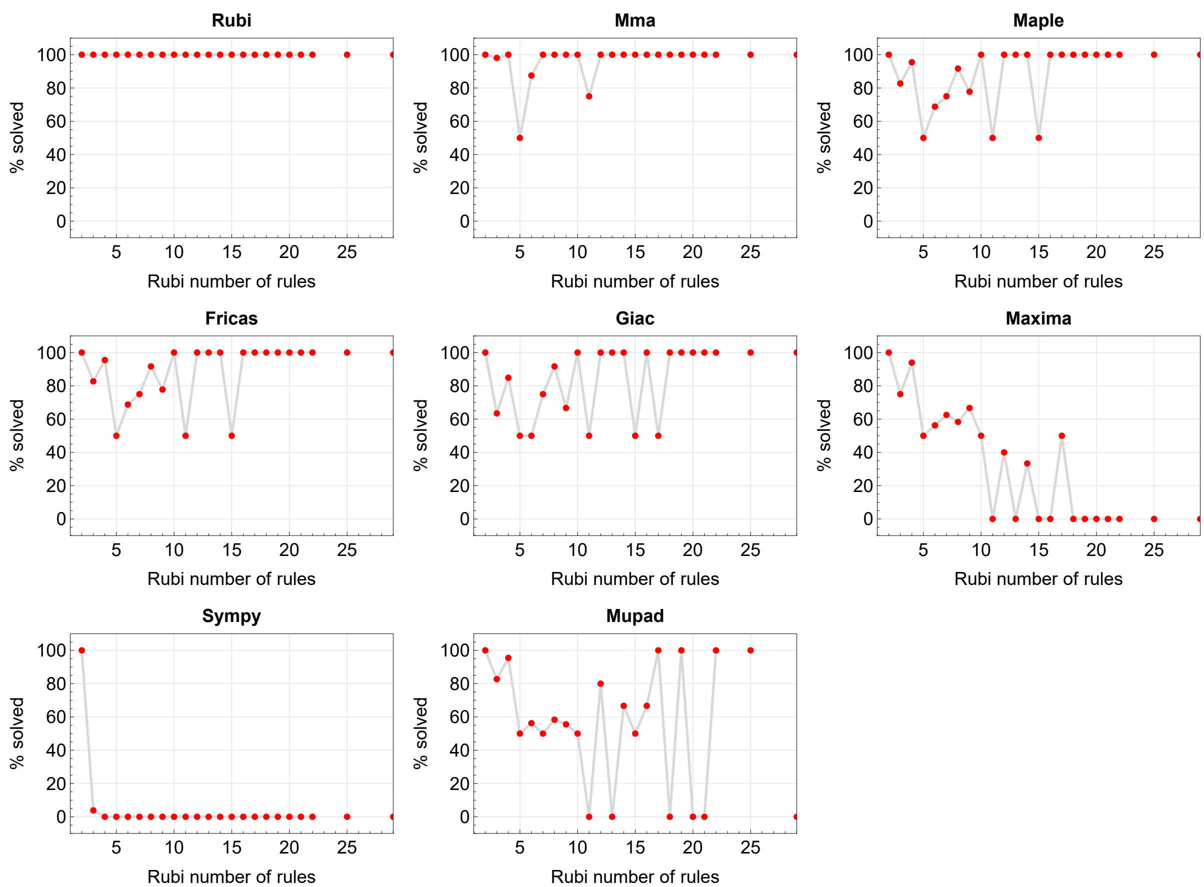


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

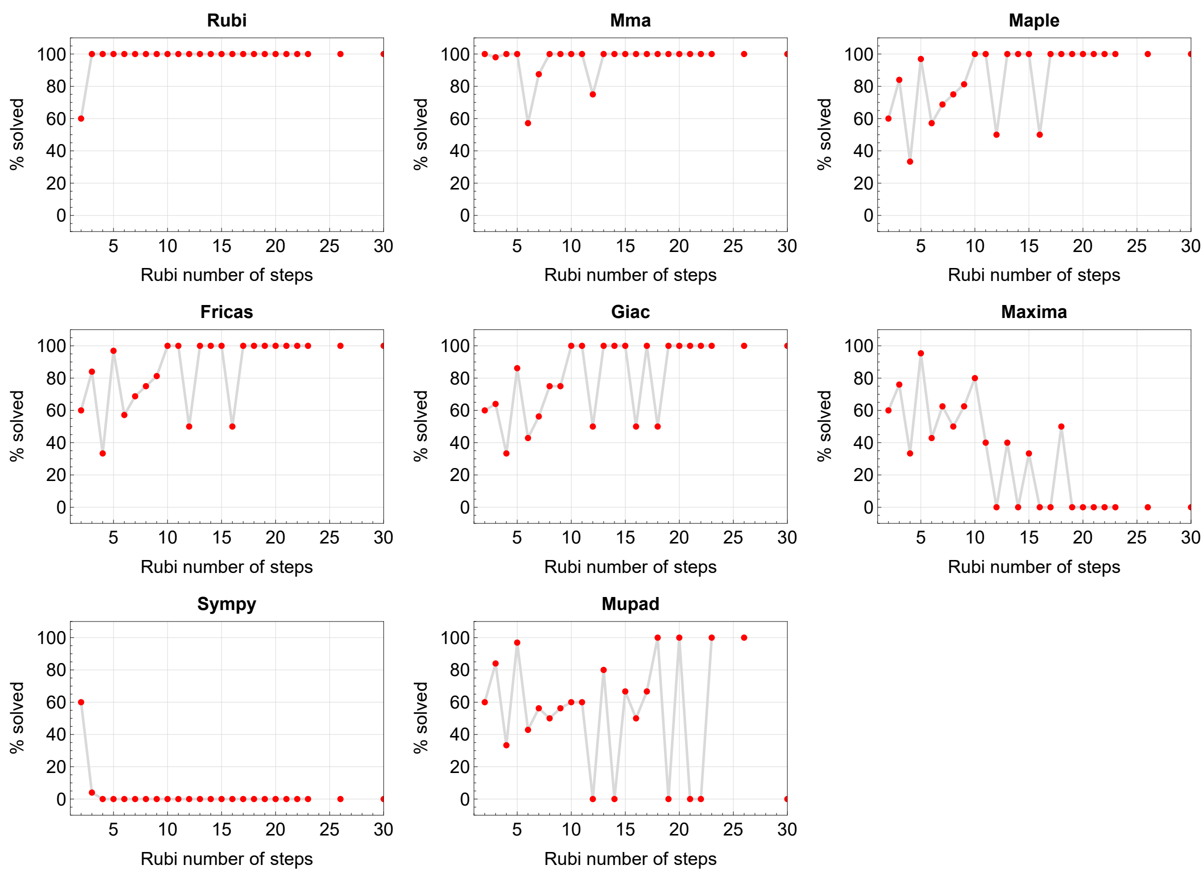


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

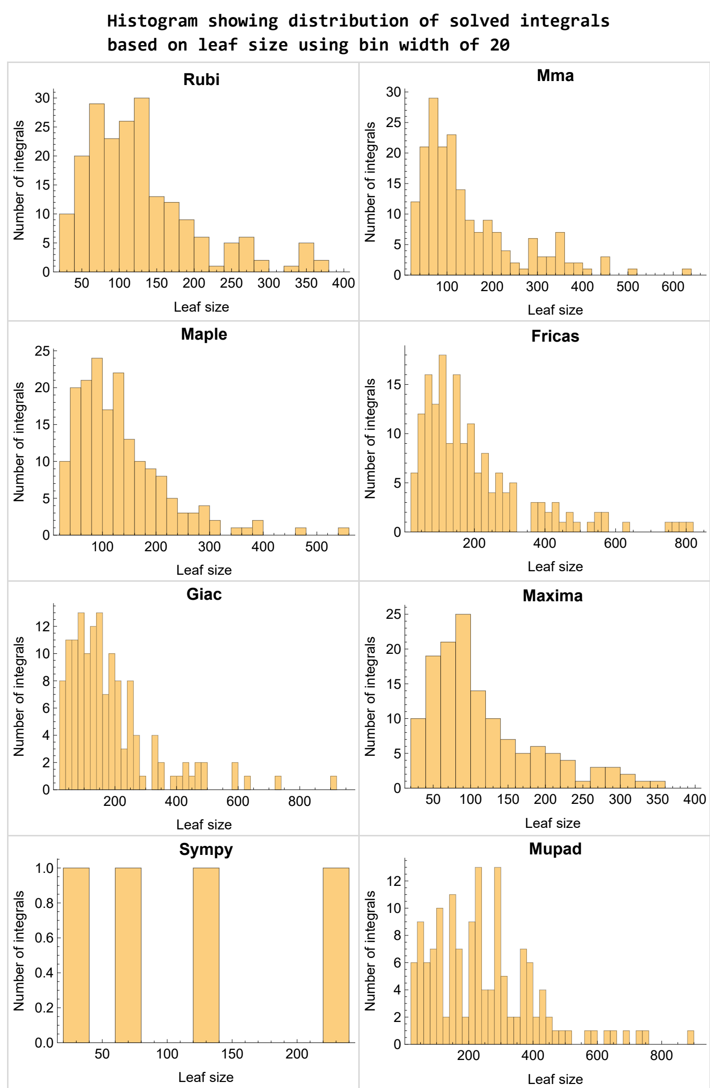


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

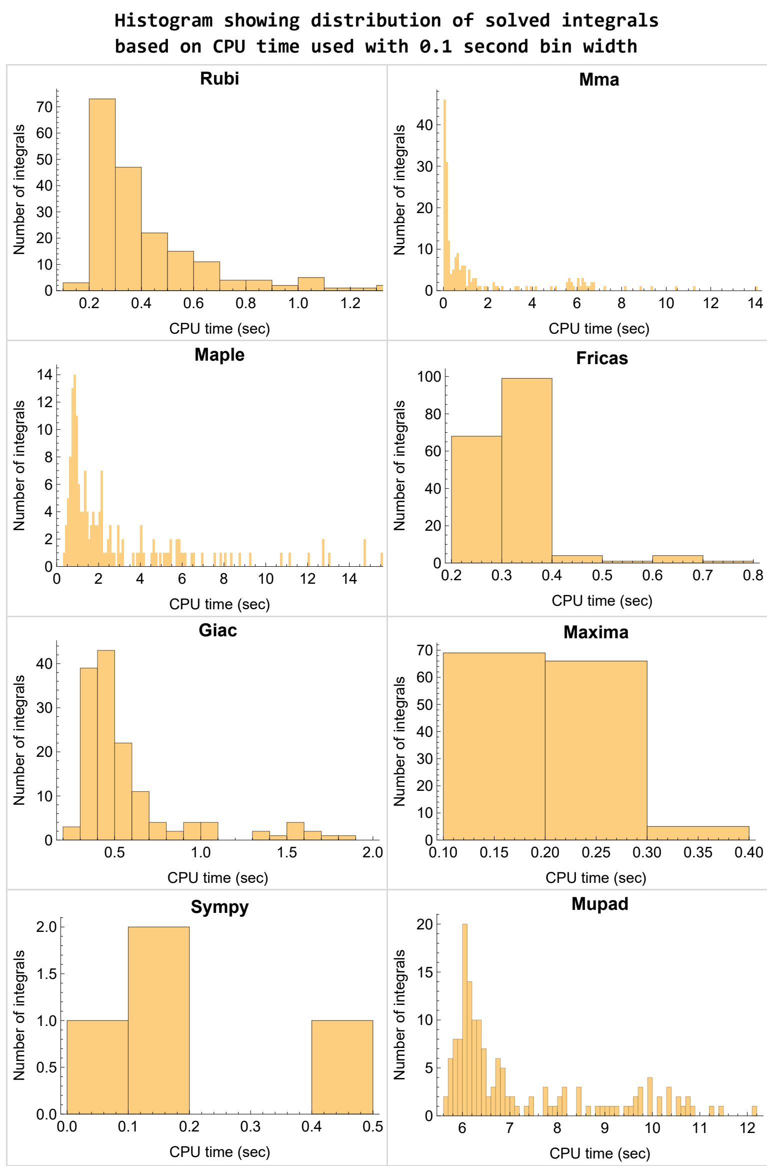


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

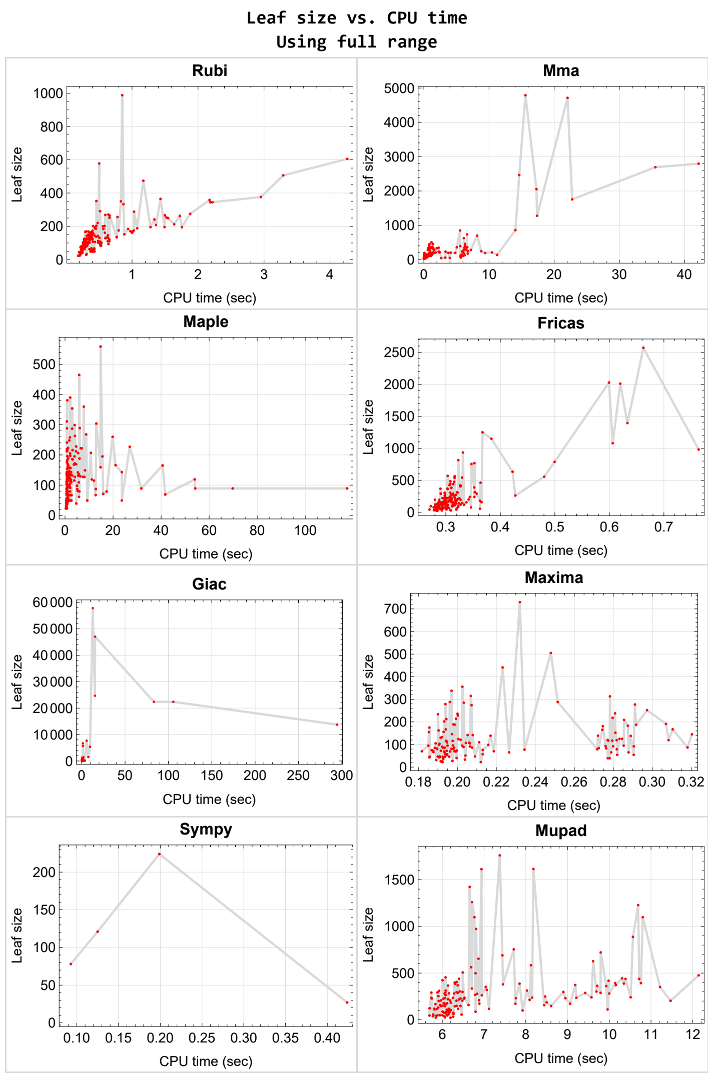


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

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1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{115, 117, 118, 119, 121, 122\}$

Mathematica $\{114, 117, 118, 123, 124, 128, 136, 203, 204, 205, 206, 207\}$

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

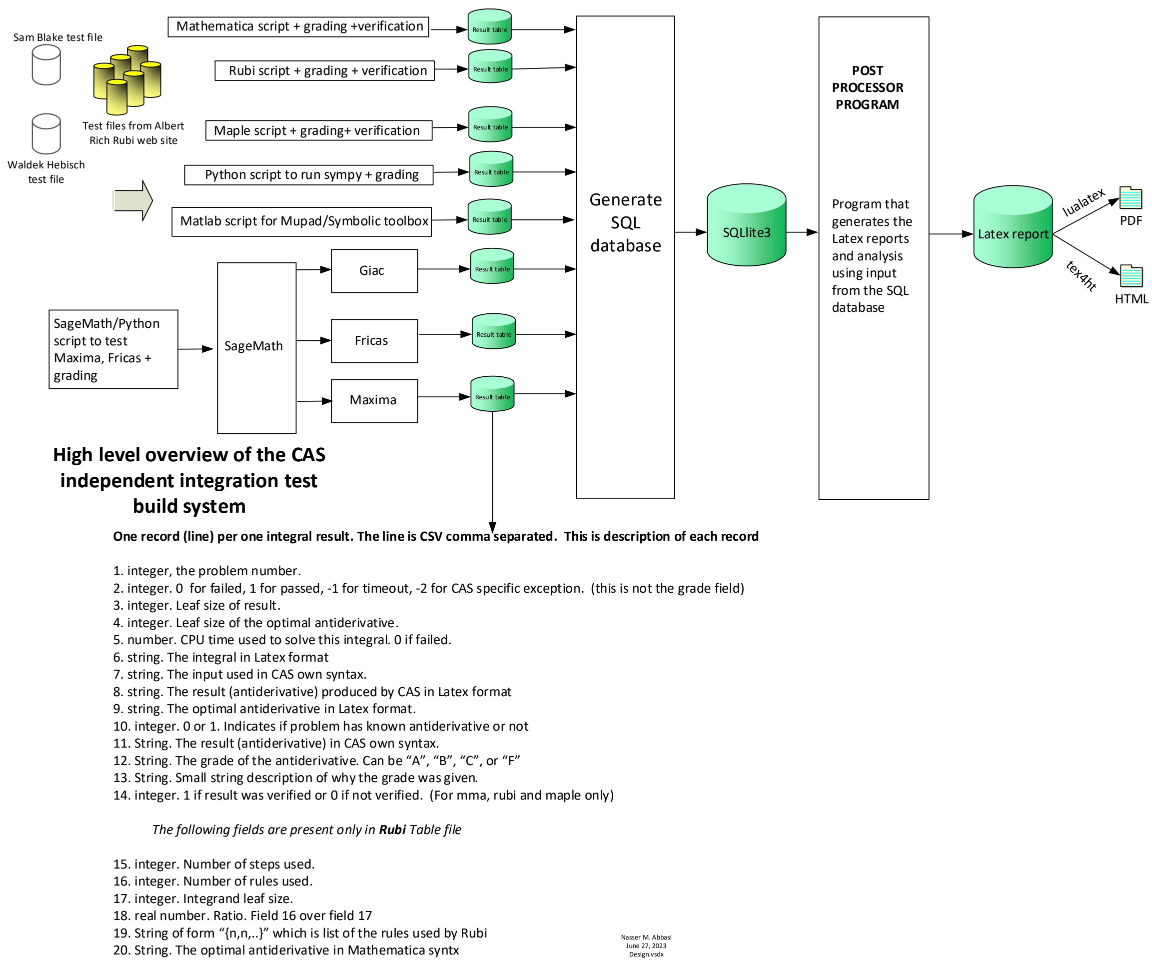
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205 }

B grade { }

C grade { }

F normal fail { 206, 207 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 94, 96, 97, 98, 99, 100, 101, 102, 116, 120, 130, 131, 132, 133, 134, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207 }

B grade { 55, 56, 57, 58, 59, 60, 88, 89, 90, 93, 105, 106, 110, 126, 127, 128, 136, 158, 179, 206 }

C grade { 11, 12, 13, 91, 92, 95, 103, 104, 107, 108, 109, 111, 112, 113, 114, 115, 117, 118, 119, 123, 124, 137, 147, 148, 149, 203, 204, 205 }

F normal fail { 121, 122, 125, 129, 135, 138, 139 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 16, 18, 22, 23, 24, 27, 28, 32, 33, 37, 40, 41, 42, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 113, 114, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202 }

B grade { 17, 107, 111, 112 }

C grade { 10, 14, 15, 19, 20, 21, 25, 26, 29, 30, 31, 34, 35, 36, 38, 39, 43, 53, 54, 59, 60, 87 }

F normal fail { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 14, 15, 16, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 91, 92, 95, 96, 99, 100, 103, 107, 111, 112, 140, 141, 142, 143, 144, 145, 146, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 183, 184, 187, 188, 198, 199 }

B grade { 10, 11, 12, 13, 17, 24, 30, 57, 58, 59, 60, 74, 84, 88, 89, 90, 93, 94, 97, 98, 101, 102, 104, 105, 106, 108, 109, 110, 113, 114, 147, 148, 149, 181, 182, 185, 186, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202 }

C grade { }

F normal fail { 115, 116, 119, 120, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

F(-1) timedout fail { 117, 118, 121, 122 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 55, 56, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 96, 100, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 182, 183, 184, 185, 186, 193, 194, 195, 196, 197 }

B grade { 53, 54, 57, 58, 59, 60, 87, 88, 89, 90, 99, 181, 192 }

C grade { }

F normal fail { 91, 92, 93, 94, 97, 98, 101, 103, 104, 105, 106, 108, 109, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

F(-1) timedout fail { 66, 95, 102, 107, 110, 111, 114 }

F(-2) exception fail { 176, 177, 178, 179, 180, 187, 188, 189, 190, 191, 198, 199, 200, 201, 202 }

2.1.6 Giac

A grade { 4, 5, 6, 7, 12, 13, 17, 18, 22, 28, 32, 33, 37, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 55, 56, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 97, 98, 101, 102, 103, 104, 106, 107, 108, 110, 111, 112, 114, 142, 143, 144, 148, 149, 152, 153, 154, 157, 158, 159, 162, 163, 164, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 195, 196, 197, 198, 199, 200, 201, 202 }

B grade { 2, 3, 10, 11, 16, 21, 23, 24, 27, 36, 42, 53, 57, 58, 59, 60, 92, 95, 96, 99, 100, 105, 109, 113, 140, 141, 145, 146, 147, 151, 156, 161, 167, 168, 181, 193 }

C grade { }

F normal fail { 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

F(-1) timedout fail { 1, 8, 9, 14, 15, 19, 20, 25, 26, 29, 30, 31, 34, 35, 38, 39, 150, 155, 160, 165, 166 }

F(-2) exception fail { 91 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202 }

C grade { }

F normal fail { }

F(-1) timedout fail { 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 203, 204, 205, 206, 207 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 22, 56 }

B grade { 32, 40 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 24, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 79, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 97, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207 }

F(-1) timedout fail { 25, 70, 71, 80, 81, 95, 98, 99, 100, 101, 102, 134 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	104	136	121	95	159	0	0	235
N.S.	1	0.90	1.18	1.05	0.83	1.38	0.00	0.00	2.04
time (sec)	N/A	0.247	0.293	1.529	0.185	0.298	0.000	0.000	5.738

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	66	82	81	51	87	0	22388	154
N.S.	1	0.93	1.15	1.14	0.72	1.23	0.00	315.32	2.17
time (sec)	N/A	0.227	0.082	0.848	0.204	0.301	0.000	105.659	6.058

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	29	38	23	25	27	0	1200	43
N.S.	1	0.97	1.27	0.77	0.83	0.90	0.00	40.00	1.43
time (sec)	N/A	0.201	0.016	0.807	0.191	0.293	0.000	0.503	5.856

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	47	23	22	24	0	23	38
N.S.	1	1.00	1.96	0.96	0.92	1.00	0.00	0.96	1.58
time (sec)	N/A	0.193	0.017	0.526	0.192	0.285	0.000	0.352	5.773

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	48	60	67	45	69	0	60	146
N.S.	1	0.89	1.11	1.24	0.83	1.28	0.00	1.11	2.70
time (sec)	N/A	0.218	0.086	1.014	0.193	0.295	0.000	0.377	5.970

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	69	96	101	69	110	0	82	207
N.S.	1	0.85	1.19	1.25	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.232	0.038	1.499	0.199	0.316	0.000	0.423	5.969

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	97	111	143	91	158	0	104	267
N.S.	1	0.84	0.97	1.24	0.79	1.37	0.00	0.90	2.32
time (sec)	N/A	0.257	0.283	2.523	0.206	0.318	0.000	0.483	6.784

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	110	135	87	116	0	0	351
N.S.	1	1.00	1.09	1.34	0.86	1.15	0.00	0.00	3.48
time (sec)	N/A	0.294	0.198	2.329	0.318	0.287	0.000	0.000	11.215

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	65	88	0	0	231
N.S.	1	1.00	1.12	1.36	0.90	1.22	0.00	0.00	3.21
time (sec)	N/A	0.265	0.128	1.510	0.288	0.285	0.000	0.000	8.949

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	48	47	56	39	80	0	1008	111
N.S.	1	1.23	1.21	1.44	1.00	2.05	0.00	25.85	2.85
time (sec)	N/A	0.406	0.103	1.092	0.277	0.288	0.000	1.092	5.812

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	75	49	54	84	0	108	108
N.S.	1	1.00	1.83	1.20	1.32	2.05	0.00	2.63	2.63
time (sec)	N/A	0.231	0.084	0.519	0.282	0.288	0.000	0.540	6.107

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	86	92	160	0	141	228
N.S.	1	1.00	1.52	1.05	1.12	1.95	0.00	1.72	2.78
time (sec)	N/A	0.290	0.145	0.806	0.280	0.303	0.000	0.554	5.746

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	164	129	125	222	0	199	291
N.S.	1	1.00	1.34	1.06	1.02	1.82	0.00	1.63	2.39
time (sec)	N/A	0.306	0.160	1.415	0.285	0.302	0.000	0.643	5.737

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	108	75	195	96	168	0	0	283
N.S.	1	0.91	0.63	1.64	0.81	1.41	0.00	0.00	2.38
time (sec)	N/A	0.272	0.179	3.947	0.198	0.309	0.000	0.000	6.159

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	65	54	125	58	90	0	0	204
N.S.	1	0.90	0.75	1.74	0.81	1.25	0.00	0.00	2.83
time (sec)	N/A	0.242	0.074	1.981	0.194	0.312	0.000	0.000	6.338

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	48	40	70	43	45	0	5671	178
N.S.	1	0.92	0.77	1.35	0.83	0.87	0.00	109.06	3.42
time (sec)	N/A	0.219	0.027	1.117	0.186	0.300	0.000	1.481	6.032

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	28	94	53	76	0	47	56
N.S.	1	1.00	0.93	3.13	1.77	2.53	0.00	1.57	1.87
time (sec)	N/A	0.203	0.032	1.204	0.195	0.278	0.000	0.502	5.780

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	114	86	228	107	206	0	121	392
N.S.	1	0.86	0.65	1.73	0.81	1.56	0.00	0.92	2.97
time (sec)	N/A	0.263	0.151	4.641	0.204	0.321	0.000	0.749	10.758

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	174	188	152	152	0	0	392
N.S.	1	1.00	1.17	1.26	1.02	1.02	0.00	0.00	2.63
time (sec)	N/A	0.373	1.129	6.026	0.281	0.317	0.000	0.000	10.142

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	128	159	132	120	196	0	0	287
N.S.	1	1.07	1.32	1.10	1.00	1.63	0.00	0.00	2.39
time (sec)	N/A	0.574	1.307	2.965	0.277	0.280	0.000	0.000	9.422

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	63	120	79	84	125	0	5370	213
N.S.	1	0.89	1.69	1.11	1.18	1.76	0.00	75.63	3.00
time (sec)	N/A	0.274	0.489	2.125	0.273	0.295	0.000	9.590	8.092

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	34	32	47	41	78	38	123
N.S.	1	1.00	0.76	0.71	1.04	0.91	1.73	0.84	2.73
time (sec)	N/A	0.185	0.250	0.460	0.190	0.319	0.093	0.523	5.695

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	78	94	80	79	105	0	143	201
N.S.	1	1.05	1.27	1.08	1.07	1.42	0.00	1.93	2.72
time (sec)	N/A	0.295	5.624	0.744	0.272	0.312	0.000	0.653	5.912

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	102	191	146	139	192	0	209	293
N.S.	1	1.04	1.95	1.49	1.42	1.96	0.00	2.13	2.99
time (sec)	N/A	0.345	9.390	1.784	0.272	0.309	0.000	0.735	5.885

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	143	99	260	133	240	0	0	398
N.S.	1	0.89	0.62	1.62	0.83	1.50	0.00	0.00	2.49
time (sec)	N/A	0.301	0.299	19.811	0.192	0.344	0.000	0.000	6.296

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	81	66	142	72	104	0	0	262
N.S.	1	0.89	0.73	1.56	0.79	1.14	0.00	0.00	2.88
time (sec)	N/A	0.256	0.137	5.375	0.188	0.311	0.000	0.000	6.946

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	63	52	49	57	61	0	24693	281
N.S.	1	0.90	0.74	0.70	0.81	0.87	0.00	352.76	4.01
time (sec)	N/A	0.228	0.038	2.673	0.195	0.295	0.000	15.223	6.281

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	86	66	116	80	118	0	94	253
N.S.	1	0.88	0.67	1.18	0.82	1.20	0.00	0.96	2.58
time (sec)	N/A	0.248	0.103	2.401	0.190	0.327	0.000	0.983	5.906

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	188	243	195	209	289	0	0	438
N.S.	1	1.04	1.35	1.08	1.16	1.61	0.00	0.00	2.43
time (sec)	N/A	1.026	8.811	15.578	0.286	0.291	0.000	0.000	10.388

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	B	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	123	177	149	165	220	0	0	371
N.S.	1	1.03	1.49	1.25	1.39	1.85	0.00	0.00	3.12
time (sec)	N/A	0.381	6.744	8.319	0.274	0.298	0.000	0.000	10.158

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	83	115	98	117	154	0	0	288
N.S.	1	0.93	1.29	1.10	1.31	1.73	0.00	0.00	3.24
time (sec)	N/A	0.301	5.639	4.996	0.280	0.314	0.000	0.000	9.791

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	44	43	72	54	121	55	156
N.S.	1	1.00	0.70	0.68	1.14	0.86	1.92	0.87	2.48
time (sec)	N/A	0.238	2.670	1.035	0.195	0.363	0.125	0.532	8.440

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	96	106	94	93	121	0	162	264
N.S.	1	1.04	1.15	1.02	1.01	1.32	0.00	1.76	2.87
time (sec)	N/A	0.317	6.002	1.894	0.276	0.281	0.000	0.557	6.079

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	113	83	227	109	154	0	0	379
N.S.	1	0.88	0.64	1.76	0.84	1.19	0.00	0.00	2.94
time (sec)	N/A	0.276	0.290	26.934	0.195	0.338	0.000	0.000	7.453

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	94	76	159	85	116	0	0	320
N.S.	1	0.88	0.71	1.49	0.79	1.08	0.00	0.00	2.99
time (sec)	N/A	0.262	0.114	14.787	0.188	0.334	0.000	0.000	7.066

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	78	62	127	70	74	0	57842	131
N.S.	1	0.89	0.70	1.44	0.80	0.84	0.00	657.30	1.49
time (sec)	N/A	0.240	0.054	7.563	0.182	0.285	0.000	12.719	5.903

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	90	68	129	82	131	0	96	298
N.S.	1	0.88	0.67	1.26	0.80	1.28	0.00	0.94	2.92
time (sec)	N/A	0.249	0.113	6.430	0.189	0.314	0.000	0.568	6.484

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	126	252	166	238	247	0	0	437
N.S.	1	0.88	1.76	1.16	1.66	1.73	0.00	0.00	3.06
time (sec)	N/A	0.381	6.723	21.033	0.281	0.315	0.000	0.000	10.717

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	117	125	115	181	179	0	0	363
N.S.	1	1.04	1.11	1.02	1.60	1.58	0.00	0.00	3.21
time (sec)	N/A	0.349	6.258	12.040	0.275	0.363	0.000	0.000	9.712

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	57	56	108	70	224	72	237
N.S.	1	1.00	0.66	0.64	1.24	0.80	2.57	0.83	2.72
time (sec)	N/A	0.265	3.290	1.352	0.206	0.283	0.199	0.392	8.151

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	120	136	136	117	135	0	194	295
N.S.	1	1.03	1.17	1.17	1.01	1.16	0.00	1.67	2.54
time (sec)	N/A	0.341	11.247	5.782	0.277	0.319	0.000	0.457	6.342

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	144	209	222	218	219	0	274	384
N.S.	1	1.03	1.49	1.59	1.56	1.56	0.00	1.96	2.74
time (sec)	N/A	0.409	10.453	6.569	0.280	0.325	0.000	0.470	6.307

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	202	283	304	313	291	0	339	454
N.S.	1	1.02	1.43	1.54	1.58	1.47	0.00	1.71	2.29
time (sec)	N/A	0.569	7.214	13.016	0.278	0.357	0.000	0.582	6.077

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	136	101	115	175	167	0	136	388
N.S.	1	1.05	0.78	0.88	1.35	1.28	0.00	1.05	2.98
time (sec)	N/A	0.725	0.698	1.613	0.186	0.305	0.000	3.520	10.349

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	110	84	91	130	147	0	116	281
N.S.	1	1.04	0.79	0.86	1.23	1.39	0.00	1.09	2.65
time (sec)	N/A	0.601	0.223	0.947	0.191	0.292	0.000	1.547	9.999

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	84	54	67	89	125	0	96	172
N.S.	1	1.02	0.66	0.82	1.09	1.52	0.00	1.17	2.10
time (sec)	N/A	0.490	0.111	0.770	0.203	0.285	0.000	0.630	9.062

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	58	28	43	47	58	0	58	61
N.S.	1	1.57	0.76	1.16	1.27	1.57	0.00	1.57	1.65
time (sec)	N/A	0.378	0.026	0.666	0.191	0.289	0.000	0.351	6.152

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	34	32	27	31	28	0	33	32
N.S.	1	1.06	1.00	0.84	0.97	0.88	0.00	1.03	1.00
time (sec)	N/A	0.208	0.015	0.529	0.189	0.309	0.000	0.366	6.260

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	24	27	26	30	0	26	23
N.S.	1	1.00	0.75	0.84	0.81	0.94	0.00	0.81	0.72
time (sec)	N/A	0.300	0.025	0.731	0.192	0.278	0.000	0.448	5.843

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	47	30	49	46	63	0	46	45
N.S.	1	0.92	0.59	0.96	0.90	1.24	0.00	0.90	0.88
time (sec)	N/A	0.379	0.042	1.475	0.194	0.284	0.000	0.568	5.692

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	57	61	67	66	96	0	66	63
N.S.	1	0.84	0.90	0.99	0.97	1.41	0.00	0.97	0.93
time (sec)	N/A	0.405	0.098	2.720	0.188	0.294	0.000	0.602	6.268

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	67	77	87	86	127	0	86	83
N.S.	1	0.80	0.92	1.04	1.02	1.51	0.00	1.02	0.99
time (sec)	N/A	0.405	0.140	5.101	0.193	0.291	0.000	0.491	6.089

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	68	146	166	338	95	0	172	99
N.S.	1	0.81	1.74	1.98	4.02	1.13	0.00	2.05	1.18
time (sec)	N/A	0.366	0.677	1.314	0.197	0.272	0.000	2.214	7.924

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	58	106	120	214	75	0	120	73
N.S.	1	0.84	1.54	1.74	3.10	1.09	0.00	1.74	1.06
time (sec)	N/A	0.362	0.557	0.780	0.199	0.346	0.000	0.901	6.263

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	48	106	70	90	47	0	68	47
N.S.	1	0.96	2.12	1.40	1.80	0.94	0.00	1.36	0.94
time (sec)	N/A	0.354	0.417	0.800	0.192	0.284	0.000	0.464	5.887

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	48	22	27	42	27	21	21
N.S.	1	1.00	2.09	0.96	1.17	1.83	1.17	0.91	0.91
time (sec)	N/A	0.176	0.125	0.404	0.197	0.270	0.424	0.292	6.195

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	69	44	70	62	0	65	25
N.S.	1	1.00	2.38	1.52	2.41	2.14	0.00	2.24	0.86
time (sec)	N/A	0.293	0.472	0.699	0.196	0.288	0.000	0.369	6.065

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	124	94	155	111	0	127	115
N.S.	1	1.00	2.14	1.62	2.67	1.91	0.00	2.19	1.98
time (sec)	N/A	0.407	0.728	1.123	0.185	0.287	0.000	0.382	5.868

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	84	189	134	234	155	0	187	183
N.S.	1	1.02	2.30	1.63	2.85	1.89	0.00	2.28	2.23
time (sec)	N/A	0.514	0.963	1.928	0.190	0.301	0.000	0.438	6.204

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	110	284	168	315	198	0	244	387
N.S.	1	1.04	2.68	1.58	2.97	1.87	0.00	2.30	3.65
time (sec)	N/A	0.622	1.600	3.674	0.207	0.300	0.000	0.442	7.849

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	164	112	127	202	218	0	146	444
N.S.	1	0.87	0.59	0.67	1.07	1.15	0.00	0.77	2.35
time (sec)	N/A	0.334	1.092	4.067	0.193	0.297	0.000	3.348	10.306

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	127	91	103	167	198	0	126	361
N.S.	1	0.87	0.62	0.71	1.14	1.36	0.00	0.86	2.47
time (sec)	N/A	0.300	0.310	2.001	0.194	0.310	0.000	1.535	9.928

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	91	70	79	110	178	0	102	240
N.S.	1	0.88	0.67	0.76	1.06	1.71	0.00	0.98	2.31
time (sec)	N/A	0.268	0.211	1.161	0.211	0.297	0.000	0.646	9.573

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	54	36	55	70	104	0	90	116
N.S.	1	0.90	0.60	0.92	1.17	1.73	0.00	1.50	1.93
time (sec)	N/A	0.234	0.060	0.917	0.219	0.315	0.000	0.391	7.120

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	50	36	37	46	59	0	45	87
N.S.	1	0.96	0.69	0.71	0.88	1.13	0.00	0.87	1.67
time (sec)	N/A	0.232	0.042	0.907	0.196	0.320	0.000	0.299	6.004

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	61	49	49	0	76	0	115	103
N.S.	1	0.94	0.75	0.75	0.00	1.17	0.00	1.77	1.58
time (sec)	N/A	0.246	0.052	2.123	0.000	0.283	0.000	0.344	6.169

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	50	38	39	36	57	0	36	36
N.S.	1	0.91	0.69	0.71	0.65	1.04	0.00	0.65	0.65
time (sec)	N/A	0.226	0.055	4.703	0.193	0.288	0.000	0.408	6.052

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	65	48	49	46	94	0	46	46
N.S.	1	0.89	0.66	0.67	0.63	1.29	0.00	0.63	0.63
time (sec)	N/A	0.240	0.071	9.280	0.193	0.303	0.000	0.431	6.121

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	110	78	79	76	127	0	76	76
N.S.	1	0.87	0.61	0.62	0.60	1.00	0.00	0.60	0.60
time (sec)	N/A	0.264	0.100	17.326	0.213	0.296	0.000	0.446	6.043

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	125	88	89	86	162	0	86	85
N.S.	1	0.86	0.61	0.61	0.59	1.12	0.00	0.59	0.59
time (sec)	N/A	0.275	0.141	31.773	0.208	0.288	0.000	0.485	6.179

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	170	118	119	116	195	0	116	116
N.S.	1	0.85	0.59	0.60	0.58	0.98	0.00	0.58	0.58
time (sec)	N/A	0.300	0.217	53.983	0.198	0.327	0.000	0.533	6.378

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	149	102	115	188	248	0	136	418
N.S.	1	0.87	0.60	0.67	1.10	1.45	0.00	0.80	2.44
time (sec)	N/A	0.315	0.544	3.859	0.199	0.308	0.000	1.664	9.982

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	110	82	91	146	226	0	114	302
N.S.	1	0.87	0.65	0.72	1.16	1.79	0.00	0.90	2.40
time (sec)	N/A	0.278	0.239	2.135	0.197	0.306	0.000	0.998	9.692

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	73	52	67	98	154	0	81	186
N.S.	1	0.89	0.63	0.82	1.20	1.88	0.00	0.99	2.27
time (sec)	N/A	0.244	0.106	1.875	0.191	0.294	0.000	0.540	8.499

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	69	52	49	72	104	0	59	148
N.S.	1	0.93	0.70	0.66	0.97	1.41	0.00	0.80	2.00
time (sec)	N/A	0.238	0.124	2.026	0.192	0.301	0.000	0.473	6.435

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	78	61	61	80	147	0	154	169
N.S.	1	0.91	0.71	0.71	0.93	1.71	0.00	1.79	1.97
time (sec)	N/A	0.258	0.136	5.838	0.198	0.318	0.000	0.806	5.835

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	86	69	67	75	131	0	174	171
N.S.	1	0.90	0.72	0.70	0.78	1.36	0.00	1.81	1.78
time (sec)	N/A	0.263	0.210	12.749	0.187	0.298	0.000	0.418	6.093

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	65	48	49	46	84	0	46	46
N.S.	1	0.89	0.66	0.67	0.63	1.15	0.00	0.63	0.63
time (sec)	N/A	0.239	0.067	23.637	0.186	0.323	0.000	0.453	5.784

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	95	68	69	66	117	0	66	66
N.S.	1	0.87	0.62	0.63	0.61	1.07	0.00	0.61	0.61
time (sec)	N/A	0.243	0.051	41.732	0.193	0.289	0.000	0.510	6.089

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	125	88	89	86	152	0	86	86
N.S.	1	0.86	0.61	0.61	0.59	1.05	0.00	0.59	0.59
time (sec)	N/A	0.288	0.087	69.787	0.198	0.286	0.000	0.530	6.797

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	125	88	89	86	185	0	86	85
N.S.	1	0.86	0.61	0.61	0.59	1.28	0.00	0.59	0.59
time (sec)	N/A	0.270	0.084	117.450	0.194	0.321	0.000	0.555	6.144

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	168	112	127	213	290	0	146	476
N.S.	1	0.86	0.57	0.65	1.09	1.49	0.00	0.75	2.44
time (sec)	N/A	0.334	0.965	8.043	0.194	0.313	0.000	1.624	12.140

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	115	50	81	95	102	0	76	172
N.S.	1	0.87	0.38	0.61	0.72	0.77	0.00	0.58	1.30
time (sec)	N/A	0.281	0.077	5.244	0.195	0.280	0.000	0.740	6.887

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	92	62	79	121	198	0	91	240
N.S.	1	0.88	0.59	0.75	1.15	1.89	0.00	0.87	2.29
time (sec)	N/A	0.257	0.192	5.977	0.202	0.311	0.000	0.481	10.512

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	94	73	71	103	196	0	185	228
N.S.	1	0.89	0.69	0.67	0.97	1.85	0.00	1.75	2.15
time (sec)	N/A	0.272	0.527	15.887	0.191	0.301	0.000	0.404	6.435

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	119	89	89	95	186	0	232	235
N.S.	1	0.88	0.66	0.66	0.70	1.38	0.00	1.72	1.74
time (sec)	N/A	0.274	0.120	54.184	0.202	0.307	0.000	0.482	6.532

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	131	124	109	356	129	0	146	231
N.S.	1	1.03	0.98	0.86	2.80	1.02	0.00	1.15	1.82
time (sec)	N/A	0.501	0.824	5.724	0.203	0.284	0.000	0.556	7.763

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	112	315	119	288	369	0	135	203
N.S.	1	1.04	2.92	1.10	2.67	3.42	0.00	1.25	1.88
time (sec)	N/A	0.487	0.741	11.110	0.196	0.290	0.000	0.366	11.468

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	106	359	143	285	445	0	179	171
N.S.	1	0.88	2.99	1.19	2.38	3.71	0.00	1.49	1.42
time (sec)	N/A	0.418	5.056	23.636	0.203	0.308	0.000	0.416	7.750

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	137	733	165	279	439	0	204	209
N.S.	1	1.03	5.51	1.24	2.10	3.30	0.00	1.53	1.57
time (sec)	N/A	0.433	6.643	40.609	0.194	0.314	0.000	0.455	6.877

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	195	394	176	0	200	0	0	0
N.S.	1	1.20	2.43	1.09	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	1.232	5.592	0.826	0.000	0.342	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	107	114	89	0	169	0	428	0
N.S.	1	1.06	1.13	0.88	0.00	1.67	0.00	4.24	0.00
time (sec)	N/A	0.494	0.505	0.627	0.000	0.309	0.000	0.752	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	99	206	129	0	279	0	149	0
N.S.	1	1.11	2.31	1.45	0.00	3.13	0.00	1.67	0.00
time (sec)	N/A	0.512	0.932	0.804	0.000	0.310	0.000	0.457	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	175	309	170	0	380	0	211	0
N.S.	1	1.07	1.90	1.04	0.00	2.33	0.00	1.29	0.00
time (sec)	N/A	0.968	1.429	1.090	0.000	0.310	0.000	0.477	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	195	141	139	0	239	0	917	0
N.S.	1	1.17	0.84	0.83	0.00	1.43	0.00	5.49	0.00
time (sec)	N/A	1.381	5.772	0.658	0.000	0.352	0.000	1.300	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	93	46	55	145	48	0	209	0
N.S.	1	1.06	0.52	0.62	1.65	0.55	0.00	2.38	0.00
time (sec)	N/A	0.479	3.981	0.652	0.320	0.292	0.000	0.571	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	134	233	154	0	315	0	183	0
N.S.	1	1.11	1.93	1.27	0.00	2.60	0.00	1.51	0.00
time (sec)	N/A	0.714	5.858	0.902	0.000	0.320	0.000	0.336	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	208	334	196	0	424	0	246	0
N.S.	1	1.06	1.70	0.99	0.00	2.15	0.00	1.25	0.00
time (sec)	N/A	1.283	1.325	0.944	0.000	0.307	0.000	0.377	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	212	112	87	277	98	0	1485	0
N.S.	1	1.40	0.74	0.58	1.83	0.65	0.00	9.83	0.00
time (sec)	N/A	1.567	5.718	12.799	0.291	0.323	0.000	7.650	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	131	60	67	191	70	0	1411	0
N.S.	1	1.11	0.51	0.57	1.62	0.59	0.00	11.96	0.00
time (sec)	N/A	0.589	5.670	2.418	0.307	0.285	0.000	1.853	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	169	261	162	0	363	0	223	0
N.S.	1	1.12	1.73	1.07	0.00	2.40	0.00	1.48	0.00
time (sec)	N/A	0.965	6.057	1.342	0.000	0.303	0.000	0.450	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	275	360	222	0	485	0	290	0
N.S.	1	1.21	1.59	0.98	0.00	2.14	0.00	1.28	0.00
time (sec)	N/A	1.787	6.663	6.971	0.000	0.308	0.000	0.439	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	241	118	241	0	229	0	211	0
N.S.	1	1.61	0.79	1.61	0.00	1.53	0.00	1.41	0.00
time (sec)	N/A	1.229	0.786	0.787	0.000	0.317	0.000	0.530	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	109	118	134	0	200	0	156	0
N.S.	1	1.02	1.10	1.25	0.00	1.87	0.00	1.46	0.00
time (sec)	N/A	0.480	0.477	0.714	0.000	0.310	0.000	0.388	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	138	103	0	263	0	132	0
N.S.	1	1.00	2.23	1.66	0.00	4.24	0.00	2.13	0.00
time (sec)	N/A	0.334	0.530	0.638	0.000	0.315	0.000	0.346	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	253	292	144	0	369	0	185	0
N.S.	1	1.87	2.16	1.07	0.00	2.73	0.00	1.37	0.00
time (sec)	N/A	1.478	0.689	0.825	0.000	0.303	0.000	0.352	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	195	334	311	0	270	0	220	0
N.S.	1	1.10	1.89	1.76	0.00	1.53	0.00	1.24	0.00
time (sec)	N/A	1.679	0.651	0.760	0.000	0.311	0.000	0.407	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	146	128	212	0	237	0	121	0
N.S.	1	1.09	0.96	1.58	0.00	1.77	0.00	0.90	0.00
time (sec)	N/A	0.598	0.624	0.729	0.000	0.298	0.000	0.395	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	121	206	134	0	421	0	205	0
N.S.	1	1.07	1.82	1.19	0.00	3.73	0.00	1.81	0.00
time (sec)	N/A	0.619	1.807	0.803	0.000	0.326	0.000	0.444	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-1)	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	262	294	144	0	383	0	168	0
N.S.	1	1.82	2.04	1.00	0.00	2.66	0.00	1.17	0.00
time (sec)	N/A	1.639	0.927	0.769	0.000	0.310	0.000	0.470	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-1)	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	344	394	381	0	307	0	184	0
N.S.	1	1.66	1.90	1.84	0.00	1.48	0.00	0.89	0.00
time (sec)	N/A	2.075	0.824	0.979	0.000	0.322	0.000	0.635	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	175	284	288	0	279	0	140	0
N.S.	1	1.05	1.70	1.72	0.00	1.67	0.00	0.84	0.00
time (sec)	N/A	0.751	0.608	0.797	0.000	0.299	0.000	0.629	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	152	451	219	0	539	0	262	0
N.S.	1	1.08	3.20	1.55	0.00	3.82	0.00	1.86	0.00
time (sec)	N/A	0.839	0.861	0.806	0.000	0.316	0.000	0.383	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-1)	B	F	A	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD
size	191	376	332	182	0	564	0	240	0
N.S.	1	1.97	1.74	0.95	0.00	2.95	0.00	1.26	0.00
time (sec)	N/A	2.807	1.975	0.902	0.000	0.316	0.000	0.348	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	982	988	232	0	0	0	0	0	0
N.S.	1	1.01	0.24	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.828	3.327	0.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	131	209	0	0	0	0	0	0
N.S.	1	1.07	1.70	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.579	2.368	0.000	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	80	136	2692	0	0	0	0	0	0
N.S.	1	1.70	33.65	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.352	35.531	0.000	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	No	N/A	TBD	TBD	TBD	TBD	TBD
size	80	136	2796	0	0	0	0	0	0
N.S.	1	1.70	34.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.374	42.172	0.000	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	551	578	128	0	0	0	0	0	0
N.S.	1	1.05	0.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.498	1.182	0.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	126	133	100	0	0	0	0	0	0
N.S.	1	1.06	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.597	0.655	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	136	0	0	0	0	0	0	0
N.S.	1	1.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	80	136	0	0	0	0	0	0	0
N.S.	1	1.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	269	269	4715	0	0	0	0	0	0
N.S.	1	1.00	17.53	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.611	22.052	0.000	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	187	187	2054	0	0	0	0	0	0
N.S.	1	1.00	10.98	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.472	17.282	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.336	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	108	108	232	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.399	2.276	0.000	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	142	626	0	0	0	0	0	0
N.S.	1	1.03	4.54	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.452	6.135	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	248	252	1276	0	0	0	0	0	0
N.S.	1	1.02	5.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.615	17.383	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	111	156	0	0	0	0	0	0	0
N.S.	1	1.41	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.366	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	167	105	0	0	0	0	0	0
N.S.	1	1.02	0.64	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.294	0.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	75	63	0	0	0	0	0	0
N.S.	1	1.04	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.053	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	43	43	43	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.213	0.045	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	83	81	68	0	0	0	0	0	0
N.S.	1	0.98	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	0.129	0.000	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	123	123	83	0	0	0	0	0	0
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.269	0.191	0.000	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	311	352	0	0	0	0	0	0	0
N.S.	1	1.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.455	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	157	161	1756	0	0	0	0	0	0
N.S.	1	1.03	11.18	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.562	22.768	0.000	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	70	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	89	116	0	0	0	0	0	0	0
N.S.	1	1.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.287	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	89	116	0	0	0	0	0	0	0
N.S.	1	1.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	96	82	81	73	90	0	22388	176
N.S.	1	1.09	0.93	0.92	0.83	1.02	0.00	254.41	2.00
time (sec)	N/A	0.293	0.083	0.809	0.200	0.283	0.000	83.239	6.116

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	46	38	41	43	45	0	1200	74
N.S.	1	0.84	0.69	0.75	0.78	0.82	0.00	21.82	1.35
time (sec)	N/A	0.215	0.012	0.712	0.191	0.302	0.000	0.448	6.261

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	47	23	22	24	0	23	47
N.S.	1	1.00	1.96	0.96	0.92	1.00	0.00	0.96	1.96
time (sec)	N/A	0.191	0.025	0.349	0.212	0.283	0.000	0.300	6.002

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	48	60	67	45	69	0	60	146
N.S.	1	0.89	1.11	1.24	0.83	1.28	0.00	1.11	2.70
time (sec)	N/A	0.220	0.140	0.964	0.191	0.291	0.000	0.361	6.065

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	69	96	101	69	110	0	82	207
N.S.	1	0.85	1.19	1.25	0.85	1.36	0.00	1.01	2.56
time (sec)	N/A	0.232	0.024	1.341	0.196	0.304	0.000	0.378	6.182

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	81	98	65	73	0	13744	110
N.S.	1	1.00	1.12	1.36	0.90	1.01	0.00	190.89	1.53
time (sec)	N/A	0.261	0.104	1.395	0.279	0.280	0.000	294.678	9.959

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	47	59	39	47	0	1008	55
N.S.	1	1.00	1.24	1.55	1.03	1.24	0.00	26.53	1.45
time (sec)	N/A	0.231	0.058	1.078	0.278	0.280	0.000	1.031	6.461

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	75	49	54	84	0	108	158
N.S.	1	1.00	1.83	1.20	1.32	2.05	0.00	2.63	3.85
time (sec)	N/A	0.228	0.065	0.531	0.290	0.285	0.000	0.312	5.948

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	125	86	92	160	0	141	225
N.S.	1	1.00	1.52	1.05	1.12	1.95	0.00	1.72	2.74
time (sec)	N/A	0.271	0.087	0.806	0.291	0.289	0.000	0.340	6.024

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	164	129	125	222	0	199	288
N.S.	1	1.00	1.34	1.06	1.02	1.82	0.00	1.63	2.36
time (sec)	N/A	0.298	0.110	1.342	0.283	0.294	0.000	0.383	6.071

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	123	108	135	105	140	0	0	232
N.S.	1	1.11	0.97	1.22	0.95	1.26	0.00	0.00	2.09
time (sec)	N/A	0.339	0.303	1.891	0.189	0.324	0.000	0.000	6.061

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	70	145	69	70	74	0	6671	150
N.S.	1	0.90	1.86	0.88	0.90	0.95	0.00	85.53	1.92
time (sec)	N/A	0.253	0.058	1.046	0.199	0.302	0.000	1.339	6.005

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	43	46	40	40	42	0	41	117
N.S.	1	0.93	1.00	0.87	0.87	0.91	0.00	0.89	2.54
time (sec)	N/A	0.215	0.019	0.473	0.200	0.292	0.000	0.336	6.156

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	75	92	93	69	115	0	99	221
N.S.	1	0.89	1.10	1.11	0.82	1.37	0.00	1.18	2.63
time (sec)	N/A	0.248	0.035	1.349	0.193	0.302	0.000	0.381	6.133

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	111	142	157	105	177	0	138	310
N.S.	1	0.88	1.13	1.25	0.83	1.40	0.00	1.10	2.46
time (sec)	N/A	0.283	0.052	2.263	0.206	0.308	0.000	0.452	5.924

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	176	185	119	118	0	0	235
N.S.	1	1.00	1.18	1.24	0.80	0.79	0.00	0.00	1.58
time (sec)	N/A	0.371	0.782	3.142	0.282	0.292	0.000	0.000	9.211

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	77	116	83	81	0	7670	147
N.S.	1	1.00	0.82	1.23	0.88	0.86	0.00	81.60	1.56
time (sec)	N/A	0.309	0.553	2.191	0.276	0.285	0.000	5.677	8.606

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	116	79	79	118	0	148	277
N.S.	1	1.00	1.49	1.01	1.01	1.51	0.00	1.90	3.55
time (sec)	N/A	0.272	0.593	0.703	0.282	0.301	0.000	0.345	6.834

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	293	145	138	218	0	241	584
N.S.	1	1.00	2.20	1.09	1.04	1.64	0.00	1.81	4.39
time (sec)	N/A	0.358	6.478	1.684	0.289	0.301	0.000	0.413	8.124

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	351	216	183	306	0	337	888
N.S.	1	1.00	1.74	1.07	0.91	1.51	0.00	1.67	4.40
time (sec)	N/A	0.397	1.134	3.130	0.287	0.298	0.000	0.514	10.566

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	157	141	206	162	194	0	0	366
N.S.	1	1.05	0.94	1.37	1.08	1.29	0.00	0.00	2.44
time (sec)	N/A	0.406	0.180	5.456	0.190	0.288	0.000	0.000	6.136

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	104	90	110	113	116	0	47033	226
N.S.	1	0.99	0.86	1.05	1.08	1.10	0.00	447.93	2.15
time (sec)	N/A	0.329	0.133	2.590	0.195	0.293	0.000	15.439	6.517

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	61	67	56	57	66	0	58	118
N.S.	1	0.91	1.00	0.84	0.85	0.99	0.00	0.87	1.76
time (sec)	N/A	0.223	0.020	0.855	0.213	0.305	0.000	0.450	5.807

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	104	130	118	98	153	0	131	312
N.S.	1	0.90	1.12	1.02	0.84	1.32	0.00	1.13	2.69
time (sec)	N/A	0.285	0.058	2.523	0.216	0.297	0.000	0.667	6.081

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	147	195	212	142	225	0	185	424
N.S.	1	0.89	1.18	1.28	0.86	1.36	0.00	1.12	2.57
time (sec)	N/A	0.327	0.080	4.564	0.208	0.310	0.000	0.678	6.006

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	226	268	167	157	0	0	297
N.S.	1	1.00	1.03	1.22	0.76	0.71	0.00	0.00	1.35
time (sec)	N/A	0.464	0.929	8.787	0.310	0.294	0.000	0.000	8.900

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	113	169	119	116	0	0	249
N.S.	1	1.00	0.77	1.16	0.82	0.79	0.00	0.00	1.71
time (sec)	N/A	0.400	0.893	4.616	0.308	0.293	0.000	0.000	8.465

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	96	95	143	0	199	289
N.S.	1	1.00	1.40	0.94	0.93	1.40	0.00	1.95	2.83
time (sec)	N/A	0.303	1.244	1.764	0.285	0.330	0.000	0.393	6.372

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	355	186	187	293	0	421	405
N.S.	1	1.00	1.83	0.96	0.96	1.51	0.00	2.17	2.09
time (sec)	N/A	0.410	6.586	3.070	0.292	0.291	0.000	0.446	6.342

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	346	289	252	412	0	471	507
N.S.	1	1.00	1.19	0.99	0.87	1.42	0.00	1.62	1.74
time (sec)	N/A	0.492	2.260	6.131	0.297	0.328	0.000	0.552	6.489

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	264	184	189	288	261	0	343	498
N.S.	1	1.29	0.90	0.93	1.41	1.28	0.00	1.68	2.44
time (sec)	N/A	0.634	0.878	1.296	0.252	0.428	0.000	1.539	6.796

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	159	117	121	142	157	0	177	217
N.S.	1	1.26	0.93	0.96	1.13	1.25	0.00	1.40	1.72
time (sec)	N/A	0.379	0.350	0.885	0.205	0.365	0.000	0.608	6.398

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	75	65	71	65	63	0	71	91
N.S.	1	1.01	0.88	0.96	0.88	0.85	0.00	0.96	1.23
time (sec)	N/A	0.244	0.047	0.605	0.227	0.294	0.000	0.341	6.139

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	33	33	31	0	35	48
N.S.	1	1.00	1.00	0.97	0.97	0.91	0.00	1.03	1.41
time (sec)	N/A	0.201	0.016	0.539	0.204	0.308	0.000	0.305	5.942

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	78	65	76	77	118	0	114	144
N.S.	1	0.93	0.77	0.90	0.92	1.40	0.00	1.36	1.71
time (sec)	N/A	0.273	0.114	0.910	0.234	0.299	0.000	0.336	6.299

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	136	115	137	139	271	0	201	281
N.S.	1	0.92	0.78	0.93	0.94	1.83	0.00	1.36	1.90
time (sec)	N/A	0.335	0.698	1.625	0.217	0.318	0.000	0.379	6.057

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	184	195	214	0	476	0	241	372
N.S.	1	1.04	1.10	1.21	0.00	2.69	0.00	1.36	2.10
time (sec)	N/A	0.921	1.103	0.947	0.000	0.315	0.000	0.945	9.185

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	102	152	112	0	305	0	107	148
N.S.	1	1.06	1.58	1.17	0.00	3.18	0.00	1.11	1.54
time (sec)	N/A	0.473	0.285	0.640	0.000	0.308	0.000	0.471	6.341

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	93	108	109	0	314	0	129	204
N.S.	1	1.16	1.35	1.36	0.00	3.92	0.00	1.61	2.55
time (sec)	N/A	0.631	0.304	0.806	0.000	0.353	0.000	0.339	6.479

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	170	350	223	0	633	0	273	654
N.S.	1	1.10	2.27	1.45	0.00	4.11	0.00	1.77	4.25
time (sec)	N/A	0.955	6.241	1.286	0.000	0.422	0.000	0.378	6.866

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	307	345	504	390	0	1079	0	490	1099
N.S.	1	1.12	1.64	1.27	0.00	3.51	0.00	1.60	3.58
time (sec)	N/A	2.181	1.174	2.147	0.000	0.606	0.000	0.393	6.770

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	288	204	205	505	555	0	494	755
N.S.	1	1.19	0.84	0.85	2.09	2.29	0.00	2.04	3.12
time (sec)	N/A	1.009	4.111	2.104	0.248	0.481	0.000	1.597	7.712

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	184	145	143	274	388	0	248	351
N.S.	1	1.14	0.90	0.89	1.70	2.41	0.00	1.54	2.18
time (sec)	N/A	0.550	0.502	1.275	0.207	0.352	0.000	0.658	7.041

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	127	162	98	124	195	0	156	158
N.S.	1	1.17	1.49	0.90	1.14	1.79	0.00	1.43	1.45
time (sec)	N/A	0.319	0.204	0.941	0.200	0.317	0.000	0.404	6.628

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	50	42	49	47	69	0	51	105
N.S.	1	0.94	0.79	0.92	0.89	1.30	0.00	0.96	1.98
time (sec)	N/A	0.232	0.061	0.798	0.209	0.297	0.000	0.377	6.202

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	105	96	105	116	259	0	165	235
N.S.	1	0.92	0.84	0.92	1.02	2.27	0.00	1.45	2.06
time (sec)	N/A	0.302	0.422	2.033	0.194	0.298	0.000	0.348	6.254

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	173	187	172	189	542	0	278	439
N.S.	1	0.92	0.99	0.91	1.01	2.88	0.00	1.48	2.34
time (sec)	N/A	0.380	6.100	4.060	0.198	0.330	0.000	0.445	6.398

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	341	248	0	815	0	406	722
N.S.	1	1.00	1.02	0.74	0.00	2.45	0.00	1.22	2.17
time (sec)	N/A	0.846	1.442	1.789	0.000	0.323	0.000	1.098	9.795

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	200	169	162	0	569	0	251	313
N.S.	1	1.00	0.84	0.81	0.00	2.84	0.00	1.26	1.56
time (sec)	N/A	0.550	0.862	1.171	0.000	0.303	0.000	0.544	8.028

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	161	139	156	0	768	0	218	1616
N.S.	1	1.40	1.21	1.36	0.00	6.68	0.00	1.90	14.05
time (sec)	N/A	0.985	0.713	1.725	0.000	0.353	0.000	0.364	8.182

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	266	403	287	0	1149	0	356	973
N.S.	1	1.12	1.69	1.21	0.00	4.83	0.00	1.50	4.09
time (sec)	N/A	1.447	6.337	2.955	0.000	0.384	0.000	0.410	6.810

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	424	506	361	465	0	2011	0	596	1424
N.S.	1	1.19	0.85	1.10	0.00	4.74	0.00	1.41	3.36
time (sec)	N/A	3.251	1.264	5.897	0.000	0.620	0.000	0.457	6.656

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	365	304	263	730	981	0	585	1229
N.S.	1	1.14	0.95	0.82	2.27	3.06	0.00	1.82	3.83
time (sec)	N/A	1.390	6.235	4.136	0.232	0.764	0.000	1.774	10.692

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	256	196	197	441	788	0	464	690
N.S.	1	1.10	0.84	0.85	1.90	3.40	0.00	2.00	2.97
time (sec)	N/A	0.777	1.507	2.188	0.223	0.500	0.000	0.813	7.440

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	163	213	134	228	462	0	257	304
N.S.	1	1.09	1.43	0.90	1.53	3.10	0.00	1.72	2.04
time (sec)	N/A	0.369	1.456	1.993	0.200	0.364	0.000	0.441	6.418

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	69	60	66	81	154	0	69	369
N.S.	1	0.92	0.80	0.88	1.08	2.05	0.00	0.92	4.92
time (sec)	N/A	0.242	0.195	1.467	0.197	0.312	0.000	0.347	6.368

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	133	121	131	156	404	0	154	334
N.S.	1	0.92	0.83	0.90	1.08	2.79	0.00	1.06	2.30
time (sec)	N/A	0.330	0.642	5.471	0.193	0.327	0.000	0.395	6.708

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	203	195	207	236	754	0	327	563
N.S.	1	0.92	0.88	0.94	1.07	3.41	0.00	1.48	2.55
time (sec)	N/A	0.442	4.828	10.754	0.200	0.347	0.000	0.440	6.690

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	474	474	351	354	0	1249	0	632	1099
N.S.	1	1.00	0.74	0.75	0.00	2.64	0.00	1.33	2.32
time (sec)	N/A	1.144	0.915	2.981	0.000	0.367	0.000	1.062	10.802

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	350	350	212	258	0	934	0	384	627
N.S.	1	1.00	0.61	0.74	0.00	2.67	0.00	1.10	1.79
time (sec)	N/A	0.804	2.450	2.027	0.000	0.331	0.000	0.572	9.613

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	249	195	299	0	1394	0	339	1762
N.S.	1	1.23	0.97	1.48	0.00	6.90	0.00	1.68	8.72
time (sec)	N/A	1.506	3.757	4.013	0.000	0.633	0.000	0.411	7.377

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	289	358	459	360	0	2027	0	451	1261
N.S.	1	1.24	1.59	1.25	0.00	7.01	0.00	1.56	4.36
time (sec)	N/A	2.109	6.321	7.813	0.000	0.599	0.000	0.412	6.712

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	492	605	448	559	0	2571	0	731	1614
N.S.	1	1.23	0.91	1.14	0.00	5.23	0.00	1.49	3.28
time (sec)	N/A	4.197	1.392	14.788	0.000	0.662	0.000	0.452	6.938

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	271	271	4791	0	0	0	0	0	0
N.S.	1	1.00	17.68	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.623	15.591	0.000	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	186	186	2464	0	0	0	0	0	0
N.S.	1	1.00	13.25	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	14.643	0.000	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD
size	129	129	849	0	0	0	0	0	0
N.S.	1	1.00	6.58	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.340	5.557	0.000	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	B	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	284	0	858	0	0	0	0	0	0
N.S.	1	0.00	3.02	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	14.046	0.000	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F	F	F	F	F	F(-1)
verified	N/A	N/A	No	N/A	TBD	TBD	TBD	TBD	TBD
size	737	0	695	0	0	0	0	0	0
N.S.	1	0.00	0.94	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	8.177	0.000	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	23	25	23	25	25	22	25	25
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.96	1.09	1.09
time (sec)	N/A	0.204	2.604	0.722	2.170	0.472	78.501	1.496	6.956

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [114] had the largest ratio of [1.26086999999999994]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	0.90	19	0.211
2	A	5	4	0.93	19	0.211
3	A	5	4	0.97	17	0.235
4	A	5	4	1.00	17	0.235
5	A	5	4	0.89	19	0.211
6	A	5	4	0.85	19	0.211
7	A	5	4	0.84	19	0.211
8	A	3	3	1.00	19	0.158
9	A	3	3	1.00	19	0.158
10	A	9	9	1.23	19	0.474
11	A	3	3	1.00	19	0.158
12	A	3	3	1.00	19	0.158
13	A	3	3	1.00	19	0.158
14	A	5	4	0.91	21	0.190
15	A	5	4	0.90	21	0.190
16	A	5	4	0.92	19	0.211
17	A	4	3	1.00	21	0.143
18	A	5	4	0.86	21	0.190
19	A	3	3	1.00	21	0.143
20	A	9	9	1.07	21	0.429
21	A	3	3	0.89	21	0.143
22	A	2	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
23	A	3	3	1.05	21	0.143
24	A	3	3	1.04	21	0.143
25	A	5	4	0.89	21	0.190
26	A	5	4	0.89	21	0.190
27	A	5	4	0.90	19	0.211
28	A	5	4	0.88	21	0.190
29	A	18	17	1.04	21	0.810
30	A	3	3	1.03	21	0.143
31	A	3	3	0.93	21	0.143
32	A	3	3	1.00	12	0.250
33	A	3	3	1.04	21	0.143
34	A	5	4	0.88	21	0.190
35	A	5	4	0.88	21	0.190
36	A	5	4	0.89	19	0.211
37	A	5	4	0.88	21	0.190
38	A	3	3	0.88	21	0.143
39	A	3	3	1.04	21	0.143
40	A	3	3	1.00	12	0.250
41	A	3	3	1.03	21	0.143
42	A	3	3	1.03	21	0.143
43	A	3	3	1.02	21	0.143
44	A	15	14	1.05	21	0.667
45	A	13	12	1.04	21	0.571
46	A	11	10	1.02	21	0.476
47	A	9	8	1.57	19	0.421
48	A	6	5	1.06	19	0.263
49	A	8	7	1.00	21	0.333
50	A	9	8	0.92	21	0.381
51	A	10	9	0.84	21	0.429
52	A	10	9	0.80	21	0.429
53	A	9	8	0.81	21	0.381
54	A	9	8	0.84	21	0.381
55	A	8	7	0.96	21	0.333
56	A	2	2	1.00	12	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	7	6	1.00	21	0.286
58	A	9	8	1.00	21	0.381
59	A	11	10	1.02	21	0.476
60	A	13	12	1.04	21	0.571
61	A	5	4	0.87	21	0.190
62	A	5	4	0.87	21	0.190
63	A	5	4	0.88	21	0.190
64	A	5	4	0.90	19	0.211
65	A	5	4	0.96	19	0.211
66	A	5	4	0.94	21	0.190
67	A	5	4	0.91	21	0.190
68	A	5	4	0.89	21	0.190
69	A	5	4	0.87	21	0.190
70	A	5	4	0.86	21	0.190
71	A	5	4	0.85	21	0.190
72	A	5	4	0.87	21	0.190
73	A	5	4	0.87	21	0.190
74	A	5	4	0.89	19	0.211
75	A	5	4	0.93	19	0.211
76	A	5	4	0.91	21	0.190
77	A	5	4	0.90	21	0.190
78	A	5	4	0.89	21	0.190
79	A	5	4	0.87	21	0.190
80	A	5	4	0.86	21	0.190
81	A	5	4	0.86	21	0.190
82	A	5	4	0.86	21	0.190
83	A	5	4	0.87	21	0.190
84	A	5	4	0.88	19	0.211
85	A	5	4	0.89	21	0.190
86	A	5	4	0.88	21	0.190
87	A	3	3	1.03	21	0.143
88	A	3	3	1.04	21	0.143
89	A	3	3	0.88	21	0.143
90	A	3	3	1.03	21	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	6	6	1.20	23	0.261
92	A	9	8	1.06	23	0.348
93	A	9	8	1.11	23	0.348
94	A	14	13	1.07	23	0.565
95	A	8	8	1.17	23	0.348
96	A	7	7	1.06	23	0.304
97	A	12	11	1.11	23	0.478
98	A	17	16	1.06	23	0.696
99	A	10	10	1.40	23	0.435
100	A	9	9	1.11	23	0.391
101	A	15	14	1.12	23	0.609
102	A	22	21	1.21	23	0.913
103	A	8	7	1.61	23	0.304
104	A	9	8	1.02	23	0.348
105	A	7	6	1.00	23	0.261
106	A	21	20	1.87	23	0.870
107	A	10	9	1.10	23	0.391
108	A	11	10	1.09	23	0.435
109	A	11	10	1.07	23	0.435
110	A	19	18	1.82	23	0.783
111	A	12	11	1.66	23	0.478
112	A	13	12	1.05	23	0.522
113	A	14	13	1.08	23	0.565
114	A	30	29	1.97	23	1.261
115	A	16	15	1.01	23	0.652
116	A	9	9	1.07	23	0.391
117	A	7	6	1.70	23	0.261
118	A	7	6	1.70	23	0.261
119	A	12	11	1.05	23	0.478
120	A	9	9	1.06	23	0.391
121	A	7	6	1.70	23	0.261
122	A	7	6	1.70	23	0.261
123	A	3	3	1.00	23	0.130
124	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	3	3	1.00	21	0.143
126	A	7	6	1.00	23	0.261
127	A	3	3	1.03	23	0.130
128	A	3	3	1.02	23	0.130
129	A	6	5	1.41	23	0.217
130	A	8	7	1.02	21	0.333
131	A	5	4	1.04	19	0.211
132	A	4	3	1.00	19	0.158
133	A	5	4	0.98	21	0.190
134	A	8	7	1.00	21	0.333
135	A	12	11	1.13	21	0.524
136	A	9	8	1.03	21	0.381
137	A	4	4	1.00	12	0.333
138	A	6	5	1.30	21	0.238
139	A	6	5	1.30	21	0.238
140	A	6	5	1.09	19	0.263
141	A	5	4	0.84	17	0.235
142	A	5	4	1.00	17	0.235
143	A	5	4	0.89	19	0.211
144	A	5	4	0.85	19	0.211
145	A	3	3	1.00	19	0.158
146	A	3	3	1.00	19	0.158
147	A	3	3	1.00	19	0.158
148	A	3	3	1.00	19	0.158
149	A	3	3	1.00	19	0.158
150	A	7	6	1.11	21	0.286
151	A	7	6	0.90	19	0.316
152	A	5	4	0.93	19	0.211
153	A	5	4	0.89	21	0.190
154	A	5	4	0.88	21	0.190
155	A	3	3	1.00	21	0.143
156	A	3	3	1.00	21	0.143
157	A	3	3	1.00	21	0.143
158	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	3	3	1.00	21	0.143
160	A	7	6	1.05	21	0.286
161	A	7	6	0.99	19	0.316
162	A	5	4	0.91	19	0.211
163	A	5	4	0.90	21	0.190
164	A	5	4	0.89	21	0.190
165	A	3	3	1.00	21	0.143
166	A	3	3	1.00	21	0.143
167	A	3	3	1.00	21	0.143
168	A	3	3	1.00	21	0.143
169	A	3	3	1.00	21	0.143
170	A	10	9	1.29	21	0.429
171	A	8	7	1.26	21	0.333
172	A	8	7	1.01	19	0.368
173	A	6	5	1.00	19	0.263
174	A	5	4	0.93	21	0.190
175	A	5	4	0.92	21	0.190
176	A	17	16	1.04	21	0.762
177	A	11	10	1.06	21	0.476
178	A	13	12	1.16	21	0.571
179	A	13	12	1.10	21	0.571
180	A	20	19	1.12	21	0.905
181	A	9	8	1.19	21	0.381
182	A	7	6	1.14	21	0.286
183	A	7	6	1.17	19	0.316
184	A	5	4	0.94	19	0.211
185	A	5	4	0.92	21	0.190
186	A	5	4	0.92	21	0.190
187	A	3	3	1.00	21	0.143
188	A	3	3	1.00	21	0.143
189	A	15	14	1.40	21	0.667
190	A	16	15	1.12	21	0.714
191	A	23	22	1.19	21	1.048
192	A	9	8	1.14	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
193	A	7	6	1.10	21	0.286
194	A	7	6	1.09	19	0.316
195	A	5	4	0.92	19	0.211
196	A	5	4	0.92	21	0.190
197	A	5	4	0.92	21	0.190
198	A	3	3	1.00	21	0.143
199	A	3	3	1.00	21	0.143
200	A	17	16	1.23	21	0.762
201	A	18	17	1.24	21	0.810
202	A	26	25	1.23	21	1.190
203	A	3	3	1.00	23	0.130
204	A	3	3	1.00	23	0.130
205	A	3	3	1.00	21	0.143
206	F	0	0	N/A	0.000	N/A
207	F	0	0	N/A	0.000	N/A
208	N/A	2	0	1.00	23	0.000

CHAPTER 3

Listing of integrals

3.1 $\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx$

3.1.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx=-\frac{23a\log(1-\sin(c+dx))}{16d}+\frac{7a\log(1+\sin(c+dx))}{16d}-\frac{a\sin(c+dx)}{d}+\frac{a^3}{8d(a-a\sin(c+dx))^2}-\frac{a^2}{d(a-a\sin(c+dx))}+\frac{a^2}{8d(a+a\sin(c+dx))}$$

output
```
-23/16*a*ln(1-sin(d*x+c))/d+7/16*a*ln(1+sin(d*x+c))/d-a*sin(d*x+c)/d+1/8*a
^3/d/(a-a*sin(d*x+c))^2-a^2/d/(a-a*sin(d*x+c))+1/8*a^2/d/(a+a*sin(d*x+c))
```

3.1.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.18

$$\int (a+a\sin(c+dx))\tan^5(c+dx)\,dx =\frac{15a\text{arctanh}(\sin(c+dx))}{8d}+\frac{15a\sec(c+dx)\tan(c+dx)}{8d}-\frac{15a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{5a\sec(c+dx)\tan^3(c+dx)}{d}-\frac{a\sin(c+dx)\tan^4(c+dx)}{d}-\frac{a(4\log(\cos(c+dx))+2\tan^2(c+dx)-\tan^4(c+dx))}{4d}$$

input
```
Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

output
```
(15*a*ArcTanh[Sin[c + d*x]])/(8*d) + (15*a*Sec[c + d*x]*Tan[c + d*x])/(8*d
) - (15*a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (5*a*Sec[c + d*x]*Tan[c + d
*x]^3)/d - (a*Sin[c + d*x]*Tan[c + d*x]^4)/d - (a*(4*Log[Cos[c + d*x]] + 2
*Tan[c + d*x]^2 - Tan[c + d*x]^4))/(4*d)
```

3.1.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^5(a\sin(c+dx)+a)dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^5\sin^5(c+dx)}{(a-a\sin(c+dx))^3(\sin(c+dx)a+a)^2}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{a^3}{4(a-a\sin(c+dx))^3}-\frac{a^2}{(a-a\sin(c+dx))^2}-\frac{a^2}{8(\sin(c+dx)a+a)^2}+\frac{23a}{16(a-a\sin(c+dx))}+\frac{7a}{16(\sin(c+dx)a+a)}-1\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3}{8(a-a\sin(c+dx))^2}-\frac{a^2}{a-a\sin(c+dx)}+\frac{a^2}{8(a\sin(c+dx)+a)}-a\sin(c+dx)-\frac{23}{16}a\log(a-a\sin(c+dx))+\frac{7}{16}a\log(a\sin(c+d}{d}$$

input
```
Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^5,x]
```

output
```
((-23*a*Log[a - a*Sin[c + d*x]])/16 + (7*a*Log[a + a*Sin[c + d*x]])/16 - a
*Sin[c + d*x] + a^3/(8*(a - a*Sin[c + d*x])^2) - a^2/(a - a*Sin[c + d*x])
+ a^2/(8*(a + a*Sin[c + d*x])))/d
```

3.1.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.1.4 Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\frac{15\sin(dx+c)}{8}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\left(\tan^4(d\right.}{4}\right.}{d}$
default	$\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c)\right)}{8}-\frac{15\sin(dx+c)}{8}+\frac{15\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+a\left(\frac{\left(\tan^4(d\right.}{4}\right.}{d}$
parts	$\frac{a\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{a\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\frac{3\left(\sin^7(dx+c)\right)}{8\cos(dx+c)^2}-\frac{3\left(\sin^5(dx+c)\right)}{8}-\frac{5\left(\sin^3(dx+c\right.}{8}\right.}{d}$
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}+\frac{i\left(2ia\,e^{2i(dx+c)}+9a\,e^{i(dx+c)}-2ia\,e^{4i(dx+c)}+6a\,e^{3i(dx+c)}+9a\,e^{5i(dx+c}\right.}{4\left(-i+e^{i(dx+c)}\right)^4\left(e^{i(dx+c)}+i\right)^2d}$

input
```
int((a+a*sin(d*x+c))*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/4*sin(d*x+c)^7/cos(d*x+c)^4-3/8*sin(d*x+c)^7/cos(d*x+c)^2-3/8*si
n(d*x+c)^5-5/8*sin(d*x+c)^3-15/8*sin(d*x+c)+15/8*ln(sec(d*x+c)+tan(d*x+c))
)+a*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.38

$$\int (a + a\sin(c + dx))\tan^5(c + dx)\,dx$$
$$= \frac{16\,a\cos{(dx+c)}^4 + 2\,a\cos{(dx+c)}^2 + 7\left(a\cos{(dx+c)}^2\sin{(dx+c)} - a\cos{(dx+c)}^2\right)\log{(\sin{(dx+c)} +}}{16\left(d\cos{(dx+}\right.}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/16*(16*a*cos(d*x + c)^4 + 2*a*cos(d*x + c)^2 + 7*(a*cos(d*x + c)^2*sin(d
*x + c) - a*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 23*(a*cos(d*x + c)^2*s
in(d*x + c) - a*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(8*a*cos(d*x +
c)^2 + a)*sin(d*x + c) - 6*a)/(d*cos(d*x + c)^2*sin(d*x + c) - d*cos(d*x +
 c)^2)
```

3.1.6 Sympy [F]

$$\int (a + a\sin(c + dx))\tan^5(c + dx)\,dx = a\left(\int \sin{(c + dx)}\tan^5{(c + dx)}\,dx + \int \tan^5{(c + dx)}\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)**5,x)
```

output
```
a*(Integral(sin(c + d*x)*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x
))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

$$\int (a + a\sin(c + dx))\tan^5(c + dx)\,dx$$
$$= \frac{7\,a\log\left(\sin\left(dx + c\right) + 1\right) - 23\,a\log\left(\sin\left(dx + c\right) - 1\right) - 16\,a\sin\left(dx + c\right) + \frac{2\left(9\,a\sin(dx+c)^2 - a\sin(dx+c) - 6\,a\right)}{\sin(dx+c)^3 - \sin(dx+c)^2 - \sin(dx+c) + 1}}{16\,d}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
1/16*(7*a*log(sin(d*x + c) + 1) - 23*a*log(sin(d*x + c) - 1) - 16*a*sin(d*
x + c) + 2*(9*a*sin(d*x + c)^2 - a*sin(d*x + c) - 6*a)/(sin(d*x + c)^3 - s
in(d*x + c)^2 - sin(d*x + c) + 1))/d
```

3.1.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))\tan^5(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
Timed out
```

3.1.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.04

$$\int (a + a\sin(c + dx))\tan^5(c + dx)\,dx$$
$$= \frac{-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2} + \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} - 5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2} - }{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 - 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 - 2\tan\left(\frac{c}{2}\right.\right.}$$
$$- \frac{23\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - 1\right)}{8\,d} + \frac{7\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) + 1\right)}{8\,d} + \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)}{d}$$

input
```
int(tan(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

output
```
((11*a*tan(c/2 + (d*x)/2)^2)/2 - (15*a*tan(c/2 + (d*x)/2))/4 + (11*a*tan(c
/2 + (d*x)/2)^3)/4 - 5*a*tan(c/2 + (d*x)/2)^4 + (11*a*tan(c/2 + (d*x)/2)^5
)/4 + (11*a*tan(c/2 + (d*x)/2)^6)/2 - (15*a*tan(c/2 + (d*x)/2)^7)/4)/(d*(2
*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^4 + 2*
tan(c/2 + (d*x)/2)^5 - 2*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1))
 - (23*a*log(tan(c/2 + (d*x)/2) - 1))/(8*d) + (7*a*log(tan(c/2 + (d*x)/2)
+ 1))/(8*d) + (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.2 $\int (a + a\sin(c + dx))\tan^3(c + dx)\,dx$

3.2.1 Optimal result

Integrand size = 19, antiderivative size = 71

$$\int (a + a\sin(c + dx))\tan^3(c + dx)\,dx = \frac{5a\log(1-\sin(c+dx))}{4d} - \frac{a\log(1+\sin(c+dx))}{4d} + \frac{a\sin(c+dx)}{d} + \frac{a^2}{2d(a-a\sin(c+dx))}$$

output
```
5/4*a*ln(1-sin(d*x+c))/d-1/4*a*ln(1+sin(d*x+c))/d+a*sin(d*x+c)/d+1/2*a^2/d
/(a-a*sin(d*x+c))
```

3.2.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15

$$\int (a + a\sin(c + dx))\tan^3(c + dx)\,dx = -\frac{3a\text{arctanh}(\sin(c+dx))}{2d} + \frac{3a\sec(c+dx)\tan(c+dx)}{2d} - \frac{a\sin(c+dx)\tan^2(c+dx)}{d} + \frac{a(2\log(\cos(c+dx)) + \tan^2(c+dx))}{2d}$$

input
```
Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

output
```
(-3*a*ArcTanh[Sin[c + d*x]])/(2*d) + (3*a*Sec[c + d*x]*Tan[c + d*x])/(2*d)
 - (a*Sin[c + d*x]*Tan[c + d*x]^2)/d + (a*(2*Log[Cos[c + d*x]] + Tan[c + d
*x]^2))/(2*d)
```

3.2.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3(a\sin(c+dx)+a)dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^3\sin^3(c+dx)}{(a-a\sin(c+dx))^2(\sin(c+dx)a+a)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{a^2}{2(a-a\sin(c+dx))^2}-\frac{5a}{4(a-a\sin(c+dx))}-\frac{a}{4(\sin(c+dx)a+a)}+1\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2}{2(a-a\sin(c+dx))}+a\sin(c+dx)+\frac{5}{4}a\log(a-a\sin(c+dx))-\frac{1}{4}a\log(a\sin(c+dx)+a)}{d}$$

input
```
Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

output
```
((5*a*Log[a - a*Sin[c + d*x]])/4 - (a*Log[a + a*Sin[c + d*x]])/4 + a*Sin[c
 + d*x] + a^2/(2*(a - a*Sin[c + d*x])))/d
```

3.2.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.2.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$
parts	$\frac{a\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{a\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-iax-\frac{ia\,e^{i(dx+c)}}{2d}+\frac{ia\,e^{-i(dx+c)}}{2d}-\frac{2iac}{d}-\frac{ia\,e^{i(dx+c)}}{\left(-i+e^{i(dx+c)}\right)^2 d}-\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{5a\ln\left(-i+e^{i(dx+c)}\right)}{2d}$

input
```
int((a+a*sin(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*
ln(sec(d*x+c)+tan(d*x+c)))+a*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx = -\frac{4\,a\cos{(dx+c)}^2+(a\sin{(dx+c)}-a)\log{(\sin{(dx+c)}+1)}-5\,(a\sin{(dx+c)}-a)\log{(-\sin{(dx+c)}}}{4\,(d\sin{(dx+c)}-d)}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(4*a*cos(d*x + c)^2 + (a*sin(d*x + c) - a)*log(sin(d*x + c) + 1) - 5*
(a*sin(d*x + c) - a)*log(-sin(d*x + c) + 1) + 4*a*sin(d*x + c) - 2*a)/(d*s
in(d*x + c) - d)
```

3.2.6 Sympy [F]

$$\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx = a\left(\int \sin{(c+dx)}\tan^3{(c+dx)}\,dx + \int \tan^3{(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)**3,x)
```

output
```
a*(Integral(sin(c + d*x)*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x
))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.72

$$\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx = -\frac{a\log{(\sin{(dx+c)}+1)}-5\,a\log{(\sin{(dx+c)}-1)}-4\,a\sin{(dx+c)}+\frac{2\,a}{\sin(dx+c)-1}}{4\,d}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*(a*log(sin(d*x + c) + 1) - 5*a*log(sin(d*x + c) - 1) - 4*a*sin(d*x +
c) + 2*a/(sin(d*x + c) - 1))/d
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22388 vs. 2(66) = 132.

Time = 105.66 (sec) , antiderivative size = 22388, normalized size of antiderivative = 315.32

$$\int (a + a\sin(c + dx))\tan^3(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(3*a*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 3*a*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 2*a*log(4*(tan(d*
x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
 tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 2*a*tan(
d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 6*a*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2
*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/
2*d*x)^6*tan(1/2*c)^6*tan(c) + 6*a*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^6*tan(1/
2*c)^6*tan(c) - 4*a*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^...
```

3.2.9 Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.17

$$\int (a+a\sin(c+dx))\tan^3(c+dx)\,dx$$
$$=\frac{5\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{2\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{2\,d}$$
$$+\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-4\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$
$$-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

output
```
(5*a*log(tan(c/2 + (d*x)/2) - 1))/(2*d) - (a*log(tan(c/2 + (d*x)/2) + 1))/
(2*d) + (3*a*tan(c/2 + (d*x)/2) - 4*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 +
 (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) - 2*tan(c/2
 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4 + 1)) - (a*log(tan(c/2 + (d*x)/2)^2 +
 1))/d
```

3.3 $\int (a + a\sin(c + dx))\tan(c + dx)\,dx$

3.3.1 Optimal result

Integrand size = 17, antiderivative size = 30

$$\int (a + a\sin(c + dx))\tan(c + dx)\,dx = -\frac{a\log(1 - \sin(c + dx))}{d} - \frac{a\sin(c + dx)}{d}$$

output
```
-a*ln(1-sin(d*x+c))/d-a*sin(d*x+c)/d
```

3.3.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27

$$\int (a + a\sin(c + dx))\tan(c + dx)\,dx = \frac{a\text{arctanh}(\sin(c + dx))}{d} - \frac{a\log(\cos(c + dx))}{d} - \frac{a\sin(c + dx)}{d}$$

input
```
Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

output
```
(a*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (a*Sin[c + d*x])/d
```

3.3.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3186, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a\sin(c+dx)+a)dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a\sin(c+dx)}{a-a\sin(c+dx)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{a}{a-a\sin(c+dx)}-1\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a\sin(c+dx)-a\log(a-a\sin(c+dx))}{d}$$

input
```
Int[(a + a*Sin[c + d*x])*Tan[c + d*x],x]
```

output
```
(-(a*Log[a - a*Sin[c + d*x]]) - a*Sin[c + d*x])/d
```

3.3.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.3.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$-\frac{a(\sin(dx+c)+\ln(\sin(dx+c)-1))}{d}$	23
default	$-\frac{a(\sin(dx+c)+\ln(\sin(dx+c)-1))}{d}$	23
parts	$\frac{a\ln(1+\tan^2(dx+c))}{2d}+\frac{a(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$	47
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2a\ln(-i+e^{i(dx+c)})}{d}$	66

input
```
int((a+a*sin(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*a*(sin(d*x+c)+ln(sin(d*x+c)-1))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int (a+a\sin(c+dx))\tan(c+dx)\,dx=-\frac{a\log(-\sin(dx+c)+1)+a\sin(dx+c)}{d}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

output
```
-(a*log(-sin(d*x + c) + 1) + a*sin(d*x + c))/d
```

3.3.6 Sympy [F]

$$\int (a+a\sin(c+dx))\tan(c+dx)\,dx = a\left(\int \sin(c+dx)\tan(c+dx)\,dx + \int \tan(c+dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c),x)
```

output
```
a*(Integral(sin(c + d*x)*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int (a+a\sin(c+dx))\tan(c+dx)\,dx = -\frac{a\log(\sin(dx+c)-1)+a\sin(dx+c)}{d}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

output
```
-(a*log(sin(d*x + c) - 1) + a*sin(d*x + c))/d
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. 2(30) = 60.

Time = 0.50 (sec) , antiderivative size = 1200, normalized size of antiderivative = 40.00

$$\int (a+a\sin(c+dx))\tan(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

output
```
-1/2*(a*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - a*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*t
an(1/2*c)^2 + a*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 a*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^2 - a*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + t
an(c)^2 + 1))*tan(1/2*d*x)^2 - 4*a*tan(1/2*d*x)^2*tan(1/2*c) + a*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/...
```

3.3.9 Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

$$\int (a+a\sin(c+dx))\tan(c+dx)\,dx$$
$$= -\frac{a\left(\sin\left(c+d\,x\right)+2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)\right)}{d}$$

input
```
int(tan(c + d*x)*(a + a*sin(c + d*x)),x)
```

output
```
-(a*(sin(c + d*x) + 2*log(tan(c/2 + (d*x)/2) - 1) - log(tan(c/2 + (d*x)/2)
^2 + 1)))/d
```

3.4 $\int \cot(c+dx)(a+a\sin(c+dx))\,dx$

3.4.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \cot(c+dx)(a+a\sin(c+dx))\,dx=\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \cot(c+dx)(a+a\sin(c+dx))\,dx=\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\tan(c+dx))}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

output
```
(a*Log[Cos[c + d*x]])/d + (a*Log[Tan[c + d*x]])/d + (a*Cos[d*x]*Sin[c])/d
+ (a*Cos[c]*Sin[d*x])/d
```

3.4.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3186, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(c+dx)+a}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc(c+dx)(\sin(c+dx)a+a)}{a}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int (\csc(c+dx)+1)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sin(c+dx)+a\log(a\sin(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]*(a + a*Sin[c + d*x]),x]
```

output
```
(a*Log[a*Sin[c + d*x]] + a*Sin[c + d*x])/d
```

3.4.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.4.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{a\sin(dx+c)+a\ln(\sin(dx+c))}{d}$	23
default	$\frac{a\sin(dx+c)+a\ln(\sin(dx+c))}{d}$	23
risch	$-iax-\frac{2iac}{d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{a\sin(dx+c)}{d}$	43

input
```
int(cot(d*x+c)*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*sin(d*x+c)+a*ln(sin(d*x+c)))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+a\sin(c+dx))\,dx=\frac{a\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+a\sin\left(dx+c\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(a*log(1/2*sin(d*x + c)) + a*sin(d*x + c))/d
```

3.4.6 Sympy [F]

$$\int \cot(c+dx)(a+a\sin(c+dx))\,dx = a\left(\int \sin(c+dx)\cot(c+dx)\,dx + \int \cot(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x)
```

output
```
a*(Integral(sin(c + d*x)*cot(c + d*x), x) + Integral(cot(c + d*x), x))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{a\log(\sin(dx+c)) + a\sin(dx+c)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sin(d*x + c)) + a*sin(d*x + c))/d
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cot(c+dx)(a+a\sin(c+dx))\,dx = \frac{a\log(|\sin(dx+c)|) + a\sin(dx+c)}{d}$$

input
```
integrate(cot(d*x+c)*(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(sin(d*x + c))) + a*sin(d*x + c))/d
```

3.4.9 Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.58

$$\int \cot(c+dx)(a+a\sin(c+dx))\,dx$$
$$=\frac{a\left(\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\sin\left(c+d\,x\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

input
```
int(cot(c + d*x)*(a + a*sin(c + d*x)),x)
```

output
```
(a*(log(tan(c/2 + (d*x)/2)) + sin(c + d*x) - log(tan(c/2 + (d*x)/2)^2 + 1)
))/d
```

3.5 $\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$

3.5.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx = -\frac{a\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\log(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d}$$

output
```
-a*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-a*ln(sin(d*x+c))/d-a*sin(d*x+c)/d
```

3.5.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx = -\frac{a\csc(c+dx)}{d} - \frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d} - \frac{a\sin(c+dx)}{d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

output
```
-((a*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) - (a*Sin[c + d*x])/d
```

3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3186, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(c+dx)+a}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))(\sin(c+dx)a+a)^2}{a^3}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{84}$$

$$\frac{\int \left(\csc^3(c+dx)+\csc^2(c+dx)-\csc(c+dx)-1\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a\sin(c+dx)-\frac{1}{2}a\csc^2(c+dx)-a\csc(c+dx)-a\log(a\sin(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x]),x]
```

output
```
(-(a*Csc[c + d*x]) - (a*Csc[c + d*x]^2)/2 - a*Log[a*Sin[c + d*x]] - a*Sin[
c + d*x])/d
```

3.5.3.1 Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.5.4 Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

method	result	size
derivativedivides	$\frac{a\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	67
default	$\frac{a\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)}{d}$	67
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2ia\left(ie^{2i(dx+c)}+e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}$	118

input
```
int(cot(d*x+c)^3*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/sin(d*x+c)*cos(d*x+c)^4-(2+cos(d*x+c)^2)*sin(d*x+c))+a*(-1/2*co
t(d*x+c)^2-ln(sin(d*x+c))))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$$
$$=-\frac{2\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(a\cos\left(dx+c\right)^2-2a\right)\sin\left(dx+c\right)-a}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(a*cos(d*x + c)^2
 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

3.5.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx = a\left(\int \sin(c+dx)\cot^3(c+dx)\,dx+\int \cot^3(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**3*(a+a*sin(d*x+c)),x)
```

output
```
a*(Integral(sin(c + d*x)*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x
))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx = -\frac{2\,a\log\left(\sin\left(dx+c\right)\right)+2\,a\sin\left(dx+c\right)+\frac{2\,a\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*a*log(sin(d*x + c)) + 2*a*sin(d*x + c) + (2*a*sin(d*x + c) + a)/si
n(d*x + c)^2)/d
```

3.5.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$$
$$= -\frac{2\,a\log\left(|\sin\left(dx+c\right)|\right)+2\,a\sin\left(dx+c\right)-\frac{3\,a\sin(dx+c)^2-2\,a\sin(dx+c)-a}{\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*a*log(abs(sin(d*x + c))) + 2*a*sin(d*x + c) - (3*a*sin(d*x + c)^2
- 2*a*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

3.5.9 Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.70

$$\int \cot^3(c+dx)(a+a\sin(c+dx))\,dx$$
$$= \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} - \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$
$$- \frac{10\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$
$$- \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + a*sin(c + d*x)),x)
```

output
```
(a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a*tan(c/2 + (d*x)/2))/(2*d) - (a/2
+ 2*a*tan(c/2 + (d*x)/2) + (a*tan(c/2 + (d*x)/2)^2)/2 + 10*a*tan(c/2 + (d*
x)/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a*tan(c/
2 + (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

3.6 $\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx$

3.6.1 Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{2a\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(c+dx))}{d}+\frac{a\sin(c+dx)}{d}$$

output
```
2*a*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*a*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/
d+a*ln(sin(d*x+c))/d+a*sin(d*x+c)/d
```

3.6.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{a\cot^2(c+dx)}{2d}-\frac{a\cot^4(c+dx)}{4d}+\frac{2a\csc(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}+\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\tan(c+dx))}{d}+\frac{a\sin(c+dx)}{d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

output
```
(a*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^4)/(4*d) + (2*a*Csc[c + d*x])/d
 - (a*Csc[c + d*x]^3)/(3*d) + (a*Log[Cos[c + d*x]])/d + (a*Log[Tan[c + d*x
]])/d + (a*Sin[c + d*x])/d
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(c+dx)+a}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^5(c+dx)(a-a\sin(c+dx))^2(\sin(c+dx)a+a)^3}{a^5}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\csc^5(c+dx)+\csc^4(c+dx)-2\csc^3(c+dx)-2\csc^2(c+dx)+\csc(c+dx)+1\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sin(c+dx)-\frac{1}{4}a\csc^4(c+dx)-\frac{1}{3}a\csc^3(c+dx)+a\csc^2(c+dx)+2a\csc(c+dx)+a\log(a\sin(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + a*Sin[c + d*x]),x]
```

output
```
(2*a*Csc[c + d*x] + a*Csc[c + d*x]^2 - (a*Csc[c + d*x]^3)/3 - (a*Csc[c + d
*x]^4)/4 + a*Log[a*Sin[c + d*x]] + a*Sin[c + d*x])/d
```

3.6.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.6.4 Maple [A] (verified)

Time = 1.50 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\mathrm{s}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)\right)+a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\mathrm{s}\right.}{d}$
risch	$-iax-\frac{ia\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{ia\,\mathrm{e}^{-i(dx+c)}}{2d}-\frac{2iac}{d}+\frac{4ia\left(3i\mathrm{e}^{6i(dx+c)}+3\,\mathrm{e}^{7i(dx+c)}-3i\mathrm{e}^{4i(dx+c)}-7\,\mathrm{e}^{5i(dx+c)}+3i\mathrm{e}^{2i(dx+c)}\right.}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}$

input
```
int(cot(d*x+c)^5*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/3/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(
d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)
^2+ln(sin(d*x+c))))
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx = -\frac{12\,a\cos(dx+c)^2 - 12\left(a\cos(dx+c)^4 - 2\,a\cos(dx+c)^2 + a\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 4\left(3\,a\cos(dx+c\right.}{12\left(d\cos(dx+c)^4 - 2\,d\cos(dx+c)^2 + d\right)}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a
)*log(1/2*sin(d*x + c)) - 4*(3*a*cos(d*x + c)^4 - 12*a*cos(d*x + c)^2 + 8*
a)*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

3.6.6 Sympy [F]

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx = a\left(\int \sin(c+dx)\cot^5(c+dx)\,dx + \int \cot^5(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**5*(a+a*sin(d*x+c)),x)
```

output
```
a*(Integral(sin(c + d*x)*cot(c + d*x)**5, x) + Integral(cot(c + d*x)**5, x
))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx = \frac{12\,a\log(\sin(dx+c)) + 12\,a\sin(dx+c) + \frac{24\,a\sin(dx+c)^3 + 12\,a\sin(dx+c)^2 - 4\,a\sin(dx+c) - 3\,a}{\sin(dx+c)^4}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(12*a*log(sin(d*x + c)) + 12*a*sin(d*x + c) + (24*a*sin(d*x + c)^3 +
12*a*sin(d*x + c)^2 - 4*a*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx$$
$$=\frac{12\,a\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^4-24\,a\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,a\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/12*(12*a*log(abs(sin(d*x + c))) + 12*a*sin(d*x + c) - (25*a*sin(d*x + c)
^4 - 24*a*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*a*sin(d*x + c) + 3*a)/s
in(d*x + c)^4)/d
```

3.6.9 Mupad [B] (verification not implemented)

Time = 5.97 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.56

$$\int \cot^5(c+dx)(a+a\sin(c+dx))\,dx=\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$
$$+\frac{46\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{40\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}$$
$$-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}$$
$$-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(cot(c + d*x)^5*(a + a*sin(c + d*x)),x)
```

output
```
(7*a*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*a*tan
(c/2 + (d*x)/2))/3 - a/4 + (40*a*tan(c/2 + (d*x)/2)^3)/3 + 3*a*tan(c/2 + (
d*x)/2)^4 + 46*a*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*ta
n(c/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2
+ (d*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x
)/2)^4)/(64*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

3.7 $\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$

3.7.1 Optimal result

Integrand size = 19, antiderivative size = 115

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = -\frac{3a\csc(c+dx)}{d}-\frac{3a\csc^2(c+dx)}{2d}+\frac{a\csc^3(c+dx)}{d}+\frac{3a\csc^4(c+dx)}{4d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^6(c+dx)}{6d}-\frac{a\log(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}$$

output
```
-3*a*csc(d*x+c)/d-3/2*a*csc(d*x+c)^2/d+a*csc(d*x+c)^3/d+3/4*a*csc(d*x+c)^4
/d-1/5*a*csc(d*x+c)^5/d-1/6*a*csc(d*x+c)^6/d-a*ln(sin(d*x+c))/d-a*sin(d*x+
c)/d
```

3.7.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.97

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = -\frac{3a\csc(c+dx)}{d}+\frac{a\csc^3(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a(6\cot^2(c+dx)-3\cot^4(c+dx)+2\cot^6(c+dx)+12\log(\cos(c+dx))+12\log(\tan(c+dx)))}{12d}-\frac{a\sin(c+dx)}{d}$$

input
```
Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

output
```
(-3*a*Csc[c + d*x])/d + (a*Csc[c + d*x]^3)/d - (a*Csc[c + d*x]^5)/(5*d) -
(a*(6*Cot[c + d*x]^2 - 3*Cot[c + d*x]^4 + 2*Cot[c + d*x]^6 + 12*Log[Cos[c
+ d*x]] + 12*Log[Tan[c + d*x]]))/(12*d) - (a*Sin[c + d*x])/d
```

3.7.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(c+dx)+a}{\tan(c+dx)^7}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^7(c+dx)(a-a\sin(c+dx))^3(\sin(c+dx)a+a)^4}{a^7}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int (\csc^7(c+dx)+\csc^6(c+dx)-3\csc^5(c+dx)-3\csc^4(c+dx)+3\csc^3(c+dx)+3\csc^2(c+dx)-\csc(c+dx) -}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a\sin(c+dx)-\frac{1}{6}a\csc^6(c+dx)-\frac{1}{5}a\csc^5(c+dx)+\frac{3}{4}a\csc^4(c+dx)+a\csc^3(c+dx)-\frac{3}{2}a\csc^2(c+dx)-3a\csc}{d}$$

input
```
Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x]),x]
```

output
```
(-3*a*Csc[c + d*x] - (3*a*Csc[c + d*x]^2)/2 + a*Csc[c + d*x]^3 + (3*a*Csc[
c + d*x]^4)/4 - (a*Csc[c + d*x]^5)/5 - (a*Csc[c + d*x]^6)/6 - a*Log[a*Sin[
c + d*x]] - a*Sin[c + d*x])/d
```

3.7.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.7.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^8(dx+c)}{5\sin(dx+c)^3}-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)\right)+a\left(-\frac{(c}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}+\frac{\cos^8(dx+c)}{5\sin(dx+c)^3}-\frac{\cos^8(dx+c)}{\sin(dx+c)}-\left(\frac{16}{5}+\cos^6(dx+c)+\frac{6\left(\cos^4(dx+c)\right)}{5}+\frac{8\left(\cos^2(dx+c)\right)}{5}\right)\sin(dx+c)\right)+a\left(-\frac{(c}{}\right.}{d}$
risch	$iax+\frac{ia\,e^{i(dx+c)}}{2d}-\frac{ia\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2ia\left(45ie^{10i(dx+c)}+45\,e^{11i(dx+c)}-90ie^{8i(dx+c)}-165\,e^{9i(dx+c)}+170ie\right.}{}$

input
```
int(cot(d*x+c)^7*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/5/sin(d*x+c)^5*cos(d*x+c)^8+1/5/sin(d*x+c)^3*cos(d*x+c)^8-1/sin
(d*x+c)*cos(d*x+c)^8-(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)
*sin(d*x+c))+a*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin
(d*x+c))))
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.37

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$$
$$= \frac{90\,a\cos(dx+c)^4 - 135\,a\cos(dx+c)^2 - 60\left(a\cos(dx+c)^6 - 3\,a\cos(dx+c)^4 + 3\,a\cos(dx+c)^2 - a\right)\mathrm{lo}}{60\left(d\cos(dx+c)^6 - 3\,d\cos(\right.}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/60*(90*a*cos(d*x + c)^4 - 135*a*cos(d*x + c)^2 - 60*(a*cos(d*x + c)^6 -
3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) - 12*(5
*a*cos(d*x + c)^6 - 30*a*cos(d*x + c)^4 + 40*a*cos(d*x + c)^2 - 16*a)*sin(
d*x + c) + 55*a)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)
^2 - d)
```

3.7.6 Sympy [F]

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = a\left(\int \sin(c+dx)\cot^7(c+dx)\,dx + \int \cot^7(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**7*(a+a*sin(d*x+c)),x)
```

output
```
a*(Integral(sin(c + d*x)*cot(c + d*x)**7, x) + Integral(cot(c + d*x)**7, x
))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.79

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx =$$
$$-\frac{60\,a\log(\sin(dx+c)) + 60\,a\sin(dx+c) + \frac{180\,a\sin(dx+c)^5+90\,a\sin(dx+c)^4-60\,a\sin(dx+c)^3-45\,a\sin(dx+c)^2+12\,a\sin(dx}{\sin(dx+c)^6}}{60\,d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/60*(60*a*log(sin(d*x + c)) + 60*a*sin(d*x + c) + (180*a*sin(d*x + c)^5
+ 90*a*sin(d*x + c)^4 - 60*a*sin(d*x + c)^3 - 45*a*sin(d*x + c)^2 + 12*a*s
in(d*x + c) + 10*a)/sin(d*x + c)^6)/d
```

3.7.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx = -\frac{60\,a\log\left(|\sin\left(dx+c\right)|\right)+60\,a\sin\left(dx+c\right)-\frac{147\,a\sin(dx+c)^6-180\,a\sin(dx+c)^5-90\,a\sin(dx+c)^4+60\,a\sin(dx+c)^3+45\,a\sin}{\sin(dx+c)^6}}{60\,d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/60*(60*a*log(abs(sin(d*x + c))) + 60*a*sin(d*x + c) - (147*a*sin(d*x +
c)^6 - 180*a*sin(d*x + c)^5 - 90*a*sin(d*x + c)^4 + 60*a*sin(d*x + c)^3 +
45*a*sin(d*x + c)^2 - 12*a*sin(d*x + c) - 10*a)/sin(d*x + c)^6)/d
```

3.7.9 Mupad [B] (verification not implemented)

Time = 6.78 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.32

$$\begin{aligned}\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx &= \frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{32\,d}-\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{128\,d}\\ &-\frac{19\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{32\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{384\,d}\\ &-\frac{a\left(1920\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-1920\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{1920\,d}\\ &-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{51\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{16}+\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{128}+\frac{35\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{32}+\frac{25\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{128}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{80}-\frac{11\,a\,t}{}\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}\end{aligned}$$

input
```
int(cot(c + d*x)^7*(a + a*sin(c + d*x)),x)
```

output
```
(3*a*tan(c/2 + (d*x)/2)^3)/(32*d) - (29*a*tan(c/2 + (d*x)/2)^2)/(128*d) -
(19*a*tan(c/2 + (d*x)/2))/(16*d) + (a*tan(c/2 + (d*x)/2)^4)/(32*d) - (a*ta
n(c/2 + (d*x)/2)^5)/(160*d) - (a*tan(c/2 + (d*x)/2)^6)/(384*d) - (a*(1920*
log(tan(c/2 + (d*x)/2)) - 1920*log(tan(c/2 + (d*x)/2)^2 + 1)))/(1920*d) -
(cot(c/2 + (d*x)/2)^6*(a/384 + (a*tan(c/2 + (d*x)/2))/160 - (11*a*tan(c/2
+ (d*x)/2)^2)/384 - (7*a*tan(c/2 + (d*x)/2)^3)/80 + (25*a*tan(c/2 + (d*x)/
2)^4)/128 + (35*a*tan(c/2 + (d*x)/2)^5)/32 + (29*a*tan(c/2 + (d*x)/2)^6)/1
28 + (51*a*tan(c/2 + (d*x)/2)^7)/16))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.7. $\int \cot^7(c+dx)(a+a\sin(c+dx))\,dx$

3.8 $\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx$

3.8.1 Optimal result

Integrand size = 19, antiderivative size = 101

$$\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx = -ax+\frac{a\cos(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-\frac{a\sec^3(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)}{d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

output
```
-a*x+a*cos(d*x+c)/d+3*a*sec(d*x+c)/d-a*sec(d*x+c)^3/d+1/5*a*sec(d*x+c)^5/d
+a*tan(d*x+c)/d-1/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5/d
```

3.8.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx = -\frac{a\arctan(\tan(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-\frac{a\sec^3(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}+\frac{a\tan(c+dx)}{d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

input
```
Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]
```

output
```
-((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d
- (a*Sec[c + d*x]^3)/d + (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (
a*Tan[c + d*x]^3)/(3*d) + (a*Tan[c + d*x]^5)/(5*d)
```

3.8.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^6(a\sin(c+dx)+a)dx$$

$$\Big\downarrow \text{3189}$$

$$\int \left(a\tan^6(c+dx)+a\sin(c+dx)\tan^6(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\cos(c+dx)}{d}+\frac{a\tan^5(c+dx)}{5d}-\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{a\sec^5(c+dx)}{5d}-\frac{a\sec^3(c+dx)}{d}+\frac{3a\sec(c+dx)}{d}-ax$$

input
```
Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^6,x]
```

output
```
-(a*x) + (a*Cos[c + d*x])/d + (3*a*Sec[c + d*x])/d - (a*Sec[c + d*x]^3)/d
+ (a*Sec[c + d*x]^5)/(5*d) + (a*Tan[c + d*x])/d - (a*Tan[c + d*x]^3)/(3*d)
 + (a*Tan[c + d*x]^5)/(5*d)
```

3.8.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.8.4 Maple [A] (verified)

Time = 2.33 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.34

method	result
derivativedivides	$\frac{a\left(\frac{\sin^8(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^8(dx+c)}{5\cos(dx+c)^3}+\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^5(\right.}{5}\right.}{d}$
default	$\frac{a\left(\frac{\sin^8(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^8(dx+c)}{5\cos(dx+c)^3}+\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx+c)\right)}{5}+\frac{8\left(\sin^2(dx+c)\right)}{5}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^5(\right.}{5}\right.}{d}$
parts	$\frac{a\left(\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{\left(\tan^3(dx+c)\right)}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{a\left(\frac{\sin^8(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^8(dx+c)}{5\cos(dx+c)^3}+\frac{\sin^8(dx+c)}{\cos(dx+c)}+\left(\frac{16}{5}\right.\right.}{}$
risch	$-ax+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}+\frac{-\frac{182ia\,e^{2i(dx+c)}}{15}+\frac{2a\,e^{i(dx+c)}}{15}+\frac{42a\,e^{3i(dx+c)}}{5}-\frac{46ia}{15}-14ia\,e^{4i(dx+c)}+10a\,e^{5i(d}}{\left(e^{i(dx+c)}+i\right)^3\left(-i+e^{i(dx+c)}\right)^5d}$

input
```
int((a+a*sin(d*x+c))*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/5*sin(d*x+c)^8/cos(d*x+c)^5-1/5*sin(d*x+c)^8/cos(d*x+c)^3+sin(d*
x+c)^8/cos(d*x+c)+(16/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*co
s(d*x+c))+a*(1/5*tan(d*x+c)^5-1/3*tan(d*x+c)^3+tan(d*x+c)-d*x-c))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15

$$\int (a + a\sin(c+dx))\tan^6(c+dx)\,dx$$
$$= \frac{15\,adx\cos(dx+c)^3 - 38\,a\cos(dx+c)^4 - 11\,a\cos(dx+c)^2 - \left(15\,adx\cos(dx+c)^3 - 15\,a\cos(dx+c)^4\right.}{15\left(d\cos(dx+c)^3\sin(dx+c) - d\cos(dx+c)^3\right)}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
1/15*(15*a*d*x*cos(d*x + c)^3 - 38*a*cos(d*x + c)^4 - 11*a*cos(d*x + c)^2
- (15*a*d*x*cos(d*x + c)^3 - 15*a*cos(d*x + c)^4 - 22*a*cos(d*x + c)^2 + 4
*a)*sin(d*x + c) + a)/(d*cos(d*x + c)^3*sin(d*x + c) - d*cos(d*x + c)^3)
```

3.8.6 Sympy [F]

$$\int (a + a\sin(c+dx))\tan^6(c+dx)\,dx = a\left(\int \sin(c+dx)\tan^6(c+dx)\,dx + \int \tan^6(c+dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)**6,x)
```

output
```
a*(Integral(sin(c + d*x)*tan(c + d*x)**6, x) + Integral(tan(c + d*x)**6, x
))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.86

$$\int (a + a\sin(c+dx))\tan^6(c+dx)\,dx$$
$$= \frac{\left(3\tan(dx+c)^5 - 5\tan(dx+c)^3 - 15\,dx - 15\,c + 15\tan(dx+c)\right)a + 3\,a\left(\frac{15\cos(dx+c)^4 - 5\cos(dx+c)^2 + 1}{\cos(dx+c)^5} + 5\right.}{15\,d}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/15*((3*tan(d*x + c)^5 - 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x +
c))*a + 3*a*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5
*cos(d*x + c)))/d
```

3.8.8 Giac [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
Timed out
```

3.8.9 Mupad [B] (verification not implemented)

Time = 11.22 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.48

$$\int (a+a\sin(c+dx))\tan^6(c+dx)\,dx$$

$$=\frac{\left(\frac{a\,(30\,c+30\,d\,x-30)}{15}-2\,a\,(c+d\,x)\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{a\,(15\,c+15\,d\,x+60)}{15}-a\,(c+d\,x)\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\left(4\,a\,(c+\right.}{}$$

$$-a\,x$$

input
```
int(tan(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

output
```
((a*(15*c + 15*d*x - 96))/15 - tan(c/2 + (d*x)/2)*((a*(30*c + 30*d*x - 162
))/15 - 2*a*(c + d*x)) - a*(c + d*x) + tan(c/2 + (d*x)/2)^8*((a*(15*c + 15
*d*x + 60))/15 - a*(c + d*x)) + tan(c/2 + (d*x)/2)^9*((a*(30*c + 30*d*x -
30))/15 - 2*a*(c + d*x)) - tan(c/2 + (d*x)/2)^4*((a*(30*c + 30*d*x - 52))/
15 - 2*a*(c + d*x)) - tan(c/2 + (d*x)/2)^7*((a*(60*c + 60*d*x - 40))/15 -
4*a*(c + d*x)) - tan(c/2 + (d*x)/2)^2*((a*(15*c + 15*d*x - 156))/15 - a*(c
 + d*x)) + tan(c/2 + (d*x)/2)^6*((a*(30*c + 30*d*x - 140))/15 - 2*a*(c + d
*x)) + tan(c/2 + (d*x)/2)^3*((a*(60*c + 60*d*x - 344))/15 - 4*a*(c + d*x))
 + (44*a*tan(c/2 + (d*x)/2)^5)/15)/(d*(tan(c/2 + (d*x)/2) - 1)^5*(tan(c/2
+ (d*x)/2) + 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)) - a*x
```

3.8. $\int (a + a\sin(c + dx))\tan^6(c + dx)\,dx$

3.9 $\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx$

3.9.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx = ax - \frac{a\cos(c + dx)}{d} - \frac{2a\sec(c + dx)}{d} + \frac{a\sec^3(c + dx)}{3d} - \frac{a\tan(c + dx)}{d} + \frac{a\tan^3(c + dx)}{3d}$$

output
```
a*x-a*cos(d*x+c)/d-2*a*sec(d*x+c)/d+1/3*a*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/
3*a*tan(d*x+c)^3/d
```

3.9.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx = \frac{a\arctan(\tan(c + dx))}{d} - \frac{a\cos(c + dx)}{d} - \frac{2a\sec(c + dx)}{d} + \frac{a\sec^3(c + dx)}{3d} - \frac{a\tan(c + dx)}{d} + \frac{a\tan^3(c + dx)}{3d}$$

input
```
Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

output
```
(a*ArcTan[Tan[c + d*x]])/d - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (
a*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

3.9.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4(a\sin(c+dx)+a)dx$$

$$\Big\downarrow \text{3189}$$

$$\int \left(a\tan^4(c+dx)+a\sin(c+dx)\tan^4(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cos(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\sec^3(c+dx)}{3d}-\frac{2a\sec(c+dx)}{d}+ax$$

input
```
Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

output
```
a*x - (a*Cos[c + d*x])/d - (2*a*Sec[c + d*x])/d + (a*Sec[c + d*x]^3)/(3*d)
 - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

3.9.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.9.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)}{d}$
default	$\frac{a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)}{d}$
parts	$\frac{a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right.\right.}{d}$
risch	$ax-\frac{a\,e^{i(dx+c)}}{2d}-\frac{a\,e^{-i(dx+c)}}{2d}-\frac{4\left(-2ia+a\,e^{i(dx+c)}-3ia\,e^{2i(dx+c)}+3a\,e^{3i(dx+c)}\right)}{3\left(e^{i(dx+c)}+i\right)\left(-i+e^{i(dx+c)}\right)^3 d}$

input
```
int((a+a*sin(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/3/cos(d*x+c)^3*sin(d*x+c)^6-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x
+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c))+a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.22

$$\int (a+a\sin(c+dx))\tan^4(c+dx)\,dx =$$
$$-\frac{3\,adx\cos(dx+c)-7\,a\cos(dx+c)^2-\left(3\,adx\cos(dx+c)-3\,a\cos(dx+c)^2-2\,a\right)\sin(dx+c)-a}{3\left(d\cos(dx+c)\sin(dx+c)-d\cos(dx+c)\right)}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/3*(3*a*d*x*cos(d*x + c) - 7*a*cos(d*x + c)^2 - (3*a*d*x*cos(d*x + c) -
3*a*cos(d*x + c)^2 - 2*a)*sin(d*x + c) - a)/(d*cos(d*x + c)*sin(d*x + c) -
 d*cos(d*x + c))
```

3.9.6 Sympy [F]

$$\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx = a\left(\int \sin(c + dx)\tan^4(c + dx)\,dx + \int \tan^4(c + dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)**4,x)
```

output
```
a*(Integral(sin(c + d*x)*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x
))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx = \frac{\left(\tan(dx + c)^3 + 3\,dx + 3\,c - 3\,\tan(dx + c)\right)a - a\left(\frac{6\,\cos(dx+c)^2 - 1}{\cos(dx+c)^3} + 3\,\cos(dx + c)\right)}{3\,d}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - a*((6*cos(d*x + c
)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

3.9.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.9.9 Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.21

$$\int (a + a\sin(c + dx))\tan^4(c + dx)\,dx = a\,x$$

$$+ \frac{\left(\frac{2\,a\,(3\,c+3\,d\,x)}{3} - \frac{a\,(6\,c+6\,d\,x-6)}{3}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 + \left(\frac{a\,(3\,c+3\,d\,x-12)}{3} - \frac{a\,(3\,c+3\,d\,x)}{3}\right)\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 + \frac{4\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right) - 1\right)^3\,\left(\mathrm{t}\right.}$$

input
```
int(tan(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

output
```
a*x + ((a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 16))/3 + tan(c/2 + (d*x)/2)
^2*((a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 4))/3) - tan(c/2 + (d*x)/2)^4*
((a*(3*c + 3*d*x))/3 - (a*(3*c + 3*d*x - 12))/3) + tan(c/2 + (d*x)/2)^5*((
2*a*(3*c + 3*d*x))/3 - (a*(6*c + 6*d*x - 6))/3) - tan(c/2 + (d*x)/2)*((2*a
*(3*c + 3*d*x))/3 - (a*(6*c + 6*d*x - 26))/3) + (4*a*tan(c/2 + (d*x)/2)^3)
/3)/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x
)/2)^2 + 1))
```

3.10 $\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx$

3.10.1 Optimal result

Integrand size = 19, antiderivative size = 39

$$\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx=-ax+\frac{a\cos(c+dx)}{d}+\frac{a\cos(c+dx)}{d(1-\sin(c+dx))}$$

output
```
-a*x+a*cos(d*x+c)/d+a*cos(d*x+c)/d/(1-sin(d*x+c))
```

3.10.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.21

$$\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx=-\frac{a\arctan(\tan(c+dx))}{d}+\frac{a\cos(c+dx)}{d}+\frac{a\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

output
```
-((a*ArcTan[Tan[c + d*x]])/d) + (a*Cos[c + d*x])/d + (a*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

3.10.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.474, Rules used = {3042, 3187, 3042, 3225, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^2(a\sin(c+dx)+a)dx$$

$$\Big\downarrow \text{ 3187}$$

$$a^2\int \frac{\sin^2(c+dx)}{a-a\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$a^2\int \frac{\sin(c+dx)^2}{a-a\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3225}$$

$$a^2\left(\frac{\cos(c+dx)}{ad}-\frac{\int -\frac{\sin(c+dx)}{1-\sin(c+dx)}dx}{a}\right)$$

$$\Big\downarrow \text{ 25}$$

$$a^2\left(\frac{\int \frac{\sin(c+dx)}{1-\sin(c+dx)}dx}{a}+\frac{\cos(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a^2\left(\frac{\int \frac{\sin(c+dx)}{1-\sin(c+dx)}dx}{a}+\frac{\cos(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{ 3214}$$

$$a^2\left(\frac{\int \frac{1}{1-\sin(c+dx)}dx-x}{a}+\frac{\cos(c+dx)}{ad}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a^2\left(\frac{\int \frac{1}{1-\sin(c+dx)}dx - x}{a} + \frac{\cos(c+dx)}{ad}\right)$$

$$\Big\downarrow \; 3127$$

$$a^2\left(\frac{\cos(c+dx)}{ad} + \frac{\frac{\cos(c+dx)}{d(1-\sin(c+dx))} - x}{a}\right)$$

input
```
Int[(a + a*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

output
```
a^2*(Cos[c + d*x]/(a*d) + (-x + Cos[c + d*x]/(d*(1 - Sin[c + d*x])))/a)
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3187
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Simp[a^p   Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ
[p, 2*m]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

3.10.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

method	result	size
risch	$-ax+\frac{a\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{a\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{2a}{d\left(-i+\mathrm{e}^{i(dx+c)}\right)}$	56
derivativedivides	$\frac{a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+a(\tan(dx+c)-dx-c)}{d}$	59
default	$\frac{a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+a(\tan(dx+c)-dx-c)}{d}$	59
parts	$\frac{a(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)}{d}$	63

input
```
int((a+a*sin(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a*x+1/2*a/d*exp(I*(d*x+c))+1/2*a/d*exp(-I*(d*x+c))+2*a/d/(-I+exp(I*(d*x+c
)))
```

3.10.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(38) = 76.

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.05

$$\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx = -\frac{adx-a\cos(dx+c)^2+(adx-2a)\cos(dx+c)-(adx-a\cos(dx+c)+a)\sin(dx+c)-a}{d\cos(dx+c)-d\sin(dx+c)+d}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(a*d*x - a*cos(d*x + c)^2 + (a*d*x - 2*a)*cos(d*x + c) - (a*d*x - a*cos(d
*x + c) + a)*sin(d*x + c) - a)/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

3.10.6 Sympy [F]

$$\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx = a\left(\int \sin(c+dx)\tan^2(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)**2,x)
```

output
```
a*(Integral(sin(c + d*x)*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x
))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int (a+a\sin(c+dx))\tan^2(c+dx)\,dx = -\frac{(dx+c-\tan(dx+c))a - a\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right)}{d}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((d*x + c - tan(d*x + c))*a - a*(1/cos(d*x + c) + cos(d*x + c)))/d
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(38) = 76$.

Time = 1.09 (sec) , antiderivative size = 1008, normalized size of antiderivative = 25.85

$$\int (a + a\sin(c + dx))\tan^2(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4
*tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*a*
tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*ta
n(1/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3
*tan(1/2*c)^3 + 2*a*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d
*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x
*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*a*tan(d*x)*tan(1/2*d*x)^3*t
an(1/2*c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan
(1/2*d*x)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*
tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*
tan(1/2*c)^3 - 8*a*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*a*
tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*
tan(c) - 8*a*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*a*tan(d*x)*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 8*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*t
an(c) - 2*a*tan(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)
^3 - a*tan(d*x)*tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^
3*tan(1/2*c)*tan(c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^
4*tan(c) + 2*a*tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*a*tan(
1/2*d*x)^3*tan(1/2*c) + 24*a*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*a*tan(1/2*...
```

3.10.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.85

$$\int (a + a\sin(c + dx))\tan^2(c + dx)\,dx$$
$$= \frac{(a\,(c + d\,x - 2) - a\,(c + d\,x))\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + (a\,(c + d\,x) - a\,(c + d\,x - 2))\,\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right) - a\,(c + d\,x) +}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right) - 1\right)\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}$$
$$- a\,x$$

input
```
int(tan(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)*(a*(c + d*x) - a*(c + d*x - 2)) - tan(c/2 + (d*x)/2)^2
*(a*(c + d*x) - a*(c + d*x - 2)) - a*(c + d*x) + a*(c + d*x - 4))/(d*(tan(
c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)) - a*x
```

3.10. $\int (a + a\sin(c + dx))\tan^2(c + dx)\,dx$

3.11 $\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx$

3.11.1 Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx = -ax-\frac{a\text{arctanh}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}$$

output
```
-a*x-a*arctanh(cos(d*x+c))/d+a*cos(d*x+c)/d-a*cot(d*x+c)/d
```

3.11.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx = \frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

output
```
(a*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[
c + d*x]^2])/d - (a*Log[Cos[(c + d*x)/2]])/d + (a*Log[Sin[(c + d*x)/2]])/d
```

3.11.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a\sin(c+dx)+a}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \ 3189$$

$$\int \left(a\cot^2(c+dx)+a\cos(c+dx)\cot(c+dx)\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{a\text{arctanh}(\cos(c+dx))}{d}+\frac{a\cos(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-ax$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x]),x]
```

output
```
-(a*x) - (a*ArcTanh[Cos[c + d*x]])/d + (a*Cos[c + d*x])/d - (a*Cot[c + d*x
])/d
```

3.11.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.11.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a(-\cot(dx+c)-dx-c)}{d}$	49
default	$\frac{a(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a(-\cot(dx+c)-dx-c)}{d}$	49
risch	$-ax+\frac{a\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{a\,\mathrm{e}^{-i(dx+c)}}{2d}-\frac{2ia}{d(\mathrm{e}^{2i(dx+c)}-1)}-\frac{a\ln(\mathrm{e}^{i(dx+c)}+1)}{d}+\frac{a\ln(\mathrm{e}^{i(dx+c)}-1)}{d}$	91

input
```
int(cot(d*x+c)^2*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a*(-cot(d*x+c)-d*x-c))
```

3.11.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05

$$\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx =$$
$$-\frac{a\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-a\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)+2}{2\,d\sin\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - a*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - a*cos(d*x + c))*sin(d
*x + c))/(d*sin(d*x + c))
```

3.11.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx = a\left(\int \sin\left(c+dx\right)\cot^2\left(c+dx\right)dx + \int \cot^2\left(c+dx\right)dx\right)$$

input
```
integrate(cot(d*x+c)**2*(a+a*sin(d*x+c)),x)
```

output
```
a*(Integral(sin(c + d*x)*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x
))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx = -\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a - a(2\cos\left(dx+c\right) - \log\left(\cos\left(dx+c\right)+1\right) + \log\left(\cos\left(dx+c\right)-1\right))}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*(d*x + c + 1/tan(d*x + c))*a - a*(2*cos(d*x + c) - log(cos(d*x + c
) + 1) + log(cos(d*x + c) - 1)))/d
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(41) = 82.

Time = 0.54 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.63

$$\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx = -\frac{6\left(dx+c\right)a - 6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 10\,a\tan}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(6*(d*x + c)*a - 6*a*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
 + 1/2*c) + (2*a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*
a*tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*
c)))/d
```

3.11.9 Mupad [B] (verification not implemented)

Time = 6.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.63

$$\int \cot^2(c+dx)(a+a\sin(c+dx))\,dx$$
$$= \frac{2\,a\,\mathrm{atan}\left(\frac{\sqrt{2}\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,\cos\left(\frac{c}{2}-\frac{\pi}{4}+\frac{d\,x}{2}\right)}\right)+a\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\,\sin(2\,c+2\,d\,x)}{2}}{d\,\sin\left(c+d\,x\right)}$$

input
```
int(cot(c + d*x)^2*(a + a*sin(c + d*x)),x)
```

output
```
(2*a*atan((2^(1/2)*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)))/(2*cos(c/2 -
 pi/4 + (d*x)/2))) + a*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*
cos(c + d*x) - (a*sin(2*c + 2*d*x))/2)/(d*sin(c + d*x))
```

3.12 $\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$

3.12.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx = ax+\frac{3a\operatorname{arctanh}(\cos(c+dx))}{2d}-\frac{3a\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}$$

output
```
a*x+3/2*a*arctanh(cos(d*x+c))/d-3/2*a*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*a*co
s(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

3.12.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx = -\frac{a\cos(c+dx)}{d}-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}+\frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

output
```
-((a*Cos[c + d*x])/d) - (a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*H
ypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*a*Log[Cos[(c
+ d*x)/2]])/(2*d) - (3*a*Log[Sin[(c + d*x)/2]])/(2*d) + (a*Sec[(c + d*x)/2
]^2)/(8*d)
```

3.12.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sin(c+dx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a\sin(c+dx)+a}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{3189}$$

$$\int \left(a\cot^4(c+dx)+a\cos(c+dx)\cot^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3a\text{arctanh}(\cos(c+dx))}{2d}-\frac{3a\cos(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^2(c+dx)}{2d}+ax$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x]),x]
```

output
```
a*x + (3*a*ArcTanh[Cos[c + d*x]])/(2*d) - (3*a*Cos[c + d*x])/(2*d) + (a*Co
t[c + d*x])/d - (a*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)
/(3*d)
```

3.12.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.12.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$
default	$\frac{a\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+a\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)}{d}$
risch	$ax-\frac{a\,e^{i(dx+c)}}{2d}-\frac{a\,e^{-i(dx+c)}}{2d}+\frac{a\left(12i e^{4i(dx+c)}+3\,e^{5i(dx+c)}-12i e^{2i(dx+c)}+8i-3\,e^{i(dx+c)}\right)}{3d\left(e^{2i(dx+c)}-1\right)^3}+\frac{3a\ln\left(e^{i(dx+c)}+1\right)}{2d}$

input
```
int(cot(d*x+c)^4*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2
*ln(csc(d*x+c)-cot(d*x+c)))+a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

3.12.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(74) = 148.

Time = 0.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$$
$$= \frac{16\,a\cos\left(dx+c\right)^3+9\,\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\,\left(a\cos\left(dx+c\right)^2-a\right)\,\mathrm{l}}{1}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(16*a*cos(d*x + c)^3 + 9*(a*cos(d*x + c)^2 - a)*log(1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) - 9*(a*cos(d*x + c)^2 - a)*log(-1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*a*cos(d
*x + c)^3 - 2*a*d*x + 3*a*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 -
 d)*sin(d*x + c))
```

3.12.6 Sympy [F]

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx = a\left(\int \sin\left(c+dx\right)\cot^4\left(c+dx\right)dx+\int \cot^4\left(c+dx\right)dx\right)$$

input
```
integrate(cot(d*x+c)**4*(a+a*sin(d*x+c)),x)
```

output
```
a*(Integral(sin(c + d*x)*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x
))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$$
$$= \frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,a\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\right.\right.}{12\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*a*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1
) - 3*log(cos(d*x + c) - 1)))/d
```

3.12.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,(dx+c)a-36\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx\right.}{24\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)
*a - 36*a*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*
a/(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*a*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1
/2*d*x + 1/2*c)^2 - 3*a*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/
d
```

3.12.9 Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.78

$$\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d} - \frac{-5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+17\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{14\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{a}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)} - \frac{5\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,d} + \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d} - \frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,d} - \frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{6\,a^2+4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}-\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6\,a^2+4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$

input
```
int(cot(c + d*x)^4*(a + a*sin(c + d*x)),x)
```

output
```
(a*tan(c/2 + (d*x)/2)^2)/(8*d) - (a/3 + a*tan(c/2 + (d*x)/2) - (14*a*tan(c
/2 + (d*x)/2)^2)/3 + 17*a*tan(c/2 + (d*x)/2)^3 - 5*a*tan(c/2 + (d*x)/2)^4)
/(d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (d
*x)/2))/(8*d) + (a*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*a*log(tan(c/2 + (d*x)
/2)))/(2*d) - (2*a*atan((4*a^2)/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a^
2*tan(c/2 + (d*x)/2))/(6*a^2 + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.12. $\int \cot^4(c+dx)(a+a\sin(c+dx))\,dx$

3.13 $\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx$

3.13.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx = -ax-\frac{15a\text{arctanh}(\cos(c+dx))}{8d}+\frac{15a\cos(c+dx)}{8d}-\frac{a\cot(c+dx)}{d}+\frac{5a\cos(c+dx)\cot^2(c+dx)}{8d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}-\frac{a\cot^5(c+dx)}{5d}$$

output
```
-a*x-15/8*a*arctanh(cos(d*x+c))/d+15/8*a*cos(d*x+c)/d-a*cot(d*x+c)/d+5/8*a
*cos(d*x+c)*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*a*cos(d*x+c)*cot(d*x+c
)^4/d-1/5*a*cot(d*x+c)^5/d
```

3.13.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.34

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx$$
$$= \frac{a\cos(c+dx)}{d} + \frac{9a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d} - \frac{15a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d} + \frac{15a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{9a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

output
```
(a*Cos[c + d*x])/d + (9*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]
^4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d
*x]^2])/(5*d) - (15*a*Log[Cos[(c + d*x)/2]])/(8*d) + (15*a*Log[Sin[(c + d*
x)/2]])/(8*d) - (9*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(
64*d)
```

3.13.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a\sin(c+dx)+a)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a\sin(c+dx)+a}{\tan(c+dx)^6}dx$$
$$\Big\downarrow \text{3189}$$
$$\int \left(a\cot^6(c+dx)+a\cos(c+dx)\cot^5(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{15a\operatorname{arctanh}(\cos(c+dx))}{8d}+\frac{15a\cos(c+dx)}{8d}-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5a\cos(c+dx)\cot^2(c+dx)}{8d}-ax$$

input
```
Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x]),x]
```

output
```
-(a*x) - (15*a*ArcTanh[Cos[c + d*x]])/(8*d) + (15*a*Cos[c + d*x])/(8*d) -
(a*Cot[c + d*x])/d + (5*a*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c +
d*x]^3)/(3*d) - (a*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)
/(5*d)
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.13.4 Maple [A] (verified)

Time = 1.42 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{a\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c)\right)}{8}+\frac{15\cos(dx+c)}{8}+\frac{15\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a\left(-\frac{(\cos}{}\right.}{d}$
default	$\frac{a\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c)\right)}{8}+\frac{15\cos(dx+c)}{8}+\frac{15\ln(\csc(dx+c)-\cot(dx+c))}{8}\right)+a\left(-\frac{(\cos}{}\right.}{d}$
risch	$-ax+\frac{a\,e^{i(dx+c)}}{2d}+\frac{a\,e^{-i(dx+c)}}{2d}-\frac{a\left(360i e^{8i(dx+c)}+135\,e^{9i(dx+c)}-720i e^{6i(dx+c)}-150\,e^{7i(dx+c)}+1120i e^{4i(dx+c)}\right.}{60d\left(e^{2i(dx+c)}-1\right)^5}$

input
```
int(cot(d*x+c)^6*(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*c
os(d*x+c)^5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)
))+a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c))
```

3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.

Time = 0.30 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.82

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx =$$
$$-\frac{368\,a\cos\left(dx+c\right)^5 - 560\,a\cos\left(dx+c\right)^3 + 225\left(a\cos\left(dx+c\right)^4 - 2\,a\cos\left(dx+c\right)^2 + a\right)\log\left(\frac{1}{2}\,\cos\left(dx+\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(a*cos(d*x + c)^
4 - 2*a*cos(d*x + c)^2 + a)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225
*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*a*cos(d
*x + c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*a*cos(d*x + c)^3 + 8*a*d*x - 15*a
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*
sin(d*x + c))
```

3.13.6 Sympy [F]

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx = a\left(\int \sin\left(c+dx\right)\cot^6\left(c+dx\right)dx + \int \cot^6\left(c+dx\right)dx\right)$$

input
```
integrate(cot(d*x+c)**6*(a+a*sin(d*x+c)),x)
```

output
```
a*(Integral(sin(c + d*x)*cot(c + d*x)**6, x) + Integral(cot(c + d*x)**6, x
))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx =$$

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+15\,a\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos\left(dx+c\right)+15\right.}{240\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan
(d*x + c)^5)*a + 15*a*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) -
15*log(cos(d*x + c) - 1)))/d
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.63

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx$$

$$=\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-960\,(dx}{}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*a*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan
(1/2*d*x + 1/2*c)^3 - 240*a*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 180
0*a*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*a/(
tan(1/2*d*x + 1/2*c)^2 + 1) - (4110*a*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1
/2*d*x + 1/2*c)^4 - 240*a*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*
c)^2 + 15*a*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.13.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.39

$$\int \cot^6(c+dx)(a+a\sin(c+dx))\,dx = \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

$$-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-72\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{59\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{15\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

$$-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}$$

$$+\frac{15\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{\frac{15\,a^2}{2}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(\frac{15\,a^2}{2}+4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}\right)}{d}$$

input
```
int(cot(c + d*x)^6*(a + a*sin(c + d*x)),x)
```

output
```
(11*a*tan(c/2 + (d*x)/2))/(16*d) - (a/5 + (a*tan(c/2 + (d*x)/2))/2 - (32*a
*tan(c/2 + (d*x)/2)^2)/15 - (15*a*tan(c/2 + (d*x)/2)^3)/2 + (59*a*tan(c/2
+ (d*x)/2)^4)/3 - 72*a*tan(c/2 + (d*x)/2)^5 + 22*a*tan(c/2 + (d*x)/2)^6)/(
d*(32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (d*x)/2)^7)) - (a*tan(c/2 + (d*x
)/2)^2)/(4*d) - (7*a*tan(c/2 + (d*x)/2)^3)/(96*d) + (a*tan(c/2 + (d*x)/2)^
4)/(64*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) + (15*a*log(tan(c/2 + (d*x)/2
)))/(8*d) + (2*a*atan((4*a^2)/((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)) - (1
5*a^2*tan(c/2 + (d*x)/2))/(2*((15*a^2)/2 + 4*a^2*tan(c/2 + (d*x)/2)))))/d
```

3.14 $\int (a + a\sin(c + dx))^2 \tan^5(c + dx)\, dx$

3.14.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (a + a\sin(c + dx))^2 \tan^5(c + dx)\, dx = -\frac{31a^2\log(1 - \sin(c + dx))}{8d} - \frac{a^2\log(1 + \sin(c + dx))}{8d} - \frac{2a^2\sin(c + dx)}{d} - \frac{a^2\sin^2(c + dx)}{2d} + \frac{a^4}{4d(a - a\sin(c + dx))^2} - \frac{9a^3}{4d(a - a\sin(c + dx))}$$

output
```
-31/8*a^2*ln(1-sin(d*x+c))/d-1/8*a^2*ln(1+sin(d*x+c))/d-2*a^2*sin(d*x+c)/d
-1/2*a^2*sin(d*x+c)^2/d+1/4*a^4/d/(a-a*sin(d*x+c))^2-9/4*a^3/d/(a-a*sin(d*
x+c))
```

3.14.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int (a + a\sin(c + dx))^2 \tan^5(c + dx)\, dx = -\frac{a^2\left(31\log(1 - \sin(c + dx)) + \log(1 + \sin(c + dx)) - \frac{2}{(-1+\sin(c+dx))^2} - \frac{18}{-1+\sin(c+dx)} + 16\sin(c + dx) + 4\,\mathrm{s}\right.}{8d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

output
```
-1/8*(a^2*(31*Log[1 - Sin[c + d*x]] + Log[1 + Sin[c + d*x]] - 2/(-1 + Sin[
c + d*x])^2 - 18/(-1 + Sin[c + d*x]) + 16*Sin[c + d*x] + 4*Sin[c + d*x]^2)
)/d
```

3.14.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^5(a\sin(c+dx)+a)^2dx$$

$$\Big\downarrow \; 3186$$

$$\frac{\int \frac{a^5\sin^5(c+dx)}{(a-a\sin(c+dx))^3(\sin(c+dx)a+a)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(\frac{a^4}{2(a-a\sin(c+dx))^3}-\frac{9a^3}{4(a-a\sin(c+dx))^2}+\frac{31a^2}{8(a-a\sin(c+dx))}-\frac{a^2}{8(\sin(c+dx)a+a)}-\sin(c+dx)a-2a\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{a^4}{4(a-a\sin(c+dx))^2}-\frac{9a^3}{4(a-a\sin(c+dx))}-\frac{1}{2}a^2\sin^2(c+dx)-2a^2\sin(c+dx)-\frac{31}{8}a^2\log(a-a\sin(c+dx))-\frac{1}{8}a^2\log(a\sin}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^5,x]
```

output
```
((-31*a^2*Log[a - a*Sin[c + d*x]])/8 - (a^2*Log[a + a*Sin[c + d*x]])/8 - 2
*a^2*Sin[c + d*x] - (a^2*Sin[c + d*x]^2)/2 + a^4/(4*(a - a*Sin[c + d*x])^2
) - (9*a^3)/(4*(a - a*Sin[c + d*x])))/d
```

3.14.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.14.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.95 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.64

method	result
risch	$4ia^2x+\frac{a^2e^{2i(dx+c)}}{8d}+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}+\frac{a^2e^{-2i(dx+c)}}{8d}+\frac{8ia^2c}{d}+\frac{i\left(-9a^2e^{i(dx+c)}-16ia^2e^{2i(dx+c)}\right.}{2\left(-i+e^{i(dx+c)}\right.}$
derivativedivides	$a^2\left(\frac{\sin^8(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}-\frac{\left(\sin^6(dx+c)\right)}{2}-\frac{3\left(\sin^4(dx+c)\right)}{4}-\frac{3\left(\sin^2(dx+c)\right)}{2}-3\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\right.$
default	$a^2\left(\frac{\sin^8(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}-\frac{\left(\sin^6(dx+c)\right)}{2}-\frac{3\left(\sin^4(dx+c)\right)}{4}-\frac{3\left(\sin^2(dx+c)\right)}{2}-3\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^7(dx+c)}{4\cos(dx+c)^4}-\right.$
parts	$\frac{a^2\left(\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{a^2\left(\frac{\sin^8(dx+c)}{4\cos(dx+c)^4}-\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}-\frac{\left(\sin^6(dx+c)\right)}{2}-\frac{3\left(\sin^4(dx+c)\right)}{4}\right.}{d}$

input
```
int((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
4*I*a^2*x+1/8*a^2/d*exp(2*I*(d*x+c))+I*a^2/d*exp(I*(d*x+c))-I*a^2/d*exp(-I
*(d*x+c))+1/8*a^2/d*exp(-2*I*(d*x+c))+8*I/d*a^2*c+1/2*I*(-9*a^2*exp(I*(d*x
+c))-16*I*a^2*exp(2*I*(d*x+c))+9*a^2*exp(3*I*(d*x+c)))/(-I+exp(I*(d*x+c)))
^4/d-1/4*a^2/d*ln(exp(I*(d*x+c))+I)-31/4*a^2/d*ln(-I+exp(I*(d*x+c)))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.41

$$\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx$$
$$= \frac{4\,a^2\cos{(dx+c)}^4+22\,a^2\cos{(dx+c)}^2-12\,a^2-\left(a^2\cos{(dx+c)}^2+2\,a^2\sin{(dx+c)}-2\,a^2\right)\log{(\sin{(dx}}{8\left(d\cos{(dx}\right.}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
1/8*(4*a^2*cos(d*x + c)^4 + 22*a^2*cos(d*x + c)^2 - 12*a^2 - (a^2*cos(d*x
+ c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(sin(d*x + c) + 1) - 31*(a^2*cos(d
*x + c)^2 + 2*a^2*sin(d*x + c) - 2*a^2)*log(-sin(d*x + c) + 1) - 2*(4*a^2*
cos(d*x + c)^2 - 5*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c)
 - 2*d)
```

3.14.6 Sympy [F]

$$\int (a+a\sin(c+dx))^2\tan^5(c+dx)\,dx = a^2\left(\int 2\sin{(c+dx)}\tan^5{(c+dx)}\,dx + \int \sin^2{(c+dx)}\tan^5{(c+dx)}\,dx + \int \tan^5{(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**5,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**5, x) + Integral(sin(c + d*x)*
*2*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5, x))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.81

$$\int (a + a\sin(c+dx))^2 \tan^5(c+dx)\,dx =$$

$$-\frac{4\,a^2\sin\left(dx+c\right)^2 + a^2\log\left(\sin\left(dx+c\right)+1\right) + 31\,a^2\log\left(\sin\left(dx+c\right)-1\right) + 16\,a^2\sin\left(dx+c\right) - \frac{2\,(9\,a^2 \mathrm{s}}{\sin(dx+c}}{8\,d}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/8*(4*a^2*sin(d*x + c)^2 + a^2*log(sin(d*x + c) + 1) + 31*a^2*log(sin(d*
x + c) - 1) + 16*a^2*sin(d*x + c) - 2*(9*a^2*sin(d*x + c) - 8*a^2)/(sin(d*
x + c)^2 - 2*sin(d*x + c) + 1))/d
```

3.14.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c+dx))^2 \tan^5(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
Timed out
```

3.14.9 Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.38

$$\int (a + a\sin(c+dx))^2 \tan^5(c+dx)\,dx$$

$$= \frac{4\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} - \frac{a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{4\,d}$$

$$- \frac{\frac{15\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2} - 22\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \frac{61\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2} - 36\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + \frac{61\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + 8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 12\tan\right.}$$

$$- \frac{31\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{4\,d}$$

input
```
int(tan(c + d*x)^5*(a + a*sin(c + d*x))^2,x)
```

output
```
(4*a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (a^2*log(tan(c/2 + (d*x)/2) + 1)
)/(4*d) - ((61*a^2*tan(c/2 + (d*x)/2)^3)/2 - 22*a^2*tan(c/2 + (d*x)/2)^2 -
 36*a^2*tan(c/2 + (d*x)/2)^4 + (61*a^2*tan(c/2 + (d*x)/2)^5)/2 - 22*a^2*ta
n(c/2 + (d*x)/2)^6 + (15*a^2*tan(c/2 + (d*x)/2)^7)/2 + (15*a^2*tan(c/2 + (
d*x)/2))/2)/(d*(8*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 12*tan(c/2
 + (d*x)/2)^3 + 14*tan(c/2 + (d*x)/2)^4 - 12*tan(c/2 + (d*x)/2)^5 + 8*tan(
c/2 + (d*x)/2)^6 - 4*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8 + 1)) - (
31*a^2*log(tan(c/2 + (d*x)/2) - 1))/(4*d)
```

3.14. $\int (a + a\sin(c + dx))^2 \tan^5(c + dx)\, dx$

3.15 $\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

3.15.1 Optimal result

Integrand size = 21, antiderivative size = 72

$$\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx=\frac{3a^2\log(1-\sin(c+dx))}{d}+\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\sin^2(c+dx)}{2d}+\frac{a^3}{d(a-a\sin(c+dx))}$$

output
```
3*a^2*ln(1-sin(d*x+c))/d+2*a^2*sin(d*x+c)/d+1/2*a^2*sin(d*x+c)^2/d+a^3/d/(
a-a*sin(d*x+c))
```

3.15.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.75

$$\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx=\frac{a^2\left(6\log(1-\sin(c+dx))+\frac{2}{1-\sin(c+dx)}+4\sin(c+dx)+\sin^2(c+dx)\right)}{2d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

output
```
(a^2*(6*Log[1 - Sin[c + d*x]] + 2/(1 - Sin[c + d*x]) + 4*Sin[c + d*x] + Si
n[c + d*x]^2))/(2*d)
```

3.15.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(c+dx)^3(a\sin(c+dx)+a)^2dx$$

$$\Big\downarrow \ 3186$$

$$\frac{\int \frac{a^3\sin^3(c+dx)}{(a-a\sin(c+dx))^2}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \ 49$$

$$\frac{\int \left(\frac{a^3}{(a-a\sin(c+dx))^2}-\frac{3a^2}{a-a\sin(c+dx)}+\sin(c+dx)a+2a\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{a^3}{a-a\sin(c+dx)}+\frac{1}{2}a^2\sin^2(c+dx)+2a^2\sin(c+dx)+3a^2\log(a-a\sin(c+dx))}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

output
```
(3*a^2*Log[a - a*Sin[c + d*x]] + 2*a^2*Sin[c + d*x] + (a^2*Sin[c + d*x]^2)
/2 + a^3/(a - a*Sin[c + d*x]))/d
```

3.15.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.15.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.98 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.74

method	result
risch	$-3ia^2x - \frac{ia^2 e^{i(dx+c)}}{d} + \frac{ia^2 e^{-i(dx+c)}}{d} - \frac{6ia^2c}{d} - \frac{2ia^2 e^{i(dx+c)}}{\left(-i+e^{i(dx+c)}\right)^2 d} + \frac{6a^2\ln\left(-i+e^{i(dx+c)}\right)}{d} - \frac{a^2\cos(2dx+2}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec}{}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)+2a^2\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec}{}\right.}{d}$
parts	$\frac{a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{a^2\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)}{d}+\frac{2a^2\left(\right.}{}$

input
```
int((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-3*I*a^2*x-I*a^2/d*exp(I*(d*x+c))+I*a^2/d*exp(-I*(d*x+c))-6*I/d*a^2*c-2*I*
a^2*exp(I*(d*x+c))/(-I+exp(I*(d*x+c)))^2/d+6*a^2/d*ln(-I+exp(I*(d*x+c)))-1
/4*a^2/d*cos(2*d*x+2*c)
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25

$$\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = -\frac{6\,a^2\cos(dx+c)^2-3\,a^2-12\,(a^2\sin(dx+c)-a^2)\log(-\sin(dx+c)+1)+\left(2\,a^2\cos(dx+c)^2+7\,a^2\right.}{4\,(d\sin(dx+c)-d)}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(6*a^2*cos(d*x + c)^2 - 3*a^2 - 12*(a^2*sin(d*x + c) - a^2)*log(-sin(
d*x + c) + 1) + (2*a^2*cos(d*x + c)^2 + 7*a^2)*sin(d*x + c))/(d*sin(d*x +
c) - d)
```

3.15.6 Sympy [F]

$$\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx = a^2\left(\int 2\sin(c+dx)\tan^3(c+dx)\,dx + \int \sin^2(c+dx)\tan^3(c+dx)\,dx + \int \tan^3(c+dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(sin(c + d*x)*
*2*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3, x))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int (a + a\sin(c + dx))^2 \tan^3(c + dx)\,dx = \frac{a^2 \sin(dx + c)^2 + 6\,a^2 \log(\sin(dx + c) - 1) + 4\,a^2 \sin(dx + c) - \frac{2\,a^2}{\sin(dx+c)-1}}{2\,d}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*(a^2*sin(d*x + c)^2 + 6*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c)
 - 2*a^2/(sin(d*x + c) - 1))/d
```

3.15.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^2 \tan^3(c + dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.15.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.83

$$\int (a + a\sin(c + dx))^2 \tan^3(c + dx)\,dx = \frac{6\,a^2 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 - 6\,a^2 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 + 8\,a^2 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 - 6\,a^2 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 6\,a^2 \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right.}{d\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^6 - 2\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^5 + 3\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 - 4\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3 + 3\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 - 2\tan\left(\frac{c}{2} + \right.\right.} + \frac{6\,a^2 \ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right) - 1\right)}{d} - \frac{3\,a^2 \ln\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

output
```
(8*a^2*tan(c/2 + (d*x)/2)^3 - 6*a^2*tan(c/2 + (d*x)/2)^2 - 6*a^2*tan(c/2 +
 (d*x)/2)^4 + 6*a^2*tan(c/2 + (d*x)/2)^5 + 6*a^2*tan(c/2 + (d*x)/2))/(d*(3
*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^3 + 3*
tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^6 + 1))
 + (6*a^2*log(tan(c/2 + (d*x)/2) - 1))/d - (3*a^2*log(tan(c/2 + (d*x)/2)^2
 + 1))/d
```

3.15. $\int (a+a\sin(c+dx))^2\tan^3(c+dx)\,dx$

3.16 $\int (a + a\sin(c + dx))^2 \tan(c + dx)\, dx$

3.16.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int (a + a\sin(c + dx))^2 \tan(c + dx)\, dx = -\frac{2a^2 \log(1 - \sin(c + dx))}{d} - \frac{2a^2 \sin(c + dx)}{d} - \frac{a^2 \sin^2(c + dx)}{2d}$$

output
```
-2*a^2*ln(1-sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*sin(d*x+c)^2/d
```

3.16.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.77

$$\int (a + a\sin(c + dx))^2 \tan(c + dx)\, dx = -\frac{a^2(4\log(1 - \sin(c + dx)) + 4\sin(c + dx) + \sin^2(c + dx))}{2d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

output
```
-1/2*(a^2*(4*Log[1 - Sin[c + d*x]] + 4*Sin[c + d*x] + Sin[c + d*x]^2))/d
```

3.16.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)(a\sin(c+dx)+a)^2dx$$

$$\Big\downarrow \text{ 3186}$$

$$\frac{\int \frac{a\sin(c+dx)(\sin(c+dx)a+a)}{a-a\sin(c+dx)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{\int \left(\frac{2a^2}{a-a\sin(c+dx)}-\sin(c+dx)a-2a\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{2}a^2\sin^2(c+dx)-2a^2\sin(c+dx)-2a^2\log(a-a\sin(c+dx))}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x],x]
```

output
```
(-2*a^2*Log[a - a*Sin[c + d*x]] - 2*a^2*Sin[c + d*x] - (a^2*Sin[c + d*x]^2
)/2)/d
```

3.16.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.16.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.35

method	result	s
derivativedivides	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-a^2\ln(\cos(dx+c))}{d}$	7
default	$\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))-a^2\ln(\cos(dx+c))}{d}$	7
parts	$\frac{a^2\ln(1+\tan^2(dx+c))}{2d}+\frac{a^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}+\frac{2a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$	7
risch	$2ia^2x+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}+\frac{4ia^2c}{d}-\frac{4a^2\ln(-i+e^{i(dx+c)})}{d}+\frac{a^2\cos(2dx+2c)}{4d}$	9

input
```
int((a+a*sin(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+2*a^2*(-sin(d*x+c)+ln(sec(d*x+
c)+tan(d*x+c)))-a^2*ln(cos(d*x+c)))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int (a+a\sin(c+dx))^2\tan(c+dx)\,dx = \frac{a^2\cos{(dx+c)}^2 - 4\,a^2\log{(-\sin{(dx+c)}+1)} - 4\,a^2\sin{(dx+c)}}{2\,d}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(a^2*cos(d*x + c)^2 - 4*a^2*log(-sin(d*x + c) + 1) - 4*a^2*sin(d*x + c
))/d
```

3.16.6 Sympy [F]

$$\int (a+a\sin(c+dx))^2\tan(c+dx)\,dx = a^2\left(\int 2\sin{(c+dx)}\tan{(c+dx)}\,dx + \int \sin^2{(c+dx)}\tan{(c+dx)}\,dx + \int \tan{(c+dx)}\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**2*tan(d*x+c),x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*tan(c + d*x), x) + Integral(sin(c + d*x)**2*
tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int (a+a\sin(c+dx))^2\tan(c+dx)\,dx = -\frac{a^2\sin{(dx+c)}^2 + 4\,a^2\log{(\sin{(dx+c)}-1)} + 4\,a^2\sin{(dx+c)}}{2\,d}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(a^2*sin(d*x + c)^2 + 4*a^2*log(sin(d*x + c) - 1) + 4*a^2*sin(d*x + c
))/d
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5671 vs. $2(50) = 100$.

Time = 1.48 (sec) , antiderivative size = 5671, normalized size of antiderivative = 109.06

$$\int (a + a\sin(c + dx))^2 \tan(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

output
```
-1/4*(4*a^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a
^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(4*
(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - a
^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a^2*log(2*(tan(1/2*
d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)
/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*
x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a^2*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^
2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*
x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 4*a^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)
/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*...
```

3.16.9 Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.42

$$\int (a+a\sin(c+dx))^2\tan(c+dx)\,dx =$$

$$-\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(4\,a^2\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)-2\,a^2\left(4\right.\right.}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

$$-\frac{2\,a^2\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{d}$$

input
```
int(tan(c + d*x)*(a + a*sin(c + d*x))^2,x)
```

output
```
- (4*a^2*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*(4*a^2*(2*log(tan(c/2
 + (d*x)/2) - 1) - log(tan(c/2 + (d*x)/2)^2 + 1)) - 2*a^2*(4*log(tan(c/2 +
 (d*x)/2) - 1) - 2*log(tan(c/2 + (d*x)/2)^2 + 1) + 1)) + 4*a^2*tan(c/2 + (
d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (2*a^2*(2*log(tan(c/2 + (d*x)/
2) - 1) - log(tan(c/2 + (d*x)/2)^2 + 1)))/d
```

3.17 $\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$

3.17.1 Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx=-\frac{\csc^2(c+dx)(a+a\sin(c+dx))^4}{2a^2d}$$

output
```
-1/2*csc(d*x+c)^2*(a+a*sin(d*x+c))^4/a^2/d
```

3.17.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx=-\frac{a^2\csc^2(c+dx)(1+\sin(c+dx))^4}{2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

output
```
-1/2*(a^2*Csc[c + d*x]^2*(1 + Sin[c + d*x])^4)/d
```

3.17.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3186, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^2}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))(\sin(c+dx)a+a)^3}{a^3}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{83}$$

$$-\frac{\csc^2(c+dx)(a\sin(c+dx)+a)^4}{2a^2d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^2,x]
```

output
```
-1/2*(Csc[c + d*x]^2*(a + a*Sin[c + d*x])^4)/(a^2*d)
```

3.17.3.1 Defintions of rubi rules used

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(28) = 56$.

Time = 1.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.13

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)+a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+$
default	$\frac{a^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)+a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+$
risch	$-\frac{a^2\left(56i e^{3i(dx+c)}-8i e^{5i(dx+c)}-e^{6i(dx+c)}+2+2\,e^{4i(dx+c)}-48i\cos(dx+c)+64\sin(dx+c)-19\cos(2dx+2c)-17i\sin(2dx}{8d\left(e^{2i(dx+c)}-1\right)^2}$

input
```
int(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cos(d*x+c)^2+ln(sin(d*x+c)))+2*a^2*(-cos(d*x+c)^4/sin(d*x+c)
-(2+cos(d*x+c)^2)*sin(d*x+c))+a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(28) = 56$.

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.53

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{2\,a^2\cos\left(dx+c\right)^4-3\,a^2\cos\left(dx+c\right)^2+3\,a^2-8\,\left(a^2\cos\left(dx+c\right)^2-2\,a^2\right)\sin\left(dx+c\right)}{4\,\left(d\cos\left(dx+c\right)^2-d\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*(2*a^2*cos(d*x + c)^4 - 3*a^2*cos(d*x + c)^2 + 3*a^2 - 8*(a^2*cos(d*x
+ c)^2 - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

3.17.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = a^2\left(\int 2\sin(c+dx)\cot^3(c+dx)\,dx + \int \sin^2(c+dx)\cot^3(c+dx)\,dx + \int \cot^3(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(sin(c + d*x)*
*2*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3, x))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.77

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{a^2\sin(dx+c)^2 + 4\,a^2\sin(dx+c) + \frac{4\,a^2\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2*sin(d*x + c)^2 + 4*a^2*sin(d*x + c) + (4*a^2*sin(d*x + c) + a^2)
/sin(d*x + c)^2)/d
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$=-\frac{a^2\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)\right)^2+4\,a^2\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(a^2*(1/sin(d*x + c) + sin(d*x + c))^2 + 4*a^2*(1/sin(d*x + c) + sin(
d*x + c)))/d
```

3.17.9 Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.87

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$=-\frac{a^2\left(2\sin(c+d\,x)^4+8\sin(c+d\,x)^3-\sin(c+d\,x)^2+8\,\sin(c+d\,x)+2\right)}{4\,d\sin(c+d\,x)^2}$$

input
```
int(cot(c + d*x)^3*(a + a*sin(c + d*x))^2,x)
```

output
```
-(a^2*(8*sin(c + d*x) - sin(c + d*x)^2 + 8*sin(c + d*x)^3 + 2*sin(c + d*x)
^4 + 2))/(4*d*sin(c + d*x)^2)
```

3.18 $\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx$

3.18.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\begin{aligned}\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx = &-\frac{6a^2\csc(c+dx)}{d}+\frac{2a^2\csc^3(c+dx)}{d}\\ &+\frac{a^2\csc^4(c+dx)}{2d}-\frac{2a^2\csc^5(c+dx)}{5d}\\ &-\frac{a^2\csc^6(c+dx)}{6d}+\frac{2a^2\log(\sin(c+dx))}{d}\\ &-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\sin^2(c+dx)}{2d}\end{aligned}$$

output
```
-6*a^2*csc(d*x+c)/d+2*a^2*csc(d*x+c)^3/d+1/2*a^2*csc(d*x+c)^4/d-2/5*a^2*cs
c(d*x+c)^5/d-1/6*a^2*csc(d*x+c)^6/d+2*a^2*ln(sin(d*x+c))/d-2*a^2*sin(d*x+c
)/d-1/2*a^2*sin(d*x+c)^2/d
```

3.18.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.65

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx =$$
$$-\frac{a^2(180\csc(c+dx)-60\csc^3(c+dx)-15\csc^4(c+dx)+12\csc^5(c+dx)+5\csc^6(c+dx)-60\log(\sin(}{30d}$$

input
```
Integrate[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

output
```
-1/30*(a^2*(180*Csc[c + d*x] - 60*Csc[c + d*x]^3 - 15*Csc[c + d*x]^4 + 12*
Csc[c + d*x]^5 + 5*Csc[c + d*x]^6 - 60*Log[Sin[c + d*x]] + 60*Sin[c + d*x]
 + 15*Sin[c + d*x]^2))/d
```

3.18.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^7(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^2}{\tan(c+dx)^7}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^7(c+dx)(a-a\sin(c+dx))^3(\sin(c+dx)a+a)^5}{a^7}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(a\csc^7(c+dx)+2a\csc^6(c+dx)-2a\csc^5(c+dx)-6a\csc^4(c+dx)+6a\csc^2(c+dx)+2a\csc(c+dx)-2a-\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^2\sin^2(c+dx)-2a^2\sin(c+dx)-\frac{1}{6}a^2\csc^6(c+dx)-\frac{2}{5}a^2\csc^5(c+dx)+\frac{1}{2}a^2\csc^4(c+dx)+2a^2\csc^3(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^7*(a + a*Sin[c + d*x])^2,x]
```

output
```
(-6*a^2*Csc[c + d*x] + 2*a^2*Csc[c + d*x]^3 + (a^2*Csc[c + d*x]^4)/2 - (2*
a^2*Csc[c + d*x]^5)/5 - (a^2*Csc[c + d*x]^6)/6 + 2*a^2*Log[a*Sin[c + d*x]]
 - 2*a^2*Sin[c + d*x] - (a^2*Sin[c + d*x]^2)/2)/d
```

3.18.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.18.4 Maple [A] (verified)

Time = 4.64 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.73

method	result
derivativedivides	$a^2\left(-\frac{\cos^8(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}+\frac{(\cos^6(dx+c))}{2}+\frac{3(\cos^4(dx+c))}{4}+\frac{3(\cos^2(dx+c))}{2}+3\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}\right.$
default	$a^2\left(-\frac{\cos^8(dx+c)}{4\sin(dx+c)^4}+\frac{\cos^8(dx+c)}{2\sin(dx+c)^2}+\frac{(\cos^6(dx+c))}{2}+\frac{3(\cos^4(dx+c))}{4}+\frac{3(\cos^2(dx+c))}{2}+3\ln(\sin(dx+c))\right)+2a^2\left(-\frac{\cos^8(dx+c)}{5\sin(dx+c)^5}\right.$
risch	$-2ia^2x+\frac{a^2e^{2i(dx+c)}}{8d}+\frac{ia^2e^{i(dx+c)}}{d}-\frac{ia^2e^{-i(dx+c)}}{d}+\frac{a^2e^{-2i(dx+c)}}{8d}-\frac{4ia^2c}{d}-\frac{4ia^2(45e^{11i(dx+c)}+30ie^{8i}}{}$

input
```
int(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/4/sin(d*x+c)^4*cos(d*x+c)^8+1/2/sin(d*x+c)^2*cos(d*x+c)^8+1/2
*cos(d*x+c)^6+3/4*cos(d*x+c)^4+3/2*cos(d*x+c)^2+3*ln(sin(d*x+c)))+2*a^2*(-
1/5/sin(d*x+c)^5*cos(d*x+c)^8+1/5/sin(d*x+c)^3*cos(d*x+c)^8-1/sin(d*x+c)*c
os(d*x+c)^8-(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+
c))+a^2*(-1/6*cot(d*x+c)^6+1/4*cot(d*x+c)^4-1/2*cot(d*x+c)^2-ln(sin(d*x+c)
)))
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.56

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$= \frac{30\,a^2\cos{(dx+c)}^8 - 105\,a^2\cos{(dx+c)}^6 + 135\,a^2\cos{(dx+c)}^4 - 45\,a^2\cos{(dx+c)}^2 - 5\,a^2 + 120\,(a^2\cos}{60}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/60*(30*a^2*cos(d*x + c)^8 - 105*a^2*cos(d*x + c)^6 + 135*a^2*cos(d*x + c
)^4 - 45*a^2*cos(d*x + c)^2 - 5*a^2 + 120*(a^2*cos(d*x + c)^6 - 3*a^2*cos(
d*x + c)^4 + 3*a^2*cos(d*x + c)^2 - a^2)*log(1/2*sin(d*x + c)) - 24*(5*a^2
*cos(d*x + c)^6 - 30*a^2*cos(d*x + c)^4 + 40*a^2*cos(d*x + c)^2 - 16*a^2)*
sin(d*x + c))/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^2
- d)
```

3.18.6 Sympy [F]

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx = a^2\left(\int 2\sin{(c+dx)}\cot^7{(c+dx)}\,dx + \int \sin^2{(c+dx)}\cot^7{(c+dx)}\,dx + \int \cot^7{(c+dx)}\,dx\right)$$

input
```
integrate(cot(d*x+c)**7*(a+a*sin(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**7, x) + Integral(sin(c + d*x)*
*2*cot(c + d*x)**7, x) + Integral(cot(c + d*x)**7, x))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.81

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx =$$

$$-\frac{15\,a^2\sin\left(dx+c\right)^2-60\,a^2\log\left(\sin\left(dx+c\right)\right)+60\,a^2\sin\left(dx+c\right)+\frac{180\,a^2\sin(dx+c)^5-60\,a^2\sin(dx+c)^3-15\,a^2\sin(dx}{\sin(dx+c)^6}}{30\,d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/30*(15*a^2*sin(d*x + c)^2 - 60*a^2*log(sin(d*x + c)) + 60*a^2*sin(d*x +
 c) + (180*a^2*sin(d*x + c)^5 - 60*a^2*sin(d*x + c)^3 - 15*a^2*sin(d*x + c
)^2 + 12*a^2*sin(d*x + c) + 5*a^2)/sin(d*x + c)^6)/d
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.92

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx =$$

$$-\frac{15\,a^2\sin\left(dx+c\right)^2-60\,a^2\log\left(\left|\sin\left(dx+c\right)\right|\right)+60\,a^2\sin\left(dx+c\right)+\frac{147\,a^2\sin(dx+c)^6+180\,a^2\sin(dx+c)^5-60\,a^2\sin}{\sin(d}}{30\,d}$$

input
```
integrate(cot(d*x+c)^7*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/30*(15*a^2*sin(d*x + c)^2 - 60*a^2*log(abs(sin(d*x + c))) + 60*a^2*sin(
d*x + c) + (147*a^2*sin(d*x + c)^6 + 180*a^2*sin(d*x + c)^5 - 60*a^2*sin(d
*x + c)^3 - 15*a^2*sin(d*x + c)^2 + 12*a^2*sin(d*x + c) + 5*a^2)/sin(d*x +
 c)^6)/d
```

3.18.9 Mupad [B] (verification not implemented)

Time = 10.76 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.97

$$\int \cot^7(c+dx)(a+a\sin(c+dx))^2\,dx =$$

$$-\frac{a^2\left(24\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-312\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-220\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3864\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\right.}{}$$

input
```
int(cot(c + d*x)^7*(a + a*sin(c + d*x))^2,x)
```

output
```
-(a^2*(24*tan(c/2 + (d*x)/2) - 20*tan(c/2 + (d*x)/2)^2 - 312*tan(c/2 + (d*
x)/2)^3 - 220*tan(c/2 + (d*x)/2)^4 + 3864*tan(c/2 + (d*x)/2)^5 - 360*tan(c
/2 + (d*x)/2)^6 + 21000*tan(c/2 + (d*x)/2)^7 + 3510*tan(c/2 + (d*x)/2)^8 +
 21000*tan(c/2 + (d*x)/2)^9 - 360*tan(c/2 + (d*x)/2)^10 + 3864*tan(c/2 + (
d*x)/2)^11 - 220*tan(c/2 + (d*x)/2)^12 - 312*tan(c/2 + (d*x)/2)^13 - 20*ta
n(c/2 + (d*x)/2)^14 + 24*tan(c/2 + (d*x)/2)^15 + 5*tan(c/2 + (d*x)/2)^16 +
 3840*tan(c/2 + (d*x)/2)^6*log(tan(c/2 + (d*x)/2)^2 + 1) + 7680*tan(c/2 +
(d*x)/2)^8*log(tan(c/2 + (d*x)/2)^2 + 1) + 3840*tan(c/2 + (d*x)/2)^10*log(
tan(c/2 + (d*x)/2)^2 + 1) - 3840*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2
)^6 - 7680*log(tan(c/2 + (d*x)/2))*tan(c/2 + (d*x)/2)^8 - 3840*log(tan(c/2
 + (d*x)/2))*tan(c/2 + (d*x)/2)^10 + 5))/(1920*d*tan(c/2 + (d*x)/2)^6*(tan
(c/2 + (d*x)/2)^2 + 1)^2)
```

3.19 $\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx$

3.19.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx = -\frac{9a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{6a^2\sec(c+dx)}{d}-\frac{2a^2\sec^3(c+dx)}{d}+\frac{2a^2\sec^5(c+dx)}{5d}+\frac{9a^2\tan(c+dx)}{2d}-\frac{3a^2\tan^3(c+dx)}{2d}+\frac{9a^2\tan^5(c+dx)}{10d}-\frac{a^2\sin^2(c+dx)\tan^5(c+dx)}{2d}$$

output
```
-9/2*a^2*x+2*a^2*cos(d*x+c)/d+6*a^2*sec(d*x+c)/d-2*a^2*sec(d*x+c)^3/d+2/5*
a^2*sec(d*x+c)^5/d+9/2*a^2*tan(d*x+c)/d-3/2*a^2*tan(d*x+c)^3/d+9/10*a^2*ta
n(d*x+c)^5/d-1/2*a^2*sin(d*x+c)^2*tan(d*x+c)^5/d
```

3.19.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.17

$$\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx =$$

$$-\frac{a^2\sec^5(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4\left(-500+10(103+90c+90dx)\cos(c+dx)-544\right.}{}$$

input
```
Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

output
```
-1/160*(a^2*Sec[c + d*x]^5*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(-500 +
 10*(103 + 90*c + 90*d*x)*Cos[c + d*x] - 544*Cos[2*(c + d*x)] - 206*Cos[3*
(c + d*x)] - 180*c*Cos[3*(c + d*x)] - 180*d*x*Cos[3*(c + d*x)] + 20*Cos[4*
(c + d*x)] + 250*Sin[c + d*x] - 824*Sin[2*(c + d*x)] - 720*c*Sin[2*(c + d*
x)] - 720*d*x*Sin[2*(c + d*x)] + 351*Sin[3*(c + d*x)] + 5*Sin[5*(c + d*x)]
))/d
```

3.19.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^6(a\sin(c+dx)+a)^2dx$$

$$\Big\downarrow \text{3189}$$

$$\int \left(a^2\tan^6(c+dx)+a^2\sin^2(c+dx)\tan^6(c+dx)+2a^2\sin(c+dx)\tan^6(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2\cos(c+dx)}{d}+\frac{9a^2\tan^5(c+dx)}{10d}-\frac{3a^2\tan^3(c+dx)}{2d}+\frac{9a^2\tan(c+dx)}{2d}+\frac{2a^2\sec^5(c+dx)}{5d}-\frac{2a^2\sec^3(c+dx)}{d}+\frac{6a^2\sec(c+dx)}{d}-\frac{a^2\sin^2(c+dx)\tan^5(c+dx)}{2d}-\frac{9a^2x}{2}$$

input
```
Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^6,x]
```

output
```
(-9*a^2*x)/2 + (2*a^2*Cos[c + d*x])/d + (6*a^2*Sec[c + d*x])/d - (2*a^2*Se
c[c + d*x]^3)/d + (2*a^2*Sec[c + d*x]^5)/(5*d) + (9*a^2*Tan[c + d*x])/(2*d
) - (3*a^2*Tan[c + d*x]^3)/(2*d) + (9*a^2*Tan[c + d*x]^5)/(10*d) - (a^2*Si
n[c + d*x]^2*Tan[c + d*x]^5)/(2*d)
```

3.19.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.19.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{9a^2x}{2} - \frac{i a^2 e^{2i(dx+c)}}{8d} + \frac{a^2 e^{i(dx+c)}}{d} + \frac{a^2 e^{-i(dx+c)}}{d} + \frac{i a^2 e^{-2i(dx+c)}}{8d} + \frac{-\frac{156 a^2 e^{i(dx+c)}}{5} - 24 i a^2 e^{2i(dx+c)} + \frac{54ia}{5}}{(e^{i(dx+c)}}$
derivativedivides	$a^2\left(\frac{\sin^9(dx+c)}{5\cos(dx+c)^5} - \frac{4\left(\sin^9(dx+c)\right)}{15\cos(dx+c)^3} + \frac{8\left(\sin^9(dx+c)\right)}{5\cos(dx+c)} + \frac{8\left(\sin^7(dx+c) + \frac{7\left(\sin^5(dx+c)\right)}{6} + \frac{35\left(\sin^3(dx+c)\right)}{24} + \frac{35\sin(dx+c)}{16}\right)\cos(dx+c}{5}\right.$
default	$a^2\left(\frac{\sin^9(dx+c)}{5\cos(dx+c)^5} - \frac{4\left(\sin^9(dx+c)\right)}{15\cos(dx+c)^3} + \frac{8\left(\sin^9(dx+c)\right)}{5\cos(dx+c)} + \frac{8\left(\sin^7(dx+c) + \frac{7\left(\sin^5(dx+c)\right)}{6} + \frac{35\left(\sin^3(dx+c)\right)}{24} + \frac{35\sin(dx+c)}{16}\right)\cos(dx+c}{5}\right.$
parts	$\frac{a^2\left(\frac{\left(\tan^5(dx+c)\right)}{5} - \frac{\left(\tan^3(dx+c)\right)}{3} + \tan(dx+c) - \arctan(\tan(dx+c))\right)}{d} + a^2\left(\frac{\sin^9(dx+c)}{5\cos(dx+c)^5} - \frac{4\left(\sin^9(dx+c)\right)}{15\cos(dx+c)^3} + \frac{8\left(\sin^9(dx+c\right.}{5\cos(dx+c)}\right.$

input
```
int((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-9/2*a^2*x-1/8*I*a^2/d*exp(2*I*(d*x+c))+a^2/d*exp(I*(d*x+c))+a^2/d*exp(-I*
(d*x+c))+1/8*I*a^2/d*exp(-2*I*(d*x+c))+2/5*(-78*a^2*exp(I*(d*x+c))-60*I*a^
2*exp(2*I*(d*x+c))+27*I*a^2-40*a^2*exp(3*I*(d*x+c))-75*I*a^2*exp(4*I*(d*x+
c))+30*a^2*exp(5*I*(d*x+c)))/(exp(I*(d*x+c))+I)/(-I+exp(I*(d*x+c)))^5/d
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

$$\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx =$$
$$-\frac{45\,a^2dx\cos\left(dx+c\right)^3-10\,a^2\cos\left(dx+c\right)^4-90\,a^2dx\cos\left(dx+c\right)+78\,a^2\cos\left(dx+c\right)^2-4\,a^2-\left(5\,a^2\,\mathrm{co}\right.}{10\left(d\cos\left(dx+c\right)^3+2\,d\cos\left(dx+c\right)\sin\left(dx+c\right.\right.}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/10*(45*a^2*d*x*cos(d*x + c)^3 - 10*a^2*cos(d*x + c)^4 - 90*a^2*d*x*cos(
d*x + c) + 78*a^2*cos(d*x + c)^2 - 4*a^2 - (5*a^2*cos(d*x + c)^4 - 90*a^2*
d*x*cos(d*x + c) + 84*a^2*cos(d*x + c)^2 - 6*a^2)*sin(d*x + c))/(d*cos(d*x
 + c)^3 + 2*d*cos(d*x + c)*sin(d*x + c) - 2*d*cos(d*x + c))
```

3.19.6 Sympy [F]

$$\int (a+a\sin(c+dx))^2\tan^6(c+dx)\,dx = a^2\left(\int 2\sin\left(c+dx\right)\tan^6\left(c+dx\right)dx + \int \sin^2\left(c+dx\right)\tan^6\left(c+dx\right)dx + \int \tan^6\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**6,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**6, x) + Integral(sin(c + d*x)*
*2*tan(c + d*x)**6, x) + Integral(tan(c + d*x)**6, x))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

$$\int (a + a\sin(c + dx))^2 \tan^6(c + dx)\, dx$$
$$= \frac{\left(6\,\tan\left(dx + c\right)^5 - 20\,\tan\left(dx + c\right)^3 - 105\,dx - 105\,c + \frac{15\,\tan(dx+c)}{\tan(dx+c)^2+1} + 90\,\tan\left(dx + c\right)\right)a^2 + 2\left(3\,\tan\left(dx \right.\right.}{}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/30*((6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d*x
 + c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 -
5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^2 + 12*a^2*((15*cos(
d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d*x + c)^5 + 5*cos(d*x + c)))/d
```

3.19.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^2 \tan^6(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
Timed out
```

3.19.9 Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.63

$$\int (a + a\sin(c + dx))^2 \tan^6(c + dx)\, dx = -\frac{9\,a^2\,x}{2}$$
$$-\frac{\frac{9\,a^2\,(c+d\,x)}{2} - \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\,\left(18\,a^2\,(c + d\,x) - \frac{a^2\,(180\,c+180\,d\,x-422)}{10}\right) + \frac{174\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5} - \frac{a^2\,(45\,c+45\,d\,x-128)}{10} + }{}$$

input
```
int(tan(c + d*x)^6*(a + a*sin(c + d*x))^2,x)
```

output
```
- (9*a^2*x)/2 - ((9*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*(18*a^2*(c + d*x
) - (a^2*(180*c + 180*d*x - 422))/10) + (174*a^2*tan(c/2 + (d*x)/2)^5)/5 -
 (a^2*(45*c + 45*d*x - 128))/10 + tan(c/2 + (d*x)/2)^9*(18*a^2*(c + d*x) -
 (a^2*(180*c + 180*d*x - 90))/10) + tan(c/2 + (d*x)/2)^4*(27*a^2*(c + d*x)
 - (a^2*(270*c + 270*d*x - 168))/10) - tan(c/2 + (d*x)/2)^8*((63*a^2*(c +
d*x))/2 - (a^2*(315*c + 315*d*x - 360))/10) - tan(c/2 + (d*x)/2)^6*(27*a^2
*(c + d*x) - (a^2*(270*c + 270*d*x - 600))/10) - tan(c/2 + (d*x)/2)^3*(36*
a^2*(c + d*x) - (a^2*(360*c + 360*d*x - 424))/10) + tan(c/2 + (d*x)/2)^2*(
(63*a^2*(c + d*x))/2 - (a^2*(315*c + 315*d*x - 536))/10) + tan(c/2 + (d*x)
/2)^7*(36*a^2*(c + d*x) - (a^2*(360*c + 360*d*x - 600))/10))/(d*(tan(c/2 +
 (d*x)/2) - 1)^5*(tan(c/2 + (d*x)/2) + 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.19. $\int (a+a\sin(c+dx))^2 \tan^6(c+dx)\,dx$

3.20 $\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$

3.20.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx=\frac{7a^2x}{2}-\frac{16a^2\cos(c+dx)}{3d}-\frac{7a^2\cos(c+dx)\sin(c+dx)}{2d}-\frac{8a^2\cos(c+dx)\sin^2(c+dx)}{3d(1-\sin(c+dx))}+\frac{a^4\cos(c+dx)\sin^3(c+dx)}{3d(a-a\sin(c+dx))^2}$$

output
```
7/2*a^2*x-16/3*a^2*cos(d*x+c)/d-7/2*a^2*cos(d*x+c)*sin(d*x+c)/d-8/3*a^2*co
s(d*x+c)*sin(d*x+c)^2/d/(1-sin(d*x+c))+1/3*a^4*cos(d*x+c)*sin(d*x+c)^3/d/(
a-a*sin(d*x+c))^2
```

3.20.2 Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.32

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx=$$
$$-\frac{a^2\left(-21(7+12c+12dx)\cos\left(\frac{1}{2}(c+dx)\right)+(239+84c+84dx)\cos\left(\frac{3}{2}(c+dx)\right)+3\left(-5\cos\left(\frac{5}{2}(c+dx)\right)\right.\right.}{48d\,(\mathrm{c}}$$

input
```
Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

output
```
-1/48*(a^2*(-21*(7 + 12*c + 12*d*x)*Cos[(c + d*x)/2] + (239 + 84*c + 84*d*
x)*Cos[(3*(c + d*x))/2] + 3*(-5*Cos[(5*(c + d*x))/2] + Cos[(7*(c + d*x))/2
] + 2*(50 + 56*c + 56*d*x + (-27 + 28*c + 28*d*x)*Cos[c + d*x] - 6*Cos[2*(
c + d*x)] - Cos[3*(c + d*x)])*Sin[(c + d*x)/2])))/(d*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^3)
```

3.20.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3187, 3042, 3244, 25, 3042, 3456, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4(a\sin(c+dx)+a)^2dx$$

$$\Big\downarrow \text{3187}$$

$$a^4\int \frac{\sin^4(c+dx)}{(a-a\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$a^4\int \frac{\sin(c+dx)^4}{(a-a\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3244}$$

$$a^4\left(\frac{\int -\frac{\sin^2(c+dx)(5\sin(c+dx)a+3a)}{a-a\sin(c+dx)}dx}{3a^2}+\frac{\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}\right)$$

$$\Big\downarrow \text{25}$$

$$a^4\left(\frac{\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\int \frac{\sin^2(c+dx)(5\sin(c+dx)a+3a)}{a-a\sin(c+dx)}dx}{3a^2}\right)$$

$$\Big\downarrow \; 3042$$

$$a^4\left(\frac{\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\int\frac{\sin(c+dx)^2(5\sin(c+dx)a+3a)}{a-a\sin(c+dx)}dx}{3a^2}\right)$$

$$\Big\downarrow \; 3456$$

$$a^4\left(\frac{\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{8\sin^2(c+dx)\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{\int\sin(c+dx)(21\sin(c+dx)a^2+16a^2)dx}{a^2}}{3a^2}\right)$$

$$\Big\downarrow \; 3042$$

$$a^4\left(\frac{\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{8\sin^2(c+dx)\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{\int\sin(c+dx)(21\sin(c+dx)a^2+16a^2)dx}{a^2}}{3a^2}\right)$$

$$\Big\downarrow \; 3213$$

$$a^4\left(\frac{\sin^3(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{8\sin^2(c+dx)\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{-\frac{16a^2\cos(c+dx)}{d}-\frac{21a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{21a^2x}{2}}{a^2}}{3a^2}\right)$$

input
```
Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

output
```
a^4*((Cos[c + d*x]*Sin[c + d*x]^3)/(3*d*(a - a*Sin[c + d*x])^2) - ((8*Cos[
c + d*x]*Sin[c + d*x]^2)/(d*(1 - Sin[c + d*x])) - ((21*a^2*x)/2 - (16*a^2*
Cos[c + d*x])/d - (21*a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2)/(3*a^2))
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3187
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Simp[a^p   Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ
[p, 2*m]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.20.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.96 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.10

method	result
risch	$\frac{7a^2x}{2}+\frac{ia^2\mathrm{e}^{2i(dx+c)}}{8d}-\frac{a^2\mathrm{e}^{i(dx+c)}}{d}-\frac{a^2\mathrm{e}^{-i(dx+c)}}{d}-\frac{ia^2\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{2(-21ia^2\mathrm{e}^{i(dx+c)}+12a^2\mathrm{e}^{2i(dx+c)}-11a^2)}{3(-i+\mathrm{e}^{i(dx+c)})^3d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+2a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{3}+\frac{5dx}{2}+\frac{5c}{2}\right)+2a^2\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}\right.}{d}$
parts	$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a^2\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}\right.}{3}\right.}{d}$

input
```
int((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
7/2*a^2*x+1/8*I*a^2/d*exp(2*I*(d*x+c))-a^2/d*exp(I*(d*x+c))-a^2/d*exp(-I*(
d*x+c))-1/8*I*a^2/d*exp(-2*I*(d*x+c))-2/3*(-21*I*a^2*exp(I*(d*x+c))+12*a^2
*exp(2*I*(d*x+c))-11*a^2)/(-I+exp(I*(d*x+c)))^3/d
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.63

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx$$
$$=\frac{3\,a^2\cos(dx+c)^4-6\,a^2\cos(dx+c)^3-42\,a^2dx+(21\,a^2dx+31\,a^2)\cos(dx+c)^2-2\,a^2-(21\,a^2dx-38\,a}{6\,(d\cos(dx+c)^2-d\cos(dx+}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(3*a^2*cos(d*x + c)^4 - 6*a^2*cos(d*x + c)^3 - 42*a^2*d*x + (21*a^2*d*
x + 31*a^2)*cos(d*x + c)^2 - 2*a^2 - (21*a^2*d*x - 38*a^2)*cos(d*x + c) -
(3*a^2*cos(d*x + c)^3 - 42*a^2*d*x + 9*a^2*cos(d*x + c)^2 + 2*a^2 - (21*a^
2*d*x - 40*a^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x
+ c) + (d*cos(d*x + c) + 2*d)*sin(d*x + c) - 2*d)
```

3.20.6 Sympy [F]

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = a^2\left(\int 2\sin(c+dx)\tan^4(c+dx)\,dx + \int \sin^2(c+dx)\tan^4(c+dx)\,dx + \int \tan^4(c+dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**4,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**4, x) + Integral(sin(c + d*x)*
*2*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4, x))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00

$$\int (a+a\sin(c+dx))^2\tan^4(c+dx)\,dx = \frac{\left(2\tan(dx+c)^3+15\,dx+15\,c-\frac{3\tan(dx+c)}{\tan(dx+c)^2+1}-12\tan(dx+c)\right)a^2+2\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\mathrm{ta}\right.}{6\,d}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6*((2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 +
1) - 12*tan(d*x + c))*a^2 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x +
c))*a^2 - 4*a^2*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/
d
```

3.20.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^2 \tan^4(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.20.9 Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.39

$$\int (a + a\sin(c + dx))^2 \tan^4(c + dx)\, dx = \frac{7a^2x}{2} + \frac{\frac{7a^2(c+dx)}{2} - \tan\left(\frac{c}{2} + \frac{dx}{2}\right)\left(\frac{21a^2(c+dx)}{2} - \frac{a^2(63c+63dx-150)}{6}\right) - \frac{a^2(21c+21dx-64)}{6} + \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^6\left(\frac{21a^2(c+dx)}{2}\right.}{}$$

input
```
int(tan(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

output
```
(7*a^2*x)/2 + ((7*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((21*a^2*(c + d*x)
)/2 - (a^2*(63*c + 63*d*x - 150))/6) - (a^2*(21*c + 21*d*x - 64))/6 + tan(
c/2 + (d*x)/2)^6*((21*a^2*(c + d*x))/2 - (a^2*(63*c + 63*d*x - 42))/6) - t
an(c/2 + (d*x)/2)^5*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x - 126))/
6) + tan(c/2 + (d*x)/2)^2*((35*a^2*(c + d*x))/2 - (a^2*(105*c + 105*d*x -
194))/6) + tan(c/2 + (d*x)/2)^4*((49*a^2*(c + d*x))/2 - (a^2*(147*c + 147*
d*x - 196))/6) - tan(c/2 + (d*x)/2)^3*((49*a^2*(c + d*x))/2 - (a^2*(147*c
+ 147*d*x - 252))/6))/(d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2)^2
+ 1)^2)
```

3.21 $\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$

3.21.1 Optimal result

Integrand size = 21, antiderivative size = 71

$$\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx=-\frac{5a^2x}{2}+\frac{2a^2\cos(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
-5/2*a^2*x+2*a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)/d/(1-sin(d*x+c))+1/2*a^2*co
s(d*x+c)*sin(d*x+c)/d
```

3.21.2 Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.69

$$\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx=-\frac{a^2\sec(c+dx)(1+\sin(c+dx))^{5/2}\left(-10\arcsin\left(\frac{\sqrt{1-\sin(c+dx)}}{\sqrt{2}}\right)\sqrt{1-\sin(c+dx)}+\sqrt{1+\sin(c+dx)}(-\right.}{2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}$$

input
```
Integrate[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

output
```
-1/2*(a^2*Sec[c + d*x]*(1 + Sin[c + d*x])^(5/2)*(-10*ArcSin[Sqrt[1 - Sin[c
 + d*x]]/Sqrt[2]]*Sqrt[1 - Sin[c + d*x]] + Sqrt[1 + Sin[c + d*x]]*(-8 + 3*
Sin[c + d*x] + Sin[c + d*x]^2)))/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^
4)
```

3.21.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a\sin(c+dx)+a)^2dx$$

$$\Big\downarrow \text{3188}$$

$$a^2\int \left(-\sin^2(c+dx)-2\sin(c+dx)+\frac{2}{1-\sin(c+dx)}-2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2\left(\frac{2\cos(c+dx)}{d}+\frac{\sin(c+dx)\cos(c+dx)}{2d}+\frac{2\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{5x}{2}\right)$$

input
```
Int[(a + a*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

output
```
a^2*((-5*x)/2 + (2*Cos[c + d*x])/d + (2*Cos[c + d*x])/(d*(1 - Sin[c + d*x]
)) + (Cos[c + d*x]*Sin[c + d*x])/(2*d))
```

3.21.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.21.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 2.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{5a^2x}{2}+\frac{a^2 e^{i(dx+c)}}{d}+\frac{a^2 e^{-i(dx+c)}}{d}+\frac{4a^2}{d\left(-i+e^{i(dx+c)}\right)}+\frac{a^2\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+a^2}{d}$
default	$\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right)+a^2}{d}$
parts	$\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)}{d}+\frac{2a^2\left(\frac{\sin}{\cos}\right.}{}$

input
```
int((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/2*a^2*x+a^2/d*exp(I*(d*x+c))+a^2/d*exp(-I*(d*x+c))+4*a^2/d/(-I+exp(I*(d
*x+c)))+1/4*a^2/d*sin(2*d*x+2*c)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.76

$$\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx$$
$$= \frac{a^2\cos\left(dx+c\right)^3 - 5\,a^2dx + 4\,a^2\cos\left(dx+c\right)^2 + 4\,a^2 - \left(5\,a^2dx - 7\,a^2\right)\cos\left(dx+c\right) + \left(5\,a^2dx + a^2\cos\left(dx\right.\right.}{2\left(d\cos\left(dx+c\right) - d\sin\left(dx+c\right) + d\right)}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
1/2*(a^2*cos(d*x + c)^3 - 5*a^2*d*x + 4*a^2*cos(d*x + c)^2 + 4*a^2 - (5*a^
2*d*x - 7*a^2)*cos(d*x + c) + (5*a^2*d*x + a^2*cos(d*x + c)^2 - 3*a^2*cos(
d*x + c) + 4*a^2)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

3.21.6 Sympy [F]

$$\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = a^2\left(\int 2\sin\left(c+dx\right)\tan^2\left(c+dx\right)dx + \int \sin^2\left(c+dx\right)\tan^2\left(c+dx\right)dx + \int \tan^2\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**2*tan(d*x+c)**2,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(sin(c + d*x)*
*2*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2, x))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18

$$\int (a + a\sin(c+dx))^2 \tan^2(c+dx)\,dx =$$
$$-\frac{\left(3\,dx + 3\,c - \frac{\tan(dx+c)}{\tan(dx+c)^2+1} - 2\,\tan\left(dx+c\right)\right)a^2 + 2\left(dx + c - \tan\left(dx+c\right)\right)a^2 - 4\,a^2\left(\frac{1}{\cos(dx+c)} + \cos\left(dx\right.\right.}{2\,d}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*((3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a
^2 + 2*(d*x + c - tan(d*x + c))*a^2 - 4*a^2*(1/cos(d*x + c) + cos(d*x + c)
))/d
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5370 vs. 2(65) = 130.

Time = 9.59 (sec) , antiderivative size = 5370, normalized size of antiderivative = 75.63

$$\int (a + a\sin(c+dx))^2 \tan^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(5*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 5*a^2*d*
x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 5*a^2*d*x*tan(d*x)^2*tan
(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*t
an(1/2*c)^3*tan(c)^3 + 5*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(
c)^3 - 8*a^2*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 5*a^2*tan(d
*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 5*a^2*tan(d*x)^2*tan(1/2*d*x)
^4*tan(1/2*c)^4*tan(c)^3 - 5*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^
4 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 5*a^2*d*x*t
an(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 8*a^2*tan(d*x)^3*tan(1/2*d*x)
^4*tan(1/2*c)^4*tan(c) + 20*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3
*tan(c)^2 - 5*a^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 8*a^2*tan(d*x
)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*d*
x)^4*tan(c)^3 - 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)*tan(c)^3 -
 20*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)*tan(1/2*c)^3*tan(c)^3 - 20*a^2*d*x*tan
(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 32*a^2*tan(d*x)^3*tan(1/2*d*x
)^3*tan(1/2*c)^3*tan(c)^3 - 5*a^2*d*x*tan(d*x)^3*tan(1/2*c)^4*tan(c)^3 - 8
*a^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 4*a^2*tan(d*x)^3*tan(
1/2*d*x)^4*tan(1/2*c)^4 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan
(c) - 20*a^2*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 2*a^2*tan(d
*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 20*a^2*tan(d*x)^2*tan(1/2*d*...
```

3.21.9 Mupad [B] (verification not implemented)

Time = 8.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.00

$$\int (a+a\sin(c+dx))^2\tan^2(c+dx)\,dx = -\frac{5\,a^2\,x}{2}$$
$$-\frac{\frac{5\,a^2\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(5\,c+5\,d\,x-6)}{2}\right)-\frac{a^2\,(5\,c+5\,d\,x-16)}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{5\,a^2\,(c+d\,x)}{2}-\frac{a^2\,(5}{}\right.}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right.}$$

input
```
int(tan(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

output
```
- (5*a^2*x)/2 - ((5*a^2*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((5*a^2*(c + d*x
))/2 - (a^2*(5*c + 5*d*x - 6))/2) - (a^2*(5*c + 5*d*x - 16))/2 + tan(c/2 +
 (d*x)/2)^4*((5*a^2*(c + d*x))/2 - (a^2*(5*c + 5*d*x - 10))/2) - tan(c/2 +
 (d*x)/2)^3*(5*a^2*(c + d*x) - (a^2*(10*c + 10*d*x - 10))/2) + tan(c/2 + (
d*x)/2)^2*(5*a^2*(c + d*x) - (a^2*(10*c + 10*d*x - 22))/2))/(d*(tan(c/2 +
(d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^2)
```

3.22 $\int (a+a\sin(c+dx))^2\,dx$

3.22.1 Optimal result

Integrand size = 12, antiderivative size = 45

$$\int (a+a\sin(c+dx))^2\,dx = \frac{3a^2x}{2} - \frac{2a^2\cos(c+dx)}{d} - \frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
3/2*a^2*x-2*a^2*cos(d*x+c)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

3.22.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int (a+a\sin(c+dx))^2\,dx = -\frac{a^2(-6(c+dx)+8\cos(c+dx)+\sin(2(c+dx)))}{4d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^2,x]
```

output
```
-1/4*(a^2*(-6*(c + d*x) + 8*Cos[c + d*x] + Sin[2*(c + d*x)]))/d
```

3.22.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(c+dx)+a)^2dx$$

$$\Big\downarrow \; 3123$$

$$-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a^2x}{2}$$

input
```
Int[(a + a*Sin[c + d*x])^2,x]
```

output
```
(3*a^2*x)/2 - (2*a^2*Cos[c + d*x])/d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*
d)
```

3.22.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

3.22.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result
parallelrisch	$-\frac{a^2(-6dx+8\cos(dx+c)+\sin(2dx+2c)-8)}{4d}$
risch	$\frac{3a^2x}{2}-\frac{2a^2\cos(dx+c)}{d}-\frac{a^2\sin(2dx+2c)}{4d}$
parts	$a^2x+\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\cos(dx+c)}{d}$
derivativedivides	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2a^2\cos(dx+c)+a^2(dx+c)}{d}$
default	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2a^2\cos(dx+c)+a^2(dx+c)}{d}$
norman	$\frac{\frac{a^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a^2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3a^2x}{2}-\frac{a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+3a^2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3a^2x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{4a^2\left(\tan^2\left(\right.\right.}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

input
```
int((a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a^2*(-6*d*x+8*cos(d*x+c)+sin(2*d*x+2*c)-8)/d
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int (a+a\sin(c+dx))^2\,dx=\frac{3\,a^2dx-a^2\cos\left(dx+c\right)\sin\left(dx+c\right)-4\,a^2\cos\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(3*a^2*d*x - a^2*cos(d*x + c)*sin(d*x + c) - 4*a^2*cos(d*x + c))/d
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.73

$$\int (a+a\sin(c+dx))^2\,dx = \begin{cases} \frac{a^2x\sin^2(c+dx)}{2}+\frac{a^2x\cos^2(c+dx)}{2}+a^2x-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{2a^2\cos(c+dx)}{d} & \text{for } d\neq 0 \\ x(a\sin(c)+a)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+a*sin(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*x -
a**2*sin(c + d*x)*cos(c + d*x)/(2*d) - 2*a**2*cos(c + d*x)/d, Ne(d, 0)), (
x*(a*sin(c) + a)**2, True))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int (a+a\sin(c+dx))^2\,dx = a^2x+\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2}{4\,d}-\frac{2\,a^2\cos(dx+c)}{d}$$

input
```
integrate((a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
a^2*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - 2*a^2*cos(d*x + c)/d
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int (a+a\sin(c+dx))^2\,dx = \frac{3}{2}\,a^2x-\frac{2\,a^2\cos(dx+c)}{d}-\frac{a^2\sin(2\,dx+2\,c)}{4\,d}$$

input
```
integrate((a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
3/2*a^2*x - 2*a^2*cos(d*x + c)/d - 1/4*a^2*sin(2*d*x + 2*c)/d
```

3.22.9 Mupad [B] (verification not implemented)

Time = 5.70 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.73

$$\int (a+a\sin(c+dx))^2\,dx=\frac{3\,a^2\,x}{2}$$
$$-\frac{a^2\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)-a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-a^2\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}-4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^2\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)-a^2\,(3\,c+3\,d\,x}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

input
```
int((a + a*sin(c + d*x))^2,x)
```

output
```
(3*a^2*x)/2 - (a^2*((3*c)/2 + (3*d*x)/2) - a^2*tan(c/2 + (d*x)/2)^3 - a^2*
((3*c)/2 + (3*d*x)/2 - 4) + tan(c/2 + (d*x)/2)^2*(2*a^2*((3*c)/2 + (3*d*x)
/2) - a^2*(3*c + 3*d*x - 4)) + a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)
/2)^2 + 1)^2)
```

3.23 $\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$

3.23.1 Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{a^2x}{2}-\frac{2a^2\text{arctanh}(\cos(c+dx))}{d}+\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
-1/2*a^2*x-2*a^2*arctanh(cos(d*x+c))/d+2*a^2*cos(d*x+c)/d-a^2*cot(d*x+c)/d
+1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

3.23.2 Mathematica [A] (verified)

Time = 5.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.27

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{a^2\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(7\cos(c+dx)+\cos(3(c+dx))+4\big(c+dx-4\cos(c+dx)+4\log\left(\right.\right.\right.}{16d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

output
```
-1/16*(a^2*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(7*Cos[c + d*x] + Cos[3*(c +
d*x)] + 4*(c + d*x - 4*Cos[c + d*x] + 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[
(c + d*x)/2]])*Sin[c + d*x]))/d
```

3.23.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^2}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{\int \left(\csc^2(c+dx)a^4-\sin^2(c+dx)a^4+2\csc(c+dx)a^4-2\sin(c+dx)a^4\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{2a^4\text{arctanh}(\cos(c+dx))}{d}+\frac{2a^4\cos(c+dx)}{d}-\frac{a^4\cot(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^4x}{2}}{a^2}$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^2,x]
```

output
```
(-1/2*(a^4*x) - (2*a^4*ArcTanh[Cos[c + d*x]])/d + (2*a^4*Cos[c + d*x])/d -
 (a^4*Cot[c + d*x])/d + (a^4*Cos[c + d*x]*Sin[c + d*x])/(2*d))/a^2
```

3.23.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.23.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^2(-\cot(dx+c)-dx-c)}{d}$
default	$\frac{a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^2(-\cot(dx+c)-dx-c)}{d}$
risch	$-\frac{a^2x}{2}-\frac{ia^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{a^2\mathrm{e}^{i(dx+c)}}{d}+\frac{a^2\mathrm{e}^{-i(dx+c)}}{d}+\frac{ia^2\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{2ia^2}{d(\mathrm{e}^{2i(dx+c)}-1)}+\frac{2a^2\ln(\mathrm{e}^{i(dx+c)}-1)}{d}$

input
```
int(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a^2*(cos(d*x+c)+ln(cs
c(d*x+c)-cot(d*x+c)))+a^2*(-cot(d*x+c)-d*x-c))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx =$$
$$-\frac{a^2\cos(dx+c)^3+2\,a^2\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2\,a^2\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c}{2\,d\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*cos(d*x + c)^3 + 2*a^2*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
- 2*a^2*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + a^2*cos(d*x + c) + (a^
2*d*x - 4*a^2*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

3.23.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = a^2\left(\int 2\sin(c+dx)\cot^2(c+dx)\,dx + \int \sin^2(c+dx)\cot^2(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(sin(c + d*x)*
*2*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2, x))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2 - 4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2 + 4\,a^2(2\,\cos(dx+c) - \log(\cos(dx+c)+1)}{4\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*a^2 - 4*(d*x + c + 1/tan(d*x + c))*a
^2 + 4*a^2*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1)
))/d
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(70) = 140.

Time = 0.65 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.93

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx =$$

$$-\frac{(dx+c)a^2 - 4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + \frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 \cdots\right.}{}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - a^2*tan(1/2*d
*x + 1/2*c) + (4*a^2*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/2*c) + 2*
(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*tan(1/2*d
*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.23.9 Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.72

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$= \frac{2\,a^2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{-3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 8\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right) - a^2}{d\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + 4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + 2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$
$$+ \frac{a^2\,\mathrm{atan}\left(\frac{a^4}{4\,a^4+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)} - \frac{4\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^4+a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

input
```
int(cot(c + d*x)^2*(a + a*sin(c + d*x))^2,x)
```

output
```
(2*a^2*log(tan(c/2 + (d*x)/2)))/d + (8*a^2*tan(c/2 + (d*x)/2)^3 - 3*a^2*ta
n(c/2 + (d*x)/2)^4 - a^2 + 8*a^2*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)
/2) + 4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (a^2*atan(a^4/(4
*a^4 + a^4*tan(c/2 + (d*x)/2)) - (4*a^4*tan(c/2 + (d*x)/2))/(4*a^4 + a^4*t
an(c/2 + (d*x)/2))))/d + (a^2*tan(c/2 + (d*x)/2))/(2*d)
```

3.24 $\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$

3.24.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = -\frac{a^2x}{2}+\frac{3a^2\operatorname{arctanh}(\cos(c+dx))}{d}-\frac{2a^2\cos(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)\csc(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
-1/2*a^2*x+3*a^2*arctanh(cos(d*x+c))/d-2*a^2*cos(d*x+c)/d-1/3*a^2*cot(d*x+
c)^3/d-a^2*cot(d*x+c)*csc(d*x+c)/d-1/2*a^2*cos(d*x+c)*sin(d*x+c)/d
```

3.24.2 Mathematica [A] (verified)

Time = 9.39 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.95

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$= \frac{a^2(1+\sin(c+dx))^2\left(-12(c+dx)-48\cos(c+dx)+4\cot\left(\frac{1}{2}(c+dx)\right)-6\csc^2\left(\frac{1}{2}(c+dx)\right)+72\log\left(\cos\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

output
```
(a^2*(1 + Sin[c + d*x])^2*(-12*(c + d*x) - 48*Cos[c + d*x] + 4*Cot[(c + d*
x)/2] - 6*Csc[(c + d*x)/2]^2 + 72*Log[Cos[(c + d*x)/2]] - 72*Log[Sin[(c +
d*x)/2]] + 6*Sec[(c + d*x)/2]^2 + 8*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (C
sc[(c + d*x)/2]^4*Sin[c + d*x])/2 - 6*Sin[2*(c + d*x)] - 4*Tan[(c + d*x)/2
]))/(24*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4)
```

3.24.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sin(c+dx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^2}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{\int \left(\csc^4(c+dx)a^6+2\csc^3(c+dx)a^6-\csc^2(c+dx)a^6+\sin^2(c+dx)a^6-4\csc(c+dx)a^6+2\sin(c+dx)a^6-a^6\right)}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3a^6\text{arctanh}(\cos(c+dx))}{d}-\frac{2a^6\cos(c+dx)}{d}-\frac{a^6\cot^3(c+dx)}{3d}-\frac{a^6\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^6\cot(c+dx)\csc(c+dx)}{d}-\frac{a^6x}{2}}{a^4}$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^2,x]
```

output
```
(-1/2*(a^6*x) + (3*a^6*ArcTanh[Cos[c + d*x]])/d - (2*a^6*Cos[c + d*x])/d -
 (a^6*Cot[c + d*x]^3)/(3*d) - (a^6*Cot[c + d*x]*Csc[c + d*x])/d - (a^6*Cos
[c + d*x]*Sin[c + d*x])/(2*d))/a^4
```

3.24.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.24.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.49

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\cdots\right.}{d}$
default	$\frac{a^2\left(-\frac{\cos^5(dx+c)}{\sin(dx+c)}-\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)+2a^2\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\cdots\right.}{d}$
risch	$-\frac{a^2x}{2}+\frac{ia^2e^{2i(dx+c)}}{8d}-\frac{a^2e^{i(dx+c)}}{d}-\frac{a^2e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2a^2\left(3ie^{4i(dx+c)}+3e^{5i(dx+c)}+i-3e^{i(dx\cdots}{3d\left(e^{2i(dx+c)}-1\right)^3}$

input
```
int(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x
+c)-3/2*d*x-3/2*c)+2*a^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-
3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+a^2*(-1/3*cot(d*x+c)^3+cot(d
*x+c)+d*x+c))
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(92) = 184.

Time = 0.31 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.96

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{3\,a^2\cos\left(dx+c\right)^5 - 4\,a^2\cos\left(dx+c\right)^3 + 3\,a^2\cos\left(dx+c\right) + 9\left(a^2\cos\left(dx+c\right)^2 - a^2\right)\log\left(\frac{1}{2}\cos\left(dx+c\right) + \right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*a^2*cos(d*x + c)^5 - 4*a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c) + 9*
(a^2*cos(d*x + c)^2 - a^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a
^2*cos(d*x + c)^2 - a^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(a^
2*d*x*cos(d*x + c)^2 + 4*a^2*cos(d*x + c)^3 - a^2*d*x - 6*a^2*cos(d*x + c)
)*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

3.24.6 Sympy [F]

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = a^2\left(\int 2\sin\left(c+dx\right)\cot^4\left(c+dx\right)dx + \int \sin^2\left(c+dx\right)\cot^4\left(c+dx\right)dx + \int \cot^4\left(c+dx\right)dx\right)$$

input
```
integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**2,x)
```

output
```
a**2*(Integral(2*sin(c + d*x)*cot(c + d*x)**4, x) + Integral(sin(c + d*x)*
*2*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4, x))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.42

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx =$$
$$-\frac{3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)a^2-2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\,a^2\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx\right.}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/6*(3*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x +
c)))*a^2 - 2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3
*a^2*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x
 + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

3.24.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(92) = 184$.

Time = 0.73 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.13

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx$$
$$=\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,(dx+c)a^2-72\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-3\,a^2\tan\left(\frac{1}{2}\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*(d*x
+ c)*a^2 - 72*a^2*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a^2*tan(1/2*d*x + 1/2
*c) + 24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2*
tan(1/2*d*x + 1/2*c) - 4*a^2)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*tan(1/2*d*x +
1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.24.9 Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.99

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^2\,dx = \frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{4d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24d}$$
$$-\frac{3a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2\operatorname{atan}\left(\frac{a^4}{6a^4-a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{6a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{6a^4-a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d}$$
$$-\frac{-9a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+34a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{19a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+36a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+2a^2}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\right)}$$
$$-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}$$

input
```
int(cot(c + d*x)^4*(a + a*sin(c + d*x))^2,x)
```

output
```
(a^2*tan(c/2 + (d*x)/2)^2)/(4*d) + (a^2*tan(c/2 + (d*x)/2)^3)/(24*d) - (3*
a^2*log(tan(c/2 + (d*x)/2)))/d - (a^2*atan(a^4/(6*a^4 - a^4*tan(c/2 + (d*x
)/2)) + (6*a^4*tan(c/2 + (d*x)/2))/(6*a^4 - a^4*tan(c/2 + (d*x)/2))))/d -
(36*a^2*tan(c/2 + (d*x)/2)^3 - (a^2*tan(c/2 + (d*x)/2)^2)/3 + (19*a^2*tan(
c/2 + (d*x)/2)^4)/3 + 34*a^2*tan(c/2 + (d*x)/2)^5 - 9*a^2*tan(c/2 + (d*x)/
2)^6 + a^2/3 + 2*a^2*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 16*t
an(c/2 + (d*x)/2)^5 + 8*tan(c/2 + (d*x)/2)^7)) - (a^2*tan(c/2 + (d*x)/2))/
(8*d)
```

3.25 $\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx$

3.25.1 Optimal result

Integrand size = 21, antiderivative size = 160

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx = \frac{209a^3\log(1-\sin(c+dx))}{16d} - \frac{a^3\log(1+\sin(c+dx))}{16d} + \frac{7a^3\sin(c+dx)}{d} + \frac{3a^3\sin^2(c+dx)}{2d} + \frac{a^3\sin^3(c+dx)}{3d} + \frac{a^6}{6d(a-a\sin(c+dx))^3} - \frac{13a^5}{8d(a-a\sin(c+dx))^2} + \frac{71a^4}{8d(a-a\sin(c+dx))}$$

output
```
209/16*a^3*ln(1-sin(d*x+c))/d-1/16*a^3*ln(1+sin(d*x+c))/d+7*a^3*sin(d*x+c)
/d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3*sin(d*x+c)^3/d+1/6*a^6/d/(a-a*sin(d*x+c)
)^3-13/8*a^5/d/(a-a*sin(d*x+c))^2+71/8*a^4/d/(a-a*sin(d*x+c))
```

3.25.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.62

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx$$
$$=\frac{a^3\left(627\log(1-\sin(c+dx))-3\log(1+\sin(c+dx))-\frac{8}{(-1+\sin(c+dx))^3}-\frac{78}{(-1+\sin(c+dx))^2}-\frac{426}{-1+\sin(c+dx)}+33\right.}{48d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

output
```
(a^3*(627*Log[1 - Sin[c + d*x]] - 3*Log[1 + Sin[c + d*x]] - 8/(-1 + Sin[c
+ d*x])^3 - 78/(-1 + Sin[c + d*x])^2 - 426/(-1 + Sin[c + d*x]) + 336*Sin[c
 + d*x] + 72*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3))/(48*d)
```

3.25.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^7(c+dx)(a\sin(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)^7(a\sin(c+dx)+a)^3dx$$
$$\Big\downarrow \text{3186}$$
$$\frac{\int \frac{a^7\sin^7(c+dx)}{(a-a\sin(c+dx))^4(\sin(c+dx)a+a)}d(a\sin(c+dx))}{d}$$
$$\Big\downarrow \text{99}$$
$$\frac{\int \left(\frac{a^6}{2(a-a\sin(c+dx))^4}-\frac{13a^5}{4(a-a\sin(c+dx))^3}+\frac{71a^4}{8(a-a\sin(c+dx))^2}-\frac{209a^3}{16(a-a\sin(c+dx))}-\frac{a^3}{16(\sin(c+dx)a+a)}+\sin^2(c+dx)a^2+3\,\mathrm{s}\right.}{d}$$
$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^6}{6(a-a\sin(c+dx))^3}-\frac{13a^5}{8(a-a\sin(c+dx))^2}+\frac{71a^4}{8(a-a\sin(c+dx))}+\frac{1}{3}a^3\sin^3(c+dx)+\frac{3}{2}a^3\sin^2(c+dx)+7a^3\sin(c+dx)+\frac{209}{16}}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^7,x]
```

output
```
((209*a^3*Log[a - a*Sin[c + d*x]])/16 - (a^3*Log[a + a*Sin[c + d*x]])/16 +
 7*a^3*Sin[c + d*x] + (3*a^3*Sin[c + d*x]^2)/2 + (a^3*Sin[c + d*x]^3)/3 +
a^6/(6*(a - a*Sin[c + d*x])^3) - (13*a^5)/(8*(a - a*Sin[c + d*x])^2) + (71
*a^4)/(8*(a - a*Sin[c + d*x])))/d
```

3.25.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.25.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 19.81 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.62

method	result
risch	$-13ia^3x+\frac{ia^3 e^{3i(dx+c)}}{24d}-\frac{3a^3 e^{2i(dx+c)}}{8d}-\frac{29ia^3 e^{i(dx+c)}}{8d}+\frac{29ia^3 e^{-i(dx+c)}}{8d}-\frac{3a^3 e^{-2i(dx+c)}}{8d}-\frac{ia^3 e^{-3i(dx}}{24d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^{11}(dx+c)}{6\cos(dx+c)^6}-\frac{5\left(\sin^{11}(dx+c)\right)}{24\cos(dx+c)^4}+\frac{35\left(\sin^{11}(dx+c)\right)}{48\cos(dx+c)^2}+\frac{35\left(\sin^9(dx+c)\right)}{48}+\frac{15\left(\sin^7(dx+c)\right)}{16}+\frac{21\left(\sin^5(dx+c)\right)}{16}+\frac{35\left(\sin^3(dx+c)\right)}{16}\right.}{}$
default	$\frac{a^3\left(\frac{\sin^{11}(dx+c)}{6\cos(dx+c)^6}-\frac{5\left(\sin^{11}(dx+c)\right)}{24\cos(dx+c)^4}+\frac{35\left(\sin^{11}(dx+c)\right)}{48\cos(dx+c)^2}+\frac{35\left(\sin^9(dx+c)\right)}{48}+\frac{15\left(\sin^7(dx+c)\right)}{16}+\frac{21\left(\sin^5(dx+c)\right)}{16}+\frac{35\left(\sin^3(dx+c)\right)}{16}\right.}{}$
parts	$\frac{a^3\left(\frac{\left(\tan^6(dx+c)\right)}{6}-\frac{\left(\tan^4(dx+c)\right)}{4}+\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{a^3\left(\frac{\sin^{11}(dx+c)}{6\cos(dx+c)^6}-\frac{5\left(\sin^{11}(dx+c)\right)}{24\cos(dx+c)^4}+\frac{35\left(\sin^{11}(c\right.}{48\cos(dx\right.}}{}$

input
```
int((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x,method=_RETURNVERBOSE)
```

output
```
-13*I*a^3*x+1/24*I/d*a^3*exp(3*I*(d*x+c))-3/8*a^3/d*exp(2*I*(d*x+c))-29/8*
I/d*a^3*exp(I*(d*x+c))+29/8*I/d*a^3*exp(-I*(d*x+c))-3/8*a^3/d*exp(-2*I*(d*
x+c))-1/24*I/d*a^3*exp(-3*I*(d*x+c))-26*I/d*a^3*c-1/12*I*(213*a^3*exp(I*(d
*x+c))+774*I*a^3*exp(2*I*(d*x+c))-1138*a^3*exp(3*I*(d*x+c))-774*I*a^3*exp(
4*I*(d*x+c))+213*a^3*exp(5*I*(d*x+c)))/(-I+exp(I*(d*x+c)))^6/d+209/8*a^3/d
*ln(-I+exp(I*(d*x+c)))-1/8*a^3/d*ln(exp(I*(d*x+c))+I)
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.50

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx =$$
$$-\frac{16\,a^3\cos{(dx+c)}^6-216\,a^3\cos{(dx+c)}^4+1002\,a^3\cos{(dx+c)}^2-482\,a^3+3\left(3\,a^3\cos{(dx+c)}^2-4\,a^3\right.}{}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="fricas")
```

output
```
-1/48*(16*a^3*cos(d*x + c)^6 - 216*a^3*cos(d*x + c)^4 + 1002*a^3*cos(d*x +
 c)^2 - 482*a^3 + 3*(3*a^3*cos(d*x + c)^2 - 4*a^3 - (a^3*cos(d*x + c)^2 -
4*a^3)*sin(d*x + c))*log(sin(d*x + c) + 1) - 627*(3*a^3*cos(d*x + c)^2 - 4
*a^3 - (a^3*cos(d*x + c)^2 - 4*a^3)*sin(d*x + c))*log(-sin(d*x + c) + 1) -
 2*(12*a^3*cos(d*x + c)^4 + 398*a^3*cos(d*x + c)^2 - 245*a^3)*sin(d*x + c)
)/(3*d*cos(d*x + c)^2 - (d*cos(d*x + c)^2 - 4*d)*sin(d*x + c) - 4*d)
```

3.25.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**7,x)
```

output
```
Timed out
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.83

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx$$
$$= \frac{16\,a^3\sin{(dx+c)}^3 + 72\,a^3\sin{(dx+c)}^2 - 3\,a^3\log{(\sin{(dx+c)}+1)} + 627\,a^3\log{(\sin{(dx+c)}-1)} + 336\,a}{48\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="maxima")
```

output
```
1/48*(16*a^3*sin(d*x + c)^3 + 72*a^3*sin(d*x + c)^2 - 3*a^3*log(sin(d*x +
c) + 1) + 627*a^3*log(sin(d*x + c) - 1) + 336*a^3*sin(d*x + c) - 2*(213*a^
3*sin(d*x + c)^2 - 387*a^3*sin(d*x + c) + 178*a^3)/(sin(d*x + c)^3 - 3*sin
(d*x + c)^2 + 3*sin(d*x + c) - 1))/d
```

3.25.8 Giac [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^7,x, algorithm="giac")
```

output
```
Timed out
```

3.25.9 Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 398, normalized size of antiderivative = 2.49

$$\int (a+a\sin(c+dx))^3\tan^7(c+dx)\,dx$$
$$= \frac{\frac{105\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{4}-\frac{263\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{2}+\frac{1301\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}-582\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{1657\,a^3\tan\left(\frac{c}{2}+\frac{d}{\cdots}\right.}{2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+18\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-38\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+63\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-84\tan\cdots\right.}$$
$$+\frac{209\,a^3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{8\,d}-\frac{a^3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{8\,d}-\frac{13\,a^3\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

input
```
int(tan(c + d*x)^7*(a + a*sin(c + d*x))^3,x)
```

output
```
((1301*a^3*tan(c/2 + (d*x)/2)^3)/4 - (263*a^3*tan(c/2 + (d*x)/2)^2)/2 - 58
2*a^3*tan(c/2 + (d*x)/2)^4 + (1657*a^3*tan(c/2 + (d*x)/2)^5)/2 - (2767*a^3
*tan(c/2 + (d*x)/2)^6)/3 + (1657*a^3*tan(c/2 + (d*x)/2)^7)/2 - 582*a^3*tan
(c/2 + (d*x)/2)^8 + (1301*a^3*tan(c/2 + (d*x)/2)^9)/4 - (263*a^3*tan(c/2 +
 (d*x)/2)^10)/2 + (105*a^3*tan(c/2 + (d*x)/2)^11)/4 + (105*a^3*tan(c/2 + (
d*x)/2))/4)/(d*(18*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) - 38*tan(c/
2 + (d*x)/2)^3 + 63*tan(c/2 + (d*x)/2)^4 - 84*tan(c/2 + (d*x)/2)^5 + 92*ta
n(c/2 + (d*x)/2)^6 - 84*tan(c/2 + (d*x)/2)^7 + 63*tan(c/2 + (d*x)/2)^8 - 3
8*tan(c/2 + (d*x)/2)^9 + 18*tan(c/2 + (d*x)/2)^10 - 6*tan(c/2 + (d*x)/2)^1
1 + tan(c/2 + (d*x)/2)^12 + 1)) + (209*a^3*log(tan(c/2 + (d*x)/2) - 1))/(8
*d) - (a^3*log(tan(c/2 + (d*x)/2) + 1))/(8*d) - (13*a^3*log(tan(c/2 + (d*x
)/2)^2 + 1))/d
```

3.26 $\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx$

3.26.1 Optimal result

Integrand size = 21, antiderivative size = 91

$$\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx=\frac{7a^3\log(1-\sin(c+dx))}{d}+\frac{5a^3\sin(c+dx)}{d}+\frac{3a^3\sin^2(c+dx)}{2d}+\frac{a^3\sin^3(c+dx)}{3d}+\frac{2a^4}{d(a-a\sin(c+dx))}$$

output
```
7*a^3*ln(1-sin(d*x+c))/d+5*a^3*sin(d*x+c)/d+3/2*a^3*sin(d*x+c)^2/d+1/3*a^3
*sin(d*x+c)^3/d+2*a^4/d/(a-a*sin(d*x+c))
```

3.26.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.73

$$\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx=\frac{a^3\left(42\log(1-\sin(c+dx))+\frac{12}{1-\sin(c+dx)}+30\sin(c+dx)+9\sin^2(c+dx)+2\sin^3(c+dx)\right)}{6d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

output
```
(a^3*(42*Log[1 - Sin[c + d*x]] + 12/(1 - Sin[c + d*x]) + 30*Sin[c + d*x] +
 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/(6*d)
```

3.26.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sin(c+dx)+a)^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(c+dx)^3(a\sin(c+dx)+a)^3dx$$

$$\Big\downarrow \ 3186$$

$$\frac{\int \frac{a^3\sin^3(c+dx)(\sin(c+dx)a+a)}{(a-a\sin(c+dx))^2}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \ 86$$

$$\frac{\int \left(\frac{2a^4}{(a-a\sin(c+dx))^2}-\frac{7a^3}{a-a\sin(c+dx)}+\sin^2(c+dx)a^2+3\sin(c+dx)a^2+5a^2\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{2a^4}{a-a\sin(c+dx)}+\frac{1}{3}a^3\sin^3(c+dx)+\frac{3}{2}a^3\sin^2(c+dx)+5a^3\sin(c+dx)+7a^3\log(a-a\sin(c+dx))}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

output
```
(7*a^3*Log[a - a*Sin[c + d*x]] + 5*a^3*Sin[c + d*x] + (3*a^3*Sin[c + d*x]^
2)/2 + (a^3*Sin[c + d*x]^3)/3 + (2*a^4)/(a - a*Sin[c + d*x]))/d
```

3.26.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.26.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.38 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.56

method	result
risch	$-7ia^3x-\frac{21ia^3\mathrm{e}^{i(dx+c)}}{8d}+\frac{21ia^3\mathrm{e}^{-i(dx+c)}}{8d}-\frac{14ia^3c}{d}-\frac{4ia^3\mathrm{e}^{i(dx+c)}}{\left(-i+\mathrm{e}^{i(dx+c)}\right)^2d}+\frac{14a^3\ln\left(-i+\mathrm{e}^{i(dx+c)}\right)}{d}-\frac{a^3\sin(}{}$
derivativedivides	$\frac{a^3\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\frac{5\left(\sin^3(dx+c)\right)}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+\right.}{2}\right.}{}$
default	$\frac{a^3\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\frac{5\left(\sin^3(dx+c)\right)}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a^3\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+\right.}{2}\right.}{}$
parts	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{a^3\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\frac{5\left(\sin^3(dx+c)\right)}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)}{2}\right.}{d}$

input
```
int((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-7*I*a^3*x-21/8*I/d*a^3*exp(I*(d*x+c))+21/8*I/d*a^3*exp(-I*(d*x+c))-14*I/d
*a^3*c-4*I*a^3*exp(I*(d*x+c))/(-I+exp(I*(d*x+c)))^2/d+14*a^3/d*ln(-I+exp(I
*(d*x+c)))-1/12*a^3/d*sin(3*d*x+3*c)-3/4/d*a^3*cos(2*d*x+2*c)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14

$$\int (a + a\sin(c + dx))^3 \tan^3(c + dx)\, dx$$
$$= \frac{4\, a^3 \cos{(dx + c)}^4 - 50\, a^3 \cos{(dx + c)}^2 + 31\, a^3 + 84\, (a^3 \sin{(dx + c)} - a^3) \log{(-\sin{(dx + c)} + 1)} - (14\, a^3}{12\, (d \sin{(dx + c)} - d)}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/12*(4*a^3*cos(d*x + c)^4 - 50*a^3*cos(d*x + c)^2 + 31*a^3 + 84*(a^3*sin(
d*x + c) - a^3)*log(-sin(d*x + c) + 1) - (14*a^3*cos(d*x + c)^2 + 55*a^3)*
sin(d*x + c))/(d*sin(d*x + c) - d)
```

3.26.6 Sympy [F]

$$\int (a + a\sin(c + dx))^3 \tan^3(c + dx)\, dx = a^3 \left(\int 3 \sin{(c + dx)} \tan^3{(c + dx)}\, dx + \int 3 \sin^2{(c + dx)} \tan^3{(c + dx)}\, dx + \int \sin^3{(c + dx)} \tan^3{(c + dx)}\, dx + \int \tan^3{(c + dx)}\, dx \right)$$

input
```
integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(3*sin(c + d*x
)**2*tan(c + d*x)**3, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**3, x) +
Integral(tan(c + d*x)**3, x))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.79

$$\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx$$
$$=\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2+42\,a^3\log\left(\sin\left(dx+c\right)-1\right)+30\,a^3\sin\left(dx+c\right)-\frac{12\,a^3}{\sin(dx+c)-1}}{6\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 42*a^3*log(sin(d*x + c)
 - 1) + 30*a^3*sin(d*x + c) - 12*a^3/(sin(d*x + c) - 1))/d
```

3.26.8 Giac [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.26.9 Mupad [B] (verification not implemented)

Time = 6.95 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.88

$$\int (a+a\sin(c+dx))^3\tan^3(c+dx)\,dx=\frac{14\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}$$
$$+\frac{14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-14\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{98\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{100\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{98\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-6\tan\left(\frac{c}{2}-\right.\right.}$$
$$-\frac{7\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

output
```
(14*a^3*log(tan(c/2 + (d*x)/2) - 1))/d + ((98*a^3*tan(c/2 + (d*x)/2)^3)/3
- 14*a^3*tan(c/2 + (d*x)/2)^2 - (100*a^3*tan(c/2 + (d*x)/2)^4)/3 + (98*a^3
*tan(c/2 + (d*x)/2)^5)/3 - 14*a^3*tan(c/2 + (d*x)/2)^6 + 14*a^3*tan(c/2 +
(d*x)/2)^7 + 14*a^3*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 - 2*tan
(c/2 + (d*x)/2) - 6*tan(c/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^4 - 6*tan(
c/2 + (d*x)/2)^5 + 4*tan(c/2 + (d*x)/2)^6 - 2*tan(c/2 + (d*x)/2)^7 + tan(c
/2 + (d*x)/2)^8 + 1)) - (7*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.26. $\int (a + a\sin(c+dx))^3 \tan^3(c+dx)\,dx$

3.27 $\int (a+a\sin(c+dx))^3\tan(c+dx)\,dx$

3.27.1 Optimal result

Integrand size = 19, antiderivative size = 70

$$\int (a+a\sin(c+dx))^3\tan(c+dx)\,dx=-\frac{4a^3\log(1-\sin(c+dx))}{d}-\frac{4a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

output
```
-4*a^3*ln(1-sin(d*x+c))/d-4*a^3*sin(d*x+c)/d-3/2*a^3*sin(d*x+c)^2/d-1/3*a^
3*sin(d*x+c)^3/d
```

3.27.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.74

$$\int (a+a\sin(c+dx))^3\tan(c+dx)\,dx=-\frac{a^3(24\log(1-\sin(c+dx))+24\sin(c+dx)+9\sin^2(c+dx)+2\sin^3(c+dx))}{6d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

output
```
-1/6*(a^3*(24*Log[1 - Sin[c + d*x]] + 24*Sin[c + d*x] + 9*Sin[c + d*x]^2 +
 2*Sin[c + d*x]^3))/d
```

3.27.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sin(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)(a\sin(c+dx)+a)^3dx$$

$$\Big\downarrow \text{ 3186}$$

$$\frac{\int \frac{a\sin(c+dx)(\sin(c+dx)a+a)^2}{a-a\sin(c+dx)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{\int \left(\frac{4a^3}{a-a\sin(c+dx)}-\sin^2(c+dx)a^2-3\sin(c+dx)a^2-4a^2\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{3}a^3\sin^3(c+dx)-\frac{3}{2}a^3\sin^2(c+dx)-4a^3\sin(c+dx)-4a^3\log(a-a\sin(c+dx))}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x],x]
```

output
```
(-4*a^3*Log[a - a*Sin[c + d*x]] - 4*a^3*Sin[c + d*x] - (3*a^3*Sin[c + d*x]
^2)/2 - (a^3*Sin[c + d*x]^3)/3)/d
```

3.27.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.27.4 Maple [A] (verified)

Time = 2.67 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.70

method	result
derivativedivides	$-\frac{a^3\left(\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{3\left(\sin^2(dx+c)\right)}{2}+4\sin(dx+c)+4\ln(\sin(dx+c)-1)\right)}{d}$
default	$-\frac{a^3\left(\frac{\left(\sin^3(dx+c)\right)}{3}+\frac{3\left(\sin^2(dx+c)\right)}{2}+4\sin(dx+c)+4\ln(\sin(dx+c)-1)\right)}{d}$
risch	$4ia^3x+\frac{17ia^3e^{i(dx+c)}}{8d}-\frac{17ia^3e^{-i(dx+c)}}{8d}+\frac{8ia^3c}{d}-\frac{8a^3\ln(-i+e^{i(dx+c)})}{d}+\frac{a^3\sin(3dx+3c)}{12d}+\frac{3a^3\cos(2dx+}{4d}$
parts	$\frac{a^3\ln(1+\tan^2(dx+c))}{2d}+\frac{a^3\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}+\frac{3a^3(-\sin(dx+c)+\ln(\sec(d}{d}$

input
```
int((a+a*sin(d*x+c))^3*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/d*a^3*(1/3*sin(d*x+c)^3+3/2*sin(d*x+c)^2+4*sin(d*x+c)+4*ln(sin(d*x+c)-1
))
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (a + a\sin(c + dx))^3 \tan(c + dx)\, dx = \frac{9\,a^3 \cos{(dx + c)}^2 - 24\,a^3 \log{(-\sin{(dx + c)} + 1)} + 2\,\left(a^3 \cos{(dx + c)}^2 - 13\,a^3\right) \sin{(dx + c)}}{6\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

output
```
1/6*(9*a^3*cos(d*x + c)^2 - 24*a^3*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x
 + c)^2 - 13*a^3)*sin(d*x + c))/d
```

3.27.6 Sympy [F]

$$\int (a + a\sin(c + dx))^3 \tan(c + dx)\, dx = a^3 \left(\int 3\sin{(c + dx)} \tan{(c + dx)}\, dx + \int 3\sin^2{(c + dx)} \tan{(c + dx)}\, dx + \int \sin^3{(c + dx)} \tan{(c + dx)}\, dx + \int \tan{(c + dx)}\, dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**3*tan(d*x+c),x)
```

output
```
a**3*(Integral(3*sin(c + d*x)*tan(c + d*x), x) + Integral(3*sin(c + d*x)**
2*tan(c + d*x), x) + Integral(sin(c + d*x)**3*tan(c + d*x), x) + Integral(
tan(c + d*x), x))
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

$$\int (a + a\sin(c + dx))^3 \tan(c + dx)\, dx = -\frac{2\, a^3 \sin\left(dx + c\right)^3 + 9\, a^3 \sin\left(dx + c\right)^2 + 24\, a^3 \log\left(\sin\left(dx + c\right) - 1\right) + 24\, a^3 \sin\left(dx + c\right)}{6\, d}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 + 24*a^3*log(sin(d*x + c
) - 1) + 24*a^3*sin(d*x + c))/d
```

3.27.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 24693 vs. 2(66) = 132.

Time = 15.22 (sec) , antiderivative size = 24693, normalized size of antiderivative = 352.76

$$\int (a + a\sin(c + dx))^3 \tan(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

output
```
-1/12*(24*a^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/
2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^
2 + tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 2
4*a^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2
*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x
) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 24*a^3*lo
g(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + t
an(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2
 - 9*a^3*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 24*a^3*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))
*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 - 24*a^3*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1
/2*d*x)^6*tan(1/2*c)^6 + 24*a^3*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*ta
n(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2...
```

3.27.9 Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 4.01

$$\int (a+a\sin(c+dx))^3\tan(c+dx)\,dx =$$

$$-\frac{\frac{56\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(2\,a^3\left(12\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d}{}\right.\right.\right.\right.}{}$$

$$-\frac{2\,a^3\left(12\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)-6\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\right)}{3\,d}$$

input
```
int(tan(c + d*x)*(a + a*sin(c + d*x))^3,x)
```

output
```
- ((56*a^3*tan(c/2 + (d*x)/2)^3)/3 + 8*a^3*tan(c/2 + (d*x)/2)^5 - tan(c/2
+ (d*x)/2)^2*(2*a^3*(12*log(tan(c/2 + (d*x)/2) - 1) - 6*log(tan(c/2 + (d*x
)/2)^2 + 1)) - (2*a^3*(36*log(tan(c/2 + (d*x)/2) - 1) - 18*log(tan(c/2 + (
d*x)/2)^2 + 1) + 9))/3) - tan(c/2 + (d*x)/2)^4*(2*a^3*(12*log(tan(c/2 + (d
*x)/2) - 1) - 6*log(tan(c/2 + (d*x)/2)^2 + 1)) - (2*a^3*(36*log(tan(c/2 +
(d*x)/2) - 1) - 18*log(tan(c/2 + (d*x)/2)^2 + 1) + 9))/3) + 8*a^3*tan(c/2
+ (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^3) - (2*a^3*(12*log(tan(c/2 + (d
*x)/2) - 1) - 6*log(tan(c/2 + (d*x)/2)^2 + 1)))/(3*d)
```

3.27. $\int (a+a\sin(c+dx))^3\tan(c+dx)\,dx$

3.28 $\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$

3.28.1 Optimal result

Integrand size = 21, antiderivative size = 98

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{3a^3\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}+\frac{2a^3\log(\sin(c+dx))}{d}-\frac{2a^3\sin(c+dx)}{d}-\frac{3a^3\sin^2(c+dx)}{2d}-\frac{a^3\sin^3(c+dx)}{3d}$$

output
```
-3*a^3*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d+2*a^3*ln(sin(d*x+c))/d-2*a^3*si
n(d*x+c)/d-3/2*a^3*sin(d*x+c)^2/d-1/3*a^3*sin(d*x+c)^3/d
```

3.28.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.67

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{a^3(18\csc(c+dx)+3\csc^2(c+dx)-12\log(\sin(c+dx))+12\sin(c+dx)+9\sin^2(c+dx)+2\sin^3(c+$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/6*(a^3*(18*Csc[c + d*x] + 3*Csc[c + d*x]^2 - 12*Log[Sin[c + d*x]] + 12*
Sin[c + d*x] + 9*Sin[c + d*x]^2 + 2*Sin[c + d*x]^3))/d
```

3.28.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sin(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^3}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))(\sin(c+dx)a+a)^4}{a^3}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{84}$$

$$\frac{\int \left(a^2\csc^3(c+dx)+3a^2\csc^2(c+dx)+2a^2\csc(c+dx)-2a^2-a^2\sin^2(c+dx)-3a^2\sin(c+dx)\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^3\sin^3(c+dx)-\frac{3}{2}a^3\sin^2(c+dx)-2a^3\sin(c+dx)-\frac{1}{2}a^3\csc^2(c+dx)-3a^3\csc(c+dx)+2a^3\log(a\sin(c+dx))}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^3,x]
```

output
```
(-3*a^3*Csc[c + d*x] - (a^3*Csc[c + d*x]^2)/2 + 2*a^3*Log[a*Sin[c + d*x]]
- 2*a^3*Sin[c + d*x] - (3*a^3*Sin[c + d*x]^2)/2 - (a^3*Sin[c + d*x]^3)/3)/
d
```

3.28.3.1 Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.28.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^3\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^3\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)+a}{d}$
default	$\frac{\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3a^3\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^3\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)+a}{d}$
risch	$-2ia^3x-\frac{ia^3e^{3i(dx+c)}}{24d}+\frac{3a^3e^{2i(dx+c)}}{8d}+\frac{9ia^3e^{i(dx+c)}}{8d}-\frac{9ia^3e^{-i(dx+c)}}{8d}+\frac{3a^3e^{-2i(dx+c)}}{8d}+\frac{ia^3e^{-3i(dx+c)}}{24d}$

input
```
int(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*(1/2*cos(d*x+c)^2+ln(sin(d*
x+c)))+3*a^3*(-cos(d*x+c)^4/sin(d*x+c)-(2+cos(d*x+c)^2)*sin(d*x+c))+a^3*(-
1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.20

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx$$
$$= \frac{18\,a^3\cos\left(dx+c\right)^4 - 27\,a^3\cos\left(dx+c\right)^2 + 15\,a^3 + 24\,\left(a^3\cos\left(dx+c\right)^2 - a^3\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right) + 4\,\left(a^3\cos\right.}{12\,\left(d\cos\left(dx+c\right)^2 - d\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(18*a^3*cos(d*x + c)^4 - 27*a^3*cos(d*x + c)^2 + 15*a^3 + 24*(a^3*cos
(d*x + c)^2 - a^3)*log(1/2*sin(d*x + c)) + 4*(a^3*cos(d*x + c)^4 - 8*a^3*c
os(d*x + c)^2 + 16*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

3.28.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = a^3\left(\int 3\sin\left(c+dx\right)\cot^3\left(c+dx\right)dx + \int 3\sin^2\left(c+dx\right)\cot^3\left(c+dx\right)dx + \int \sin^3\left(c+dx\right)\cot^3\left(c+dx\right)dx + \int \cot^3\left(c+dx\right)dx\right)$$

input
```
integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(3*sin(c + d*x
)**2*cot(c + d*x)**3, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**3, x) +
Integral(cot(c + d*x)**3, x))
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx =$$
$$-\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2-12\,a^3\log\left(\sin\left(dx+c\right)\right)+12\,a^3\sin\left(dx+c\right)+\frac{3\,\left(6\,a^3\sin(dx+c)+a^3\right)}{\sin(dx+c)^2}}{6\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(sin(d*x + c
)) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c) + a^3)/sin(d*x + c)^2)/d
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx =$$
$$-\frac{2\,a^3\sin\left(dx+c\right)^3+9\,a^3\sin\left(dx+c\right)^2-12\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+12\,a^3\sin\left(dx+c\right)+\frac{3\,\left(6\,a^3\sin(dx+c)^2+6\,a\ldots\right.}{\sin(dx+\ldots}}{6\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(2*a^3*sin(d*x + c)^3 + 9*a^3*sin(d*x + c)^2 - 12*a^3*log(abs(sin(d*x
 + c))) + 12*a^3*sin(d*x + c) + 3*(6*a^3*sin(d*x + c)^2 + 6*a^3*sin(d*x +
c) + a^3)/sin(d*x + c)^2)/d
```

3.28.9 Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.58

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{2\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}$$
$$-\frac{22\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\frac{49\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{2}+\frac{182\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}+\frac{51\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{2}+34\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{3\,a^3\tan}{}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$
$$-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d} - \frac{2\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + a*sin(c + d*x))^3,x)
```

output
```
(2*a^3*log(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^2)/(8*d) - ((3
*a^3*tan(c/2 + (d*x)/2)^2)/2 + 34*a^3*tan(c/2 + (d*x)/2)^3 + (51*a^3*tan(c
/2 + (d*x)/2)^4)/2 + (182*a^3*tan(c/2 + (d*x)/2)^5)/3 + (49*a^3*tan(c/2 +
(d*x)/2)^6)/2 + 22*a^3*tan(c/2 + (d*x)/2)^7 + a^3/2 + 6*a^3*tan(c/2 + (d*x
)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + 12*tan(c/2 +
(d*x)/2)^6 + 4*tan(c/2 + (d*x)/2)^8)) - (3*a^3*tan(c/2 + (d*x)/2))/(2*d) -
 (2*a^3*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.29 $\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx$

3.29.1 Optimal result

Integrand size = 21, antiderivative size = 180

$$\begin{aligned}\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx =& -\frac{23a^3x}{2}+\frac{136a^3\cos(c+dx)}{5d}-\frac{136a^3\cos^3(c+dx)}{15d}\\ &+\frac{23a^3\cos(c+dx)\sin(c+dx)}{2d}\\ &+\frac{a^6\cos(c+dx)\sin^5(c+dx)}{5d(a-a\sin(c+dx))^3}\\ &-\frac{13a^5\cos(c+dx)\sin^4(c+dx)}{15d(a-a\sin(c+dx))^2}\\ &+\frac{23a^6\cos(c+dx)\sin^3(c+dx)}{3d\left(a^3-a^3\sin(c+dx)\right)}\end{aligned}$$

output
```
-23/2*a^3*x+136/5*a^3*cos(d*x+c)/d-136/15*a^3*cos(d*x+c)^3/d+23/2*a^3*cos(
d*x+c)*sin(d*x+c)/d+1/5*a^6*cos(d*x+c)*sin(d*x+c)^5/d/(a-a*sin(d*x+c))^3-1
3/15*a^5*cos(d*x+c)*sin(d*x+c)^4/d/(a-a*sin(d*x+c))^2+23/3*a^6*cos(d*x+c)*
sin(d*x+c)^3/d/(a^3-a^3*sin(d*x+c))
```

3.29.2 Mathematica [A] (verified)

Time = 8.81 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.35

$$\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx$$
$$=\frac{(a+a\sin(c+dx))^3\left(-690(c+dx)+405\cos(c+dx)-5\cos(3(c+dx))+\frac{12}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4}-\frac{}{(c}\right.}{60d\left(\cos\left(\right.\right.}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]
```

output
```
((a + a*Sin[c + d*x])^3*(-690*(c + d*x) + 405*Cos[c + d*x] - 5*Cos[3*(c +
d*x)] + 12/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 - 112/(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^2 + (24*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])^5 - (224*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)^3 + (1576*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 45*S
in[2*(c + d*x)]))/(60*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

3.29.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.04, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {3042, 3187, 3042, 3244, 25, 3042, 3456, 3042, 3456, 27, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^6(c+dx)(a\sin(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \tan(c+dx)^6(a\sin(c+dx)+a)^3dx$$
$$\Big\downarrow \text{3187}$$
$$a^6\int \frac{\sin^6(c+dx)}{(a-a\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3042}$$

$$a^6 \int \frac{\sin(c+dx)^6}{(a-a\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3244}$$

$$a^6 \left(\frac{\int -\frac{\sin^4(c+dx)(8\sin(c+dx)a+5a)}{(a-a\sin(c+dx))^2}dx}{5a^2}+\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}\right)$$

$$\Big\downarrow \text{25}$$

$$a^6 \left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\int \frac{\sin^4(c+dx)(8\sin(c+dx)a+5a)}{(a-a\sin(c+dx))^2}dx}{5a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^6 \left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\int \frac{\sin(c+dx)^4(8\sin(c+dx)a+5a)}{(a-a\sin(c+dx))^2}dx}{5a^2}\right)$$

$$\Big\downarrow \text{3456}$$

$$a^6 \left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\int \frac{\sin^3(c+dx)\left(63\sin(c+dx)a^2+52a^2\right)}{a-a\sin(c+dx)}dx}{3a^2}}{5a^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$a^6 \left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\int \frac{\sin(c+dx)^3\left(63\sin(c+dx)a^2+52a^2\right)}{a-a\sin(c+dx)}dx}{3a^2}}{5a^2}\right)$$

$$\Big\downarrow \text{3456}$$

$$a^6 \left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{\int 3\sin^2(c+dx)\left(136\sin(c+dx)a^3+115a^3\right)dx}{a^2}}{3a^2}}{5a^2}\right)$$

$$\Big\downarrow \text{27}$$

$$a^6 \left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{3\int \sin^2(c+dx)\left(136\sin(c+dx)a^3+115a^3\right)dx}{a^2}}{3a^2}}{5a^2}\right)$$

3.29. $\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx$

$$\Big\downarrow \text{ 3042}$$

$$a^6\left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{3\int \sin(c+dx)^2\left(136\sin(c+dx)a^3+115a^3\right)dx}{a^2}}{3a^2}}{5a^2}\right)$$

$$\Big\downarrow \text{ 3227}$$

$$a^6\left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{3\left(136a^3\int \sin^3(c+dx)dx+115a^3\int \sin^2(c+dx)dx\right)}{a^2}}{3a^2}}{5a^2}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$a^6\left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{3\left(115a^3\int \sin(c+dx)^2dx+136a^3\int \sin(c+dx)^3dx\right)}{a^2}}{3a^2}}{5a^2}\right)$$

$$\Big\downarrow \text{ 3113}$$

$$a^6\left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{3\left(115a^3\int \sin(c+dx)^2dx-\frac{136a^3\int\left(1-\cos^2(c+dx)\right)\cdots}{d}\right)}{a^2}}{3a^2}}{5a^2}\right)$$

$$\Big\downarrow \text{ 2009}$$

$$a^6\left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{3\left(115a^3\int \sin(c+dx)^2dx-\frac{136a^3\left(\cos(c+dx)-\frac{1}{3}\cos^3\cdots\right)}{d}\right)}{a^2}}{3a^2}}{5a^2}\right)$$

$$\Big\downarrow \text{ 3115}$$

3.29. $\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx$

$$a^6\left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{3\left(115a^3\left(\frac{\int 1dx}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{136a^3(\cos \cdots}{\cdots}\right)}{a^2}}{3a^2}}{5a^2}\right.$$

$\downarrow$ 24

$$a^6\left(\frac{\sin^5(c+dx)\cos(c+dx)}{5d(a-a\sin(c+dx))^3}-\frac{\frac{13a\sin^4(c+dx)\cos(c+dx)}{3d(a-a\sin(c+dx))^2}-\frac{\frac{115a^2\sin^3(c+dx)\cos(c+dx)}{d(a-a\sin(c+dx))}-\frac{3\left(115a^3\left(\frac{x}{2}-\frac{\sin(c+dx)\cos(c+dx)}{2d}\right)-\frac{136a^3(\cos(c \cdots}{\cdots}\right)}{a^2}}{3a^2}}{5a^2}\right.$$

input
```
Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^6,x]
```

output
```
a^6*((Cos[c + d*x]*Sin[c + d*x]^5)/(5*d*(a - a*Sin[c + d*x])^3) - ((13*a*C
os[c + d*x]*Sin[c + d*x]^4)/(3*d*(a - a*Sin[c + d*x])^2) - ((115*a^2*Cos[c
 + d*x]*Sin[c + d*x]^3)/(d*(a - a*Sin[c + d*x])) - (3*((-136*a^3*(Cos[c +
d*x] - Cos[c + d*x]^3/3))/d + 115*a^3*(x/2 - (Cos[c + d*x]*Sin[c + d*x])/(
2*d))))/a^2)/(3*a^2))/(5*a^2))
```

3.29.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3187
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_), x_Symbol] :> Simp[a^p   Int[Sin[e + f*x]^p/(a - b*Sin[e + f*x])^m, x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ
[p, 2*m]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3244
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e
+ f*x])^m*((c + d*Sin[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Simp[1/(a*b*
(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 2)*
Simp[b*(c^2*(m + 1) + d^2*(n - 1)) + a*c*d*(m - n + 1) + d*(a*d*(m - n + 1)
 + b*c*(m + n))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
 NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1]
&& GtQ[n, 1] && (IntegersQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

rule 3456
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Simp[1/(a*b*(2*m + 1))   Int[(a + b*Sin[e + f*x])^(m
+ 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
 b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Fre
eQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (In
tegerQ[2*n] || EqQ[c, 0])
```

3.29. $\int (a + a\sin(c+dx))^3 \tan^6(c+dx)\,dx$

3.29.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 15.58 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{23a^3x}{2}-\frac{a^3\mathrm{e}^{3i(dx+c)}}{24d}-\frac{3ia^3\mathrm{e}^{2i(dx+c)}}{8d}+\frac{27a^3\mathrm{e}^{i(dx+c)}}{8d}+\frac{27a^3\mathrm{e}^{-i(dx+c)}}{8d}+\frac{3ia^3\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{a^3\mathrm{e}^{-3i(dx+c)}}{24d}$
derivativedivides	$a^3\left(\frac{\sin^{10}(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^{10}(dx+c)}{3\cos(dx+c)^3}+\frac{7(\sin^{10}(dx+c))}{3\cos(dx+c)}+\frac{7\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8(\sin^6(dx+c))}{7}+\frac{48(\sin^4(dx+c))}{35}+\frac{64(\sin^2(dx+c))}{35}\right)c}{3}\right.$
default	$a^3\left(\frac{\sin^{10}(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^{10}(dx+c)}{3\cos(dx+c)^3}+\frac{7(\sin^{10}(dx+c))}{3\cos(dx+c)}+\frac{7\left(\frac{128}{35}+\sin^8(dx+c)+\frac{8(\sin^6(dx+c))}{7}+\frac{48(\sin^4(dx+c))}{35}+\frac{64(\sin^2(dx+c))}{35}\right)c}{3}\right.$
parts	$\frac{a^3\left(\frac{(\tan^5(dx+c))}{5}-\frac{(\tan^3(dx+c))}{3}+\tan(dx+c)-\arctan(\tan(dx+c))\right)}{d}+\frac{a^3\left(\frac{\sin^{10}(dx+c)}{5\cos(dx+c)^5}-\frac{\sin^{10}(dx+c)}{3\cos(dx+c)^3}+\frac{7(\sin^{10}(dx+c)}{3\cos(dx+c)}\right.}{}$

input
```
int((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x,method=_RETURNVERBOSE)
```

output
```
-23/2*a^3*x-1/24*a^3/d*exp(3*I*(d*x+c))-3/8*I/d*a^3*exp(2*I*(d*x+c))+27/8*
a^3/d*exp(I*(d*x+c))+27/8*a^3/d*exp(-I*(d*x+c))+3/8*I/d*a^3*exp(-2*I*(d*x+
c))-1/24*a^3/d*exp(-3*I*(d*x+c))+2/15*(-1160*a^3*exp(2*I*(d*x+c))-810*I*a^
3*exp(3*I*(d*x+c))+760*I*a^3*exp(I*(d*x+c))+225*a^3*exp(4*I*(d*x+c))+197*a
^3)/(-I+exp(I*(d*x+c)))^5/d
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.61

$$\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx =$$

$$-\frac{10\,a^3\cos{(dx+c)}^6-15\,a^3\cos{(dx+c)}^5-140\,a^3\cos{(dx+c)}^4-1380\,a^3dx+(345\,a^3dx-839\,a^3)\cos{(c}}{}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="fricas")
```

output
```
-1/30*(10*a^3*cos(d*x + c)^6 - 15*a^3*cos(d*x + c)^5 - 140*a^3*cos(d*x + c
)^4 - 1380*a^3*d*x + (345*a^3*d*x - 839*a^3)*cos(d*x + c)^3 + 6*a^3 + (103
5*a^3*d*x + 668*a^3)*cos(d*x + c)^2 - 6*(115*a^3*d*x - 233*a^3)*cos(d*x +
c) - (10*a^3*cos(d*x + c)^5 + 25*a^3*cos(d*x + c)^4 - 115*a^3*cos(d*x + c)
^3 - 1380*a^3*d*x - 6*a^3 + (345*a^3*d*x + 724*a^3)*cos(d*x + c)^2 - 6*(11
5*a^3*d*x - 232*a^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + 3*d*c
os(d*x + c)^2 - 2*d*cos(d*x + c) - (d*cos(d*x + c)^2 - 2*d*cos(d*x + c) -
4*d)*sin(d*x + c) - 4*d)
```

3.29.6 Sympy [F]

$$\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx = a^3\left(\int 3\sin(c+dx)\tan^6(c+dx)\,dx + \int 3\sin^2(c+dx)\tan^6(c+dx)\,dx + \int \sin^3(c+dx)\tan^6(c+dx)\,dx + \int \tan^6(c+dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**6,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**6, x) + Integral(3*sin(c + d*x
)**2*tan(c + d*x)**6, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**6, x) +
Integral(tan(c + d*x)**6, x))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.16

$$\int (a+a\sin(c+dx))^3\tan^6(c+dx)\,dx = \frac{3\left(6\tan(dx+c)^5-20\tan(dx+c)^3-105\,dx-105\,c+\frac{15\tan(dx+c)}{\tan(dx+c)^2+1}+90\tan(dx+c)\right)a^3+2\left(3\tan(dx\right.}{}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="maxima")
```

output
```
1/30*(3*(6*tan(d*x + c)^5 - 20*tan(d*x + c)^3 - 105*d*x - 105*c + 15*tan(d
*x + c)/(tan(d*x + c)^2 + 1) + 90*tan(d*x + c))*a^3 + 2*(3*tan(d*x + c)^5
- 5*tan(d*x + c)^3 - 15*d*x - 15*c + 15*tan(d*x + c))*a^3 - 2*(5*cos(d*x +
 c)^3 - (90*cos(d*x + c)^4 - 20*cos(d*x + c)^2 + 3)/cos(d*x + c)^5 - 60*co
s(d*x + c))*a^3 + 18*a^3*((15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 1)/cos(d
*x + c)^5 + 5*cos(d*x + c)))/d
```

3.29.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^3 \tan^6(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^6,x, algorithm="giac")
```

output
```
Timed out
```

3.29.9 Mupad [B] (verification not implemented)

Time = 10.39 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.43

$$\int (a + a\sin(c + dx))^3 \tan^6(c + dx)\, dx = -\frac{23\, a^3\, x}{2}$$
$$-\frac{\frac{23\, a^3\, (c+d\, x)}{2} - \tan\left(\frac{c}{2} + \frac{d\, x}{2}\right)\, \left(\frac{115\, a^3\, (c+d\, x)}{2} - \frac{a^3\, (1725\, c+1725\, d\, x-4750)}{30}\right) - \frac{a^3\, (345\, c+345\, d\, x-1088)}{30} + \tan\left(\frac{c}{2} + \frac{d\, x}{2}\right)^{10}\, (}{}$$

input
```
int(tan(c + d*x)^6*(a + a*sin(c + d*x))^3,x)
```

output
```
- (23*a^3*x)/2 - ((23*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((115*a^3*(c +
 d*x))/2 - (a^3*(1725*c + 1725*d*x - 4750))/30) - (a^3*(345*c + 345*d*x -
1088))/30 + tan(c/2 + (d*x)/2)^10*((115*a^3*(c + d*x))/2 - (a^3*(1725*c +
1725*d*x - 690))/30) - tan(c/2 + (d*x)/2)^9*((299*a^3*(c + d*x))/2 - (a^3*
(4485*c + 4485*d*x - 3450))/30) + tan(c/2 + (d*x)/2)^2*((299*a^3*(c + d*x)
)/2 - (a^3*(4485*c + 4485*d*x - 10694))/30) + tan(c/2 + (d*x)/2)^8*((575*a
^3*(c + d*x))/2 - (a^3*(8625*c + 8625*d*x - 8740))/30) - tan(c/2 + (d*x)/2
)^3*((575*a^3*(c + d*x))/2 - (a^3*(8625*c + 8625*d*x - 18460))/30) - tan(c
/2 + (d*x)/2)^7*(437*a^3*(c + d*x) - (a^3*(13110*c + 13110*d*x - 16100))/3
0) + tan(c/2 + (d*x)/2)^4*(437*a^3*(c + d*x) - (a^3*(13110*c + 13110*d*x -
 25244))/30) + tan(c/2 + (d*x)/2)^6*(529*a^3*(c + d*x) - (a^3*(15870*c + 1
5870*d*x - 23368))/30) - tan(c/2 + (d*x)/2)^5*(529*a^3*(c + d*x) - (a^3*(1
5870*c + 15870*d*x - 26680))/30))/(d*(tan(c/2 + (d*x)/2) - 1)^5*(tan(c/2 +
 (d*x)/2)^2 + 1)^3)
```

3.29. $\int (a + a\sin(c + dx))^3 \tan^6(c + dx)\, dx$

3.30 $\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$

3.30.1 Optimal result

Integrand size = 21, antiderivative size = 119

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx=\frac{17a^3x}{2}-\frac{6a^3\cos(c+dx)}{d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{2a^3\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{25a^3\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
17/2*a^3*x-6*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d+2/3*a^3*cos(d*x+c)/d/
(1-sin(d*x+c))^2-25/3*a^3*cos(d*x+c)/d/(1-sin(d*x+c))-3/2*a^3*cos(d*x+c)*s
in(d*x+c)/d
```

3.30.2 Mathematica [A] (verified)

Time = 6.74 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.49

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$$

$$=\frac{(a+a\sin(c+dx))^3\left(102(c+dx)-69\cos(c+dx)+\cos(3(c+dx))+\frac{8}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{\ldots}{\left(\cos\left(\frac{1}{2}(\ldots\right.\right.}\right.}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

output
```
((a + a*Sin[c + d*x])^3*(102*(c + d*x) - 69*Cos[c + d*x] + Cos[3*(c + d*x)
] + 8/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (16*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] - Sin[(c + d*x)/2])^3 - (200*Sin[(c + d*x)/2])/(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2]) - 9*Sin[2*(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] +
Sin[(c + d*x)/2])^6)
```

3.30.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sin(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4(a\sin(c+dx)+a)^3dx$$

$$\Big\downarrow \text{3188}$$

$$a^4\int \left(\frac{\sin^3(c+dx)}{a}+\frac{3\sin^2(c+dx)}{a}+\frac{5\sin(c+dx)}{a}+\frac{7}{a}-\frac{9}{a(1-\sin(c+dx))}+\frac{2}{a(1-\sin(c+dx))^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^4\left(\frac{\cos^3(c+dx)}{3ad}-\frac{6\cos(c+dx)}{ad}-\frac{3\sin(c+dx)\cos(c+dx)}{2ad}-\frac{25\cos(c+dx)}{3ad(1-\sin(c+dx))}+\frac{2\cos(c+dx)}{3ad(1-\sin(c+dx))^2}+\right.$$

input
```
Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

output
```
a^4*((17*x)/(2*a) - (6*Cos[c + d*x])/(a*d) + Cos[c + d*x]^3/(3*a*d) + (2*C
os[c + d*x])/(3*a*d*(1 - Sin[c + d*x])^2) - (25*Cos[c + d*x])/(3*a*d*(1 -
Sin[c + d*x])) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a*d))
```

3.30.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.30.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 8.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.25

method	result
risch	$\frac{17a^3x}{2}+\frac{3ia^3e^{2i(dx+c)}}{8d}-\frac{23a^3e^{i(dx+c)}}{8d}-\frac{23a^3e^{-i(dx+c)}}{8d}-\frac{3ia^3e^{-2i(dx+c)}}{8d}-\frac{2(-48ia^3e^{i(dx+c)}+27a^3e^{2i(dx+c}}{3(-i+e^{i(dx+c)})^3 d}$
derivativedivides	$a^3\left(\frac{\sin^8(dx+c)}{3\cos(dx+c)^3}-\frac{5(\sin^8(dx+c))}{3\cos(dx+c)}-\frac{5\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6(\sin^4(dx+c))}{5}+\frac{8(\sin^2(dx+c))}{5}\right)\cos(dx+c)}{3}\right)+3a^3\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-4\right.$
default	$a^3\left(\frac{\sin^8(dx+c)}{3\cos(dx+c)^3}-\frac{5(\sin^8(dx+c))}{3\cos(dx+c)}-\frac{5\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6(\sin^4(dx+c))}{5}+\frac{8(\sin^2(dx+c))}{5}\right)\cos(dx+c)}{3}\right)+3a^3\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-4\right.$
parts	$\frac{a^3\left(\frac{(\tan^3(dx+c))}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a^3\left(\frac{\sin^8(dx+c)}{3\cos(dx+c)^3}-\frac{5(\sin^8(dx+c))}{3\cos(dx+c)}-\frac{5\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6(\sin^4(d\right.}{5}}{}\right.}{d}$

input
```
int((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
17/2*a^3*x+3/8*I/d*a^3*exp(2*I*(d*x+c))-23/8*a^3/d*exp(I*(d*x+c))-23/8*a^3
/d*exp(-I*(d*x+c))-3/8*I/d*a^3*exp(-2*I*(d*x+c))-2/3*(-48*I*a^3*exp(I*(d*x
+c))+27*a^3*exp(2*I*(d*x+c))-25*a^3)/(-I+exp(I*(d*x+c)))^3/d+1/12*a^3/d*co
s(3*d*x+3*c)
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(105) = 210$.

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.85

$$\int (a + a\sin(c + dx))^3 \tan^4(c + dx)\,dx = \frac{2\,a^3\cos(dx+c)^5 + 7\,a^3\cos(dx+c)^4 - 22\,a^3\cos(dx+c)^3 - 102\,a^3dx - 4\,a^3 + (51\,a^3dx + 77\,a^3)\cos(dx}{6\,(d\cos}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(2*a^3*cos(d*x + c)^5 + 7*a^3*cos(d*x + c)^4 - 22*a^3*cos(d*x + c)^3 -
 102*a^3*d*x - 4*a^3 + (51*a^3*d*x + 77*a^3)*cos(d*x + c)^2 - (51*a^3*d*x
- 100*a^3)*cos(d*x + c) + (2*a^3*cos(d*x + c)^4 - 5*a^3*cos(d*x + c)^3 + 1
02*a^3*d*x - 27*a^3*cos(d*x + c)^2 - 4*a^3 + (51*a^3*d*x - 104*a^3)*cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c)
 + 2*d)*sin(d*x + c) - 2*d)
```

3.30.6 Sympy [F]

$$\int (a + a\sin(c + dx))^3 \tan^4(c + dx)\,dx = a^3\left(\int 3\sin(c + dx)\tan^4(c + dx)\,dx + \int 3\sin^2(c + dx)\tan^4(c + dx)\,dx + \int \sin^3(c + dx)\tan^4(c + dx)\,dx + \int \tan^4(c + dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**4,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**4, x) + Integral(3*sin(c + d*x
)**2*tan(c + d*x)**4, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**4, x) +
Integral(tan(c + d*x)**4, x))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.39

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx$$
$$= \frac{2\left(\cos(dx+c)^3 - \frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3} - 9\cos(dx+c)\right)a^3 + 3\left(2\tan(dx+c)^3 + 15\,dx + 15\,c - \frac{3\tan(dx+c)}{\tan(dx+c)^2+1} - \right.}{}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6*(2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d*x
 + c))*a^3 + 3*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x
 + c)^2 + 1) - 12*tan(d*x + c))*a^3 + 2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*
tan(d*x + c))*a^3 - 6*a^3*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d
*x + c)))/d
```

3.30.8 Giac [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.30.9 Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 3.12

$$\int (a+a\sin(c+dx))^3\tan^4(c+dx)\,dx=\frac{17\,a^3\,x}{2}$$
$$+\frac{\frac{17\,a^3\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{51\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(153\,c+153\,d\,x-378)}{6}\right)-\frac{a^3\,(51\,c+51\,d\,x-160)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(\frac{51\,a^3\,(c\cdots}{2}\right.}{}$$

input
```
int(tan(c + d*x)^4*(a + a*sin(c + d*x))^3,x)
```

output
```
(17*a^3*x)/2 + ((17*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((51*a^3*(c + d*
x))/2 - (a^3*(153*c + 153*d*x - 378))/6) - (a^3*(51*c + 51*d*x - 160))/6 +
 tan(c/2 + (d*x)/2)^8*((51*a^3*(c + d*x))/2 - (a^3*(153*c + 153*d*x - 102)
)/6) - tan(c/2 + (d*x)/2)^7*(51*a^3*(c + d*x) - (a^3*(306*c + 306*d*x - 30
6))/6) + tan(c/2 + (d*x)/2)^2*(51*a^3*(c + d*x) - (a^3*(306*c + 306*d*x -
654))/6) + tan(c/2 + (d*x)/2)^6*(85*a^3*(c + d*x) - (a^3*(510*c + 510*d*x
- 578))/6) - tan(c/2 + (d*x)/2)^3*(85*a^3*(c + d*x) - (a^3*(510*c + 510*d*
x - 1022))/6) - tan(c/2 + (d*x)/2)^5*(102*a^3*(c + d*x) - (a^3*(612*c + 61
2*d*x - 918))/6) + tan(c/2 + (d*x)/2)^4*(102*a^3*(c + d*x) - (a^3*(612*c +
 612*d*x - 1002))/6))/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2 + tan(
c/2 + (d*x)/2)^3 - 1)^3)
```

3.31 $\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$

3.31.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = -\frac{11a^3x}{2}+\frac{5a^3\cos(c+dx)}{d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{4a^3\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
-11/2*a^3*x+5*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c)^3/d+4*a^3*cos(d*x+c)/d/(
1-sin(d*x+c))+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

3.31.2 Mathematica [A] (verified)

Time = 5.64 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.29

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx$$
$$=\frac{(a+a\sin(c+dx))^3\left(-66(c+dx)+57\cos(c+dx)-\cos(3(c+dx))+\frac{96\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+9\sin(2(c\right.}{12d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^6}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

output
```
((a + a*Sin[c + d*x])^3*(-66*(c + d*x) + 57*Cos[c + d*x] - Cos[3*(c + d*x)
] + (96*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) + 9*Sin[2*
(c + d*x)]))/(12*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6)
```

3.31.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sin(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a\sin(c+dx)+a)^3dx$$

$$\Big\downarrow \text{3188}$$

$$a^2\int \left(-a\sin^3(c+dx)-3a\sin^2(c+dx)-4a\sin(c+dx)-4a+\frac{4a}{1-\sin(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2\left(-\frac{a\cos^3(c+dx)}{3d}+\frac{5a\cos(c+dx)}{d}+\frac{3a\sin(c+dx)\cos(c+dx)}{2d}+\frac{4a\cos(c+dx)}{d(1-\sin(c+dx))}-\frac{11ax}{2}\right)$$

input
```
Int[(a + a*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

output
```
a^2*((-11*a*x)/2 + (5*a*Cos[c + d*x])/d - (a*Cos[c + d*x]^3)/(3*d) + (4*a*
Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(2*
d))
```

3.31.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.31.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.00 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{11a^3x}{2}+\frac{19a^3e^{i(dx+c)}}{8d}+\frac{19a^3e^{-i(dx+c)}}{8d}+\frac{8a^3}{d(-i+e^{i(dx+c)})}-\frac{a^3\cos(3dx+3c)}{12d}+\frac{3\sin(2dx+2c)a^3}{4d}$
derivativedivides	$\frac{a^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+\right.}{d}$
default	$\frac{a^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)+3a^3\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+\right.}{d}$
parts	$\frac{a^3(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}+\frac{3a^3\left(\frac{\sin^4(}{\cos(}\right.}{}$

input
```
int((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-11/2*a^3*x+19/8*a^3/d*exp(I*(d*x+c))+19/8*a^3/d*exp(-I*(d*x+c))+8*a^3/d/(
-I+exp(I*(d*x+c)))-1/12*a^3/d*cos(3*d*x+3*c)+3/4/d*sin(2*d*x+2*c)*a^3
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.73

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx =$$
$$-\frac{2\,a^3\cos(dx+c)^4-7\,a^3\cos(dx+c)^3+33\,a^3dx-30\,a^3\cos(dx+c)^2-24\,a^3+3\,(11\,a^3dx-15\,a^3)\cos(}{6\,(d\cos(dx+c)-d}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*a^3*cos(d*x + c)^4 - 7*a^3*cos(d*x + c)^3 + 33*a^3*d*x - 30*a^3*co
s(d*x + c)^2 - 24*a^3 + 3*(11*a^3*d*x - 15*a^3)*cos(d*x + c) - (2*a^3*cos(
d*x + c)^3 + 33*a^3*d*x + 9*a^3*cos(d*x + c)^2 - 21*a^3*cos(d*x + c) + 24*
a^3)*sin(d*x + c))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

3.31.6 Sympy [F]

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = a^3\left(\int 3\sin(c+dx)\tan^2(c+dx)\,dx + \int 3\sin^2(c+dx)\tan^2(c+dx)\,dx + \int \sin^3(c+dx)\tan^2(c+dx)\,dx + \int \tan^2(c+dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**3*tan(d*x+c)**2,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(3*sin(c + d*x
)**2*tan(c + d*x)**2, x) + Integral(sin(c + d*x)**3*tan(c + d*x)**2, x) +
Integral(tan(c + d*x)**2, x))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.31

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = -\frac{2\left(\cos(dx+c)^3-\frac{3}{\cos(dx+c)}-6\cos(dx+c)\right)a^3+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\tan(dx+c)\right)a^3+6}{6\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^3 + 9*(3*d*x
+ 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan(d*x + c))*a^3 + 6*(d*x +
 c - tan(d*x + c))*a^3 - 18*a^3*(1/cos(d*x + c) + cos(d*x + c)))/d
```

3.31.8 Giac [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.31.9 Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.24

$$\int (a+a\sin(c+dx))^3\tan^2(c+dx)\,dx = -\frac{11\,a^3\,x}{2}$$
$$-\frac{\frac{11\,a^3\,(c+d\,x)}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{11\,a^3\,(c+d\,x)}{2}-\frac{a^3\,(33\,c+33\,d\,x-38)}{6}\right)-\frac{a^3\,(33\,c+33\,d\,x-104)}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{11\,a^3\,(c+d\,x)}{2}\cdots\right.}{\cdots}$$

input
```
int(tan(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

output
```
- (11*a^3*x)/2 - ((11*a^3*(c + d*x))/2 - tan(c/2 + (d*x)/2)*((11*a^3*(c +
d*x))/2 - (a^3*(33*c + 33*d*x - 38))/6) - (a^3*(33*c + 33*d*x - 104))/6 +
tan(c/2 + (d*x)/2)^6*((11*a^3*(c + d*x))/2 - (a^3*(33*c + 33*d*x - 66))/6)
 - tan(c/2 + (d*x)/2)^5*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 66))
/6) - tan(c/2 + (d*x)/2)^3*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x - 1
20))/6) + tan(c/2 + (d*x)/2)^4*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99*d*x
 - 192))/6) + tan(c/2 + (d*x)/2)^2*((33*a^3*(c + d*x))/2 - (a^3*(99*c + 99
*d*x - 246))/6))/(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^3)
```

3.32 $\int (a + a\sin(c + dx))^3 \, dx$

3.32.1 Optimal result

Integrand size = 12, antiderivative size = 63

$$\int (a + a\sin(c + dx))^3 \, dx = \frac{5a^3x}{2} - \frac{4a^3\cos(c + dx)}{d} + \frac{a^3\cos^3(c + dx)}{3d} - \frac{3a^3\cos(c + dx)\sin(c + dx)}{2d}$$

output
```
5/2*a^3*x-4*a^3*cos(d*x+c)/d+1/3*a^3*cos(d*x+c)^3/d-3/2*a^3*cos(d*x+c)*sin
(d*x+c)/d
```

3.32.2 Mathematica [A] (verified)

Time = 2.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int (a+a\sin(c+dx))^3 \, dx = \frac{a^3(30c + 30dx - 45\cos(c + dx) + \cos(3(c + dx)) - 9\sin(2(c + dx)))}{12d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^3,x]
```

output
```
(a^3*(30*c + 30*d*x - 45*Cos[c + d*x] + Cos[3*(c + d*x)] - 9*Sin[2*(c + d*
x)]))/(12*d)
```

3.32.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (a\sin(c+dx)+a)^3dx$$
$$\Big\downarrow \text{3124}$$
$$\int \left(a^3\sin^3(c+dx)+3a^3\sin^2(c+dx)+3a^3\sin(c+dx)+a^3\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{4a^3\cos(c+dx)}{d}-\frac{3a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{5a^3x}{2}$$

input
```
Int[(a + a*Sin[c + d*x])^3,x]
```

output
```
(5*a^3*x)/2 - (4*a^3*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^3)/(3*d) - (3*a^3
*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.32.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

3.32.4 Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

method	result
parallelrisch	$\frac{a^3(30dx-45\cos(dx+c)+\cos(3dx+3c)-9\sin(2dx+2c)-44)}{12d}$
risch	$\frac{5a^3x}{2}-\frac{15a^3\cos(dx+c)}{4d}+\frac{a^3\cos(3dx+3c)}{12d}-\frac{3\sin(2dx+2c)a^3}{4d}$
derivativedivides	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-3a^3\cos(dx+c)+a^3(dx+c)}{d}$
default	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-3a^3\cos(dx+c)+a^3(dx+c)}{d}$
parts	$a^3x-\frac{a^3(2+\sin^2(dx+c))\cos(dx+c)}{3d}-\frac{3a^3\cos(dx+c)}{d}+\frac{3a^3\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\frac{5a^3x}{2}-\frac{22a^3}{3d}-\frac{3a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{3a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{15a^3x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{15a^3x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{5a^3x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

input
```
int((a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*a^3*(30*d*x-45*cos(d*x+c)+cos(3*d*x+3*c)-9*sin(2*d*x+2*c)-44)/d
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int (a+a\sin(c+dx))^3\,dx = \frac{2\,a^3\cos{(dx+c)}^3+15\,a^3dx-9\,a^3\cos{(dx+c)}\sin{(dx+c)}-24\,a^3\cos{(dx+c)}}{6\,d}$$

input
```
integrate((a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/6*(2*a^3*cos(d*x + c)^3 + 15*a^3*d*x - 9*a^3*cos(d*x + c)*sin(d*x + c) -
 24*a^3*cos(d*x + c))/d
```

3.32.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.92

$$\int (a + a\sin(c + dx))^3 \, dx$$
$$= \begin{cases} \frac{3a^3x\sin^2(c+dx)}{2} + \frac{3a^3x\cos^2(c+dx)}{2} + a^3x - \frac{a^3\sin^2(c+dx)\cos(c+dx)}{d} - \frac{3a^3\sin(c+dx)\cos(c+dx)}{2d} - \frac{2a^3\cos^3(c+dx)}{3d} - \frac{3a^3\cos(}{d} \\ x(a\sin(c) + a)^3 \end{cases}$$

input
```
integrate((a+a*sin(d*x+c))**3,x)
```

output
```
Piecewise((3*a**3*x*sin(c + d*x)**2/2 + 3*a**3*x*cos(c + d*x)**2/2 + a**3*
x - a**3*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**3*sin(c + d*x)*cos(c + d*x)
/(2*d) - 2*a**3*cos(c + d*x)**3/(3*d) - 3*a**3*cos(c + d*x)/d, Ne(d, 0)),
(x*(a*sin(c) + a)**3, True))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14

$$\int (a + a\sin(c + dx))^3 \, dx = a^3x + \frac{(\cos(dx + c)^3 - 3\cos(dx + c))a^3}{3d} + \frac{3(2dx + 2c - \sin(2dx + 2c))a^3}{4d} - \frac{3a^3\cos(dx + c)}{d}$$

input
```
integrate((a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
a^3*x + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d + 3/4*(2*d*x + 2*c - s
in(2*d*x + 2*c))*a^3/d - 3*a^3*cos(d*x + c)/d
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int (a + a\sin(c + dx))^3\, dx = \frac{5}{2}\, a^3 x + \frac{a^3 \cos\left(3\, dx + 3\, c\right)}{12\, d} - \frac{15\, a^3 \cos\left(dx + c\right)}{4\, d} - \frac{3\, a^3 \sin\left(2\, dx + 2\, c\right)}{4\, d}$$

input
```
integrate((a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
5/2*a^3*x + 1/12*a^3*cos(3*d*x + 3*c)/d - 15/4*a^3*cos(d*x + c)/d - 3/4*a^
3*sin(2*d*x + 2*c)/d
```

3.32.9 Mupad [B] (verification not implemented)

Time = 8.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.48

$$\int (a + a\sin(c + dx))^3\, dx = \frac{5\, a^3\, x}{2} - \frac{\frac{5\, a^3\, (c+d\, x)}{2} - 3\, a^3 \tan\left(\frac{c}{2} + \frac{d\, x}{2}\right)^5 - \frac{a^3\, (15\, c+15\, d\, x-44)}{6} + \tan\left(\frac{c}{2} + \frac{d\, x}{2}\right)^4 \left(\frac{15\, a^3\, (c+d\, x)}{2} - \frac{a^3\, (45\, c+45\, d\, x-36)}{6}\right) + \tan(\cdots}{d\left(\tan\left(\frac{c}{2} + \frac{d\, x}{2}\right)^2 + 1\right)^3}$$

input
```
int((a + a*sin(c + d*x))^3,x)
```

output
```
(5*a^3*x)/2 - ((5*a^3*(c + d*x))/2 - 3*a^3*tan(c/2 + (d*x)/2)^5 - (a^3*(15
*c + 15*d*x - 44))/6 + tan(c/2 + (d*x)/2)^4*((15*a^3*(c + d*x))/2 - (a^3*(
45*c + 45*d*x - 36))/6) + tan(c/2 + (d*x)/2)^2*((15*a^3*(c + d*x))/2 - (a^
3*(45*c + 45*d*x - 96))/6) + 3*a^3*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)
/2)^2 + 1)^3)
```

3.33 $\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$

3.33.1 Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{a^3x}{2} - \frac{3a^3\operatorname{arctanh}(\cos(c+dx))}{d} + \frac{3a^3\cos(c+dx)}{d} - \frac{a^3\cos^3(c+dx)}{3d} - \frac{a^3\cot(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
1/2*a^3*x-3*a^3*arctanh(cos(d*x+c))/d+3*a^3*cos(d*x+c)/d-1/3*a^3*cos(d*x+c
)^3/d-a^3*cot(d*x+c)/d+3/2*a^3*cos(d*x+c)*sin(d*x+c)/d
```

3.33.2 Mathematica [A] (verified)

Time = 6.00 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{a^3\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)(15-66\sin(c+dx))-12\left(c+dx-6\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{48d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/48*(a^3*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x]*(15 - 66*Sin[c
+ d*x]) - 12*(c + d*x - 6*Log[Cos[(c + d*x)/2]] + 6*Log[Sin[(c + d*x)/2]])
*Sin[c + d*x] + Cos[3*(c + d*x)]*(9 + 2*Sin[c + d*x])))/d
```

3.33.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sin(c+dx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^3}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{\int\left(-\sin^3(c+dx)a^5+\csc^2(c+dx)a^5-3\sin^2(c+dx)a^5+3\csc(c+dx)a^5-2\sin(c+dx)a^5+2a^5\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3a^5\text{arctanh}(\cos(c+dx))}{d}-\frac{a^5\cos^3(c+dx)}{3d}+\frac{3a^5\cos(c+dx)}{d}-\frac{a^5\cot(c+dx)}{d}+\frac{3a^5\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^5x}{2}}{a^2}$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^3,x]
```

output
```
((a^5*x)/2 - (3*a^5*ArcTanh[Cos[c + d*x]])/d + (3*a^5*Cos[c + d*x])/d - (a
^5*Cos[c + d*x]^3)/(3*d) - (a^5*Cot[c + d*x])/d + (3*a^5*Cos[c + d*x]*Sin[
c + d*x])/(2*d))/a^2
```

3.33.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.33.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{-\frac{a^3(\cos^3(dx+c))}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^3(-\cot(dx+c)-d}{d}$
default	$\frac{-\frac{a^3(\cos^3(dx+c))}{3}+3a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^3(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+a^3(-\cot(dx+c)-d}{d}$
risch	$\frac{a^3x}{2}-\frac{3ia^3e^{2i(dx+c)}}{8d}+\frac{11a^3e^{i(dx+c)}}{8d}+\frac{11a^3e^{-i(dx+c)}}{8d}+\frac{3ia^3e^{-2i(dx+c)}}{8d}-\frac{2ia^3}{d(e^{2i(dx+c)}-1)}-\frac{3a^3\ln(e^{i(dx+c)}}{d}$

input
```
int(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3*a^3*cos(d*x+c)^3+3*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
+3*a^3*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a^3*(-cot(d*x+c)-d*x-c))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.32

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{9\,a^3\cos(dx+c)^3+9\,a^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-9\,a^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+
}{6\,d\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/6*(9*a^3*cos(d*x + c)^3 + 9*a^3*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) - 9*a^3*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*a^3*cos(d*x + c) +
 (2*a^3*cos(d*x + c)^3 - 3*a^3*d*x - 18*a^3*cos(d*x + c))*sin(d*x + c))/(d
*sin(d*x + c))
```

3.33.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = a^3\left(\int 3\sin(c+dx)\cot^2(c+dx)\,dx + \int 3\sin^2(c+dx)\cot^2(c+dx)\,dx + \int \sin^3(c+dx)\cot^2(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**3,x)
```

output
```
a**3*(Integral(3*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(3*sin(c + d*x
)**2*cot(c + d*x)**2, x) + Integral(sin(c + d*x)**3*cot(c + d*x)**2, x) +
Integral(cot(c + d*x)**2, x))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = -\frac{4\,a^3\cos\left(dx+c\right)^3 - 9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3 + 12\left(dx+c+\frac{1}{\tan\left(dx+c\right)}\right)a^3 - 18\,a^3(2\,\cos\left(dx+c\right) \dots}{12\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(4*a^3*cos(d*x + c)^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 12*
(d*x + c + 1/tan(d*x + c))*a^3 - 18*a^3*(2*cos(d*x + c) - log(cos(d*x + c)
 + 1) + log(cos(d*x + c) - 1)))/d
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.76

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx = \frac{3\left(dx+c\right)a^3 + 18\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) + 3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{3\left(6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} - \frac{2\left(9\,a^3\tan\left(\frac{1}{2}\,dx \dots\right.}{\dots}}{6\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(3*(d*x + c)*a^3 + 18*a^3*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1
/2*d*x + 1/2*c) - 3*(6*a^3*tan(1/2*d*x + 1/2*c) + a^3)/tan(1/2*d*x + 1/2*c
) - 2*(9*a^3*tan(1/2*d*x + 1/2*c)^5 - 12*a^3*tan(1/2*d*x + 1/2*c)^4 - 36*a
^3*tan(1/2*d*x + 1/2*c)^2 - 9*a^3*tan(1/2*d*x + 1/2*c) - 16*a^3)/(tan(1/2*
d*x + 1/2*c)^2 + 1)^3)/d
```

3.33.9 Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.87

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^3\,dx$$

$$= \frac{3\,a^3\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} + \frac{a^3\,\mathrm{atan}\left(\frac{a^6}{6\,a^6-a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{6\,a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6\,a^6-a^6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$
$$+ \frac{-7\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+24\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

input
```
int(cot(c + d*x)^2*(a + a*sin(c + d*x))^3,x)
```

output
```
(3*a^3*log(tan(c/2 + (d*x)/2)))/d + (a^3*atan(a^6/(6*a^6 - a^6*tan(c/2 + (
d*x)/2)) + (6*a^6*tan(c/2 + (d*x)/2))/(6*a^6 - a^6*tan(c/2 + (d*x)/2))))/d
 + (a^3*tan(c/2 + (d*x)/2))/(2*d) + (3*a^3*tan(c/2 + (d*x)/2)^2 + 24*a^3*t
an(c/2 + (d*x)/2)^3 - 3*a^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*tan(c/2 + (d*x)/2
)^5 - 7*a^3*tan(c/2 + (d*x)/2)^6 - a^3 + (32*a^3*tan(c/2 + (d*x)/2))/3)/(d
*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3 + 6*tan(c/2 + (d*x)/2)^5 +
 2*tan(c/2 + (d*x)/2)^7))
```

3.34 $\int (a + a\sin(c + dx))^4 \tan^5(c + dx)\, dx$

3.34.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\begin{aligned}\int (a + a\sin(c + dx))^4 \tan^5(c + dx)\, dx = & -\frac{25a^4\log(1 - \sin(c + dx))}{d} \\ & - \frac{16a^4\sin(c + dx)}{d} - \frac{9a^4\sin^2(c + dx)}{2d} \\ & - \frac{4a^4\sin^3(c + dx)}{3d} - \frac{a^4\sin^4(c + dx)}{4d} \\ & + \frac{a^6}{d(a - a\sin(c + dx))^2} - \frac{11a^5}{d(a - a\sin(c + dx))}\end{aligned}$$

output
```
-25*a^4*ln(1-sin(d*x+c))/d-16*a^4*sin(d*x+c)/d-9/2*a^4*sin(d*x+c)^2/d-4/3*
a^4*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d+a^6/d/(a-a*sin(d*x+c))^2-11*a^5/
d/(a-a*sin(d*x+c))
```

3.34.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.64

$$\int (a + a\sin(c + dx))^4 \tan^5(c + dx)\, dx =$$

$$-\frac{a^4\left(300\log(1 - \sin(c + dx)) + \frac{120 - 132\sin(c+dx)}{(-1+\sin(c+dx))^2} + 192\sin(c + dx) + 54\sin^2(c + dx) + 16\sin^3(c + dx) + 3\right.}{12d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]
```

output
```
-1/12*(a^4*(300*Log[1 - Sin[c + d*x]] + (120 - 132*Sin[c + d*x])/(-1 + Sin
[c + d*x])^2 + 192*Sin[c + d*x] + 54*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 +
3*Sin[c + d*x]^4))/d
```

3.34.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^5(c+dx)(a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^5(a\sin(c+dx)+a)^4dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^5\sin^5(c+dx)(\sin(c+dx)a+a)}{(a-a\sin(c+dx))^3}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{2a^6}{(a-a\sin(c+dx))^3}-\frac{11a^5}{(a-a\sin(c+dx))^2}+\frac{25a^4}{a-a\sin(c+dx)}-\sin^3(c+dx)a^3-4\sin^2(c+dx)a^3-9\sin(c+dx)a^3-16a^3\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^6}{(a-a\sin(c+dx))^2}-\frac{11a^5}{a-a\sin(c+dx)}-\frac{1}{4}a^4\sin^4(c+dx)-\frac{4}{3}a^4\sin^3(c+dx)-\frac{9}{2}a^4\sin^2(c+dx)-16a^4\sin(c+dx)-25a^4}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^5,x]
```

output
```
(-25*a^4*Log[a - a*Sin[c + d*x]] - 16*a^4*Sin[c + d*x] - (9*a^4*Sin[c + d*
x]^2)/2 - (4*a^4*Sin[c + d*x]^3)/3 - (a^4*Sin[c + d*x]^4)/4 + a^6/(a - a*S
in[c + d*x])^2 - (11*a^5)/(a - a*Sin[c + d*x]))/d
```

3.34.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.34.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 26.93 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.76

method	result
risch	$25ia^4x - \frac{ia^4 e^{3i(dx+c)}}{6d} + \frac{19a^4 e^{2i(dx+c)}}{16d} + \frac{17ia^4 e^{i(dx+c)}}{2d} - \frac{17ia^4 e^{-i(dx+c)}}{2d} + \frac{19a^4 e^{-2i(dx+c)}}{16d} + \frac{ia^4 e^{-3i(dx}}{6d}$
derivativedivides	$\frac{a^4\left(\frac{\sin^{10}(dx+c)}{4\cos(dx+c)^4} - \frac{3\left(\sin^{10}(dx+c)\right)}{4\cos(dx+c)^2} - \frac{3\left(\sin^8(dx+c)\right)}{4} - \left(\sin^6(dx+c)\right) - \frac{3\left(\sin^4(dx+c)\right)}{2} - 3\left(\sin^2(dx+c)\right) - 6\ln(\cos(dx+c))\right) + 4}{}$
default	$\frac{a^4\left(\frac{\sin^{10}(dx+c)}{4\cos(dx+c)^4} - \frac{3\left(\sin^{10}(dx+c)\right)}{4\cos(dx+c)^2} - \frac{3\left(\sin^8(dx+c)\right)}{4} - \left(\sin^6(dx+c)\right) - \frac{3\left(\sin^4(dx+c)\right)}{2} - 3\left(\sin^2(dx+c)\right) - 6\ln(\cos(dx+c))\right) + 4}{}$
parts	$\frac{a^4\left(\frac{\left(\tan^4(dx+c)\right)}{4} - \frac{\left(\tan^2(dx+c)\right)}{2} + \frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d} + \frac{a^4\left(\frac{\sin^{10}(dx+c)}{4\cos(dx+c)^4} - \frac{3\left(\sin^{10}(dx+c)\right)}{4\cos(dx+c)^2} - \frac{3\left(\sin^8(dx+c)\right)}{4} - \left(\sin^6(dx-\right.\right.}{d}$

input
```
int((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x,method=_RETURNVERBOSE)
```

output
```
25*I*a^4*x-1/6*I/d*a^4*exp(3*I*(d*x+c))+19/16*a^4/d*exp(2*I*(d*x+c))+17/2*
I/d*a^4*exp(I*(d*x+c))-17/2*I/d*a^4*exp(-I*(d*x+c))+19/16*a^4/d*exp(-2*I*(
d*x+c))+1/6*I/d*a^4*exp(-3*I*(d*x+c))+50*I/d*a^4*c+2*I*(-11*exp(I*(d*x+c))
*a^4-20*I*a^4*exp(2*I*(d*x+c))+11*a^4*exp(3*I*(d*x+c)))/(-I+exp(I*(d*x+c))
)^4/d-50/d*a^4*ln(-I+exp(I*(d*x+c)))-1/32/d*a^4*cos(4*d*x+4*c)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.19

$$\int (a + a\sin(c + dx))^4 \tan^5(c + dx)\, dx =$$
$$-\frac{24\, a^4 \cos(dx + c)^6 - 272\, a^4 \cos(dx + c)^4 - 2393\, a^4 \cos(dx + c)^2 + 1906\, a^4 + 2400\, \left(a^4 \cos(dx + c)^2 + 2\right.}{96\, \left(d \cos(dx + \right.}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="fricas")
```

output
```
-1/96*(24*a^4*cos(d*x + c)^6 - 272*a^4*cos(d*x + c)^4 - 2393*a^4*cos(d*x +
 c)^2 + 1906*a^4 + 2400*(a^4*cos(d*x + c)^2 + 2*a^4*sin(d*x + c) - 2*a^4)*
log(-sin(d*x + c) + 1) - 10*(8*a^4*cos(d*x + c)^4 - 96*a^4*cos(d*x + c)^2
+ 181*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2 + 2*d*sin(d*x + c) - 2*d)
```

3.34.6 Sympy [F]

$$\int (a+a\sin(c+dx))^4\tan^5(c+dx)\,dx = a^4\left(\int 4\sin(c+dx)\tan^5(c+dx)\,dx + \int 6\sin^2(c+dx)\tan^5(c+dx)\,dx + \int 4\sin^3(c+dx)\tan^5(c+dx)\,dx + \int \sin^4(c+dx)\tan^5(c+dx)\,dx + \int \tan^5(c+dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**5,x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**5, x) + Integral(6*sin(c + d*x
)**2*tan(c + d*x)**5, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**5, x)
+ Integral(sin(c + d*x)**4*tan(c + d*x)**5, x) + Integral(tan(c + d*x)**5,
 x))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.84

$$\int (a+a\sin(c+dx))^4\tan^5(c+dx)\,dx = -\frac{3\,a^4\sin(dx+c)^4 + 16\,a^4\sin(dx+c)^3 + 54\,a^4\sin(dx+c)^2 + 300\,a^4\log(\sin(dx+c)-1) + 192\,a^4\sin(}{12\,d}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="maxima")
```

output
```
-1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 54*a^4*sin(d*x + c)^
2 + 300*a^4*log(sin(d*x + c) - 1) + 192*a^4*sin(d*x + c) - 12*(11*a^4*sin(
d*x + c) - 10*a^4)/(sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/d
```

3.34.8 Giac [F(-1)]

Timed out.

$$\int (a+a\sin(c+dx))^4\tan^5(c+dx)\,dx=\text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^5,x, algorithm="giac")
```

output
```
Timed out
```

3.34.9 Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 379, normalized size of antiderivative = 2.94

$$\int (a+a\sin(c+dx))^4\tan^5(c+dx)\,dx = \frac{25\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}-\frac{50\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}-\frac{50\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-150\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{950\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{3}-\frac{1700\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+\frac{2180\,a^4\tan\left(\frac{c}{2}\right.}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-20\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-40\right.}$$

input
```
int(tan(c + d*x)^5*(a + a*sin(c + d*x))^4,x)
```

output
```
(25*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (50*a^4*log(tan(c/2 + (d*x)/2)
- 1))/d - ((950*a^4*tan(c/2 + (d*x)/2)^3)/3 - 150*a^4*tan(c/2 + (d*x)/2)^2
 - (1700*a^4*tan(c/2 + (d*x)/2)^4)/3 + (2180*a^4*tan(c/2 + (d*x)/2)^5)/3 -
 (2452*a^4*tan(c/2 + (d*x)/2)^6)/3 + (2180*a^4*tan(c/2 + (d*x)/2)^7)/3 - (
1700*a^4*tan(c/2 + (d*x)/2)^8)/3 + (950*a^4*tan(c/2 + (d*x)/2)^9)/3 - 150*
a^4*tan(c/2 + (d*x)/2)^10 + 50*a^4*tan(c/2 + (d*x)/2)^11 + 50*a^4*tan(c/2
+ (d*x)/2))/(d*(10*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2) - 20*tan(c/
2 + (d*x)/2)^3 + 31*tan(c/2 + (d*x)/2)^4 - 40*tan(c/2 + (d*x)/2)^5 + 44*ta
n(c/2 + (d*x)/2)^6 - 40*tan(c/2 + (d*x)/2)^7 + 31*tan(c/2 + (d*x)/2)^8 - 2
0*tan(c/2 + (d*x)/2)^9 + 10*tan(c/2 + (d*x)/2)^10 - 4*tan(c/2 + (d*x)/2)^1
1 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.35 $\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx$

3.35.1 Optimal result

Integrand size = 21, antiderivative size = 107

$$\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx=\frac{16a^4\log(1-\sin(c+dx))}{d}+\frac{12a^4\sin(c+dx)}{d}+\frac{4a^4\sin^2(c+dx)}{d}+\frac{4a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^4(c+dx)}{4d}+\frac{4a^5}{d(a-a\sin(c+dx))}$$

output
```
16*a^4*ln(1-sin(d*x+c))/d+12*a^4*sin(d*x+c)/d+4*a^4*sin(d*x+c)^2/d+4/3*a^4
*sin(d*x+c)^3/d+1/4*a^4*sin(d*x+c)^4/d+4*a^5/d/(a-a*sin(d*x+c))
```

3.35.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.71

$$\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx=\frac{a^4\left(192\log(1-\sin(c+dx))+\frac{48}{1-\sin(c+dx)}+144\sin(c+dx)+48\sin^2(c+dx)+16\sin^3(c+dx)+3\sin^4(c$$

input
```
Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]
```

output
```
(a^4*(192*Log[1 - Sin[c + d*x]] + 48/(1 - Sin[c + d*x]) + 144*Sin[c + d*x]
 + 48*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/(12*d)
```

3.35.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3(a\sin(c+dx)+a)^4dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^3\sin^3(c+dx)(\sin(c+dx)a+a)^2}{(a-a\sin(c+dx))^2}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{4a^5}{(a-a\sin(c+dx))^2}-\frac{16a^4}{a-a\sin(c+dx)}+\sin^3(c+dx)a^3+4\sin^2(c+dx)a^3+8\sin(c+dx)a^3+12a^3\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{4a^5}{a-a\sin(c+dx)}+\frac{1}{4}a^4\sin^4(c+dx)+\frac{4}{3}a^4\sin^3(c+dx)+4a^4\sin^2(c+dx)+12a^4\sin(c+dx)+16a^4\log(a-a\sin(c+}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^3,x]
```

output
```
(16*a^4*Log[a - a*Sin[c + d*x]] + 12*a^4*Sin[c + d*x] + 4*a^4*Sin[c + d*x]
^2 + (4*a^4*Sin[c + d*x]^3)/3 + (a^4*Sin[c + d*x]^4)/4 + (4*a^5)/(a - a*Si
n[c + d*x]))/d
```

3.35.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.35.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 14.79 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.49

method	result
risch	$-16ia^4x-\frac{13ia^4\mathrm{e}^{i(dx+c)}}{2d}+\frac{13ia^4\mathrm{e}^{-i(dx+c)}}{2d}-\frac{32ia^4c}{d}-\frac{8ia^4\mathrm{e}^{i(dx+c)}}{\left(-i+\mathrm{e}^{i(dx+c)}\right)^2d}+\frac{32a^4\ln\left(-i+\mathrm{e}^{i(dx+c)}\right)}{d}+\frac{a^4\mathrm{co}}{}$
derivativedivides	$a^4\left(\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^6(dx+c)\right)}{2}+\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\ln(\cos(dx+c))\right)+4a^4\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\right.$
default	$a^4\left(\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^6(dx+c)\right)}{2}+\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\ln(\cos(dx+c))\right)+4a^4\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\right.$
parts	$\frac{a^4\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{a^4\left(\frac{\sin^8(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^6(dx+c)\right)}{2}+\frac{3\left(\sin^4(dx+c)\right)}{4}+\frac{3\left(\sin^2(dx+c)\right)}{2}+3\ln(\cos(d\right.}{d}$

input
```
int((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-16*I*a^4*x-13/2*I/d*a^4*exp(I*(d*x+c))+13/2*I/d*a^4*exp(-I*(d*x+c))-32*I/
d*a^4*c-8*I*a^4*exp(I*(d*x+c))/(-I+exp(I*(d*x+c)))^2/d+32/d*a^4*ln(-I+exp(
I*(d*x+c)))+1/32/d*a^4*cos(4*d*x+4*c)-1/3*a^4/d*sin(3*d*x+3*c)-17/8/d*a^4*
cos(2*d*x+2*c)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.08

$$\int (a + a\sin(c + dx))^4 \tan^3(c + dx)\,dx$$
$$= \frac{104\,a^4\cos(dx + c)^4 - 976\,a^4\cos(dx + c)^2 + 689\,a^4 + 1536\,(a^4\sin(dx + c) - a^4)\log(-\sin(dx + c) + 1) -}{96\,(d\sin(dx + c) - d)}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/96*(104*a^4*cos(d*x + c)^4 - 976*a^4*cos(d*x + c)^2 + 689*a^4 + 1536*(a^
4*sin(d*x + c) - a^4)*log(-sin(d*x + c) + 1) + (24*a^4*cos(d*x + c)^4 - 30
4*a^4*cos(d*x + c)^2 - 1073*a^4)*sin(d*x + c))/(d*sin(d*x + c) - d)
```

3.35.6 Sympy [F]

$$\int (a + a\sin(c + dx))^4 \tan^3(c + dx)\,dx = a^4\left(\int 4\sin(c + dx)\tan^3(c + dx)\,dx + \int 6\sin^2(c + dx)\tan^3(c + dx)\,dx + \int 4\sin^3(c + dx)\tan^3(c + dx)\,dx + \int \sin^4(c + dx)\tan^3(c + dx)\,dx + \int \tan^3(c + dx)\,dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**3,x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**3, x) + Integral(6*sin(c + d*x
)**2*tan(c + d*x)**3, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**3, x)
+ Integral(sin(c + d*x)**4*tan(c + d*x)**3, x) + Integral(tan(c + d*x)**3,
 x))
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.79

$$\int (a + a\sin(c + dx))^4 \tan^3(c + dx)\, dx$$
$$= \frac{3\,a^4 \sin\left(dx + c\right)^4 + 16\,a^4 \sin\left(dx + c\right)^3 + 48\,a^4 \sin\left(dx + c\right)^2 + 192\,a^4 \log\left(\sin\left(dx + c\right) - 1\right) + 144\,a^4 \sin\left(dx\right.}{12\,d}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 48*a^4*sin(d*x + c)^2
 + 192*a^4*log(sin(d*x + c) - 1) + 144*a^4*sin(d*x + c) - 48*a^4/(sin(d*x
+ c) - 1))/d
```

3.35.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^4 \tan^3(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.35.9 Mupad [B] (verification not implemented)

Time = 7.07 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.99

$$\int (a+a\sin(c+dx))^4\tan^3(c+dx)\,dx = \frac{32\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d}$$
$$+\frac{32\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-32\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\frac{320\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{340\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{424\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-12\tan\right.}$$
$$-\frac{16\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + a*sin(c + d*x))^4,x)
```

output
```
(32*a^4*log(tan(c/2 + (d*x)/2) - 1))/d + ((320*a^4*tan(c/2 + (d*x)/2)^3)/3
 - 32*a^4*tan(c/2 + (d*x)/2)^2 - (340*a^4*tan(c/2 + (d*x)/2)^4)/3 + (424*a
^4*tan(c/2 + (d*x)/2)^5)/3 - (340*a^4*tan(c/2 + (d*x)/2)^6)/3 + (320*a^4*t
an(c/2 + (d*x)/2)^7)/3 - 32*a^4*tan(c/2 + (d*x)/2)^8 + 32*a^4*tan(c/2 + (d
*x)/2)^9 + 32*a^4*tan(c/2 + (d*x)/2))/(d*(5*tan(c/2 + (d*x)/2)^2 - 2*tan(c
/2 + (d*x)/2) - 8*tan(c/2 + (d*x)/2)^3 + 10*tan(c/2 + (d*x)/2)^4 - 12*tan(
c/2 + (d*x)/2)^5 + 10*tan(c/2 + (d*x)/2)^6 - 8*tan(c/2 + (d*x)/2)^7 + 5*ta
n(c/2 + (d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^9 + tan(c/2 + (d*x)/2)^10 + 1))
- (16*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.36 $\int (a + a\sin(c + dx))^4 \tan(c + dx)\,dx$

3.36.1 Optimal result

Integrand size = 19, antiderivative size = 88

$$\int (a + a\sin(c + dx))^4 \tan(c + dx)\,dx = -\frac{8a^4\log(1 - \sin(c + dx))}{d} - \frac{8a^4\sin(c + dx)}{d} - \frac{7a^4\sin^2(c + dx)}{2d} - \frac{4a^4\sin^3(c + dx)}{3d} - \frac{a^4\sin^4(c + dx)}{4d}$$

output
```
-8*a^4*ln(1-sin(d*x+c))/d-8*a^4*sin(d*x+c)/d-7/2*a^4*sin(d*x+c)^2/d-4/3*a^
4*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d
```

3.36.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int (a + a\sin(c + dx))^4 \tan(c + dx)\,dx = -\frac{a^4(96\log(1 - \sin(c + dx)) + 96\sin(c + dx) + 42\sin^2(c + dx) + 16\sin^3(c + dx) + 3\sin^4(c + dx))}{12d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]
```

output
```
-1/12*(a^4*(96*Log[1 - Sin[c + d*x]] + 96*Sin[c + d*x] + 42*Sin[c + d*x]^2
 + 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/d
```

3.36.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a\sin(c+dx)+a)^4dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a\sin(c+dx)(\sin(c+dx)a+a)^3}{a-a\sin(c+dx)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{8a^4}{a-a\sin(c+dx)}-\sin^3(c+dx)a^3-4\sin^2(c+dx)a^3-7\sin(c+dx)a^3-8a^3\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{4}a^4\sin^4(c+dx)-\frac{4}{3}a^4\sin^3(c+dx)-\frac{7}{2}a^4\sin^2(c+dx)-8a^4\sin(c+dx)-8a^4\log(a-a\sin(c+dx))}{d}$$

input
```
Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x],x]
```

output
```
(-8*a^4*Log[a - a*Sin[c + d*x]] - 8*a^4*Sin[c + d*x] - (7*a^4*Sin[c + d*x]
^2)/2 - (4*a^4*Sin[c + d*x]^3)/3 - (a^4*Sin[c + d*x]^4)/4)/d
```

3.36.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.36.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 7.56 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.44

method	result
risch	$8ia^4x+\frac{9ia^4 e^{i(dx+c)}}{2d}-\frac{9ia^4 e^{-i(dx+c)}}{2d}+\frac{16ia^4c}{d}-\frac{16a^4\ln\left(-i+e^{i(dx+c)}\right)}{d}-\frac{a^4\cos(4dx+4c)}{32d}+\frac{a^4\sin(3dx+3}{3d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+4a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6}{d}$
default	$\frac{a^4\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+4a^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)+6}{d}$
parts	$\frac{a^4\ln\left(1+\tan^2(dx+c)\right)}{2d}+\frac{a^4\left(-\frac{\left(\sin^4(dx+c)\right)}{4}-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}+\frac{4a^4(-\sin(dx+c)+\ln(\sec(dx+c)+\tan}{d}$

input
```
int((a+a*sin(d*x+c))^4*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
8*I*a^4*x+9/2*I/d*a^4*exp(I*(d*x+c))-9/2*I/d*a^4*exp(-I*(d*x+c))+16*I/d*a^
4*c-16/d*a^4*ln(-I+exp(I*(d*x+c)))-1/32/d*a^4*cos(4*d*x+4*c)+1/3*a^4/d*sin
(3*d*x+3*c)+15/8/d*a^4*cos(2*d*x+2*c)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int (a+a\sin(c+dx))^4\tan(c+dx)\,dx = -\frac{3\,a^4\cos\left(dx+c\right)^4-48\,a^4\cos\left(dx+c\right)^2+96\,a^4\log\left(-\sin\left(dx+c\right)+1\right)-16\,\left(a^4\cos\left(dx+c\right)^2-7\,a^4\right)\sin}{12\,d}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="fricas")
```

output
```
-1/12*(3*a^4*cos(d*x + c)^4 - 48*a^4*cos(d*x + c)^2 + 96*a^4*log(-sin(d*x
+ c) + 1) - 16*(a^4*cos(d*x + c)^2 - 7*a^4)*sin(d*x + c))/d
```

3.36.6 Sympy [F]

$$\int (a+a\sin(c+dx))^4\tan(c+dx)\,dx = a^4\left(\int 4\sin\left(c+dx\right)\tan\left(c+dx\right)dx + \int 6\sin^2\left(c+dx\right)\tan\left(c+dx\right)dx + \int 4\sin^3\left(c+dx\right)\tan\left(c+dx\right)dx + \int \sin^4\left(c+dx\right)\tan\left(c+dx\right)dx + \int \tan\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**4*tan(d*x+c),x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*tan(c + d*x), x) + Integral(6*sin(c + d*x)**
2*tan(c + d*x), x) + Integral(4*sin(c + d*x)**3*tan(c + d*x), x) + Integra
l(sin(c + d*x)**4*tan(c + d*x), x) + Integral(tan(c + d*x), x))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int (a + a\sin(c + dx))^4 \tan(c + dx)\, dx =$$
$$-\frac{3\,a^4 \sin\left(dx + c\right)^4 + 16\,a^4 \sin\left(dx + c\right)^3 + 42\,a^4 \sin\left(dx + c\right)^2 + 96\,a^4 \log\left(\sin\left(dx + c\right) - 1\right) + 96\,a^4 \sin\left(dx\right.}{12\,d}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 42*a^4*sin(d*x + c)^
2 + 96*a^4*log(sin(d*x + c) - 1) + 96*a^4*sin(d*x + c))/d
```

3.36.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57842 vs. 2(82) = 164.

Time = 12.72 (sec) , antiderivative size = 57842, normalized size of antiderivative = 657.30

$$\int (a + a\sin(c + dx))^4 \tan(c + dx)\, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c),x, algorithm="giac")
```

output
```
-1/96*(384*a^4*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1
/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)
^2 + tan(1/2*c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 -
384*a^4*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) -
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d
*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 384*a^4
*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2
+ tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c
)^4 - 177*a^4*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 768*a^4*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2
*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 768*a^4*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c
) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*
tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 768*a^4*log(4*(tan(d*x)^
2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +...
```

3.36.9 Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.49

$$\int (a+a\sin(c+dx))^4\tan(c+dx)\,dx = \frac{8\,a^4\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{d} - \frac{28\,a^4\,\sin\left(c+d\,x\right)}{3\,d} - \frac{16\,a^4\,\ln\left(\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} + \frac{4\,a^4\cos\left(c+d\,x\right)^2}{d} - \frac{a^4\cos\left(c+d\,x\right)^4}{4\,d} + \frac{4\,a^4\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}$$

input
```
int(tan(c + d*x)*(a + a*sin(c + d*x))^4,x)
```

output
```
(8*a^4*log(1/cos(c/2 + (d*x)/2)^2))/d - (28*a^4*sin(c + d*x))/(3*d) - (16*
a^4*log((cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))/cos(c/2 + (d*x)/2)))/d +
 (4*a^4*cos(c + d*x)^2)/d - (a^4*cos(c + d*x)^4)/(4*d) + (4*a^4*cos(c + d*
x)^2*sin(c + d*x))/(3*d)
```

3.36. $\int (a + a\sin(c + dx))^4 \tan(c + dx)\,dx$

3.37 $\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx$

3.37.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{4a^4\csc(c+dx)}{d}-\frac{a^4\csc^2(c+dx)}{2d}+\frac{5a^4\log(\sin(c+dx))}{d}-\frac{5a^4\sin^2(c+dx)}{2d}-\frac{4a^4\sin^3(c+dx)}{3d}-\frac{a^4\sin^4(c+dx)}{4d}$$

output
```
-4*a^4*csc(d*x+c)/d-1/2*a^4*csc(d*x+c)^2/d+5*a^4*ln(sin(d*x+c))/d-5/2*a^4*
sin(d*x+c)^2/d-4/3*a^4*sin(d*x+c)^3/d-1/4*a^4*sin(d*x+c)^4/d
```

3.37.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.67

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{a^4(48\csc(c+dx)+6\csc^2(c+dx)-60\log(\sin(c+dx))+30\sin^2(c+dx)+16\sin^3(c+dx)+3\sin^4(c}{12d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

output
```
-1/12*(a^4*(48*Csc[c + d*x] + 6*Csc[c + d*x]^2 - 60*Log[Sin[c + d*x]] + 30
*Sin[c + d*x]^2 + 16*Sin[c + d*x]^3 + 3*Sin[c + d*x]^4))/d
```

3.37.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a\sin(c+dx)+a)^4 \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(c+dx)+a)^4}{\tan(c+dx)^3} dx$$

$$\Big\downarrow \; 3186$$

$$\frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))(\sin(c+dx)a+a)^5}{a^3} d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 84$$

$$\frac{\int \left(\csc^3(c+dx)a^3-\sin^3(c+dx)a^3+4\csc^2(c+dx)a^3-4\sin^2(c+dx)a^3+5\csc(c+dx)a^3-5\sin(c+dx)a^3\right) d(a}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{4}a^4\sin^4(c+dx)-\frac{4}{3}a^4\sin^3(c+dx)-\frac{5}{2}a^4\sin^2(c+dx)-\frac{1}{2}a^4\csc^2(c+dx)-4a^4\csc(c+dx)+5a^4\log(a\sin(c+}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + a*Sin[c + d*x])^4,x]
```

output
```
(-4*a^4*Csc[c + d*x] - (a^4*Csc[c + d*x]^2)/2 + 5*a^4*Log[a*Sin[c + d*x]]
- (5*a^4*Sin[c + d*x]^2)/2 - (4*a^4*Sin[c + d*x]^3)/3 - (a^4*Sin[c + d*x]^
4)/4)/d
```

3.37.3.1 Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.37.4 Maple [A] (verified)

Time = 6.43 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{-\frac{a^4\left(\cos^4(dx+c)\right)}{4}+\frac{4a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^4\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+4a^4\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx\right.}{d}$
default	$\frac{-\frac{a^4\left(\cos^4(dx+c)\right)}{4}+\frac{4a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+6a^4\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+4a^4\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx\right.}{d}$
risch	$-5ia^4x-\frac{ia^4e^{3i(dx+c)}}{6d}+\frac{11a^4e^{2i(dx+c)}}{16d}+\frac{ia^4e^{i(dx+c)}}{2d}-\frac{ia^4e^{-i(dx+c)}}{2d}+\frac{11a^4e^{-2i(dx+c)}}{16d}+\frac{ia^4e^{-3i(dx+c)}}{6d}$

input
```
int(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*a^4*cos(d*x+c)^4+4/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+6*a^4*(1/2*
cos(d*x+c)^2+ln(sin(d*x+c)))+4*a^4*(-1/sin(d*x+c)*cos(d*x+c)^4-(2+cos(d*x+
c)^2)*sin(d*x+c))+a^4*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c))))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.28

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx =$$

$$-\frac{24\,a^4\cos\left(dx+c\right)^6-312\,a^4\cos\left(dx+c\right)^4+423\,a^4\cos\left(dx+c\right)^2-183\,a^4-480\,\left(a^4\cos\left(dx+c\right)^2-a^4\right)\mathrm{l}}{96\,\left(d\cos\left(dx+c\right)^2-c\right.}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/96*(24*a^4*cos(d*x + c)^6 - 312*a^4*cos(d*x + c)^4 + 423*a^4*cos(d*x +
c)^2 - 183*a^4 - 480*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*sin(d*x + c)) - 12
8*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + 4*a^4)*sin(d*x + c))/(d*cos
(d*x + c)^2 - d)
```

3.37.6 Sympy [F]

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx = a^4\left(\int 4\sin\left(c+dx\right)\cot^3\left(c+dx\right)dx + \int 6\sin^2\left(c+dx\right)\cot^3\left(c+dx\right)dx + \int 4\sin^3\left(c+dx\right)\cot^3\left(c+dx\right)dx + \int \sin^4\left(c+dx\right)\cot^3\left(c+dx\right)dx + \int \cot^3\left(c+dx\right)dx\right)$$

input
```
integrate(cot(d*x+c)**3*(a+a*sin(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**3, x) + Integral(6*sin(c + d*x
)**2*cot(c + d*x)**3, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**3, x)
+ Integral(sin(c + d*x)**4*cot(c + d*x)**3, x) + Integral(cot(c + d*x)**3,
 x))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx =$$
$$-\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\sin\left(dx+c\right)\right)+\frac{6\,\left(8\,a^4\sin(dx+c)+a^4\right.}{\sin(dx+c)^2}}{12\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^
2 - 60*a^4*log(sin(d*x + c)) + 6*(8*a^4*sin(d*x + c) + a^4)/sin(d*x + c)^2
)/d
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx =$$
$$-\frac{3\,a^4\sin\left(dx+c\right)^4+16\,a^4\sin\left(dx+c\right)^3+30\,a^4\sin\left(dx+c\right)^2-60\,a^4\log\left(\left|\sin\left(dx+c\right)\right|\right)+\frac{6\,\left(15\,a^4\sin(dx+c)^2\right.}{\sin(\ldots}}{12\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/12*(3*a^4*sin(d*x + c)^4 + 16*a^4*sin(d*x + c)^3 + 30*a^4*sin(d*x + c)^
2 - 60*a^4*log(abs(sin(d*x + c))) + 6*(15*a^4*sin(d*x + c)^2 + 8*a^4*sin(d
*x + c) + a^4)/sin(d*x + c)^2)/d
```

3.37.9 Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.92

$$\int \cot^3(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{5\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d} - \frac{8\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 + \frac{81\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{2} + \frac{224\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3} + 98\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \frac{272\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3} + 43\,a^4 t}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10} + 16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 24\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \right.} - \frac{2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d} - \frac{5\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + a*sin(c + d*x))^4,x)
```

output
```
(5*a^4*log(tan(c/2 + (d*x)/2)))/d - (a^4*tan(c/2 + (d*x)/2)^2)/(8*d) - (2*
a^4*tan(c/2 + (d*x)/2)^2 + 32*a^4*tan(c/2 + (d*x)/2)^3 + 43*a^4*tan(c/2 +
(d*x)/2)^4 + (272*a^4*tan(c/2 + (d*x)/2)^5)/3 + 98*a^4*tan(c/2 + (d*x)/2)^
6 + (224*a^4*tan(c/2 + (d*x)/2)^7)/3 + (81*a^4*tan(c/2 + (d*x)/2)^8)/2 + 8
*a^4*tan(c/2 + (d*x)/2)^9 + a^4/2 + 8*a^4*tan(c/2 + (d*x)/2))/(d*(4*tan(c/
2 + (d*x)/2)^2 + 16*tan(c/2 + (d*x)/2)^4 + 24*tan(c/2 + (d*x)/2)^6 + 16*ta
n(c/2 + (d*x)/2)^8 + 4*tan(c/2 + (d*x)/2)^10)) - (2*a^4*tan(c/2 + (d*x)/2)
)/d - (5*a^4*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.38 $\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx$

3.38.1 Optimal result

Integrand size = 21, antiderivative size = 143

$$\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx = \frac{163a^4x}{8}-\frac{16a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}+\frac{4a^4\cos(c+dx)}{3d(1-\sin(c+dx))^2}-\frac{56a^4\cos(c+dx)}{3d(1-\sin(c+dx))}-\frac{35a^4\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d}$$

output
```
163/8*a^4*x-16*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)^3/d+4/3*a^4*cos(d*x+c)/
d/(1-sin(d*x+c))^2-56/3*a^4*cos(d*x+c)/d/(1-sin(d*x+c))-35/8*a^4*cos(d*x+c
)*sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

3.38.2 Mathematica [A] (verified)

Time = 6.72 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.76

$$\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx = \frac{a^4\left(24(209+489c+489dx)\cos\left(\frac{1}{2}(c+dx)\right)-24(453+163c+163dx)\cos\left(\frac{3}{2}(c+dx)\right)+885\cos\left(\frac{5}{2}(c+\right.\right.}{}$$

input
```
Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

output
```
(a^4*(24*(209 + 489*c + 489*d*x)*Cos[(c + d*x)/2] - 24*(453 + 163*c + 163*
d*x)*Cos[(3*(c + d*x))/2] + 885*Cos[(5*(c + d*x))/2] - 129*Cos[(7*(c + d*x
))/2] - 23*Cos[(9*(c + d*x))/2] + 3*Cos[(11*(c + d*x))/2] - 16488*Sin[(c +
 d*x)/2] - 11736*c*Sin[(c + d*x)/2] - 11736*d*x*Sin[(c + d*x)/2] + 3704*Si
n[(3*(c + d*x))/2] - 3912*c*Sin[(3*(c + d*x))/2] - 3912*d*x*Sin[(3*(c + d*
x))/2] + 885*Sin[(5*(c + d*x))/2] + 129*Sin[(7*(c + d*x))/2] - 23*Sin[(9*(
c + d*x))/2] - 3*Sin[(11*(c + d*x))/2]))/(384*d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^3)
```

3.38.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^4(a\sin(c+dx)+a)^4dx$$

$$\Big\downarrow \text{3188}$$

$$a^4\int \left(\sin^4(c+dx)+4\sin^3(c+dx)+8\sin^2(c+dx)+12\sin(c+dx)-\frac{20}{1-\sin(c+dx)}+\frac{4}{(1-\sin(c+dx))^2}+1\right.$$

$$\Big\downarrow \text{2009}$$

$$a^4\left(\frac{4\cos^3(c+dx)}{3d}-\frac{16\cos(c+dx)}{d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{35\sin(c+dx)\cos(c+dx)}{8d}-\frac{56\cos(c+dx)}{3d(1-\sin(c+dx\right.$$

input
```
Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^4,x]
```

output
```
a^4*((163*x)/8 - (16*Cos[c + d*x])/d + (4*Cos[c + d*x]^3)/(3*d) + (4*Cos[c
 + d*x])/(3*d*(1 - Sin[c + d*x])^2) - (56*Cos[c + d*x])/(3*d*(1 - Sin[c +
d*x])) - (35*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (Cos[c + d*x]*Sin[c + d*x]
^3)/(4*d))
```

3.38.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.38.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 21.03 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.16

method	result
risch	$\frac{163a^4x}{8}+\frac{9ia^4\mathrm{e}^{2i(dx+c)}}{8d}-\frac{15a^4\mathrm{e}^{i(dx+c)}}{2d}-\frac{15a^4\mathrm{e}^{-i(dx+c)}}{2d}-\frac{9ia^4\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{8\left(-27ia^4\mathrm{e}^{i(dx+c)}+15a^4\mathrm{e}^{2i(dx+}\right.}{3\left(-i+\mathrm{e}^{i(dx+c)}\right)^3d}$
derivativedivides	$$a^4\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3}-\frac{2\left(\sin^9(dx+c)\right)}{\cos(dx+c)}-2\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right.$
default	$$a^4\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3}-\frac{2\left(\sin^9(dx+c)\right)}{\cos(dx+c)}-2\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+c)\right)}{6}+\frac{35\left(\sin^3(dx+c)\right)}{24}+\frac{35\sin(dx+c)}{16}\right)\cos(dx+c)+\frac{35dx}{8}+\frac{35c}{8}\right.$
parts	$\frac{a^4\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{a^4\left(\frac{\sin^9(dx+c)}{3\cos(dx+c)^3}-\frac{2\left(\sin^9(dx+c)\right)}{\cos(dx+c)}-2\left(\sin^7(dx+c)+\frac{7\left(\sin^5(dx+\right.\right.}{6}\right.\right.}{d}$

input
```
int((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
163/8*a^4*x+9/8*I/d*a^4*exp(2*I*(d*x+c))-15/2/d*a^4*exp(I*(d*x+c))-15/2*a^
4/d*exp(-I*(d*x+c))-9/8*I/d*a^4*exp(-2*I*(d*x+c))-8/3*(-27*I*a^4*exp(I*(d*
x+c))+15*a^4*exp(2*I*(d*x+c))-14*a^4)/(-I+exp(I*(d*x+c)))^3/d+1/32/d*a^4*s
in(4*d*x+4*c)+1/3*a^4/d*cos(3*d*x+3*c)
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.73

$$\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx =$$
$$-\frac{6\,a^4\cos\left(dx+c\right)^6-20\,a^4\cos\left(dx+c\right)^5-85\,a^4\cos\left(dx+c\right)^4+214\,a^4\cos\left(dx+c\right)^3+978\,a^4dx+32\,a^4-}{}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/24*(6*a^4*cos(d*x + c)^6 - 20*a^4*cos(d*x + c)^5 - 85*a^4*cos(d*x + c)^
4 + 214*a^4*cos(d*x + c)^3 + 978*a^4*d*x + 32*a^4 - (489*a^4*d*x + 721*a^4
)*cos(d*x + c)^2 + (489*a^4*d*x - 962*a^4)*cos(d*x + c) - (6*a^4*cos(d*x +
 c)^5 + 26*a^4*cos(d*x + c)^4 - 59*a^4*cos(d*x + c)^3 + 978*a^4*d*x - 273*
a^4*cos(d*x + c)^2 - 32*a^4 + (489*a^4*d*x - 994*a^4)*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^2 - d*cos(d*x + c) + (d*cos(d*x + c) + 2*d)*sin(d*
x + c) - 2*d)
```

3.38.6 Sympy [F]

$$\int (a+a\sin(c+dx))^4\tan^4(c+dx)\,dx = a^4\left(\int 4\sin\left(c+dx\right)\tan^4\left(c+dx\right)dx + \int 6\sin^2\left(c+dx\right)\tan^4\left(c+dx\right)dx + \int 4\sin^3\left(c+dx\right)\tan^4\left(c+dx\right)dx + \int \sin^4\left(c+dx\right)\tan^4\left(c+dx\right)dx + \int \tan^4\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**4,x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**4, x) + Integral(6*sin(c + d*x
)**2*tan(c + d*x)**4, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**4, x)
+ Integral(sin(c + d*x)**4*tan(c + d*x)**4, x) + Integral(tan(c + d*x)**4,
 x))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.66

$$\int (a + a\sin(c + dx))^4 \tan^4(c + dx)\, dx$$

$$= \frac{32\left(\cos(dx+c)^3 - \frac{9\cos(dx+c)^2-1}{\cos(dx+c)^3} - 9\cos(dx+c)\right)a^4 + \left(8\tan(dx+c)^3 + 105\,dx + 105\,c - \frac{3\left(13\tan(dx+c)\right.}{\tan(dx+c)^4+2}\right.}{}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/24*(32*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 - 9*cos(d
*x + c))*a^4 + (8*tan(d*x + c)^3 + 105*d*x + 105*c - 3*(13*tan(d*x + c)^3
+ 11*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1) - 72*tan(d*x +
c))*a^4 + 24*(2*tan(d*x + c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x +
 c)^2 + 1) - 12*tan(d*x + c))*a^4 + 8*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*ta
n(d*x + c))*a^4 - 32*a^4*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*
x + c)))/d
```

3.38.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^4 \tan^4(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.38.9 Mupad [B] (verification not implemented)

Time = 10.72 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.06

$$\int (a + a\sin(c+dx))^4 \tan^4(c+dx)\,dx = \frac{163\,a^4\,x}{8} + \frac{\frac{163\,a^4\,(c+d\,x)}{8} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{489\,a^4\,(c+d\,x)}{8} - \frac{a^4\,(1467\,c+1467\,d\,x-3630)}{24}\right) - \frac{a^4\,(489\,c+489\,d\,x-1536)}{24} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{}$$

input
```
int(tan(c + d*x)^4*(a + a*sin(c + d*x))^4,x)
```

output
```
(163*a^4*x)/8 + ((163*a^4*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((489*a^4*(c +
 d*x))/8 - (a^4*(1467*c + 1467*d*x - 3630))/24) - (a^4*(489*c + 489*d*x -
1536))/24 + tan(c/2 + (d*x)/2)^10*((489*a^4*(c + d*x))/8 - (a^4*(1467*c +
1467*d*x - 978))/24) - tan(c/2 + (d*x)/2)^9*((1141*a^4*(c + d*x))/8 - (a^4
*(3423*c + 3423*d*x - 2934))/24) + tan(c/2 + (d*x)/2)^2*((1141*a^4*(c + d*
x))/8 - (a^4*(3423*c + 3423*d*x - 7818))/24) + tan(c/2 + (d*x)/2)^8*((2119
*a^4*(c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 6520))/24) - tan(c/2 + (d*x)
/2)^3*((2119*a^4*(c + d*x))/8 - (a^4*(6357*c + 6357*d*x - 13448))/24) - ta
n(c/2 + (d*x)/2)^7*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8802*d*x - 117
36))/24) + tan(c/2 + (d*x)/2)^4*((1467*a^4*(c + d*x))/4 - (a^4*(8802*c + 8
802*d*x - 15912))/24) + tan(c/2 + (d*x)/2)^6*((1793*a^4*(c + d*x))/4 - (a^
4*(10758*c + 10758*d*x - 15364))/24) - tan(c/2 + (d*x)/2)^5*((1793*a^4*(c
+ d*x))/4 - (a^4*(10758*c + 10758*d*x - 18428))/24))/(d*(tan(c/2 + (d*x)/2
) - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.39 $\int (a+a\sin(c+dx))^4\tan^2(c+dx)\,dx$

3.39.1 Optimal result

Integrand size = 21, antiderivative size = 113

$$\int (a+a\sin(c+dx))^4\tan^2(c+dx)\,dx = -\frac{95a^4x}{8}+\frac{12a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}+\frac{8a^4\cos(c+dx)}{d(1-\sin(c+dx))}+\frac{31a^4\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d}$$

output
```
-95/8*a^4*x+12*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x+c)^3/d+8*a^4*cos(d*x+c)/d/
(1-sin(d*x+c))+31/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*sin(d*x
+c)^3/d
```

3.39.2 Mathematica [A] (verified)

Time = 6.26 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11

$$\int (a+a\sin(c+dx))^4\tan^2(c+dx)\,dx$$
$$= \frac{(a+a\sin(c+dx))^4\left(-1140(c+dx)+1056\cos(c+dx)-32\cos(3(c+dx))+\frac{1536\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+1\right.}{96d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}$$

input
```
Integrate[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

output
```
((a + a*Sin[c + d*x])^4*(-1140*(c + d*x) + 1056*Cos[c + d*x] - 32*Cos[3*(c
 + d*x)] + (1536*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) +
 192*Sin[2*(c + d*x)] - 3*Sin[4*(c + d*x)]))/(96*d*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^8)
```

3.39.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^2(a\sin(c+dx)+a)^4dx$$

$$\Big\downarrow \text{ 3188}$$

$$a^2\int \left(-a^2\sin^4(c+dx)-4a^2\sin^3(c+dx)-7a^2\sin^2(c+dx)-8a^2\sin(c+dx)-8a^2+\frac{8a^2}{1-\sin(c+dx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$a^2\left(-\frac{4a^2\cos^3(c+dx)}{3d}+\frac{12a^2\cos(c+dx)}{d}+\frac{a^2\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{31a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{8a^2 c}{d(1-s}\right.$$

input
```
Int[(a + a*Sin[c + d*x])^4*Tan[c + d*x]^2,x]
```

output
```
a^2*((-95*a^2*x)/8 + (12*a^2*Cos[c + d*x])/d - (4*a^2*Cos[c + d*x]^3)/(3*d
) + (8*a^2*Cos[c + d*x])/(d*(1 - Sin[c + d*x])) + (31*a^2*Cos[c + d*x]*Sin
[c + d*x])/(8*d) + (a^2*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d))
```

3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.39.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 12.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{95a^4x}{8}+\frac{11a^4e^{i(dx+c)}}{2d}+\frac{11a^4e^{-i(dx+c)}}{2d}+\frac{16a^4}{d(-i+e^{i(dx+c)})}-\frac{a^4\sin(4dx+4c)}{32d}-\frac{a^4\cos(3dx+3c)}{3d}+\frac{2a^4\sin($
derivativedivides	$a^4\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5(\sin^3(dx+c))}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx\right.\right.$
default	$a^4\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5(\sin^3(dx+c))}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx\right.\right.$
parts	$\frac{a^4(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{a^4\left(\frac{\sin^7(dx+c)}{\cos(dx+c)}+\left(\sin^5(dx+c)+\frac{5(\sin^3(dx+c))}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)-\frac{15dx}{8}\right.}{d}$

input
```
int((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-95/8*a^4*x+11/2/d*a^4*exp(I*(d*x+c))+11/2*a^4/d*exp(-I*(d*x+c))+16*a^4/d/
(-I+exp(I*(d*x+c)))-1/32/d*a^4*sin(4*d*x+4*c)-1/3*a^4/d*cos(3*d*x+3*c)+2/d
*a^4*sin(2*d*x+2*c)
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.58

$$\int (a + a\sin(c + dx))^4 \tan^2(c + dx)\, dx =$$
$$-\frac{6\,a^4\cos\left(dx+c\right)^5 + 32\,a^4\cos\left(dx+c\right)^4 - 73\,a^4\cos\left(dx+c\right)^3 + 285\,a^4dx - 288\,a^4\cos\left(dx+c\right)^2 - 192\,a^4}{}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/24*(6*a^4*cos(d*x + c)^5 + 32*a^4*cos(d*x + c)^4 - 73*a^4*cos(d*x + c)^
3 + 285*a^4*d*x - 288*a^4*cos(d*x + c)^2 - 192*a^4 + 3*(95*a^4*d*x - 127*a
^4)*cos(d*x + c) + (6*a^4*cos(d*x + c)^4 - 26*a^4*cos(d*x + c)^3 - 285*a^4
*d*x - 99*a^4*cos(d*x + c)^2 + 189*a^4*cos(d*x + c) - 192*a^4)*sin(d*x + c
))/(d*cos(d*x + c) - d*sin(d*x + c) + d)
```

3.39.6 Sympy [F]

$$\int (a + a\sin(c + dx))^4 \tan^2(c + dx)\, dx = a^4\left(\int 4\sin\left(c+dx\right)\tan^2\left(c+dx\right)dx + \int 6\sin^2\left(c+dx\right)\tan^2\left(c+dx\right)dx + \int 4\sin^3\left(c+dx\right)\tan^2\left(c+dx\right)dx + \int \sin^4\left(c+dx\right)\tan^2\left(c+dx\right)dx + \int \tan^2\left(c+dx\right)dx\right)$$

input
```
integrate((a+a*sin(d*x+c))**4*tan(d*x+c)**2,x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*tan(c + d*x)**2, x) + Integral(6*sin(c + d*x
)**2*tan(c + d*x)**2, x) + Integral(4*sin(c + d*x)**3*tan(c + d*x)**2, x)
+ Integral(sin(c + d*x)**4*tan(c + d*x)**2, x) + Integral(tan(c + d*x)**2,
 x))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.60

$$\int (a + a\sin(c + dx))^4 \tan^2(c + dx)\, dx =$$

$$-\frac{32\left(\cos\left(dx+c\right)^{3} - \frac{3}{\cos(dx+c)} - 6\,\cos\left(dx+c\right)\right)a^{4} + 3\left(15\,dx + 15\,c - \frac{9\,\tan(dx+c)^{3}+7\,\tan(dx+c)}{\tan(dx+c)^{4}+2\,\tan(dx+c)^{2}+1} - 8\,\tan\left(d\right.\right.}{}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/24*(32*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*a^4 + 3*(15*d
*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*
x + c)^2 + 1) - 8*tan(d*x + c))*a^4 + 72*(3*d*x + 3*c - tan(d*x + c)/(tan(
d*x + c)^2 + 1) - 2*tan(d*x + c))*a^4 + 24*(d*x + c - tan(d*x + c))*a^4 -
96*a^4*(1/cos(d*x + c) + cos(d*x + c)))/d
```

3.39.8 Giac [F(-1)]

Timed out.

$$\int (a + a\sin(c + dx))^4 \tan^2(c + dx)\, dx = \text{Timed out}$$

input
```
integrate((a+a*sin(d*x+c))^4*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.39.9 Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 363, normalized size of antiderivative = 3.21

$$\int (a + a\sin(c + dx))^4 \tan^2(c + dx)\, dx = -\frac{95\,a^4\,x}{8}$$

$$-\frac{\frac{95\,a^4\,(c+d\,x)}{8} - \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{95\,a^4\,(c+d\,x)}{8} - \frac{a^4\,(285\,c+285\,d\,x-326)}{24}\right) - \frac{a^4\,(285\,c+285\,d\,x-896)}{24} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(\frac{95\,a^4}{}\right.}{}$$

input
```
int(tan(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

output
```
- (95*a^4*x)/8 - ((95*a^4*(c + d*x))/8 - tan(c/2 + (d*x)/2)*((95*a^4*(c +
d*x))/8 - (a^4*(285*c + 285*d*x - 326))/24) - (a^4*(285*c + 285*d*x - 896)
)/24 + tan(c/2 + (d*x)/2)^8*((95*a^4*(c + d*x))/8 - (a^4*(285*c + 285*d*x
- 570))/24) - tan(c/2 + (d*x)/2)^7*((95*a^4*(c + d*x))/2 - (a^4*(1140*c +
1140*d*x - 570))/24) - tan(c/2 + (d*x)/2)^3*((95*a^4*(c + d*x))/2 - (a^4*(
1140*c + 1140*d*x - 1430))/24) + tan(c/2 + (d*x)/2)^6*((95*a^4*(c + d*x))/
2 - (a^4*(1140*c + 1140*d*x - 2154))/24) + tan(c/2 + (d*x)/2)^2*((95*a^4*(
c + d*x))/2 - (a^4*(1140*c + 1140*d*x - 3014))/24) - tan(c/2 + (d*x)/2)^5*
((285*a^4*(c + d*x))/4 - (a^4*(1710*c + 1710*d*x - 1770))/24) + tan(c/2 +
(d*x)/2)^4*((285*a^4*(c + d*x))/4 - (a^4*(1710*c + 1710*d*x - 3606))/24))/
(d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.39. $\int (a + a\sin(c + dx))^4 \tan^2(c + dx)\, dx$

3.40 $\int (a+a\sin(c+dx))^4\,dx$

3.40.1 Optimal result

Integrand size = 12, antiderivative size = 87

$$\int (a+a\sin(c+dx))^4\,dx=\frac{35a^4x}{8}-\frac{8a^4\cos(c+dx)}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{27a^4\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d}$$

output
```
35/8*a^4*x-8*a^4*cos(d*x+c)/d+4/3*a^4*cos(d*x+c)^3/d-27/8*a^4*cos(d*x+c)*s
in(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

3.40.2 Mathematica [A] (verified)

Time = 3.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.66

$$\int (a+a\sin(c+dx))^4\,dx=\frac{a^4(-672\cos(c+dx)+32\cos(3(c+dx))+3(140c+140dx-56\sin(2(c+dx))+\sin(4(c+dx))))}{96d}$$

input
```
Integrate[(a + a*Sin[c + d*x])^4,x]
```

output
```
(a^4*(-672*Cos[c + d*x] + 32*Cos[3*(c + d*x)] + 3*(140*c + 140*d*x - 56*Si
n[2*(c + d*x)] + Sin[4*(c + d*x)])))/(96*d)
```

3.40.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3124, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(c+dx)+a)^4dx$$

$$\Big\downarrow \text{3124}$$

$$\int \left(a^4\sin^4(c+dx)+4a^4\sin^3(c+dx)+6a^4\sin^2(c+dx)+4a^4\sin(c+dx)+a^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4a^4\cos^3(c+dx)}{3d}-\frac{8a^4\cos(c+dx)}{d}-\frac{a^4\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{27a^4\sin(c+dx)\cos(c+dx)}{8d}+\frac{35a^4x}{8}$$

input
```
Int[(a + a*Sin[c + d*x])^4,x]
```

output
```
(35*a^4*x)/8 - (8*a^4*Cos[c + d*x])/d + (4*a^4*Cos[c + d*x]^3)/(3*d) - (27
*a^4*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^4*Cos[c + d*x]*Sin[c + d*x]^3)/
(4*d)
```

3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3124
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Int[ExpandTri
g[(a + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 -
b^2, 0] && IGtQ[n, 0]
```

3.40.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

method	result
parallelrisch	$-\frac{a^4(-420dx+672\cos(dx+c)-3\sin(4dx+4c)-32\cos(3dx+3c)+168\sin(2dx+2c)+640)}{96d}$
risch	$\frac{35a^4x}{8}-\frac{7a^4\cos(dx+c)}{d}+\frac{a^4\sin(4dx+4c)}{32d}+\frac{a^4\cos(3dx+3c)}{3d}-\frac{7a^4\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{4a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+6a^4\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
default	$\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{4a^4\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+6a^4\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right.}{d}$
parts	$a^4x+\frac{a^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}-\frac{4a^4\cos(dx+c)}{d}+\frac{6a^4\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\right.}{d}$
norman	$\frac{\frac{35a^4x}{8}-\frac{40a^4}{3d}-\frac{27a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{35a^4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{35a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{27a^4\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{35a^4x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}}{(1}$

input
```
int((a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/96*a^4*(-420*d*x+672*cos(d*x+c)-3*sin(4*d*x+4*c)-32*cos(3*d*x+3*c)+168*
sin(2*d*x+2*c)+640)/d
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

$$\int (a+a\sin(c+dx))^4\,dx$$
$$=\frac{32\,a^4\cos(dx+c)^3+105\,a^4dx-192\,a^4\cos(dx+c)+3\left(2\,a^4\cos(dx+c)^3-29\,a^4\cos(dx+c)\right)\sin(dx+}{24\,d}$$

input
```
integrate((a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(32*a^4*cos(d*x + c)^3 + 105*a^4*d*x - 192*a^4*cos(d*x + c) + 3*(2*a^
4*cos(d*x + c)^3 - 29*a^4*cos(d*x + c))*sin(d*x + c))/d
```

3.40.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(82) = 164.

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.57

$$\int (a + a\sin(c + dx))^4\,dx$$
$$= \begin{cases} \frac{3a^4x\sin^4(c+dx)}{8} + \frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4} + 3a^4x\sin^2(c+dx) + \frac{3a^4x\cos^4(c+dx)}{8} + 3a^4x\cos^2(c+dx) + a^4x - \\ x(a\sin(c)+a)^4 \end{cases}$$

input
```
integrate((a+a*sin(d*x+c))**4,x)
```

output
```
Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d
*x)**2/4 + 3*a**4*x*sin(c + d*x)**2 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*
x*cos(c + d*x)**2 + a**4*x - 5*a**4*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 4
*a**4*sin(c + d*x)**2*cos(c + d*x)/d - 3*a**4*sin(c + d*x)*cos(c + d*x)**3
/(8*d) - 3*a**4*sin(c + d*x)*cos(c + d*x)/d - 8*a**4*cos(c + d*x)**3/(3*d)
 - 4*a**4*cos(c + d*x)/d, Ne(d, 0)), (x*(a*sin(c) + a)**4, True))
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.24

$$\int (a + a\sin(c + dx))^4\,dx = a^4x + \frac{4\left(\cos(dx + c)^3 - 3\cos(dx + c)\right)a^4}{3d} + \frac{(12\,dx + 12\,c + \sin(4\,dx + 4\,c) - 8\sin(2\,dx + 2\,c))a^4}{32\,d} + \frac{3\,(2\,dx + 2\,c - \sin(2\,dx + 2\,c))a^4}{2\,d} - \frac{4\,a^4\cos(dx + c)}{d}$$

input
```
integrate((a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
a^4*x + 4/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^4/d + 1/32*(12*d*x + 12*c
+ sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^4/d + 3/2*(2*d*x + 2*c - sin(2*
d*x + 2*c))*a^4/d - 4*a^4*cos(d*x + c)/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.83

$$\int (a+a\sin(c+dx))^4\,dx=\frac{35}{8}\,a^4x+\frac{a^4\cos\left(3\,dx+3\,c\right)}{3\,d}-\frac{7\,a^4\cos\left(dx+c\right)}{d}+\frac{a^4\sin\left(4\,dx+4\,c\right)}{32\,d}-\frac{7\,a^4\sin\left(2\,dx+2\,c\right)}{4\,d}$$

input
```
integrate((a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
35/8*a^4*x + 1/3*a^4*cos(3*d*x + 3*c)/d - 7*a^4*cos(d*x + c)/d + 1/32*a^4*
sin(4*d*x + 4*c)/d - 7/4*a^4*sin(2*d*x + 2*c)/d
```

3.40.9 Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.72

$$\int (a+a\sin(c+dx))^4\,dx=\frac{35\,a^4\,x}{8}-\frac{\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}-\frac{35\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{27\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{a^4\,(105\,c+105\,d\,x)}{24}-\frac{a^4\,(105\,c+105\,d\,x-320)}{24}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{}$$

input
```
int((a + a*sin(c + d*x))^4,x)
```

output
```
(35*a^4*x)/8 - ((35*a^4*tan(c/2 + (d*x)/2)^3)/4 - (35*a^4*tan(c/2 + (d*x)/
2)^5)/4 - (27*a^4*tan(c/2 + (d*x)/2)^7)/4 + (a^4*(105*c + 105*d*x))/24 - (
a^4*(105*c + 105*d*x - 320))/24 + tan(c/2 + (d*x)/2)^6*((a^4*(105*c + 105*
d*x))/6 - (a^4*(420*c + 420*d*x - 192))/24) + tan(c/2 + (d*x)/2)^2*((a^4*(
105*c + 105*d*x))/6 - (a^4*(420*c + 420*d*x - 1088))/24) + tan(c/2 + (d*x)
/2)^4*((a^4*(105*c + 105*d*x))/4 - (a^4*(630*c + 630*d*x - 960))/24) + (27
*a^4*tan(c/2 + (d*x)/2))/4)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.41 $\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$

3.41.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\begin{aligned}\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx &= \frac{17a^4x}{8}-\frac{4a^4\text{arctanh}(\cos(c+dx))}{d}\\ &\quad+\frac{4a^4\cos(c+dx)}{d}-\frac{4a^4\cos^3(c+dx)}{3d}\\ &\quad-\frac{a^4\cot(c+dx)}{d}+\frac{23a^4\cos(c+dx)\sin(c+dx)}{8d}\\ &\quad+\frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d}\end{aligned}$$

output
```
17/8*a^4*x-4*a^4*arctanh(cos(d*x+c))/d+4*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*x+
c)^3/d-a^4*cot(d*x+c)/d+23/8*a^4*cos(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c
)*sin(d*x+c)^3/d
```

3.41.2 Mathematica [A] (verified)

Time = 11.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.17

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$=\frac{a^4\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-48\cos(c+dx)-147\cos(3(c+dx))+3\cos(5(c+dx))+408c\sin(c\right.}{}$$

input
```
Integrate[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

output
```
(a^4*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-48*Cos[c + d*x] - 147*Cos[3*(c +
d*x)] + 3*Cos[5*(c + d*x)] + 408*c*Sin[c + d*x] + 408*d*x*Sin[c + d*x] - 7
68*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 768*Log[Sin[(c + d*x)/2]]*Sin[c +
d*x] + 320*Sin[2*(c + d*x)] - 32*Sin[4*(c + d*x)]))/(384*d)
```

3.41.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^4}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{\int \left(-\sin^4(c+dx)a^6-4\sin^3(c+dx)a^6+\csc^2(c+dx)a^6-5\sin^2(c+dx)a^6+4\csc(c+dx)a^6+5a^6\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{4a^6\text{arctanh}(\cos(c+dx))}{d}-\frac{4a^6\cos^3(c+dx)}{3d}+\frac{4a^6\cos(c+dx)}{d}-\frac{a^6\cot(c+dx)}{d}+\frac{a^6\sin^3(c+dx)\cos(c+dx)}{4d}+\frac{23a^6\sin(c+dx)\cos(c+dx)}{8d}}{a^2}$$

input
```
Int[Cot[c + d*x]^2*(a + a*Sin[c + d*x])^4,x]
```

output
```
((17*a^6*x)/8 - (4*a^6*ArcTanh[Cos[c + d*x]])/d + (4*a^6*Cos[c + d*x])/d -
 (4*a^6*Cos[c + d*x]^3)/(3*d) - (a^6*Cot[c + d*x])/d + (23*a^6*Cos[c + d*x
]*Sin[c + d*x])/(8*d) + (a^6*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d))/a^2
```

3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.41.4 Maple [A] (verified)

Time = 5.78 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{a^4\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{4a^4\left(\cos^3(dx+c)\right)}{3}+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4(}{d}$
default	$\frac{a^4\left(-\frac{\left(\cos^3(dx+c)\right)\sin(dx+c)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{4a^4\left(\cos^3(dx+c)\right)}{3}+6a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4a^4(}{d}$
risch	$\frac{17a^4x}{8}-\frac{3ia^4e^{2i(dx+c)}}{4d}+\frac{3a^4e^{i(dx+c)}}{2d}+\frac{3a^4e^{-i(dx+c)}}{2d}+\frac{3ia^4e^{-2i(dx+c)}}{4d}-\frac{2ia^4}{d\left(e^{2i(dx+c)}-1\right)}-\frac{4a^4\ln(e^{i(dx+c)}}{d}$

input
```
int(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(-1/4*cos(d*x+c)^3*sin(d*x+c)+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1
/8*c)-4/3*a^4*cos(d*x+c)^3+6*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
+4*a^4*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+a^4*(-cot(d*x+c)-d*x-c))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.16

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$= \frac{6\,a^4\cos(dx+c)^5 - 81\,a^4\cos(dx+c)^3 - 48\,a^4\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) + 48\,a^4\log\left(-\frac{1}{2}\cos(d\right.}{24\,d\,\mathrm{si}}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*(6*a^4*cos(d*x + c)^5 - 81*a^4*cos(d*x + c)^3 - 48*a^4*log(1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) + 48*a^4*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x +
 c) + 51*a^4*cos(d*x + c) - (32*a^4*cos(d*x + c)^3 - 51*a^4*d*x - 96*a^4*c
os(d*x + c))*sin(d*x + c))/(d*sin(d*x + c))
```

3.41.6 Sympy [F]

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx = a^4\left(\int 4\sin(c+dx)\cot^2(c+dx)\,dx + \int 6\sin^2(c+dx)\cot^2(c+dx)\,dx + \int 4\sin^3(c+dx)\cot^2(c+dx)\,dx + \int \sin^4(c+dx)\cot^2(c+dx)\,dx + \int \cot^2(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**2*(a+a*sin(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**2, x) + Integral(6*sin(c + d*x
)**2*cot(c + d*x)**2, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**2, x)
+ Integral(sin(c + d*x)**4*cot(c + d*x)**2, x) + Integral(cot(c + d*x)**2,
 x))
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.01

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx =$$
$$-\frac{128\,a^4\cos\left(dx+c\right)^3-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^4-144\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4+96\left(dx\right.}{96\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/96*(128*a^4*cos(d*x + c)^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^4 - 1
44*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 96*(d*x + c + 1/tan(d*x + c))*a^
4 - 192*a^4*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1
)))/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.67

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$=\frac{51\left(dx+c\right)a^4+96\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{12\left(8\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(69\,a^4\tan\right.}{}}{24\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(51*(d*x + c)*a^4 + 96*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 12*a^4*ta
n(1/2*d*x + 1/2*c) - 12*(8*a^4*tan(1/2*d*x + 1/2*c) + a^4)/tan(1/2*d*x + 1
/2*c) - 2*(69*a^4*tan(1/2*d*x + 1/2*c)^7 + 93*a^4*tan(1/2*d*x + 1/2*c)^5 -
 192*a^4*tan(1/2*d*x + 1/2*c)^4 - 93*a^4*tan(1/2*d*x + 1/2*c)^3 - 256*a^4*
tan(1/2*d*x + 1/2*c)^2 - 69*a^4*tan(1/2*d*x + 1/2*c) - 64*a^4)/(tan(1/2*d*
x + 1/2*c)^2 + 1)^4)/d
```

3.41.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.54

$$\int \cot^2(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$= \frac{4a^4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{17a^4\,\mathrm{atan}\left(\frac{289a^8}{16\left(34a^8-\frac{289a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}\right)}+\frac{34a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{34a^8-\frac{289a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}}\right)}{4d}$$
$$+ \frac{-\frac{25a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{2}-\frac{39a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{2}+32a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\frac{19a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}+\frac{128a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{15a^4\tan(}{2}}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+12\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\right.}$$
$$+ \frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}$$

input
```
int(cot(c + d*x)^2*(a + a*sin(c + d*x))^4,x)
```

output
```
(4*a^4*log(tan(c/2 + (d*x)/2)))/d + (17*a^4*atan((289*a^8)/(16*(34*a^8 - (
289*a^8*tan(c/2 + (d*x)/2))/16)) + (34*a^8*tan(c/2 + (d*x)/2))/(34*a^8 - (
289*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + ((15*a^4*tan(c/2 + (d*x)/2)^2)/2
 + (128*a^4*tan(c/2 + (d*x)/2)^3)/3 + (19*a^4*tan(c/2 + (d*x)/2)^4)/2 + 32
*a^4*tan(c/2 + (d*x)/2)^5 - (39*a^4*tan(c/2 + (d*x)/2)^6)/2 - (25*a^4*tan(
c/2 + (d*x)/2)^8)/2 - a^4 + (32*a^4*tan(c/2 + (d*x)/2))/3)/(d*(2*tan(c/2 +
 (d*x)/2) + 8*tan(c/2 + (d*x)/2)^3 + 12*tan(c/2 + (d*x)/2)^5 + 8*tan(c/2 +
 (d*x)/2)^7 + 2*tan(c/2 + (d*x)/2)^9)) + (a^4*tan(c/2 + (d*x)/2))/(2*d)
```

3.42 $\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

3.42.1 Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{61a^4x}{8}+\frac{2a^4\text{arctanh}(\cos(c+dx))}{d}+\frac{4a^4\cos^3(c+dx)}{3d}-\frac{5a^4\cot(c+dx)}{d}-\frac{a^4\cot^3(c+dx)}{3d}-\frac{2a^4\cot(c+dx)\csc(c+dx)}{d}-\frac{19a^4\cos(c+dx)\sin(c+dx)}{8d}-\frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d}$$

output
```
-61/8*a^4*x+2*a^4*arctanh(cos(d*x+c))/d+4/3*a^4*cos(d*x+c)^3/d-5*a^4*cot(d
*x+c)/d-1/3*a^4*cot(d*x+c)^3/d-2*a^4*cot(d*x+c)*csc(d*x+c)/d-19/8*a^4*cos(
d*x+c)*sin(d*x+c)/d-1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

3.42.2 Mathematica [A] (verified)

Time = 10.45 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.49

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$= \frac{a^4(1+\sin(c+dx))^4\left(-732(c+dx)+96\cos(c+dx)+32\cos(3(c+dx))-224\cot\left(\frac{1}{2}(c+dx)\right)-48\csc^2\right.}{}$$

input
```
Integrate[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

output
```
(a^4*(1 + Sin[c + d*x])^4*(-732*(c + d*x) + 96*Cos[c + d*x] + 32*Cos[3*(c
+ d*x)] - 224*Cot[(c + d*x)/2] - 48*Csc[(c + d*x)/2]^2 + 192*Log[Cos[(c +
d*x)/2]] - 192*Log[Sin[(c + d*x)/2]] + 48*Sec[(c + d*x)/2]^2 + 32*Csc[c +
d*x]^3*Sin[(c + d*x)/2]^4 - 2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 120*Sin[2*
(c + d*x)] + 3*Sin[4*(c + d*x)] + 224*Tan[(c + d*x)/2]))/(96*d*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^8)
```

3.42.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(c+dx)+a)^4}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{\int (\csc^4(c+dx)a^8+\sin^4(c+dx)a^8+4\csc^3(c+dx)a^8+4\sin^3(c+dx)a^8+4\csc^2(c+dx)a^8+4\sin^2(c+dx)a^8-}{a^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2a^8\operatorname{arctanh}(\cos(c+dx))}{d}+\frac{4a^8\cos^3(c+dx)}{3d}-\frac{a^8\cot^3(c+dx)}{3d}-\frac{5a^8\cot(c+dx)}{d}-\frac{a^8\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{19a^8\sin(c+dx)\cos(c+dx)}{8d}-}{a^4}$$

input
```
Int[Cot[c + d*x]^4*(a + a*Sin[c + d*x])^4,x]
```

output
```
((-61*a^8*x)/8 + (2*a^8*ArcTanh[Cos[c + d*x]])/d + (4*a^8*Cos[c + d*x]^3)/
(3*d) - (5*a^8*Cot[c + d*x])/d - (a^8*Cot[c + d*x]^3)/(3*d) - (2*a^8*Cot[c
 + d*x]*Csc[c + d*x])/d - (19*a^8*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (a^8*
Cos[c + d*x]*Sin[c + d*x]^3)/(4*d))/a^4
```

3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.42.4 Maple [A] (verified)

Time = 6.57 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.59

method	result
derivativedivides	$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+6a}{}$
default	$\frac{a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+4a^4\left(\frac{\left(\cos^3(dx+c)\right)}{3}+\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c))\right)+6a}{}$
risch	$-\frac{61a^4x}{8}-\frac{ia^4e^{4i(dx+c)}}{64d}+\frac{5ia^4e^{2i(dx+c)}}{8d}+\frac{a^4e^{i(dx+c)}}{2d}+\frac{a^4e^{-i(dx+c)}}{2d}-\frac{5ia^4e^{-2i(dx+c)}}{8d}+\frac{ia^4e^{-4i(dx+c)}}{64d}+$

input
```
int(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+4*a^
4*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+6*a^4*(-1/sin(d*
x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/2*d*x-3/2*c)+
4*a^4*(-1/2/sin(d*x+c)^2*cos(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*
ln(csc(d*x+c)-cot(d*x+c)))+a^4*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.56

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{6\,a^4\cos{(dx+c)}^7-75\,a^4\cos{(dx+c)}^5+244\,a^4\cos{(dx+c)}^3-183\,a^4\cos{(dx+c)}-24\,\left(a^4\cos{(dx+c)}\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/24*(6*a^4*cos(d*x + c)^7 - 75*a^4*cos(d*x + c)^5 + 244*a^4*cos(d*x + c)
^3 - 183*a^4*cos(d*x + c) - 24*(a^4*cos(d*x + c)^2 - a^4)*log(1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) + 24*(a^4*cos(d*x + c)^2 - a^4)*log(-1/2*cos(d*x
+ c) + 1/2)*sin(d*x + c) - (32*a^4*cos(d*x + c)^5 - 183*a^4*d*x*cos(d*x +
c)^2 - 32*a^4*cos(d*x + c)^3 + 183*a^4*d*x + 48*a^4*cos(d*x + c))*sin(d*x
+ c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

3.42.6 Sympy [F]

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx = a^4\left(\int 4\sin{(c+dx)}\cot^4{(c+dx)}\,dx + \int 6\sin^2{(c+dx)}\cot^4{(c+dx)}\,dx + \int 4\sin^3{(c+dx)}\cot^4{(c+dx)}\,dx + \int \sin^4{(c+dx)}\cot^4{(c+dx)}\,dx + \int \cot^4{(c+dx)}\,dx\right)$$

input
```
integrate(cot(d*x+c)**4*(a+a*sin(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**4, x) + Integral(6*sin(c + d*x
)**2*cot(c + d*x)**4, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**4, x)
+ Integral(sin(c + d*x)**4*cot(c + d*x)**4, x) + Integral(cot(c + d*x)**4,
 x))
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.56

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$= \frac{64\left(2\cos\left(dx+c\right)^3+6\cos\left(dx+c\right)-3\log\left(\cos\left(dx+c\right)+1\right)+3\log\left(\cos\left(dx+c\right)-1\right)\right)a^4+3\left(12\,dx+\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/96*(64*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*
log(cos(d*x + c) - 1))*a^4 + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2
*d*x + 2*c))*a^4 - 288*(3*d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)
^3 + tan(d*x + c)))*a^4 + 32*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x
 + c)^3)*a^4 + 96*a^4*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c
) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

3.42.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(130) = 260.

Time = 0.47 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.96

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$= \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-183\,(dx+c)a^4-48\,a^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+57\,a^4\tan}{}$$

input
```
integrate(cot(d*x+c)^4*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/24*(a^4*tan(1/2*d*x + 1/2*c)^3 + 12*a^4*tan(1/2*d*x + 1/2*c)^2 - 183*(d*
x + c)*a^4 - 48*a^4*log(abs(tan(1/2*d*x + 1/2*c))) + 57*a^4*tan(1/2*d*x +
1/2*c) + (88*a^4*tan(1/2*d*x + 1/2*c)^3 - 57*a^4*tan(1/2*d*x + 1/2*c)^2 -
12*a^4*tan(1/2*d*x + 1/2*c) - a^4)/tan(1/2*d*x + 1/2*c)^3 + 2*(57*a^4*tan(
1/2*d*x + 1/2*c)^7 + 96*a^4*tan(1/2*d*x + 1/2*c)^6 + 81*a^4*tan(1/2*d*x +
1/2*c)^5 + 96*a^4*tan(1/2*d*x + 1/2*c)^4 - 81*a^4*tan(1/2*d*x + 1/2*c)^3 +
 32*a^4*tan(1/2*d*x + 1/2*c)^2 - 57*a^4*tan(1/2*d*x + 1/2*c) + 32*a^4)/(ta
n(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.42.9 Mupad [B] (verification not implemented)

Time = 6.31 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.74

$$\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$= \frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2d} + \frac{a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24d} - \frac{2a^4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$
$$- \frac{61a^4\,\mathrm{atan}\left(\frac{3721a^8}{16\left(61a^8-\frac{3721a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}\right)} + \frac{61a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{61a^8-\frac{3721a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16}}\right)}{4d} + \frac{19a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}$$
$$- \frac{-19a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 60a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 + \frac{67a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3} - 48a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 + \frac{508a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3} +}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11} + 32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 + 48\tan\left(\frac{c}{2}\right.\right.}$$

input
```
int(cot(c + d*x)^4*(a + a*sin(c + d*x))^4,x)
```

output
```
(a^4*tan(c/2 + (d*x)/2)^2)/(2*d) + (a^4*tan(c/2 + (d*x)/2)^3)/(24*d) - (2*
a^4*log(tan(c/2 + (d*x)/2)))/d - (61*a^4*atan((3721*a^8)/(16*(61*a^8 - (37
21*a^8*tan(c/2 + (d*x)/2))/16)) + (61*a^8*tan(c/2 + (d*x)/2))/(61*a^8 - (3
721*a^8*tan(c/2 + (d*x)/2))/16)))/(4*d) + (19*a^4*tan(c/2 + (d*x)/2))/(8*d
) - ((61*a^4*tan(c/2 + (d*x)/2)^2)/3 - (16*a^4*tan(c/2 + (d*x)/2)^3)/3 + 1
16*a^4*tan(c/2 + (d*x)/2)^4 + (8*a^4*tan(c/2 + (d*x)/2)^5)/3 + (508*a^4*ta
n(c/2 + (d*x)/2)^6)/3 - 48*a^4*tan(c/2 + (d*x)/2)^7 + (67*a^4*tan(c/2 + (d
*x)/2)^8)/3 - 60*a^4*tan(c/2 + (d*x)/2)^9 - 19*a^4*tan(c/2 + (d*x)/2)^10 +
 a^4/3 + 4*a^4*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 32*tan(c/2
 + (d*x)/2)^5 + 48*tan(c/2 + (d*x)/2)^7 + 32*tan(c/2 + (d*x)/2)^9 + 8*tan(
c/2 + (d*x)/2)^11))
```

3.42. $\int \cot^4(c+dx)(a+a\sin(c+dx))^4\,dx$

3.43 $\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx$

3.43.1 Optimal result

Integrand size = 21, antiderivative size = 198

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{97a^4x}{8} + \frac{5a^4\text{arctanh}(\cos(c+dx))}{2d} - \frac{4a^4\cos(c+dx)}{d} - \frac{4a^4\cos^3(c+dx)}{3d} + \frac{10a^4\cot(c+dx)}{d} - \frac{5a^4\cot^3(c+dx)}{3d} - \frac{a^4\cot^5(c+dx)}{5d} + \frac{5a^4\cot(c+dx)\csc(c+dx)}{2d} - \frac{a^4\cot(c+dx)\csc^3(c+dx)}{d} + \frac{15a^4\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^4\cos(c+dx)\sin^3(c+dx)}{4d}$$

output
```
97/8*a^4*x+5/2*a^4*arctanh(cos(d*x+c))/d-4*a^4*cos(d*x+c)/d-4/3*a^4*cos(d*
x+c)^3/d+10*a^4*cot(d*x+c)/d-5/3*a^4*cot(d*x+c)^3/d-1/5*a^4*cot(d*x+c)^5/d
+5/2*a^4*cot(d*x+c)*csc(d*x+c)/d-a^4*cot(d*x+c)*csc(d*x+c)^3/d+15/8*a^4*co
s(d*x+c)*sin(d*x+c)/d+1/4*a^4*cos(d*x+c)*sin(d*x+c)^3/d
```

3.43.2 Mathematica [A] (verified)

Time = 7.21 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.43

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx$$
$$= \frac{a^4(1+\sin(c+dx))^4\left(5820(c+dx)-2400\cos(c+dx)-160\cos(3(c+dx))+2752\cot\left(\frac{1}{2}(c+dx)\right)+300\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

output
```
(a^4*(1 + Sin[c + d*x])^4*(5820*(c + d*x) - 2400*Cos[c + d*x] - 160*Cos[3*
(c + d*x)] + 2752*Cot[(c + d*x)/2] + 300*Csc[(c + d*x)/2]^2 - 30*Csc[(c +
d*x)/2]^4 + 1200*Log[Cos[(c + d*x)/2]] - 1200*Log[Sin[(c + d*x)/2]] - 300*
Sec[(c + d*x)/2]^2 + 30*Sec[(c + d*x)/2]^4 + 632*Csc[c + d*x]^3*Sin[(c + d
*x)/2]^4 + 96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 - (79*Csc[(c + d*x)/2]^4*S
in[c + d*x])/2 - (3*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 480*Sin[2*(c + d*
x)] - 15*Sin[4*(c + d*x)] - 2752*Tan[(c + d*x)/2]))/(480*d*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^8)
```

3.43.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a\sin(c+dx)+a)^4\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(c+dx)+a)^4}{\tan(c+dx)^6}dx$$

$$\Big\downarrow \text{ 3188}$$

$$\frac{\int \left(\csc^6(c+dx)a^{10}+4\csc^5(c+dx)a^{10}+3\csc^4(c+dx)a^{10}-\sin^4(c+dx)a^{10}-8\csc^3(c+dx)a^{10}-4\sin^3(c+dx)\right.}{a^6}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{5a^{10}\text{arctanh}(\cos(c+dx))}{2d} - \frac{4a^{10}\cos^3(c+dx)}{3d} - \frac{4a^{10}\cos(c+dx)}{d} - \frac{a^{10}\cot^5(c+dx)}{5d} - \frac{5a^{10}\cot^3(c+dx)}{3d} + \frac{10a^{10}\cot(c+dx)}{d} + \frac{a^{10}\sin^3(c+\cdots}{4\cdots}}{a^6}$$

input
```
Int[Cot[c + d*x]^6*(a + a*Sin[c + d*x])^4,x]
```

output
```
((97*a^10*x)/8 + (5*a^10*ArcTanh[Cos[c + d*x]])/(2*d) - (4*a^10*Cos[c + d*
x])/d - (4*a^10*Cos[c + d*x]^3)/(3*d) + (10*a^10*Cot[c + d*x])/d - (5*a^10
*Cot[c + d*x]^3)/(3*d) - (a^10*Cot[c + d*x]^5)/(5*d) + (5*a^10*Cot[c + d*x
]*Csc[c + d*x])/(2*d) - (a^10*Cot[c + d*x]*Csc[c + d*x]^3)/d + (15*a^10*Co
s[c + d*x]*Sin[c + d*x])/(8*d) + (a^10*Cos[c + d*x]*Sin[c + d*x]^3)/(4*d))
/a^6
```

3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.43.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 13.02 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.54

method	result
risch	$\frac{97a^4x}{8}+\frac{ia^4e^{4i(dx+c)}}{64d}-\frac{a^4e^{3i(dx+c)}}{6d}-\frac{ia^4e^{2i(dx+c)}}{2d}-\frac{5a^4e^{i(dx+c)}}{2d}-\frac{5a^4e^{-i(dx+c)}}{2d}+\frac{ia^4e^{-2i(dx+c)}}{2d}-\frac{a^4e}{}$
derivativedivides	$\frac{a^4\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(d\right.}{2}\right.}{}$
default	$\frac{a^4\left(-\frac{\cos^7(dx+c)}{\sin(dx+c)}-\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)-\frac{15dx}{8}-\frac{15c}{8}\right)+4a^4\left(-\frac{\cos^7(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^5(d\right.}{2}\right.}{}$

input
```
int(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
97/8*a^4*x+1/64*I/d*a^4*exp(4*I*(d*x+c))-1/6*a^4/d*exp(3*I*(d*x+c))-1/2*I/
d*a^4*exp(2*I*(d*x+c))-5/2/d*a^4*exp(I*(d*x+c))-5/2*a^4/d*exp(-I*(d*x+c))+
1/2*I/d*a^4*exp(-2*I*(d*x+c))-1/6*a^4/d*exp(-3*I*(d*x+c))-1/64*I/d*a^4*exp
(-4*I*(d*x+c))-1/15*a^4*(-420*I*exp(8*I*(d*x+c))+75*exp(9*I*(d*x+c))+1500*
I*exp(6*I*(d*x+c))-30*exp(7*I*(d*x+c))-1940*I*exp(4*I*(d*x+c))+1300*I*exp(
2*I*(d*x+c))+30*exp(3*I*(d*x+c))-344*I-75*exp(I*(d*x+c)))/d/(exp(2*I*(d*x+
c))-1)^5+5/2*a^4/d*ln(exp(I*(d*x+c))+1)-5/2*a^4/d*ln(exp(I*(d*x+c))-1)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.47

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx$$

$$=\frac{30\,a^4\cos{(dx+c)}^9-345\,a^4\cos{(dx+c)}^7+2231\,a^4\cos{(dx+c)}^5-3395\,a^4\cos{(dx+c)}^3+1455\,a^4\cos{(dx}}{}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/120*(30*a^4*cos(d*x + c)^9 - 345*a^4*cos(d*x + c)^7 + 2231*a^4*cos(d*x +
 c)^5 - 3395*a^4*cos(d*x + c)^3 + 1455*a^4*cos(d*x + c) + 150*(a^4*cos(d*x
 + c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
+ c) - 150*(a^4*cos(d*x + c)^4 - 2*a^4*cos(d*x + c)^2 + a^4)*log(-1/2*cos(
d*x + c) + 1/2)*sin(d*x + c) - 5*(32*a^4*cos(d*x + c)^7 - 291*a^4*d*x*cos(
d*x + c)^4 + 32*a^4*cos(d*x + c)^5 + 582*a^4*d*x*cos(d*x + c)^2 - 100*a^4*
cos(d*x + c)^3 - 291*a^4*d*x + 60*a^4*cos(d*x + c))*sin(d*x + c))/((d*cos(
d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

3.43.6 Sympy [F]

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx = a^4\left(\int 4\sin(c+dx)\cot^6(c+dx)\,dx + \int 6\sin^2(c+dx)\cot^6(c+dx)\,dx + \int 4\sin^3(c+dx)\cot^6(c+dx)\,dx + \int \sin^4(c+dx)\cot^6(c+dx)\,dx + \int \cot^6(c+dx)\,dx\right)$$

input
```
integrate(cot(d*x+c)**6*(a+a*sin(d*x+c))**4,x)
```

output
```
a**4*(Integral(4*sin(c + d*x)*cot(c + d*x)**6, x) + Integral(6*sin(c + d*x
)**2*cot(c + d*x)**6, x) + Integral(4*sin(c + d*x)**3*cot(c + d*x)**6, x)
+ Integral(sin(c + d*x)**4*cot(c + d*x)**6, x) + Integral(cot(c + d*x)**6,
 x))
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.58

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx = -\frac{40\left(4\cos(dx+c)^3 - \frac{6\cos(dx+c)}{\cos(dx+c)^2-1} + 24\cos(dx+c) - 15\log(\cos(dx+c)+1) + 15\log(\cos(dx+c)-\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/120*(40*(4*cos(d*x + c)^3 - 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*co
s(d*x + c) - 15*log(cos(d*x + c) + 1) + 15*log(cos(d*x + c) - 1))*a^4 + 15
*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 25*tan(d*x + c)^2 + 8)/(tan(d*x + c
)^5 + 2*tan(d*x + c)^3 + tan(d*x + c)))*a^4 - 120*(15*d*x + 15*c + (15*tan
(d*x + c)^4 + 10*tan(d*x + c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a^
4 + 8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(d*x
+ c)^5)*a^4 + 30*a^4*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)^
4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) - 1
5*log(cos(d*x + c) - 1)))/d
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.71

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx$$

$$= \frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+85\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+5820}{}$$

input
```
integrate(cot(d*x+c)^6*(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/480*(3*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*a^4*tan(1/2*d*x + 1/2*c)^4 + 85*a
^4*tan(1/2*d*x + 1/2*c)^3 - 240*a^4*tan(1/2*d*x + 1/2*c)^2 + 5820*(d*x + c
)*a^4 - 1200*a^4*log(abs(tan(1/2*d*x + 1/2*c))) - 2670*a^4*tan(1/2*d*x + 1
/2*c) - 40*(45*a^4*tan(1/2*d*x + 1/2*c)^7 + 192*a^4*tan(1/2*d*x + 1/2*c)^6
 + 69*a^4*tan(1/2*d*x + 1/2*c)^5 + 384*a^4*tan(1/2*d*x + 1/2*c)^4 - 69*a^4
*tan(1/2*d*x + 1/2*c)^3 + 320*a^4*tan(1/2*d*x + 1/2*c)^2 - 45*a^4*tan(1/2*
d*x + 1/2*c) + 128*a^4)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4 + (2740*a^4*tan(1/2
*d*x + 1/2*c)^5 + 2670*a^4*tan(1/2*d*x + 1/2*c)^4 + 240*a^4*tan(1/2*d*x +
1/2*c)^3 - 85*a^4*tan(1/2*d*x + 1/2*c)^2 - 30*a^4*tan(1/2*d*x + 1/2*c) - 3
*a^4)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.43.9 Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.29

$$\int \cot^6(c+dx)(a+a\sin(c+dx))^4\,dx = \frac{17\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d} - \frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\,d} + \frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{16\,d} + \frac{a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d} - \frac{5\,a^4\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d} - \frac{97\,a^4\,\mathrm{atan}\left(\frac{9409\,a^8}{16\left(\frac{485\,a^8}{4}+\frac{9409\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)} - \frac{485\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\left(\frac{485\,a^8}{4}+\frac{9409\,a^8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}\right)}\right)}{4\,d} - \frac{-58\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12} + 496\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11} - \frac{1567\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3} + 962\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9 - \frac{18437\,a^4\tan\left(\frac{c}{2}\right.}{15}}{d\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13} + 128\tan\right.} - \frac{89\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$

input
```
int(cot(c + d*x)^6*(a + a*sin(c + d*x))^4,x)
```

output
```
(17*a^4*tan(c/2 + (d*x)/2)^3)/(96*d) - (a^4*tan(c/2 + (d*x)/2)^2)/(2*d) +
(a^4*tan(c/2 + (d*x)/2)^4)/(16*d) + (a^4*tan(c/2 + (d*x)/2)^5)/(160*d) - (
5*a^4*log(tan(c/2 + (d*x)/2)))/(2*d) - (97*a^4*atan((9409*a^8)/(16*((485*a
^8)/4 + (9409*a^8*tan(c/2 + (d*x)/2))/16)) - (485*a^8*tan(c/2 + (d*x)/2))/
(4*((485*a^8)/4 + (9409*a^8*tan(c/2 + (d*x)/2))/16))))/(4*d) - ((97*a^4*ta
n(c/2 + (d*x)/2)^2)/15 - 8*a^4*tan(c/2 + (d*x)/2)^3 - (2312*a^4*tan(c/2 +
(d*x)/2)^4)/15 + (868*a^4*tan(c/2 + (d*x)/2)^5)/3 - (3986*a^4*tan(c/2 + (d
*x)/2)^6)/5 + (2296*a^4*tan(c/2 + (d*x)/2)^7)/3 - (18437*a^4*tan(c/2 + (d*
x)/2)^8)/15 + 962*a^4*tan(c/2 + (d*x)/2)^9 - (1567*a^4*tan(c/2 + (d*x)/2)^
10)/3 + 496*a^4*tan(c/2 + (d*x)/2)^11 - 58*a^4*tan(c/2 + (d*x)/2)^12 + a^4
/5 + 2*a^4*tan(c/2 + (d*x)/2))/(d*(32*tan(c/2 + (d*x)/2)^5 + 128*tan(c/2 +
 (d*x)/2)^7 + 192*tan(c/2 + (d*x)/2)^9 + 128*tan(c/2 + (d*x)/2)^11 + 32*ta
n(c/2 + (d*x)/2)^13)) - (89*a^4*tan(c/2 + (d*x)/2))/(16*d)
```

3.44 $\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

3.44.1 Optimal result

Integrand size = 21, antiderivative size = 130

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{35\text{arctanh}(\sin(c+dx))}{128ad}+\frac{35\sec(c+dx)\tan(c+dx)}{128ad}-\frac{35\sec(c+dx)\tan^3(c+dx)}{192ad}+\frac{7\sec(c+dx)\tan^5(c+dx)}{48ad}-\frac{\sec(c+dx)\tan^7(c+dx)}{8ad}+\frac{\tan^8(c+dx)}{8ad}$$

output
```
-35/128*arctanh(sin(d*x+c))/a/d+35/128*sec(d*x+c)*tan(d*x+c)/a/d-35/192*se
c(d*x+c)*tan(d*x+c)^3/a/d+7/48*sec(d*x+c)*tan(d*x+c)^5/a/d-1/8*sec(d*x+c)*
tan(d*x+c)^7/a/d+1/8*tan(d*x+c)^8/a/d
```

3.44.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.78

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{105\text{arctanh}(\sin(c+dx))+\frac{-48+57\sin(c+dx)+249\sin^2(c+dx)-136\sin^3(c+dx)-424\sin^4(c+dx)+87\sin^5(c+dx)+279\sin^6(c+dx)}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^4}}{384ad}$$

input
```
Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

output
```
-1/384*(105*ArcTanh[Sin[c + d*x]] + (-48 + 57*Sin[c + d*x] + 249*Sin[c + d
*x]^2 - 136*Sin[c + d*x]^3 - 424*Sin[c + d*x]^4 + 87*Sin[c + d*x]^5 + 279*
Sin[c + d*x]^6)/((-1 + Sin[c + d*x])^3*(1 + Sin[c + d*x])^4))/(a*d)
```

3.44.3 Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3185, 3042, 3087, 15, 3091, 3042, 3091, 3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^7(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^7}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \sec^2(c+dx)\tan^7(c+dx)dx}{a}-\frac{\int \sec(c+dx)\tan^8(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^7dx}{a}-\frac{\int \sec(c+dx)\tan(c+dx)^8dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^7(c+dx)d\tan(c+dx)}{ad}-\frac{\int \sec(c+dx)\tan(c+dx)^8dx}{a}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\int \sec(c+dx)\tan(c+dx)^8dx}{a}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\int \sec(c+dx)\tan^6(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\int \sec(c+dx)\tan(c+dx)^6dx}{a}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\left(\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\int \sec(c+dx)\tan^4(c+dx)dx\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\left(\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\int \sec(c+dx)\tan(c+dx)^4dx\right)}{a}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\left(\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\int \sec(c+dx)\tan^2(c+dx)dx\right)\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\left(\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\int \sec(c+dx)\tan(c+dx)^2dx\right)\right)}{a}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\left(\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int \sec(c+dx)dx\right)\right)\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\left(\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)\right)\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan^8(c+dx)}{8ad}-\frac{\frac{\tan^7(c+dx)\sec(c+dx)}{8d}-\frac{7}{8}\left(\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{\text{arctanh}(\sin(c+dx))}{2d}\right)\right)\right)}{a}$$

input
```
Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

3.44. $\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx$

output
```
Tan[c + d*x]^8/(8*a*d) - ((Sec[c + d*x]*Tan[c + d*x]^7)/(8*d) - (7*((Sec[c
 + d*x]*Tan[c + d*x]^5)/(6*d) - (5*((Sec[c + d*x]*Tan[c + d*x]^3)/(4*d) -
(3*(-1/2*ArcTanh[Sin[c + d*x]]/d + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4))
/6))/8)/a
```

3.44.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.44.4 Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{\frac{1}{64(1+\sin(dx+c))^4}-\frac{5}{48(1+\sin(dx+c))^3}+\frac{19}{64(1+\sin(dx+c))^2}-\frac{1}{2(1+\sin(dx+c))}-\frac{35\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin(}}{da}$
default	$\frac{\frac{1}{64(1+\sin(dx+c))^4}-\frac{5}{48(1+\sin(dx+c))^3}+\frac{19}{64(1+\sin(dx+c))^2}-\frac{1}{2(1+\sin(dx+c))}-\frac{35\ln(1+\sin(dx+c))}{256}-\frac{1}{96(\sin(dx+c)-1)^3}-\frac{}{128(\sin(}}{da}$
risch	$-\frac{i(279\,e^{13i(dx+c)}+22\,e^{11i(dx+c)}+1385\,e^{9i(dx+c)}+1385\,e^{5i(dx+c)}+22\,e^{3i(dx+c)}+279\,e^{i(dx+c)}-300\,e^{7i(dx+c)}-174ie^{2i(}}{192(e^{i(dx+c)}+i)^8(-i+e^{i(dx+c)})^6}$

input
```
int(tan(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/64/(1+sin(d*x+c))^4-5/48/(1+sin(d*x+c))^3+19/64/(1+sin(d*x+c))^2-
1/2/(1+sin(d*x+c))-35/256*ln(1+sin(d*x+c))-1/96/(sin(d*x+c)-1)^3-9/128/(si
n(d*x+c)-1)^2-29/128/(sin(d*x+c)-1)+35/256*ln(sin(d*x+c)-1))
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.28

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{558\,\cos(dx+c)^6-826\,\cos(dx+c)^4+476\,\cos(dx+c)^2+105\,(\cos(dx+c)^6\sin(dx+c)+\cos(dx+}{}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/768*(558*cos(d*x + c)^6 - 826*cos(d*x + c)^4 + 476*cos(d*x + c)^2 + 105
*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(sin(d*x + c) + 1) - 10
5*(cos(d*x + c)^6*sin(d*x + c) + cos(d*x + c)^6)*log(-sin(d*x + c) + 1) -
2*(87*cos(d*x + c)^4 - 38*cos(d*x + c)^2 + 8)*sin(d*x + c) - 112)/(a*d*cos
(d*x + c)^6*sin(d*x + c) + a*d*cos(d*x + c)^6)
```

3.44.6 Sympy [F]

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\tan^7(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**7/(sin(c + d*x) + 1), x)/a
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.35

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{2\left(279\sin(dx+c)^6+87\sin(dx+c)^5-424\sin(dx+c)^4-136\sin(dx+c)^3+249\sin(dx+c)^2+57\sin(dx+c)-48\right)}{a\sin(dx+c)^7+a\sin(dx+c)^6-3a\sin(dx+c)^5-3a\sin(dx+c)^4+3a\sin(dx+c)^3+3a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{105\log(\sin(dx+c)+1)}{a}-}{768\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/768*(2*(279*sin(d*x + c)^6 + 87*sin(d*x + c)^5 - 424*sin(d*x + c)^4 - 1
36*sin(d*x + c)^3 + 249*sin(d*x + c)^2 + 57*sin(d*x + c) - 48)/(a*sin(d*x
+ c)^7 + a*sin(d*x + c)^6 - 3*a*sin(d*x + c)^5 - 3*a*sin(d*x + c)^4 + 3*a*
sin(d*x + c)^3 + 3*a*sin(d*x + c)^2 - a*sin(d*x + c) - a) + 105*log(sin(d*
x + c) + 1)/a - 105*log(sin(d*x + c) - 1)/a)/d
```

3.44.8 Giac [A] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{420\log(|\sin(dx+c)+1|)}{a}-\frac{420\log(|\sin(dx+c)-1|)}{a}+\frac{2\left(385\sin(dx+c)^3-807\sin(dx+c)^2+567\sin(dx+c)-129\right)}{a(\sin(dx+c)-1)^3}-\frac{875\sin(dx+c)^4+196}{}}{3072\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/3072*(420*log(abs(sin(d*x + c) + 1))/a - 420*log(abs(sin(d*x + c) - 1))
/a + 2*(385*sin(d*x + c)^3 - 807*sin(d*x + c)^2 + 567*sin(d*x + c) - 129)/
(a*(sin(d*x + c) - 1)^3) - (875*sin(d*x + c)^4 + 1964*sin(d*x + c)^3 + 155
4*sin(d*x + c)^2 + 396*sin(d*x + c) - 21)/(a*(sin(d*x + c) + 1)^4))/d
```

3.44.9 Mupad [B] (verification not implemented)

Time = 10.35 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.98

$$\int \frac{\tan^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{245\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}-\frac{595\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{1}}{96}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-12a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+9a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\right.} - \frac{35\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a\,d}$$

input
```
int(tan(c + d*x)^7/(a + a*sin(c + d*x)),x)
```

output
```
((35*tan(c/2 + (d*x)/2))/64 + (35*tan(c/2 + (d*x)/2)^2)/32 - (245*tan(c/2
+ (d*x)/2)^3)/96 - (595*tan(c/2 + (d*x)/2)^4)/96 + (791*tan(c/2 + (d*x)/2)
^5)/192 + (231*tan(c/2 + (d*x)/2)^6)/16 - (25*tan(c/2 + (d*x)/2)^7)/16 + (
231*tan(c/2 + (d*x)/2)^8)/16 + (791*tan(c/2 + (d*x)/2)^9)/192 - (595*tan(c
/2 + (d*x)/2)^10)/96 - (245*tan(c/2 + (d*x)/2)^11)/96 + (35*tan(c/2 + (d*x
)/2)^12)/32 + (35*tan(c/2 + (d*x)/2)^13)/64)/(d*(a + 2*a*tan(c/2 + (d*x)/2
) - 5*a*tan(c/2 + (d*x)/2)^2 - 12*a*tan(c/2 + (d*x)/2)^3 + 9*a*tan(c/2 + (
d*x)/2)^4 + 30*a*tan(c/2 + (d*x)/2)^5 - 5*a*tan(c/2 + (d*x)/2)^6 - 40*a*ta
n(c/2 + (d*x)/2)^7 - 5*a*tan(c/2 + (d*x)/2)^8 + 30*a*tan(c/2 + (d*x)/2)^9
+ 9*a*tan(c/2 + (d*x)/2)^10 - 12*a*tan(c/2 + (d*x)/2)^11 - 5*a*tan(c/2 + (
d*x)/2)^12 + 2*a*tan(c/2 + (d*x)/2)^13 + a*tan(c/2 + (d*x)/2)^14)) - (35*a
tanh(tan(c/2 + (d*x)/2)))/(64*a*d)
```

3.45 $\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

3.45.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{16ad} - \frac{5\sec(c+dx)\tan(c+dx)}{16ad} + \frac{5\sec(c+dx)\tan^3(c+dx)}{24ad} - \frac{\sec(c+dx)\tan^5(c+dx)}{6ad} + \frac{\tan^6(c+dx)}{6ad}$$

output
```
5/16*arctanh(sin(d*x+c))/a/d-5/16*sec(d*x+c)*tan(d*x+c)/a/d+5/24*sec(d*x+c
)*tan(d*x+c)^3/a/d-1/6*sec(d*x+c)*tan(d*x+c)^5/a/d+1/6*tan(d*x+c)^6/a/d
```

3.45.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

$$\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{30\text{arctanh}(\sin(c+dx)) + \frac{3}{(1-\sin(c+dx))^2} - \frac{18}{1-\sin(c+dx)} + \frac{4}{(1+\sin(c+dx))^3} - \frac{21}{(1+\sin(c+dx))^2} + \frac{48}{1+\sin(c+dx)}}{96ad}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

output
```
(30*ArcTanh[Sin[c + d*x]] + 3/(1 - Sin[c + d*x])^2 - 18/(1 - Sin[c + d*x])
 + 4/(1 + Sin[c + d*x])^3 - 21/(1 + Sin[c + d*x])^2 + 48/(1 + Sin[c + d*x]
))/(96*a*d)
```

3.45.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3185, 3042, 3087, 15, 3091, 3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^5}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \sec^2(c+dx)\tan^5(c+dx)dx}{a}-\frac{\int \sec(c+dx)\tan^6(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^5dx}{a}-\frac{\int \sec(c+dx)\tan(c+dx)^6dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^5(c+dx)d\tan(c+dx)}{ad}-\frac{\int \sec(c+dx)\tan(c+dx)^6dx}{a}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^6(c+dx)}{6ad}-\frac{\int \sec(c+dx)\tan(c+dx)^6dx}{a}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^6(c+dx)}{6ad}-\frac{\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\int \sec(c+dx)\tan^4(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^6(c+dx)}{6ad}-\frac{\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\int \sec(c+dx)\tan(c+dx)^4dx}{a}$$

$$\Big\downarrow \; 3091$$

$$\frac{\tan^6(c+dx)}{6ad}-\frac{\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\int\sec(c+dx)\tan^2(c+dx)dx\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan^6(c+dx)}{6ad}-\frac{\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\int\sec(c+dx)\tan(c+dx)^2dx\right)}{a}$$

$$\Big\downarrow \; 3091$$

$$\frac{\tan^6(c+dx)}{6ad}-\frac{\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int\sec(c+dx)dx\right)\right)}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\tan^6(c+dx)}{6ad}-\frac{\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int\csc\left(c+dx+\frac{\pi}{2}\right)dx\right)\right)}{a}$$

$$\Big\downarrow \; 4257$$

$$\frac{\tan^6(c+dx)}{6ad}-\frac{\frac{\tan^5(c+dx)\sec(c+dx)}{6d}-\frac{5}{6}\left(\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{\text{arctanh}(\sin(c+dx))}{2d}\right)\right)}{a}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

output
```
Tan[c + d*x]^6/(6*a*d) - ((Sec[c + d*x]*Tan[c + d*x]^5)/(6*d) - (5*((Sec[c
 + d*x]*Tan[c + d*x]^3)/(4*d) - (3*(-1/2*ArcTanh[Sin[c + d*x]]/d + (Sec[c
+ d*x]*Tan[c + d*x])/(2*d)))/4))/6)/a
```

3.45. $\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

3.45.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.45. $\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx$

3.45.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{\frac{1}{32(\sin(dx+c)-1)^2}+\frac{3}{16(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{32}+\frac{1}{24(1+\sin(dx+c))^3}-\frac{7}{32(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}+\frac{5\ln(1+\sin(d}{32}}{da}$
default	$\frac{\frac{1}{32(\sin(dx+c)-1)^2}+\frac{3}{16(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{32}+\frac{1}{24(1+\sin(dx+c))^3}-\frac{7}{32(1+\sin(dx+c))^2}+\frac{1}{2+2\sin(dx+c)}+\frac{5\ln(1+\sin(d}{32}}{da}$
risch	$\frac{i(-8\,e^{7i(dx+c)}+2ie^{6i(dx+c)}+78\,e^{5i(dx+c)}+18ie^{8i(dx+c)}+33\,e^{9i(dx+c)}-2ie^{4i(dx+c)}-8\,e^{3i(dx+c)}-18ie^{2i(dx+c)}+33\,e^{i(dx}}{24(e^{i(dx+c)}+i)^6(-i+e^{i(dx+c)})^4da}$

input
```
int(tan(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/32/(sin(d*x+c)-1)^2+3/16/(sin(d*x+c)-1)-5/32*ln(sin(d*x+c)-1)+1/2
4/(1+sin(d*x+c))^3-7/32/(1+sin(d*x+c))^2+1/2/(1+sin(d*x+c))+5/32*ln(1+sin(
d*x+c)))
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.39

$$\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{66\,\cos\left(dx+c\right)^4-70\,\cos\left(dx+c\right)^2+15\,\left(\cos\left(dx+c\right)^4\sin\left(dx+c\right)+\cos\left(dx+c\right)^4\right)\log\left(\sin\left(dx+c\right)+1\right)}{96\,\left(ad\cos\left(dx+c\right)^4\mathrm{s}\right.}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/96*(66*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 15*(cos(d*x + c)^4*sin(d*x +
 c) + cos(d*x + c)^4)*log(sin(d*x + c) + 1) - 15*(cos(d*x + c)^4*sin(d*x +
 c) + cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(9*cos(d*x + c)^2 - 2)*si
n(d*x + c) + 20)/(a*d*cos(d*x + c)^4*sin(d*x + c) + a*d*cos(d*x + c)^4)
```

3.45.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.23

$$\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{2\left(33\sin(dx+c)^4+9\sin(dx+c)^3-31\sin(dx+c)^2-7\sin(dx+c)+8\right)}{a\sin(dx+c)^5+a\sin(dx+c)^4-2a\sin(dx+c)^3-2a\sin(dx+c)^2+a\sin(dx+c)+a}+\frac{15\log(\sin(dx+c)+1)}{a}-\frac{15\log(\sin(dx+c)-1)}{a}}{96\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/96*(2*(33*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 31*sin(d*x + c)^2 - 7*sin(
d*x + c) + 8)/(a*sin(d*x + c)^5 + a*sin(d*x + c)^4 - 2*a*sin(d*x + c)^3 -
2*a*sin(d*x + c)^2 + a*sin(d*x + c) + a) + 15*log(sin(d*x + c) + 1)/a - 15
*log(sin(d*x + c) - 1)/a)/d
```

3.45.8 Giac [A] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09

$$\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{30\log(|\sin(dx+c)+1|)}{a}-\frac{30\log(|\sin(dx+c)-1|)}{a}+\frac{3\left(15\sin(dx+c)^2-18\sin(dx+c)+5\right)}{a(\sin(dx+c)-1)^2}-\frac{55\sin(dx+c)^3+69\sin(dx+c)^2+15\sin(dx+c)-7}{a(\sin(dx+c)+1)^3}}{192\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/192*(30*log(abs(sin(d*x + c) + 1))/a - 30*log(abs(sin(d*x + c) - 1))/a +
 3*(15*sin(d*x + c)^2 - 18*sin(d*x + c) + 5)/(a*(sin(d*x + c) - 1)^2) - (5
5*sin(d*x + c)^3 + 69*sin(d*x + c)^2 + 15*sin(d*x + c) - 7)/(a*(sin(d*x +
c) + 1)^3))/d
```

3.45.9 Mupad [B] (verification not implemented)

Time = 10.00 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.65

$$\int \frac{\tan^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,a\,d} - \frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{8}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{4}-\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{55\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{12}+\frac{3\tan\left(\ldots\right.}{}}{d\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-8\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\ldots\right.}$$

input
```
int(tan(c + d*x)^5/(a + a*sin(c + d*x)),x)
```

output
```
(5*atanh(tan(c/2 + (d*x)/2)))/(8*a*d) - ((5*tan(c/2 + (d*x)/2))/8 + (5*tan
(c/2 + (d*x)/2)^2)/4 - (5*tan(c/2 + (d*x)/2)^3)/3 - (55*tan(c/2 + (d*x)/2)
^4)/12 + (3*tan(c/2 + (d*x)/2)^5)/4 - (55*tan(c/2 + (d*x)/2)^6)/12 - (5*ta
n(c/2 + (d*x)/2)^7)/3 + (5*tan(c/2 + (d*x)/2)^8)/4 + (5*tan(c/2 + (d*x)/2)
^9)/8)/(d*(a + 2*a*tan(c/2 + (d*x)/2) - 3*a*tan(c/2 + (d*x)/2)^2 - 8*a*tan
(c/2 + (d*x)/2)^3 + 2*a*tan(c/2 + (d*x)/2)^4 + 12*a*tan(c/2 + (d*x)/2)^5 +
 2*a*tan(c/2 + (d*x)/2)^6 - 8*a*tan(c/2 + (d*x)/2)^7 - 3*a*tan(c/2 + (d*x)
/2)^8 + 2*a*tan(c/2 + (d*x)/2)^9 + a*tan(c/2 + (d*x)/2)^10))
```

3.46 $\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$

3.46.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{3\text{arctanh}(\sin(c+dx))}{8ad} + \frac{3\sec(c+dx)\tan(c+dx)}{8ad} - \frac{\sec(c+dx)\tan^3(c+dx)}{4ad} + \frac{\tan^4(c+dx)}{4ad}$$

output
```
-3/8*arctanh(sin(d*x+c))/a/d+3/8*sec(d*x+c)*tan(d*x+c)/a/d-1/4*sec(d*x+c)*
tan(d*x+c)^3/a/d+1/4*tan(d*x+c)^4/a/d
```

3.46.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.66

$$\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{3\text{arctanh}(\sin(c+dx)) + \frac{1}{-1+\sin(c+dx)} - \frac{1}{(1+\sin(c+dx))^2} + \frac{4}{1+\sin(c+dx)}}{8ad}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
-1/8*(3*ArcTanh[Sin[c + d*x]] + (-1 + Sin[c + d*x])^(-1) - (1 + Sin[c + d*
x])^(-2) + 4/(1 + Sin[c + d*x]))/(a*d)
```

3.46.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3185, 3042, 3087, 15, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \sec^2(c+dx)\tan^3(c+dx)dx}{a}-\frac{\int \sec(c+dx)\tan^4(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^3dx}{a}-\frac{\int \sec(c+dx)\tan(c+dx)^4dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^3(c+dx)d\tan(c+dx)}{ad}-\frac{\int \sec(c+dx)\tan(c+dx)^4dx}{a}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^4(c+dx)}{4ad}-\frac{\int \sec(c+dx)\tan(c+dx)^4dx}{a}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^4(c+dx)}{4ad}-\frac{\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\int \sec(c+dx)\tan^2(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^4(c+dx)}{4ad}-\frac{\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\int \sec(c+dx)\tan(c+dx)^2dx}{a}$$

$$\Big\downarrow \text{3091}$$

$$\frac{\tan^4(c+dx)}{4ad}-\frac{\frac{\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int \sec(c+dx)dx\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tan^4(c+dx)}{4ad} - \frac{\frac{\tan^3(c+dx)\sec(c+dx)}{4d} - \frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d} - \frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx\right)}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tan^4(c+dx)}{4ad} - \frac{\frac{\tan^3(c+dx)\sec(c+dx)}{4d} - \frac{3}{4}\left(\frac{\tan(c+dx)\sec(c+dx)}{2d} - \frac{\operatorname{arctanh}(\sin(c+dx))}{2d}\right)}{a}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
Tan[c + d*x]^4/(4*a*d) - ((Sec[c + d*x]*Tan[c + d*x]^3)/(4*d) - (3*(-1/2*A
rcTanh[Sin[c + d*x]]/d + (Sec[c + d*x]*Tan[c + d*x])/(2*d)))/4)/a
```

3.46.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.46.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{2(1+\sin(dx+c))}-\frac{3\ln(1+\sin(dx+c))}{16}-\frac{1}{8(\sin(dx+c)-1)}+\frac{3\ln(\sin(dx+c)-1)}{16}}{da}$	67
default	$\frac{\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{2(1+\sin(dx+c))}-\frac{3\ln(1+\sin(dx+c))}{16}-\frac{1}{8(\sin(dx+c)-1)}+\frac{3\ln(\sin(dx+c)-1)}{16}}{da}$	67
risch	$-\frac{i(2i e^{4i(dx+c)}-2\,e^{3i(dx+c)}-2i e^{2i(dx+c)}+5\,e^{5i(dx+c)}+5\,e^{i(dx+c)})}{4(e^{i(dx+c)}+i)^4(-i+e^{i(dx+c)})^2 da}+\frac{3\ln(-i+e^{i(dx+c)})}{8ad}-\frac{3\ln(e^{i(dx+c)}+i)}{8ad}$	139

input
```
int(tan(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/8/(1+sin(d*x+c))^2-1/2/(1+sin(d*x+c))-3/16*ln(1+sin(d*x+c))-1/8/(
sin(d*x+c)-1)+3/16*ln(sin(d*x+c)-1))
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{10\,\cos(dx+c)^2+3\left(\cos(dx+c)^2\sin(dx+c)+\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left(\cos(dx+c)^2\right.}{16\left(ad\cos(dx+c)^2\sin(dx+c)+ad\cos\right.}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/16*(10*cos(d*x + c)^2 + 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2
)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^2*sin(d*x + c) + cos(d*x + c)^2)
*log(-sin(d*x + c) + 1) - 2*sin(d*x + c) - 6)/(a*d*cos(d*x + c)^2*sin(d*x
+ c) + a*d*cos(d*x + c)^2)
```

3.46.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\tan^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09

$$\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{2\left(5\sin(dx+c)^2+\sin(dx+c)-2\right)}{a\sin(dx+c)^3+a\sin(dx+c)^2-a\sin(dx+c)-a}+\frac{3\log(\sin(dx+c)+1)}{a}-\frac{3\log(\sin(dx+c)-1)}{a}}{16\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/16*(2*(5*sin(d*x + c)^2 + sin(d*x + c) - 2)/(a*sin(d*x + c)^3 + a*sin(d
*x + c)^2 - a*sin(d*x + c) - a) + 3*log(sin(d*x + c) + 1)/a - 3*log(sin(d*
x + c) - 1)/a)/d
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17

$$\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{6\log(|\sin(dx+c)+1|)}{a}-\frac{6\log(|\sin(dx+c)-1|)}{a}+\frac{2(3\sin(dx+c)-1)}{a(\sin(dx+c)-1)}-\frac{9\sin(dx+c)^2+2\sin(dx+c)-3}{a(\sin(dx+c)+1)^2}}{32\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/32*(6*log(abs(sin(d*x + c) + 1))/a - 6*log(abs(sin(d*x + c) - 1))/a + 2
*(3*sin(d*x + c) - 1)/(a*(sin(d*x + c) - 1)) - (9*sin(d*x + c)^2 + 2*sin(d
*x + c) - 3)/(a*(sin(d*x + c) + 1)^2))/d
```

3.46.9 Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.10

$$\int \frac{\tan^3(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a\tan\right.}$$
$$-\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4\,a\,d}$$

input
```
int(tan(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

output
```
((3*tan(c/2 + (d*x)/2))/4 + (3*tan(c/2 + (d*x)/2)^2)/2 - tan(c/2 + (d*x)/2
)^3/2 + (3*tan(c/2 + (d*x)/2)^4)/2 + (3*tan(c/2 + (d*x)/2)^5)/4)/(d*(a + 2
*a*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2 - 4*a*tan(c/2 + (d*x)/2)^3
- a*tan(c/2 + (d*x)/2)^4 + 2*a*tan(c/2 + (d*x)/2)^5 + a*tan(c/2 + (d*x)/2)
^6)) - (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d)
```

3.47 $\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx$

3.47.1 Optimal result

Integrand size = 19, antiderivative size = 37

$$\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{2ad} + \frac{1}{2d(a+a\sin(c+dx))}$$

output
```
1/2*arctanh(sin(d*x+c))/a/d+1/2/d/(a+a*sin(d*x+c))
```

3.47.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx)) + \frac{1}{1+\sin(c+dx)}}{2ad}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
(ArcTanh[Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(2*a*d)
```

3.47.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {3042, 3185, 3042, 3086, 15, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3185}$$
$$\frac{\int \sec^2(c+dx)\tan(c+dx)dx}{a}-\frac{\int \sec(c+dx)\tan^2(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sec(c+dx)^2\tan(c+dx)dx}{a}-\frac{\int \sec(c+dx)\tan(c+dx)^2dx}{a}$$
$$\Big\downarrow \text{3086}$$
$$\frac{\int \sec(c+dx)d\sec(c+dx)}{ad}-\frac{\int \sec(c+dx)\tan(c+dx)^2dx}{a}$$
$$\Big\downarrow \text{15}$$
$$\frac{\sec^2(c+dx)}{2ad}-\frac{\int \sec(c+dx)\tan(c+dx)^2dx}{a}$$
$$\Big\downarrow \text{3091}$$
$$\frac{\sec^2(c+dx)}{2ad}-\frac{\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int \sec(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sec^2(c+dx)}{2ad}-\frac{\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{1}{2}\int \csc\left(c+dx+\frac{\pi}{2}\right)dx}{a}$$
$$\Big\downarrow \text{4257}$$
$$\frac{\sec^2(c+dx)}{2ad}-\frac{\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{\text{arctanh}(\sin(c+dx))}{2d}}{a}$$

input
```
Int[Tan[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
Sec[c + d*x]^2/(2*a*d) - (-1/2*ArcTanh[Sin[c + d*x]]/d + (Sec[c + d*x]*Tan
[c + d*x])/(2*d))/a
```

3.47.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.47.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

method	result	size
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{4}+\frac{1}{2+2\sin(dx+c)}+\frac{\ln(1+\sin(dx+c))}{4}}{da}$	43
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{4}+\frac{1}{2+2\sin(dx+c)}+\frac{\ln(1+\sin(dx+c))}{4}}{da}$	43
risch	$\frac{i\,e^{i(dx+c)}}{da\left(e^{i(dx+c)}+i\right)^2}-\frac{\ln\left(-i+e^{i(dx+c)}\right)}{2ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{2ad}$	76

input
```
int(tan(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-1/4*ln(sin(d*x+c)-1)+1/2/(1+sin(d*x+c))+1/4*ln(1+sin(d*x+c)))
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{(\sin(dx+c)+1)\log(\sin(dx+c)+1)-(\sin(dx+c)+1)\log(-\sin(dx+c)+1)+2}{4\,(ad\sin(dx+c)+ad)}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*((sin(d*x + c) + 1)*log(sin(d*x + c) + 1) - (sin(d*x + c) + 1)*log(-si
n(d*x + c) + 1) + 2)/(a*d*sin(d*x + c) + a*d)
```

3.47.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\tan(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)/(sin(c + d*x) + 1), x)/a
```

3.47.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

$$\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{\log(\sin(dx+c)+1)}{a} - \frac{\log(\sin(dx+c)-1)}{a} + \frac{2}{a\sin(dx+c)+a}}{4\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*(log(sin(d*x + c) + 1)/a - log(sin(d*x + c) - 1)/a + 2/(a*sin(d*x + c)
 + a))/d
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)-1|)}{a} - \frac{\sin(dx+c)-1}{a(\sin(dx+c)+1)}}{4\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c) - 1))/a - (sin(d*
x + c) - 1)/(a*(sin(d*x + c) + 1)))/d
```

3.47.9 Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.65

$$\int \frac{\tan(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

input
```
int(tan(c + d*x)/(a + a*sin(c + d*x)),x)
```

output
```
atanh(tan(c/2 + (d*x)/2))/(a*d) - tan(c/2 + (d*x)/2)/(d*(a + 2*a*tan(c/2 +
 (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))
```

3.48 $\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx$

3.48.1 Optimal result

Integrand size = 19, antiderivative size = 32

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}$$

output
```
ln(sin(d*x+c))/a/d-ln(1+sin(d*x+c))/a/d
```

3.48.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\log(\sin(c+dx))}{ad}-\frac{\log(1+\sin(c+dx))}{ad}$$

input
```
Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
Log[Sin[c + d*x]]/(a*d) - Log[1 + Sin[c + d*x]]/(a*d)
```

3.48.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3186, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(c+dx)(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \; 3186$$

$$\frac{\int \frac{\csc(c+dx)}{a(\sin(c+dx)a+a)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 47$$

$$\frac{\frac{\int \frac{\csc(c+dx)}{a}d(a\sin(c+dx))}{a}-\frac{\int \frac{1}{\sin(c+dx)a+a}d(a\sin(c+dx))}{a}}{d}$$

$$\Big\downarrow \; 14$$

$$\frac{\frac{\log(a\sin(c+dx))}{a}-\frac{\int \frac{1}{\sin(c+dx)a+a}d(a\sin(c+dx))}{a}}{d}$$

$$\Big\downarrow \; 16$$

$$\frac{\frac{\log(a\sin(c+dx))}{a}-\frac{\log(a\sin(c+dx)+a)}{a}}{d}$$

input
```
Int[Cot[c + d*x]/(a + a*Sin[c + d*x]),x]
```

output
```
(Log[a*Sin[c + d*x]]/a - Log[a + a*Sin[c + d*x]]/a)/d
```

3.48.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.48.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{-\ln(1+\sin(dx+c))+\ln(\sin(dx+c))}{da}$	27
default	$\frac{-\ln(1+\sin(dx+c))+\ln(\sin(dx+c))}{da}$	27
risch	$-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{ad}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{da}$	42

input
```
int(cot(d*x+c)/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(-ln(1+sin(d*x+c))+ln(sin(d*x+c)))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-\log\left(\sin\left(dx+c\right)+1\right)}{ad}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(log(1/2*sin(d*x + c)) - log(sin(d*x + c) + 1))/(a*d)
```

3.48.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot\left(c+dx\right)}{\sin\left(c+dx\right)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)/(sin(c + d*x) + 1), x)/a
```

3.48.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{\log(\sin(dx+c)+1)}{a}-\frac{\log(\sin(dx+c))}{a}}{d}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(log(sin(d*x + c) + 1)/a - log(sin(d*x + c))/a)/d
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{\log(|\sin(dx+c)+1|)}{a} - \frac{\log(|\sin(dx+c)|)}{a}}{d}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-(log(abs(sin(d*x + c) + 1))/a - log(abs(sin(d*x + c)))/a)/d
```

3.48.9 Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right) - 2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a\,d}$$

input
```
int(cot(c + d*x)/(a + a*sin(c + d*x)),x)
```

output
```
(log(tan(c/2 + (d*x)/2)) - 2*log(tan(c/2 + (d*x)/2) + 1))/(a*d)
```

3.49 $\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx$

3.49.1 Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

output
```
csc(d*x+c)/a/d-1/2*csc(d*x+c)^2/a/d
```

3.49.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.75

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{(-2+\csc(c+dx))\csc(c+dx)}{2ad}$$

input
```
Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*((-2 + Csc[c + d*x])*Csc[c + d*x])/(a*d)
```

3.49.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3185, 3042, 25, 3086, 15, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^3(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{ 3185}$$

$$\frac{\int \cot(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int \cot(c+dx)\csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^2\tan\left(c+dx-\frac{\pi}{2}\right)dx}{a}-\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)\tan\left(\frac{1}{2}(2c-\pi)+dx\right)dx}{a}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)dx}{a}$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\int 1d\csc(c+dx)}{ad}-\frac{\int \csc(c+dx)d\csc(c+dx)}{ad}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{\int 1d\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\csc(c+dx)}{ad}-\frac{\csc^2(c+dx)}{2ad}$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x]),x]
```

output
```
Csc[c + d*x]/(a*d) - Csc[c + d*x]^2/(2*a*d)
```

3.49.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.49.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}}{da}$	27
default	$\frac{\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}}{da}$	27
risch	$\frac{2i\left(-i\mathrm{e}^{2i(dx+c)}+\mathrm{e}^{3i(dx+c)}-\mathrm{e}^{i(dx+c)}\right)}{da\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}$	56

input
```
int(cot(d*x+c)^3/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/sin(d*x+c)-1/2/sin(d*x+c)^2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{2\,\sin\left(dx+c\right)-1}{2\left(ad\cos\left(dx+c\right)^2-ad\right)}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*sin(d*x + c) - 1)/(a*d*cos(d*x + c)^2 - a*d)
```

3.49.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**3/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**3/(sin(c + d*x) + 1), x)/a
```

3.49.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\,\sin\left(dx+c\right)-1}{2\,ad\sin\left(dx+c\right)^2}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{2\,\sin\left(dx+c\right)-1}{2\,ad\sin\left(dx+c\right)^2}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*sin(d*x + c) - 1)/(a*d*sin(d*x + c)^2)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.72

$$\int \frac{\cot^3(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\sin\left(c+d\,x\right)-\frac{1}{2}}{a\,d\sin\left(c+d\,x\right)^2}$$

input
```
int(cot(c + d*x)^3/(a + a*sin(c + d*x)),x)
```

output
```
(sin(c + d*x) - 1/2)/(a*d*sin(c + d*x)^2)
```

3.50 $\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx$

3.50.1 Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\cot^4(c+dx)}{4ad} - \frac{\csc(c+dx)}{ad} + \frac{\csc^3(c+dx)}{3ad}$$

output
```
-1/4*cot(d*x+c)^4/a/d-csc(d*x+c)/a/d+1/3*csc(d*x+c)^3/a/d
```

3.50.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.59

$$\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{(-1+\csc(c+dx))^3(5+3\csc(c+dx))}{12ad}$$

input
```
Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

output
```
-1/12*((-1 + Csc[c + d*x])^3*(5 + 3*Csc[c + d*x]))/(a*d)
```

3.50.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3185, 3042, 25, 3086, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^5(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \cot^3(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int \cot^3(c+dx)\csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^2\tan\left(c+dx-\frac{\pi}{2}\right)^3dx}{a}-\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3dx}{a}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3dx}{a}$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \left(\csc^2(c+dx)-1\right)d\csc(c+dx)}{ad}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)}{ad}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^3dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)}{ad}-\frac{\int -\cot^3(c+dx)d(-\cot(c+dx))}{ad}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{1}{3}\csc^3(c+dx)-\csc(c+dx)}{ad}-\frac{\cot^4(c+dx)}{4ad}$$

input
```
Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x]),x]
```

output
```
-1/4*Cot[c + d*x]^4/(a*d) + (-Csc[c + d*x] + Csc[c + d*x]^3/3)/(a*d)
```

3.50.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.50.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{1}{2\sin(dx+c)^2}+\frac{1}{3\sin(dx+c)^3}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{\sin(dx+c)}}{da}$	49
default	$\frac{\frac{1}{2\sin(dx+c)^2}+\frac{1}{3\sin(dx+c)^3}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{\sin(dx+c)}}{da}$	49
risch	$-\frac{2i\left(-3i e^{6i(dx+c)}+3\,e^{7i(dx+c)}-5\,e^{5i(dx+c)}-3i e^{2i(dx+c)}+5\,e^{3i(dx+c)}-3\,e^{i(dx+c)}\right)}{3da\left(e^{2i(dx+c)}-1\right)^4}$	92

input
```
int(cot(d*x+c)^5/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/2/sin(d*x+c)^2+1/3/sin(d*x+c)^3-1/4/sin(d*x+c)^4-1/sin(d*x+c))
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

$$\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{6\,\cos\left(dx+c\right)^2-4\left(3\,\cos\left(dx+c\right)^2-2\right)\sin\left(dx+c\right)-3}{12\left(ad\cos\left(dx+c\right)^4-2\,ad\cos\left(dx+c\right)^2+ad\right)}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(6*cos(d*x + c)^2 - 4*(3*cos(d*x + c)^2 - 2)*sin(d*x + c) - 3)/(a*d*
cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2 + a*d)
```

3.50.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**5/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**5/(sin(c + d*x) + 1), x)/a
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{12\,\sin\left(dx+c\right)^3-6\,\sin\left(dx+c\right)^2-4\,\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin
(d*x + c)^4)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{12\,\sin\left(dx+c\right)^3-6\,\sin\left(dx+c\right)^2-4\,\sin\left(dx+c\right)+3}{12\,ad\sin\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/12*(12*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 4*sin(d*x + c) + 3)/(a*d*sin
(d*x + c)^4)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 5.69 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int \frac{\cot^5(c+dx)}{a+a\sin(c+dx)}\,dx=\frac{-\sin\left(c+d\,x\right)^3+\frac{\sin(c+d\,x)^2}{2}+\frac{\sin(c+d\,x)}{3}-\frac{1}{4}}{a\,d\sin\left(c+d\,x\right)^4}$$

input
```
int(cot(c + d*x)^5/(a + a*sin(c + d*x)),x)
```

output
```
(sin(c + d*x)/3 + sin(c + d*x)^2/2 - sin(c + d*x)^3 - 1/4)/(a*d*sin(c + d*
x)^4)
```

3.51 $\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$

3.51.1 Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{\cot^6(c+dx)}{6ad}+\frac{\csc(c+dx)}{ad}-\frac{2\csc^3(c+dx)}{3ad}+\frac{\csc^5(c+dx)}{5ad}$$

output
```
-1/6*cot(d*x+c)^6/a/d+csc(d*x+c)/a/d-2/3*csc(d*x+c)^3/a/d+1/5*csc(d*x+c)^5
/a/d
```

3.51.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{\csc^6(c+dx)(-15\cos(4(c+dx))+78\sin(c+dx)-5(5+7\sin(3(c+dx))-3\sin(5(c+dx))))}{240ad}$$

input
```
Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

output
```
(Csc[c + d*x]^6*(-15*Cos[4*(c + d*x)] + 78*Sin[c + d*x] - 5*(5 + 7*Sin[3*(
c + d*x)] - 3*Sin[5*(c + d*x)])))/(240*a*d)
```

3.51.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3185, 3042, 25, 3086, 210, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^7(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^7(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \cot^5(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int \cot^5(c+dx)\csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^2\tan\left(c+dx-\frac{\pi}{2}\right)^5dx}{a}-\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^5dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^5dx}{a}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^5dx}{a}$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \left(\csc^2(c+dx)-1\right)^2d\csc(c+dx)}{ad}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^5dx}{a}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(\csc^4(c+dx)-2\csc^2(c+dx)+1\right)d\csc(c+dx)}{ad}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^5dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{5}\csc^5(c+dx)-\frac{2}{3}\csc^3(c+dx)+\csc(c+dx)}{ad}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^5dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\frac{1}{5}\csc^5(c+dx)-\frac{2}{3}\csc^3(c+dx)+\csc(c+dx)}{ad}-\frac{\int -\cot^5(c+dx)d(-\cot(c+dx))}{ad}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{1}{5}\csc^5(c+dx)-\frac{2}{3}\csc^3(c+dx)+\csc(c+dx)}{ad}-\frac{\cot^6(c+dx)}{6ad}$$

input
```
Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x]),x]
```

output
```
-1/6*Cot[c + d*x]^6/(a*d) + (Csc[c + d*x] - (2*Csc[c + d*x]^3)/3 + Csc[c +
 d*x]^5/5)/(a*d)
```

3.51.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.51.4 Maple [A] (verified)

Time = 2.72 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{\frac{1}{5\sin(dx+c)^5}-\frac{2}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^4}+\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}-\frac{1}{6\sin(dx+c)^6}}{da}$
default	$\frac{\frac{1}{5\sin(dx+c)^5}-\frac{2}{3\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^4}+\frac{1}{\sin(dx+c)}-\frac{1}{2\sin(dx+c)^2}-\frac{1}{6\sin(dx+c)^6}}{da}$
risch	$\frac{2i\left(-15i e^{10i(dx+c)}+15\,e^{11i(dx+c)}-35\,e^{9i(dx+c)}-50i e^{6i(dx+c)}+78\,e^{7i(dx+c)}-78\,e^{5i(dx+c)}-15i e^{2i(dx+c)}+35\,e^{3i(dx+c)}-\right.}{15da\left(e^{2i(dx+c)}-1\right)^6}$

input
```
int(cot(d*x+c)^7/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/5/sin(d*x+c)^5-2/3/sin(d*x+c)^3+1/2/sin(d*x+c)^4+1/sin(d*x+c)-1/2
/sin(d*x+c)^2-1/6/sin(d*x+c)^6)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.41

$$\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{15\,\cos\left(dx+c\right)^4-15\,\cos\left(dx+c\right)^2-2\left(15\,\cos\left(dx+c\right)^4-20\,\cos\left(dx+c\right)^2+8\right)\sin\left(dx+c\right)+5}{30\left(ad\cos\left(dx+c\right)^6-3\,ad\cos\left(dx+c\right)^4+3\,ad\cos\left(dx+c\right)^2-ad\right)}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/30*(15*cos(d*x + c)^4 - 15*cos(d*x + c)^2 - 2*(15*cos(d*x + c)^4 - 20*co
s(d*x + c)^2 + 8)*sin(d*x + c) + 5)/(a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x +
c)^4 + 3*a*d*cos(d*x + c)^2 - a*d)
```

3.51.6 Sympy [F]

$$\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot^7(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**7/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**7/(sin(c + d*x) + 1), x)/a
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{30\sin(dx+c)^5-15\sin(dx+c)^4-20\sin(dx+c)^3+15\sin(dx+c)^2+6\sin(dx+c)-5}{30\,ad\sin(dx+c)^6}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d
*x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97

$$\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{30\sin(dx+c)^5 - 15\sin(dx+c)^4 - 20\sin(dx+c)^3 + 15\sin(dx+c)^2 + 6\sin(dx+c) - 5}{30\,ad\sin(dx+c)^6}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/30*(30*sin(d*x + c)^5 - 15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 15*sin(d
*x + c)^2 + 6*sin(d*x + c) - 5)/(a*d*sin(d*x + c)^6)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 6.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int \frac{\cot^7(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\sin(c+d\,x)^5 - \frac{\sin(c+d\,x)^4}{2} - \frac{2\sin(c+d\,x)^3}{3} + \frac{\sin(c+d\,x)^2}{2} + \frac{\sin(c+d\,x)}{5} - \frac{1}{6}}{a\,d\sin(c+d\,x)^6}$$

input
```
int(cot(c + d*x)^7/(a + a*sin(c + d*x)),x)
```

output
```
(sin(c + d*x)/5 + sin(c + d*x)^2/2 - (2*sin(c + d*x)^3)/3 - sin(c + d*x)^4
/2 + sin(c + d*x)^5 - 1/6)/(a*d*sin(c + d*x)^6)
```

3.52 $\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx$

3.52.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\cot^8(c+dx)}{8ad}-\frac{\csc(c+dx)}{ad}+\frac{\csc^3(c+dx)}{ad}-\frac{3\csc^5(c+dx)}{5ad}+\frac{\csc^7(c+dx)}{7ad}$$

output
```
-1/8*cot(d*x+c)^8/a/d-csc(d*x+c)/a/d+csc(d*x+c)^3/a/d-3/5*csc(d*x+c)^5/a/d
+1/7*csc(d*x+c)^7/a/d
```

3.52.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\csc^8(c+dx)(-245\cos(2(c+dx))-35\cos(6(c+dx))-513\sin(c+dx)+371\sin(3(c+dx))-105\sin(5}{2240ad}$$

input
```
Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

output
```
(Csc[c + d*x]^8*(-245*Cos[2*(c + d*x)] - 35*Cos[6*(c + d*x)] - 513*Sin[c +
 d*x] + 371*Sin[3*(c + d*x)] - 105*Sin[5*(c + d*x)] + 35*Sin[7*(c + d*x)])
)/(2240*a*d)
```

3.52.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.80, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3185, 3042, 25, 3086, 210, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^9(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^9(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \cot^7(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int \cot^7(c+dx)\csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)^2\tan\left(c+dx-\frac{\pi}{2}\right)^7dx}{a}-\frac{\int -\sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^7dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^7dx}{a}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^7dx}{a}$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \left(\csc^2(c+dx)-1\right)^3d\csc(c+dx)}{ad}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^7dx}{a}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(\csc^6(c+dx)-3\csc^4(c+dx)+3\csc^2(c+dx)-1\right)d\csc(c+dx)}{ad}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^7dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{7}\csc^7(c+dx)-\frac{3}{5}\csc^5(c+dx)+\csc^3(c+dx)-\csc(c+dx)}{ad}-\frac{\int \sec\left(\frac{1}{2}(2c-\pi)+dx\right)^2\tan\left(\frac{1}{2}(2c-\pi)+dx\right)^7dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\frac{1}{7}\csc^7(c+dx)-\frac{3}{5}\csc^5(c+dx)+\csc^3(c+dx)-\csc(c+dx)}{ad}-\frac{\int -\cot^7(c+dx)d(-\cot(c+dx))}{ad}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{1}{7}\csc^7(c+dx)-\frac{3}{5}\csc^5(c+dx)+\csc^3(c+dx)-\csc(c+dx)}{ad}-\frac{\cot^8(c+dx)}{8ad}$$

input
```
Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x]),x]
```

output
```
-1/8*Cot[c + d*x]^8/(a*d) + (-Csc[c + d*x] + Csc[c + d*x]^3 - (3*Csc[c + d
*x]^5)/5 + Csc[c + d*x]^7/7)/(a*d)
```

3.52.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.52.4 Maple [A] (verified)

Time = 5.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{\frac{1}{2\sin(dx+c)^6}-\frac{1}{\sin(dx+c)}+\frac{1}{7\sin(dx+c)^7}+\frac{1}{\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^2}-\frac{1}{8\sin(dx+c)^8}-\frac{3}{5\sin(dx+c)^5}-\frac{3}{4\sin(dx+c)^4}}{da}$
default	$\frac{\frac{1}{2\sin(dx+c)^6}-\frac{1}{\sin(dx+c)}+\frac{1}{7\sin(dx+c)^7}+\frac{1}{\sin(dx+c)^3}+\frac{1}{2\sin(dx+c)^2}-\frac{1}{8\sin(dx+c)^8}-\frac{3}{5\sin(dx+c)^5}-\frac{3}{4\sin(dx+c)^4}}{da}$
risch	$-\frac{2i\left(-35i e^{14i(dx+c)}+35\,e^{15i(dx+c)}-105\,e^{13i(dx+c)}-245i e^{10i(dx+c)}+371\,e^{11i(dx+c)}-513\,e^{9i(dx+c)}-245i e^{6i(dx+c)}+51}{35da\left(e^{2i(dx+c)}-1\right)^8}$

input
```
int(cot(d*x+c)^9/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d/a*(1/2/sin(d*x+c)^6-1/sin(d*x+c)+1/7/sin(d*x+c)^7+1/sin(d*x+c)^3+1/2/s
in(d*x+c)^2-1/8/sin(d*x+c)^8-3/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4)
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.51

$$\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx =$$

$$-\frac{140\,\cos\left(dx+c\right)^6-210\,\cos\left(dx+c\right)^4+140\,\cos\left(dx+c\right)^2-8\left(35\,\cos\left(dx+c\right)^6-70\,\cos\left(dx+c\right)^4+56}{280\left(ad\cos\left(dx+c\right)^8-4\,ad\cos\left(dx+c\right)^6+6\,ad\cos\left(dx+c\right)^4-4\,ad\cos\left(dx\right.\right.}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/280*(140*cos(d*x + c)^6 - 210*cos(d*x + c)^4 + 140*cos(d*x + c)^2 - 8*(
35*cos(d*x + c)^6 - 70*cos(d*x + c)^4 + 56*cos(d*x + c)^2 - 16)*sin(d*x +
c) - 35)/(a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^6 + 6*a*d*cos(d*x + c)^4
 - 4*a*d*cos(d*x + c)^2 + a*d)
```

3.52.6 Sympy [F]

$$\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot^9(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**9/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**9/(sin(c + d*x) + 1), x)/a
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{280 \sin(dx+c)^7 - 140 \sin(dx+c)^6 - 280 \sin(dx+c)^5 + 210 \sin(dx+c)^4 + 168 \sin(dx+c)^3 - 140}{280\,ad\sin(dx+c)^8}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210
*sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c
) + 35)/(a*d*sin(d*x + c)^8)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.02

$$\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{280\sin(dx+c)^7 - 140\sin(dx+c)^6 - 280\sin(dx+c)^5 + 210\sin(dx+c)^4 + 168\sin(dx+c)^3 - 140}{280\,ad\sin(dx+c)^8}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/280*(280*sin(d*x + c)^7 - 140*sin(d*x + c)^6 - 280*sin(d*x + c)^5 + 210
*sin(d*x + c)^4 + 168*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 40*sin(d*x + c
) + 35)/(a*d*sin(d*x + c)^8)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{\cot^9(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{-\sin(c+d\,x)^7 + \frac{\sin(c+d\,x)^6}{2} + \sin(c+d\,x)^5 - \frac{3\sin(c+d\,x)^4}{4} - \frac{3\sin(c+d\,x)^3}{5} + \frac{\sin(c+d\,x)^2}{2} + \frac{\sin(c+d\,x)}{7} - \frac{1}{8}}{a\,d\sin(c+d\,x)^8}$$

input
```
int(cot(c + d*x)^9/(a + a*sin(c + d*x)),x)
```

output
```
(sin(c + d*x)/7 + sin(c + d*x)^2/2 - (3*sin(c + d*x)^3)/5 - (3*sin(c + d*x
)^4)/4 + sin(c + d*x)^5 + sin(c + d*x)^6/2 - sin(c + d*x)^7 - 1/8)/(a*d*si
n(c + d*x)^8)
```

3.53 $\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$

3.53.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{ad} + \frac{3\sec^5(c+dx)}{5ad} - \frac{\sec^7(c+dx)}{7ad} + \frac{\tan^7(c+dx)}{7ad}$$

output
```
sec(d*x+c)/a/d-sec(d*x+c)^3/a/d+3/5*sec(d*x+c)^5/a/d-1/7*sec(d*x+c)^7/a/d+
1/7*tan(d*x+c)^7/a/d
```

3.53.2 Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.74

$$\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{\sec^5(c+dx)(2912 - 7620\cos(c+dx) + 3760\cos(2(c+dx)) - 3810\cos(3(c+dx)) + 1440\cos(4(c+dx))}{}$$

input
```
Integrate[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

output
```
(Sec[c + d*x]^5*(2912 - 7620*Cos[c + d*x] + 3760*Cos[2*(c + d*x)] - 3810*C
os[3*(c + d*x)] + 1440*Cos[4*(c + d*x)] - 762*Cos[5*(c + d*x)] + 80*Cos[6*
(c + d*x)] + 2432*Sin[c + d*x] - 1905*Sin[2*(c + d*x)] + 320*Sin[3*(c + d*
x)] - 1524*Sin[4*(c + d*x)] + 960*Sin[5*(c + d*x)] - 381*Sin[6*(c + d*x)])
)/(17920*a*d*(1 + Sin[c + d*x]))
```

3.53.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3185, 3042, 3086, 210, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^6(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^6}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \; 3185$$

$$\frac{\int \sec^2(c+dx)\tan^6(c+dx)dx}{a}-\frac{\int \sec(c+dx)\tan^7(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^6dx}{a}-\frac{\int \sec(c+dx)\tan(c+dx)^7dx}{a}$$

$$\Big\downarrow \; 3086$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^6dx}{a}-\frac{\int \left(\sec^2(c+dx)-1\right)^3d\sec(c+dx)}{ad}$$

$$\Big\downarrow \; 210$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^6dx}{a}-\frac{\int \left(\sec^6(c+dx)-3\sec^4(c+dx)+3\sec^2(c+dx)-1\right)d\sec(c+dx)}{ad}$$

$$\Big\downarrow \; 2009$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^6dx}{a}-\frac{\frac{1}{7}\sec^7(c+dx)-\frac{3}{5}\sec^5(c+dx)+\sec^3(c+dx)-\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^6(c+dx)d\tan(c+dx)}{ad}-\frac{\frac{1}{7}\sec^7(c+dx)-\frac{3}{5}\sec^5(c+dx)+\sec^3(c+dx)-\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^7(c+dx)}{7ad}-\frac{\frac{1}{7}\sec^7(c+dx)-\frac{3}{5}\sec^5(c+dx)+\sec^3(c+dx)-\sec(c+dx)}{ad}$$

input
```
Int[Tan[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

output
```
-((-Sec[c + d*x] + Sec[c + d*x]^3 - (3*Sec[c + d*x]^5)/5 + Sec[c + d*x]^7/
7)/(a*d)) + Tan[c + d*x]^7/(7*a*d)
```

3.53.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.53.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.31 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.98

method	result
risch	$\frac{\frac{2i}{7}-\frac{10\,\mathrm{e}^{i(dx+c)}}{7}+2\,\mathrm{e}^{11i(dx+c)}+2\,\mathrm{e}^{9i(dx+c)}-\frac{52\,\mathrm{e}^{5i(dx+c)}}{35}+\frac{6\,\mathrm{e}^{3i(dx+c)}}{7}+\frac{36\,\mathrm{e}^{7i(dx+c)}}{5}+2i\mathrm{e}^{10i(dx+c)}+\frac{22i\mathrm{e}^{2i(dx+c)}}{7}+\frac{52i\mathrm{e}^{4i}}{}}{\left(-i+\mathrm{e}^{i(dx+c)}\right)^{5}\left(\mathrm{e}^{i(dx+c)}+i\right)^{7}da}$
derivativedivides	$\frac{-\frac{2}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{9}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{da}$
default	$\frac{-\frac{2}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{9}{10\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}+\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{da}$

input
```
int(tan(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/35*(5*I-25*exp(I*(d*x+c))+35*exp(11*I*(d*x+c))+35*exp(9*I*(d*x+c))-26*ex
p(5*I*(d*x+c))+15*exp(3*I*(d*x+c))+126*exp(7*I*(d*x+c))+35*I*exp(10*I*(d*x
+c))+55*I*exp(2*I*(d*x+c))+130*I*exp(4*I*(d*x+c))+182*I*exp(6*I*(d*x+c))+1
05*I*exp(8*I*(d*x+c)))/(-I+exp(I*(d*x+c)))^5/(exp(I*(d*x+c))+I)^7/d/a
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13

$$\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{5\,\cos\left(dx+c\right)^{6}+15\,\cos\left(dx+c\right)^{4}-5\,\cos\left(dx+c\right)^{2}+2\left(15\,\cos\left(dx+c\right)^{4}-10\,\cos\left(dx+c\right)^{2}+3\right)\sin\left(dx\right.}{35\left(ad\cos\left(dx+c\right)^{5}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{5}\right)}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/35*(5*cos(d*x + c)^6 + 15*cos(d*x + c)^4 - 5*cos(d*x + c)^2 + 2*(15*cos(
d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*sin(d*x + c) + 1)/(a*d*cos(d*x + c)^5*
sin(d*x + c) + a*d*cos(d*x + c)^5)
```

3.53.6 Sympy [F]

$$\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\tan^6(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(78) = 156$.

Time = 0.20 (sec) , antiderivative size = 338, normalized size of antiderivative = 4.02

$$\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{32\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1} - \frac{4\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5\sin(dx+c}{(\cos(dx+c)+}\right.}{35\left(a + \frac{2a\sin(dx+c)}{\cos(dx+c)+1} - \frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{10a\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{20a\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{20a\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{5a\sin(d}{(\cos(dx+}\right.}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
32/35*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 4*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4 + 20*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1)/((a + 2*a*sin(
d*x + c)/(cos(d*x + c) + 1) - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 10
*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 20*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 20*a*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 5*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 10*a*sin(d*
x + c)^9/(cos(d*x + c) + 1)^9 + 4*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10
- 2*a*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - a*sin(d*x + c)^12/(cos(d*x +
 c) + 1)^12)*d)
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(78) = 156.

Time = 2.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.05

$$\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{7\left(25\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-120\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+210\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-140\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+33\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)^5}-\frac{175\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+1260\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+\cdots}{\cdots}}{560\,d}$$

input
```
integrate(tan(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/560*(7*(25*tan(1/2*d*x + 1/2*c)^4 - 120*tan(1/2*d*x + 1/2*c)^3 + 210*ta
n(1/2*d*x + 1/2*c)^2 - 140*tan(1/2*d*x + 1/2*c) + 33)/(a*(tan(1/2*d*x + 1/
2*c) - 1)^5) - (175*tan(1/2*d*x + 1/2*c)^6 + 1260*tan(1/2*d*x + 1/2*c)^5 +
 3815*tan(1/2*d*x + 1/2*c)^4 + 6020*tan(1/2*d*x + 1/2*c)^3 + 4641*tan(1/2*
d*x + 1/2*c)^2 + 1792*tan(1/2*d*x + 1/2*c) + 281)/(a*(tan(1/2*d*x + 1/2*c)
 + 1)^7))/d
```

3.53.9 Mupad [B] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \frac{\tan^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{32\left(20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{35\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^7}$$

input
```
int(tan(c + d*x)^6/(a + a*sin(c + d*x)),x)
```

output
```
-(32*(2*tan(c/2 + (d*x)/2) - 4*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2
)^3 + 5*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^5 + 1))/(35*a*d*(tan(
c/2 + (d*x)/2) - 1)^5*(tan(c/2 + (d*x)/2) + 1)^7)
```

3.54 $\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$

3.54.1 Optimal result

Integrand size = 21, antiderivative size = 69

$$\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{\sec(c+dx)}{ad}+\frac{2\sec^3(c+dx)}{3ad}-\frac{\sec^5(c+dx)}{5ad}+\frac{\tan^5(c+dx)}{5ad}$$

output
```
-sec(d*x+c)/a/d+2/3*sec(d*x+c)^3/a/d-1/5*sec(d*x+c)^5/a/d+1/5*tan(d*x+c)^5
/a/d
```

3.54.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.54

$$\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\sec^3(c+dx)(200-534\cos(c+dx)+288\cos(2(c+dx))-178\cos(3(c+dx))+24\cos(4(c+dx))-6}{960ad(1+\sin(c+dx))}$$

input
```
Integrate[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

output
```
-1/960*(Sec[c + d*x]^3*(200 - 534*Cos[c + d*x] + 288*Cos[2*(c + d*x)] - 17
8*Cos[3*(c + d*x)] + 24*Cos[4*(c + d*x)] - 64*Sin[c + d*x] - 178*Sin[2*(c
+ d*x)] + 192*Sin[3*(c + d*x)] - 89*Sin[4*(c + d*x)]))/(a*d*(1 + Sin[c + d
*x]))
```

3.54.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3185, 3042, 3086, 210, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^4}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \sec^2(c+dx)\tan^4(c+dx)dx}{a}-\frac{\int \sec(c+dx)\tan^5(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^4dx}{a}-\frac{\int \sec(c+dx)\tan(c+dx)^5dx}{a}$$

$$\Big\downarrow \text{3086}$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^4dx}{a}-\frac{\int \left(\sec^2(c+dx)-1\right)^2d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^4dx}{a}-\frac{\int \left(\sec^4(c+dx)-2\sec^2(c+dx)+1\right)d\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \sec(c+dx)^2\tan(c+dx)^4dx}{a}-\frac{\frac{1}{5}\sec^5(c+dx)-\frac{2}{3}\sec^3(c+dx)+\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \tan^4(c+dx)d\tan(c+dx)}{ad}-\frac{\frac{1}{5}\sec^5(c+dx)-\frac{2}{3}\sec^3(c+dx)+\sec(c+dx)}{ad}$$

$$\Big\downarrow \text{15}$$

$$\frac{\tan^5(c+dx)}{5ad}-\frac{\frac{1}{5}\sec^5(c+dx)-\frac{2}{3}\sec^3(c+dx)+\sec(c+dx)}{ad}$$

input
```
Int[Tan[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

output
```
-((Sec[c + d*x] - (2*Sec[c + d*x]^3)/3 + Sec[c + d*x]^5/5)/(a*d)) + Tan[c
+ d*x]^5/(5*a*d)
```

3.54.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.54.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{2\left(25i\,\mathrm{e}^{4i(dx+c)}+5\,\mathrm{e}^{5i(dx+c)}+21i\,\mathrm{e}^{2i(dx+c)}+13\,\mathrm{e}^{3i(dx+c)}+15i\,\mathrm{e}^{6i(dx+c)}+15\,\mathrm{e}^{7i(dx+c)}-9\,\mathrm{e}^{i(dx+c)}+3i\right)}{15\left(\mathrm{e}^{i(dx+c)}+i\right)^{5}\left(-i+\mathrm{e}^{i(dx+c)}\right)^{3}da}$
derivativedivides	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{da}$
default	$\frac{-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{5}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}-\frac{1}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{da}$

input
```
int(tan(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(25*I*exp(4*I*(d*x+c))+5*exp(5*I*(d*x+c))+21*I*exp(2*I*(d*x+c))+13*e
xp(3*I*(d*x+c))+15*I*exp(6*I*(d*x+c))+15*exp(7*I*(d*x+c))-9*exp(I*(d*x+c))
+3*I)/(exp(I*(d*x+c))+I)^5/(-I+exp(I*(d*x+c)))^3/d/a
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= -\frac{3\,\cos\left(dx+c\right)^{4}+6\,\cos\left(dx+c\right)^{2}+4\,\left(3\,\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)-1}{15\,\left(ad\cos\left(dx+c\right)^{3}\sin\left(dx+c\right)+ad\cos\left(dx+c\right)^{3}\right)}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/15*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 4*(3*cos(d*x + c)^2 - 1)*sin(
d*x + c) - 1)/(a*d*cos(d*x + c)^3*sin(d*x + c) + a*d*cos(d*x + c)^3)
```

3.54.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\tan^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(63) = 126$.

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 3.10

$$\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx =$$

$$-\frac{16\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1} - \frac{2\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{6\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + 1\right)}{15\left(a + \frac{2a\sin(dx+c)}{\cos(dx+c)+1} - \frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{6a\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{6a\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{2a\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{2a\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{a\sin}{(\cos(d}\right.}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-16/15*(2*sin(d*x + c)/(cos(d*x + c) + 1) - 2*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 6*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1)/((a + 2*a*sin(d*x + c
)/(cos(d*x + c) + 1) - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 6*a*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 + 6*a*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 2*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*a*sin(d*x + c)^7/(cos(d*x + c
) + 1)^7 - a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.74

$$\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{5\left(9\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\right)}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)^3}-\frac{45\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+240\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+490\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+320\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+73}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^5}}{120\,d}$$

input
```
integrate(tan(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/120*(5*(9*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 11)/(a*(tan
(1/2*d*x + 1/2*c) - 1)^3) - (45*tan(1/2*d*x + 1/2*c)^4 + 240*tan(1/2*d*x +
 1/2*c)^3 + 490*tan(1/2*d*x + 1/2*c)^2 + 320*tan(1/2*d*x + 1/2*c) + 73)/(a
*(tan(1/2*d*x + 1/2*c) + 1)^5))/d
```

3.54.9 Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{\tan^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{16\left(-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)^5}$$

input
```
int(tan(c + d*x)^4/(a + a*sin(c + d*x)),x)
```

output
```
(16*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2)^
3 + 1))/(15*a*d*(tan(c/2 + (d*x)/2) - 1)^3*(tan(c/2 + (d*x)/2) + 1)^5)
```

3.55 $\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx$

3.55.1 Optimal result

Integrand size = 21, antiderivative size = 50

$$\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\sec(c+dx)}{ad} - \frac{\sec^3(c+dx)}{3ad} + \frac{\tan^3(c+dx)}{3ad}$$

output
```
sec(d*x+c)/a/d-1/3*sec(d*x+c)^3/a/d+1/3*tan(d*x+c)^3/a/d
```

3.55.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 106 vs. 2(50) = 100.

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.12

$$\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{6-10\cos(c+dx)+2\cos(2(c+dx))+8\sin(c+dx)-5\sin(2(c+dx))}{12ad\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(1+\sin(c+dx))}$$

input
```
Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
(6 - 10*Cos[c + d*x] + 2*Cos[2*(c + d*x)] + 8*Sin[c + d*x] - 5*Sin[2*(c +
d*x)])/(12*a*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])*(1 + Sin[c + d*x]))
```

3.55.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3185, 3042, 3086, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^2}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3185}$$
$$\frac{\int \sec^2(c+dx)\tan^2(c+dx)dx}{a}-\frac{\int \sec(c+dx)\tan^3(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sec(c+dx)^2\tan(c+dx)^2dx}{a}-\frac{\int \sec(c+dx)\tan(c+dx)^3dx}{a}$$
$$\Big\downarrow \text{3086}$$
$$\frac{\int \sec(c+dx)^2\tan(c+dx)^2dx}{a}-\frac{\int \left(\sec^2(c+dx)-1\right)d\sec(c+dx)}{ad}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\int \sec(c+dx)^2\tan(c+dx)^2dx}{a}-\frac{\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)}{ad}$$
$$\Big\downarrow \text{3087}$$
$$\frac{\int \tan^2(c+dx)d\tan(c+dx)}{ad}-\frac{\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)}{ad}$$
$$\Big\downarrow \text{15}$$
$$\frac{\tan^3(c+dx)}{3ad}-\frac{\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)}{ad}$$

input
```
Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
-((-Sec[c + d*x] + Sec[c + d*x]^3/3)/(a*d)) + Tan[c + d*x]^3/(3*a*d)
```

3.55.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.55.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{da}$	70
default	$\frac{-\frac{2}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{1}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{8}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{da}$	70
risch	$\frac{2i\,\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{3i(dx+c)}+\frac{2i}{3}-\frac{2\,\mathrm{e}^{i(dx+c)}}{3}}{\left(-i+\mathrm{e}^{i(dx+c)}\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{3}da}$	74

input
```
int(tan(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
8/d/a*(-1/12/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan(1/2*d*x+1/2*c)+1)^2+1/16/(t
an(1/2*d*x+1/2*c)+1)-1/16/(tan(1/2*d*x+1/2*c)-1))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\cos(dx+c)^2 + 2\sin(dx+c) + 1}{3\,(ad\cos(dx+c)\sin(dx+c) + ad\cos(dx+c))}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/3*(cos(d*x + c)^2 + 2*sin(d*x + c) + 1)/(a*d*cos(d*x + c)*sin(d*x + c) +
 a*d*cos(d*x + c))
```

3.55.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\tan^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(tan(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{4\left(\frac{2\sin(dx+c)}{\cos(dx+c)+1} + 1\right)}{3\left(a + \frac{2a\sin(dx+c)}{\cos(dx+c)+1} - \frac{2a\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
4/3*(2*sin(d*x + c)/(cos(d*x + c) + 1) + 1)/((a + 2*a*sin(d*x + c)/(cos(d*
x + c) + 1) - 2*a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4)*d)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.36

$$\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)} - \frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^3}}{6\,d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(3/(a*(tan(1/2*d*x + 1/2*c) - 1)) - (3*tan(1/2*d*x + 1/2*c)^2 + 12*ta
n(1/2*d*x + 1/2*c) + 5)/(a*(tan(1/2*d*x + 1/2*c) + 1)^3))/d
```

3.55.9 Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94

$$\int \frac{\tan^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{4\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^3}$$

input
```
int(tan(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

output
```
-(4*(2*tan(c/2 + (d*x)/2) + 1))/(3*a*d*(tan(c/2 + (d*x)/2) - 1)*(tan(c/2 +
 (d*x)/2) + 1)^3)
```

3.56 $\int \frac{1}{a+a\sin(c+dx)}\,dx$

3.56.1 Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \frac{1}{a+a\sin(c+dx)}\,dx=-\frac{\cos(c+dx)}{d(a+a\sin(c+dx))}$$

output
```
-cos(d*x+c)/d/(a+a*sin(d*x+c))
```

3.56.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.09

$$\int \frac{1}{a+a\sin(c+dx)}\,dx=\frac{2\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a+a\sin(c+dx))}$$

input
```
Integrate[(a + a*Sin[c + d*x])^(-1),x]
```

output
```
(2*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(d*(a + a*Sin[c
 + d*x]))
```

3.56.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a\sin(c+dx)+a}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \ 3127$$

$$-\frac{\cos(c+dx)}{d(a\sin(c+dx)+a)}$$

input
```
Int[(a + a*Sin[c + d*x])^(-1),x]
```

output
```
-(Cos[c + d*x]/(d*(a + a*Sin[c + d*x])))
```

3.56.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

3.56.4 Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$-\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	22
default	$-\frac{2}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	22
risch	$-\frac{2}{da\left(e^{i(dx+c)}+i\right)}$	23
norman	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	31
parallelrisch	$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{da\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}$	31

input
```
int(1/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-2/d/a/(tan(1/2*d*x+1/2*c)+1)
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{1}{a+a\sin(c+dx)}\,dx = -\frac{\cos(dx+c)-\sin(dx+c)+1}{ad\cos(dx+c)+ad\sin(dx+c)+ad}$$

input
```
integrate(1/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-(cos(d*x + c) - sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) +
a*d)
```

3.56.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{a + a\sin(c + dx)}\,dx = \begin{cases} -\frac{2}{ad\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + ad} & \text{for } d \neq 0 \\ \frac{x}{a\sin(c) + a} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+a*sin(d*x+c)),x)
```

output
```
Piecewise((-2/(a*d*tan(c/2 + d*x/2) + a*d), Ne(d, 0)), (x/(a*sin(c) + a),
True))
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int \frac{1}{a + a\sin(c + dx)}\,dx = -\frac{2}{\left(a + \frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

input
```
integrate(1/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-2/((a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{a + a\sin(c + dx)}\,dx = -\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right)}$$

input
```
integrate(1/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-2/(a*d*(tan(1/2*d*x + 1/2*c) + 1))
```

3.56.9 Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{a+a\sin(c+dx)}\,dx=-\frac{2}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}$$

input
```
int(1/(a + a*sin(c + d*x)),x)
```

output
```
-2/(a*d*(tan(c/2 + (d*x)/2) + 1))
```

3.57 $\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx$

3.57.1 Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\operatorname{arctanh}(\cos(c+dx))}{ad} - \frac{\cot(c+dx)}{ad}$$

output
```
arctanh(cos(d*x+c))/a/d-cot(d*x+c)/a/d
```

3.57.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.

Time = 0.47 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.38

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx)+\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin(c+\right.}{2ad}$$

input
```
Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
-1/2*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Cos[c + d*x] + (-Log[Cos[(c + d*x
)/2]] + Log[Sin[(c + d*x)/2]])*Sin[c + d*x]))/(a*d)
```

3.57.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3185, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^2(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \csc^2(c+dx)dx}{a}-\frac{\int \csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc(c+dx)^2dx}{a}-\frac{\int \csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\int 1d\cot(c+dx)}{ad}-\frac{\int \csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\int \csc(c+dx)dx}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\text{arctanh}(\cos(c+dx))}{ad}-\frac{\cot(c+dx)}{ad}$$

input
```
Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x]),x]
```

output
```
ArcTanh[Cos[c + d*x]]/(a*d) - Cot[c + d*x]/(a*d)
```

3.57.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.57.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

method	result	size
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2da}$	44
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2da}$	44
risch	$-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}-\frac{\ln\left(e^{i(dx+c)}-1\right)}{da}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{da}$	63

input
```
int(cot(d*x+c)^2/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a*(tan(1/2*d*x+1/2*c)-2*ln(tan(1/2*d*x+1/2*c))-1/tan(1/2*d*x+1/2*c))
```

3.57.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. $2(29) = 58$.

Time = 0.29 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.14

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - \log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 2\cos(dx+c)}{2\,ad\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - log(-1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))
```

3.57.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot^2(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**2/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**2/(sin(c + d*x) + 1), x)/a
```

3.57.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(29) = 58$.

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.41

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{2\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\cos(dx+c)+1}{a\sin(dx+c)} - \frac{\sin(dx+c)}{a(\cos(dx+c)+1)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*log(sin(d*x + c)/(cos(d*x + c) + 1))/a + (cos(d*x + c) + 1)/(a*sin
(d*x + c)) - sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

3.57.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. 2(29) = 58.

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.24

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*log(abs(tan(1/2*d*x + 1/2*c)))/a - tan(1/2*d*x + 1/2*c)/a - (2*tan
(1/2*d*x + 1/2*c) - 1)/(a*tan(1/2*d*x + 1/2*c)))/d
```

3.57.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{\cot^2(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)+\cot(c+d\,x)}{a\,d}$$

input
```
int(cot(c + d*x)^2/(a + a*sin(c + d*x)),x)
```

output
```
-(log(tan(c/2 + (d*x)/2)) + cot(c + d*x))/(a*d)
```

3.58 $\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx$

3.58.1 Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cos(c+dx))}{2ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

output
```
-1/2*arctanh(cos(d*x+c))/a/d-1/3*cot(d*x+c)^3/a/d+1/2*cot(d*x+c)*csc(d*x+c
)/a/d
```

3.58.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(58) = 116.

Time = 0.73 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.14

$$\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2(\cos(3(c+dx))+\cos(c+dx)(3-}{96ad(1+\sin(c+dx))}$$

input
```
Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

output
```
-1/96*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Csc[(c + d*x)/2] + Sec[(c + d*x)
/2])^2*(Cos[3*(c + d*x)] + Cos[c + d*x]*(3 - 6*Sin[c + d*x]) + 6*(Log[Cos[
(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])*Sin[c + d*x]^3))/(a*d*(1 + Sin[c +
d*x]))
```

3.58.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3185, 3042, 3087, 15, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{a\sin(c+dx)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(c+dx)^4(a\sin(c+dx)+a)}dx$$
$$\Big\downarrow \text{3185}$$
$$\frac{\int \cot^2(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int \cot^2(c+dx)\csc(c+dx)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^2\tan\left(c+dx-\frac{\pi}{2}\right)^2dx}{a}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^2dx}{a}$$
$$\Big\downarrow \text{3087}$$
$$\frac{\int \cot^2(c+dx)d(-\cot(c+dx))}{ad}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^2dx}{a}$$
$$\Big\downarrow \text{15}$$
$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^2dx}{a}-\frac{\cot^3(c+dx)}{3ad}$$
$$\Big\downarrow \text{3091}$$
$$-\frac{-\frac{1}{2}\int \csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}}{a}-\frac{\cot^3(c+dx)}{3ad}$$
$$\Big\downarrow \text{3042}$$
$$-\frac{-\frac{1}{2}\int \csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}}{a}-\frac{\cot^3(c+dx)}{3ad}$$
$$\Big\downarrow \text{4257}$$
$$-\frac{\frac{\text{arctanh}(\cos(c+dx))}{2d}-\frac{\cot(c+dx)\csc(c+dx)}{2d}}{a}-\frac{\cot^3(c+dx)}{3ad}$$

input
```
Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x]),x]
```

output
```
-1/3*Cot[c + d*x]^3/(a*d) - (ArcTanh[Cos[c + d*x]]/(2*d) - (Cot[c + d*x]*C
sc[c + d*x])/(2*d))/a
```

3.58.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.58.4 Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.62

method	result	s
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8da}$	9
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+4\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8da}$	9
risch	$-\frac{-6i\,e^{4i(dx+c)}+3\,e^{5i(dx+c)}-2i-3\,e^{i(dx+c)}}{3da\left(e^{2i(dx+c)}-1\right)^3}-\frac{\ln\left(e^{i(dx+c)}+1\right)}{2da}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{2da}$	1

input
```
int(cot(d*x+c)^4/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a*(1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)+
1/tan(1/2*d*x+1/2*c)^2+4*ln(tan(1/2*d*x+1/2*c))-1/3/tan(1/2*d*x+1/2*c)^3+1
/tan(1/2*d*x+1/2*c))
```

3.58.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(52) = 104.

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.91

$$\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{4\cos(dx+c)^3-3\left(\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+3\left(\cos(dx+c)^2-1\right)\log\left(-\right.}{12\left(ad\cos(dx+c)^2-ad\right)\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(4*cos(d*x + c)^3 - 3*(cos(d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) + 3*(cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d
*x + c) - 6*cos(d*x + c)*sin(d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x
 + c))
```

3.58.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot^4(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**4/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**4/(sin(c + d*x) + 1), x)/a
```

3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(52) = 104$.

Time = 0.19 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.67

$$\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{24\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/24*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a - 12*log(sin(d*x + c)/(co
s(d*x + c) + 1))/a - (3*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^3/(a*sin(d*x + c)^3))/d
```

3.58.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(52) = 104$.

Time = 0.38 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.19

$$\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{12\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{22\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\tan(}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}}{24\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(12*log(abs(tan(1/2*d*x + 1/2*c)))/a + (a^2*tan(1/2*d*x + 1/2*c)^3 -
3*a^2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*tan(1/2*d*x + 1/2*c))/a^3 - (22*tan(1
/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x + 1/2*c) + 1)
/(a*tan(1/2*d*x + 1/2*c)^3))/d
```

3.58.9 Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.98

$$\int \frac{\cot^4(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a\,d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a\,d} + \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{3}\right)}{8\,a\,d}$$

input
```
int(cot(c + d*x)^4/(a + a*sin(c + d*x)),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(24*a*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) + log(tan(c/2
 + (d*x)/2))/(2*a*d) - tan(c/2 + (d*x)/2)/(8*a*d) + (cot(c/2 + (d*x)/2)^3*
(tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2 - 1/3))/(8*a*d)
```

3.59 $\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$

3.59.1 Optimal result

Integrand size = 21, antiderivative size = 82

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{3\text{arctanh}(\cos(c+dx))}{8ad} - \frac{\cot^5(c+dx)}{5ad} - \frac{3\cot(c+dx)\csc(c+dx)}{8ad} + \frac{\cot^3(c+dx)\csc(c+dx)}{4ad}$$

output
```
3/8*arctanh(cos(d*x+c))/a/d-1/5*cot(d*x+c)^5/a/d-3/8*cot(d*x+c)*csc(d*x+c)
/a/d+1/4*cot(d*x+c)^3*csc(d*x+c)/a/d
```

3.59.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 189 vs. 2(82) = 164.

Time = 0.96 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.30

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\csc^5(c+dx)\left(80\cos(c+dx)+40\cos(3(c+dx))+8\cos(5(c+dx))-150\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\sin(c+\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

output
```
-1/640*(Csc[c + d*x]^5*(80*Cos[c + d*x] + 40*Cos[3*(c + d*x)] + 8*Cos[5*(c
 + d*x)] - 150*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] + 150*Log[Sin[(c + d*x)/
2]]*Sin[c + d*x] + 20*Sin[2*(c + d*x)] + 75*Log[Cos[(c + d*x)/2]]*Sin[3*(c
 + d*x)] - 75*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 50*Sin[4*(c + d*x)]
 - 15*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*Log[Sin[(c + d*x)/2]]*Si
n[5*(c + d*x)]))/(a*d)
```

3.59.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3185, 3042, 3087, 15, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(c+dx)^6(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \; 3185$$

$$\frac{\int \cot^4(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int \cot^4(c+dx)\csc(c+dx)dx}{a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^2\tan\left(c+dx-\frac{\pi}{2}\right)^4dx}{a}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^4dx}{a}$$

$$\Big\downarrow \; 3087$$

$$\frac{\int \cot^4(c+dx)d(-\cot(c+dx))}{ad}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^4dx}{a}$$

$$\Big\downarrow \; 15$$

$$-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^4dx}{a}-\frac{\cot^5(c+dx)}{5ad}$$

$$\Big\downarrow \; 3091$$

$$-\frac{-\frac{3}{4}\int \cot^2(c+dx)\csc(c+dx)dx-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}}{a}-\frac{\cot^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{3}{4}\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^2dx-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}}{a}-\frac{\cot^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{3091}$$

$$-\frac{-\frac{3}{4}\left(-\frac{1}{2}\int \csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}}{a}-\frac{\cot^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{3}{4}\left(-\frac{1}{2}\int \csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}}{a}-\frac{\cot^5(c+dx)}{5ad}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{-\frac{3}{4}\left(\frac{\text{arctanh}(\cos(c+dx))}{2d}-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}}{a}-\frac{\cot^5(c+dx)}{5ad}$$

input
```
Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x]),x]
```

output
```
-1/5*Cot[c + d*x]^5/(a*d) - (-1/4*(Cot[c + d*x]^3*Csc[c + d*x])/d - (3*(Ar
cTanh[Cos[c + d*x]]/(2*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*d)))/4)/a
```

3.59.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.59.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.93 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

method	result
risch	$\frac{-40i e^{8i(dx+c)}+25\,e^{9i(dx+c)}-10\,e^{7i(dx+c)}-80i e^{4i(dx+c)}+10\,e^{3i(dx+c)}-8i-25\,e^{i(dx+c)}}{20da\left(e^{2i(dx+c)}-1\right)^5}-\frac{3\ln\left(e^{i(dx+c)}-1\right)}{8da}+\frac{3\ln(e$
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-12\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{32da}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-12\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{32da}$

input
```
int(cot(d*x+c)^6/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/20*(-40*I*exp(8*I*(d*x+c))+25*exp(9*I*(d*x+c))-10*exp(7*I*(d*x+c))-80*I*
exp(4*I*(d*x+c))+10*exp(3*I*(d*x+c))-8*I-25*exp(I*(d*x+c)))/d/a/(exp(2*I*(
d*x+c))-1)^5-3/8/d/a*ln(exp(I*(d*x+c))-1)+3/8/d/a*ln(exp(I*(d*x+c))+1)
```

3.59.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(74) = 148$.

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.89

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{16\,\cos\left(dx+c\right)^5 - 15\,\left(\cos\left(dx+c\right)^4 - 2\,\cos\left(dx+c\right)^2 + 1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right) + \frac{1}{2}\right)\sin\left(dx+c\right) + 15\,(c}{80\,\left(ad\cos\left(dx+c\right)\right.}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/80*(16*cos(d*x + c)^5 - 15*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(
1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 15*(cos(d*x + c)^4 - 2*cos(d*x + c)
^2 + 1)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 10*(5*cos(d*x + c)^3 -
 3*cos(d*x + c))*sin(d*x + c))/((a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^2
 + a*d)*sin(d*x + c))
```

3.59.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot^6(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**6/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**6/(sin(c + d*x) + 1), x)/a
```

3.59.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(74) = 148$.

Time = 0.19 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.85

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\frac{\frac{20\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{40\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{5\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{2\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a} - \frac{120\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1} + \frac{10\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \right.}{}}{320\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/320*((20*sin(d*x + c)/(cos(d*x + c) + 1) + 40*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 5*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 + 2*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a - 120*log(sin(
d*x + c)/(cos(d*x + c) + 1))/a + (5*sin(d*x + c)/(cos(d*x + c) + 1) + 10*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2 - 40*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3 - 20*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2)*(cos(d*x + c) + 1)^5/(a*si
n(d*x + c)^5))/d
```

3.59.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(74) = 148$.

Time = 0.44 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.28

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} - \frac{2a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 5a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 10a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 40a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 20a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{a^5}}{320\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/320*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a - (2*a^4*tan(1/2*d*x + 1/2*c)
^5 - 5*a^4*tan(1/2*d*x + 1/2*c)^4 - 10*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^4
*tan(1/2*d*x + 1/2*c)^2 + 20*a^4*tan(1/2*d*x + 1/2*c))/a^5 - (274*tan(1/2*
d*x + 1/2*c)^5 - 20*tan(1/2*d*x + 1/2*c)^4 - 40*tan(1/2*d*x + 1/2*c)^3 + 1
0*tan(1/2*d*x + 1/2*c)^2 + 5*tan(1/2*d*x + 1/2*c) - 2)/(a*tan(1/2*d*x + 1/
2*c)^5))/d
```

3.59.9 Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.23

$$\int \frac{\cot^6(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32\,a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,a\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,a\,d} - \frac{3\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,a\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,a\,d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}+\frac{1}{5}\right)}{32\,a\,d}$$

input
```
int(cot(c + d*x)^6/(a + a*sin(c + d*x)),x)
```

output
```
tan(c/2 + (d*x)/2)^2/(8*a*d) - tan(c/2 + (d*x)/2)^3/(32*a*d) - tan(c/2 + (
d*x)/2)^4/(64*a*d) + tan(c/2 + (d*x)/2)^5/(160*a*d) - (3*log(tan(c/2 + (d*
x)/2)))/(8*a*d) + tan(c/2 + (d*x)/2)/(16*a*d) - (cot(c/2 + (d*x)/2)^5*(4*t
an(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)/2 + 2*tan(
c/2 + (d*x)/2)^4 + 1/5))/(32*a*d)
```

3.60 $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

3.60.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{5\text{arctanh}(\cos(c+dx))}{16ad} - \frac{\cot^7(c+dx)}{7ad} + \frac{5\cot(c+dx)\csc(c+dx)}{16ad} - \frac{5\cot^3(c+dx)\csc(c+dx)}{24ad} + \frac{\cot^5(c+dx)\csc(c+dx)}{6ad}$$

output
```
-5/16*arctanh(cos(d*x+c))/a/d-1/7*cot(d*x+c)^7/a/d+5/16*cot(d*x+c)*csc(d*x
+c)/a/d-5/24*cot(d*x+c)^3*csc(d*x+c)/a/d+1/6*cot(d*x+c)^5*csc(d*x+c)/a/d
```

3.60.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 284 vs. 2(106) = 212.

Time = 1.60 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.68

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\csc^5(c+dx)\left(\csc\left(\frac{1}{2}(c+dx)\right)+\sec\left(\frac{1}{2}(c+dx)\right)\right)^2(1680\cos(c+dx)+1008\cos(3(c+dx))+336\cos(5\ldots}{\ldots}$$

input
```
Integrate[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

output
```
-1/86016*(Csc[c + d*x]^5*(Csc[(c + d*x)/2] + Sec[(c + d*x)/2])^2*(1680*Cos
[c + d*x] + 1008*Cos[3*(c + d*x)] + 336*Cos[5*(c + d*x)] + 48*Cos[7*(c + d
*x)] + 3675*Log[Cos[(c + d*x)/2]]*Sin[c + d*x] - 3675*Log[Sin[(c + d*x)/2]
]*Sin[c + d*x] - 1190*Sin[2*(c + d*x)] - 2205*Log[Cos[(c + d*x)/2]]*Sin[3*
(c + d*x)] + 2205*Log[Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 392*Sin[4*(c +
d*x)] + 735*Log[Cos[(c + d*x)/2]]*Sin[5*(c + d*x)] - 735*Log[Sin[(c + d*x)
/2]]*Sin[5*(c + d*x)] - 462*Sin[6*(c + d*x)] - 105*Log[Cos[(c + d*x)/2]]*S
in[7*(c + d*x)] + 105*Log[Sin[(c + d*x)/2]]*Sin[7*(c + d*x)]))/(a*d*(1 + S
in[c + d*x]))
```

3.60.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3185, 3042, 3087, 15, 3091, 3042, 3091, 3042, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^8(c+dx)}{a\sin(c+dx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^8(a\sin(c+dx)+a)}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \cot^6(c+dx)\csc^2(c+dx)dx}{a}-\frac{\int \cot^6(c+dx)\csc(c+dx)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)^2\tan\left(c+dx-\frac{\pi}{2}\right)^6dx}{a}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^6dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int \cot^6(c+dx)d(-\cot(c+dx))}{ad}-\frac{\int \sec\left(c+dx-\frac{\pi}{2}\right)\tan\left(c+dx-\frac{\pi}{2}\right)^6dx}{a}$$

$$\Big\downarrow \text{15}$$

$$-\frac{\int \sec \left(c+dx-\frac{\pi}{2}\right) \tan \left(c+dx-\frac{\pi}{2}\right)^6 dx}{a}-\frac{\cot^7(c+dx)}{7ad}$$

$$\Big\downarrow \textbf{3091}$$

$$-\frac{-\frac{5}{6}\int \cot^4(c+dx)\csc(c+dx)dx-\frac{\cot^5(c+dx)\csc(c+dx)}{6d}}{a}-\frac{\cot^7(c+dx)}{7ad}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{-\frac{5}{6}\int \sec \left(c+dx-\frac{\pi}{2}\right) \tan \left(c+dx-\frac{\pi}{2}\right)^4 dx-\frac{\cot^5(c+dx)\csc(c+dx)}{6d}}{a}-\frac{\cot^7(c+dx)}{7ad}$$

$$\Big\downarrow \textbf{3091}$$

$$-\frac{-\frac{5}{6}\left(-\frac{3}{4}\int \cot^2(c+dx)\csc(c+dx)dx-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}\right)-\frac{\cot^5(c+dx)\csc(c+dx)}{6d}}{a}-\frac{\cot^7(c+dx)}{7ad}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{-\frac{5}{6}\left(-\frac{3}{4}\int \sec \left(c+dx-\frac{\pi}{2}\right) \tan \left(c+dx-\frac{\pi}{2}\right)^2 dx-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}\right)-\frac{\cot^5(c+dx)\csc(c+dx)}{6d}}{a}-\frac{\cot^7(c+dx)}{7ad}$$

$$\Big\downarrow \textbf{3091}$$

$$-\frac{-\frac{5}{6}\left(-\frac{3}{4}\left(-\frac{1}{2}\int \csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}\right)-\frac{\cot^5(c+dx)\csc(c+dx)}{6d}}{a}-\frac{\cot^7(c+dx)}{7ad}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{-\frac{5}{6}\left(-\frac{3}{4}\left(-\frac{1}{2}\int \csc(c+dx)dx-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}\right)-\frac{\cot^5(c+dx)\csc(c+dx)}{6d}}{a}-\frac{\cot^7(c+dx)}{7ad}$$

$$\Big\downarrow \textbf{4257}$$

$$-\frac{-\frac{5}{6}\left(-\frac{3}{4}\left(\frac{\text{arctanh}(\cos(c+dx))}{2d}-\frac{\cot(c+dx)\csc(c+dx)}{2d}\right)-\frac{\cot^3(c+dx)\csc(c+dx)}{4d}\right)-\frac{\cot^5(c+dx)\csc(c+dx)}{6d}}{a}-\frac{\cot^7(c+dx)}{7ad}$$

input
```
Int[Cot[c + d*x]^8/(a + a*Sin[c + d*x]),x]
```

3.60. $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

output
```
-1/7*Cot[c + d*x]^7/(a*d) - (-1/6*(Cot[c + d*x]^5*Csc[c + d*x])/d - (5*(-1
/4*(Cot[c + d*x]^3*Csc[c + d*x])/d - (3*(ArcTanh[Cos[c + d*x]]/(2*d) - (Co
t[c + d*x]*Csc[c + d*x])/(2*d)))/4))/6)/a
```

3.60.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.60. $\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$

3.60.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.67 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{-336i\,e^{12i(dx+c)}+231\,e^{13i(dx+c)}-196\,e^{11i(dx+c)}-1680i\,e^{8i(dx+c)}+595\,e^{9i(dx+c)}-1008i\,e^{4i(dx+c)}-595\,e^{5i(dx+c)}+196}{168da\left(e^{2i(dx+c)}-1\right)^{7}}$
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-15\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\tan(}{}$
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-15\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-5\tan(}{}$

input
```
int(cot(d*x+c)^8/(a+a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/168*(-336*I*exp(12*I*(d*x+c))+231*exp(13*I*(d*x+c))-196*exp(11*I*(d*x+c
))-1680*I*exp(8*I*(d*x+c))+595*exp(9*I*(d*x+c))-1008*I*exp(4*I*(d*x+c))-59
5*exp(5*I*(d*x+c))+196*exp(3*I*(d*x+c))-48*I-231*exp(I*(d*x+c)))/d/a/(exp(
2*I*(d*x+c))-1)^7-5/16/d/a*ln(exp(I*(d*x+c))+1)+5/16/d/a*ln(exp(I*(d*x+c))
-1)
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. 2(96) = 192.

Time = 0.30 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.87

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$=\frac{96\cos(dx+c)^7-105\left(\cos(dx+c)^6-3\cos(dx+c)^4+3\cos(dx+c)^2-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin}{6}$$

input
```
integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/672*(96*cos(d*x + c)^7 - 105*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(
d*x + c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 105*(cos(d*x +
c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(-1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) - 14*(33*cos(d*x + c)^5 - 40*cos(d*x + c)^3 + 15*cos(d*x +
 c))*sin(d*x + c))/((a*d*cos(d*x + c)^6 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos
(d*x + c)^2 - a*d)*sin(d*x + c))
```

3.60.6 Sympy [F]

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx = \frac{\int \frac{\cot^8(c+dx)}{\sin(c+dx)+1}\,dx}{a}$$

input
```
integrate(cot(d*x+c)**8/(a+a*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**8/(sin(c + d*x) + 1), x)/a
```

3.60.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(96) = 192$.

Time = 0.21 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.97

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx = -\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{315\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{63\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{7\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}-\frac{840\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{2688\,d}$$

input
```
integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2688*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 315*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 63*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 7*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a -
840*log(sin(d*x + c)/(cos(d*x + c) + 1))/a - (7*sin(d*x + c)/(cos(d*x + c)
 + 1) + 21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 63*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 - 63*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 315*sin(d*x + c)^
5/(cos(d*x + c) + 1)^5 + 105*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3)*(cos
(d*x + c) + 1)^7/(a*sin(d*x + c)^7))/d
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(96) = 192$.

Time = 0.44 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.30

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{\frac{840\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a} + \frac{3a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 7a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 - 21a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 + 63a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^7}}{}$$

input
```
integrate(cot(d*x+c)^8/(a+a*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2688*(840*log(abs(tan(1/2*d*x + 1/2*c)))/a + (3*a^6*tan(1/2*d*x + 1/2*c)
^7 - 7*a^6*tan(1/2*d*x + 1/2*c)^6 - 21*a^6*tan(1/2*d*x + 1/2*c)^5 + 63*a^6
*tan(1/2*d*x + 1/2*c)^4 + 63*a^6*tan(1/2*d*x + 1/2*c)^3 - 315*a^6*tan(1/2*
d*x + 1/2*c)^2 - 105*a^6*tan(1/2*d*x + 1/2*c))/a^7 - (2178*tan(1/2*d*x + 1
/2*c)^7 - 105*tan(1/2*d*x + 1/2*c)^6 - 315*tan(1/2*d*x + 1/2*c)^5 + 63*tan
(1/2*d*x + 1/2*c)^4 + 63*tan(1/2*d*x + 1/2*c)^3 - 21*tan(1/2*d*x + 1/2*c)^
2 - 7*tan(1/2*d*x + 1/2*c) + 3)/(a*tan(1/2*d*x + 1/2*c)^7))/d
```

3.60.9 Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 387, normalized size of antiderivative = 3.65

$$\int \frac{\cot^8(c+dx)}{a+a\sin(c+dx)}\,dx$$
$$= \frac{3\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-3\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}-7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+7\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-21}{}$$

input
```
int(cot(c + d*x)^8/(a + a*sin(c + d*x)),x)
```

output
```
(3*sin(c/2 + (d*x)/2)^14 - 3*cos(c/2 + (d*x)/2)^14 - 7*cos(c/2 + (d*x)/2)*
sin(c/2 + (d*x)/2)^13 + 7*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2) - 21*co
s(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^12 + 63*cos(c/2 + (d*x)/2)^3*sin(c/2
 + (d*x)/2)^11 + 63*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^10 - 315*cos(c
/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2)^9 - 105*cos(c/2 + (d*x)/2)^6*sin(c/2 +
(d*x)/2)^8 + 105*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^6 + 315*cos(c/2 +
 (d*x)/2)^9*sin(c/2 + (d*x)/2)^5 - 63*cos(c/2 + (d*x)/2)^10*sin(c/2 + (d*x
)/2)^4 - 63*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^3 + 21*cos(c/2 + (d*x
)/2)^12*sin(c/2 + (d*x)/2)^2 + 840*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^7)/(2688*a*d*cos(c/2 + (d*x)/2
)^7*sin(c/2 + (d*x)/2)^7)
```

3.61 $\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.61.1 Optimal result

Integrand size = 21, antiderivative size = 189

$$\begin{aligned}\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = &-\frac{7\operatorname{arctanh}(\sin(c+dx))}{128a^2d}+\frac{a}{192d(a-a\sin(c+dx))^3}\\&-\frac{1}{32d(a-a\sin(c+dx))^2}+\frac{a^3}{80d(a+a\sin(c+dx))^5}\\&-\frac{5a^2}{64d(a+a\sin(c+dx))^4}+\frac{19a}{96d(a+a\sin(c+dx))^3}\\&-\frac{1}{4d(a+a\sin(c+dx))^2}+\frac{21}{256d\,(a^2-a^2\sin(c+dx))}\\&+\frac{35}{256d\,(a^2+a^2\sin(c+dx))}\end{aligned}$$

output
```
-7/128*arctanh(sin(d*x+c))/a^2/d+1/192*a/d/(a-a*sin(d*x+c))^3-1/32/d/(a-a*
sin(d*x+c))^2+1/80*a^3/d/(a+a*sin(d*x+c))^5-5/64*a^2/d/(a+a*sin(d*x+c))^4+
19/96*a/d/(a+a*sin(d*x+c))^3-1/4/d/(a+a*sin(d*x+c))^2+21/256/d/(a^2-a^2*si
n(d*x+c))+35/256/d/(a^2+a^2*sin(d*x+c))
```

3.61.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.59

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{210\text{arctanh}(\sin(c+dx)) - \frac{2(-144-393\sin(c+dx)+78\sin^2(c+dx)+1039\sin^3(c+dx)+560\sin^4(c+dx)-815\sin^5(c+dx)-750\sin^6(c}{(-1+\sin(c+dx))^3(1+\sin(c+dx))^5}}{3840a^2d}$$

input
```
Integrate[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

output
```
-1/3840*(210*ArcTanh[Sin[c + d*x]] - (2*(-144 - 393*Sin[c + d*x] + 78*Sin[
c + d*x]^2 + 1039*Sin[c + d*x]^3 + 560*Sin[c + d*x]^4 - 815*Sin[c + d*x]^5
 - 750*Sin[c + d*x]^6 + 105*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^3*(1 + S
in[c + d*x])^5))/(a^2*d)
```

3.61.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^7(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^7}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^7\sin^7(c+dx)}{(a-a\sin(c+dx))^4(\sin(c+dx)a+a)^6}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{a^3}{16(\sin(c+dx)a+a)^6}+\frac{5a^2}{16(\sin(c+dx)a+a)^5}+\frac{a}{64(a-a\sin(c+dx))^4}-\frac{19a}{32(\sin(c+dx)a+a)^4}-\frac{1}{16(a-a\sin(c+dx))^3}+\frac{1}{2(\sin(c+dx)a+a)}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3}{80(a\sin(c+dx)+a)^5}-\frac{7\text{arctanh}(\sin(c+dx))}{128a^2}-\frac{5a^2}{64(a\sin(c+dx)+a)^4}+\frac{a}{192(a-a\sin(c+dx))^3}+\frac{19a}{96(a\sin(c+dx)+a)^3}-\frac{1}{32(a-a\sin(c+dx))}}{d}$$

input
```
Int[Tan[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

output
```
((-7*ArcTanh[Sin[c + d*x]])/(128*a^2) + a/(192*(a - a*Sin[c + d*x])^3) - 1
/(32*(a - a*Sin[c + d*x])^2) + 21/(256*a*(a - a*Sin[c + d*x])) + a^3/(80*(
a + a*Sin[c + d*x])^5) - (5*a^2)/(64*(a + a*Sin[c + d*x])^4) + (19*a)/(96*
(a + a*Sin[c + d*x])^3) - 1/(4*(a + a*Sin[c + d*x])^2) + 35/(256*a*(a + a*
Sin[c + d*x])))/d
```

3.61.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.61.4 Maple [A] (verified)

Time = 4.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\frac{1}{80(1+\sin(dx+c))^5}-\frac{5}{64(1+\sin(dx+c))^4}+\frac{19}{96(1+\sin(dx+c))^3}-\frac{1}{4(1+\sin(dx+c))^2}+\frac{35}{256(1+\sin(dx+c))}-\frac{7\ln(1+\sin(dx+c))}{256}-\frac{}{192(\sin(}}{d\,a^2}$
default	$\frac{\frac{1}{80(1+\sin(dx+c))^5}-\frac{5}{64(1+\sin(dx+c))^4}+\frac{19}{96(1+\sin(dx+c))^3}-\frac{1}{4(1+\sin(dx+c))^2}+\frac{35}{256(1+\sin(dx+c))}-\frac{7\ln(1+\sin(dx+c))}{256}-\frac{}{192(\sin(}}{d\,a^2}$
risch	$\frac{i(-2084i e^{10i(dx+c)}-1500i e^{2i(dx+c)}-105\,e^{i(dx+c)}+2529\,e^{11i(dx+c)}+105\,e^{15i(dx+c)}-2525\,e^{3i(dx+c)}-2529\,e^{5i(dx+c)}+4}{960(e^{i(dx+c}}$

input
```
int(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/80/(1+sin(d*x+c))^5-5/64/(1+sin(d*x+c))^4+19/96/(1+sin(d*x+c))^
3-1/4/(1+sin(d*x+c))^2+35/256/(1+sin(d*x+c))-7/256*ln(1+sin(d*x+c))-1/192/
(sin(d*x+c)-1)^3-1/32/(sin(d*x+c)-1)^2-21/256/(sin(d*x+c)-1)+7/256*ln(sin(
d*x+c)-1))
```

3.61.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.15

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{1500\,\cos(dx+c)^6-3380\,\cos(dx+c)^4+2104\,\cos(dx+c)^2-105\,\left(\cos(dx+c)^8-2\,\cos(dx+c)^6\sin(\right.}{}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/3840*(1500*cos(d*x + c)^6 - 3380*cos(d*x + c)^4 + 2104*cos(d*x + c)^2 -
105*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c) - 2*cos(d*x + c)^6)*lo
g(sin(d*x + c) + 1) + 105*(cos(d*x + c)^8 - 2*cos(d*x + c)^6*sin(d*x + c)
- 2*cos(d*x + c)^6)*log(-sin(d*x + c) + 1) - 2*(105*cos(d*x + c)^6 + 500*c
os(d*x + c)^4 - 276*cos(d*x + c)^2 + 64)*sin(d*x + c) - 512)/(a^2*d*cos(d*
x + c)^8 - 2*a^2*d*cos(d*x + c)^6*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^6)
```

3.61.6 Sympy [F]

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\tan^7(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**7/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.07

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\frac{2\left(105\sin(dx+c)^7-750\sin(dx+c)^6-815\sin(dx+c)^5+560\sin(dx+c)^4+1039\sin(dx+c)^3+78\sin(dx+c)^2-393\sin(dx+c)-144\right)}{a^2\sin(dx+c)^8+2a^2\sin(dx+c)^7-2a^2\sin(dx+c)^6-6a^2\sin(dx+c)^5+6a^2\sin(dx+c)^3+2a^2\sin(dx+c)^2-2a^2\sin(dx+c)-a^2} - \frac{105\log(\sin}{a}}{3840\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/3840*(2*(105*sin(d*x + c)^7 - 750*sin(d*x + c)^6 - 815*sin(d*x + c)^5 +
560*sin(d*x + c)^4 + 1039*sin(d*x + c)^3 + 78*sin(d*x + c)^2 - 393*sin(d*x
 + c) - 144)/(a^2*sin(d*x + c)^8 + 2*a^2*sin(d*x + c)^7 - 2*a^2*sin(d*x +
c)^6 - 6*a^2*sin(d*x + c)^5 + 6*a^2*sin(d*x + c)^3 + 2*a^2*sin(d*x + c)^2
- 2*a^2*sin(d*x + c) - a^2) - 105*log(sin(d*x + c) + 1)/a^2 + 105*log(sin(
d*x + c) - 1)/a^2)/d
```

3.61.8 Giac [A] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.77

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\frac{420\log(|\sin(dx+c)+1|)}{a^2} - \frac{420\log(|\sin(dx+c)-1|)}{a^2} + \frac{10\left(77\sin(dx+c)^3-105\sin(dx+c)^2+27\sin(dx+c)+9\right)}{a^2(\sin(dx+c)-1)^3} - \frac{959\sin(dx+c)^5+6895\sin}{}}{15360\,d}$$

input
```
integrate(tan(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/15360*(420*log(abs(sin(d*x + c) + 1))/a^2 - 420*log(abs(sin(d*x + c) -
1))/a^2 + 10*(77*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 27*sin(d*x + c) + 9
)/(a^2*(sin(d*x + c) - 1)^3) - (959*sin(d*x + c)^5 + 6895*sin(d*x + c)^4 +
 14150*sin(d*x + c)^3 + 13710*sin(d*x + c)^2 + 6555*sin(d*x + c) + 1251)/(
a^2*(sin(d*x + c) + 1)^5))/d
```

3.61.9 Mupad [B] (verification not implemented)

Time = 10.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.35

$$\int \frac{\tan^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$$

$$= \frac{\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{16}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192}-\frac{49\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{24}-\frac{693}{}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-20a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-20a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+36a^2\tan\left(\frac{c}{2}+\right.\right.}$$

$$-\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64a^2d}$$

input
```
int(tan(c + d*x)^7/(a + a*sin(c + d*x))^2,x)
```

output
```
((7*tan(c/2 + (d*x)/2))/64 + (7*tan(c/2 + (d*x)/2)^2)/16 + (7*tan(c/2 + (d
*x)/2)^3)/192 - (49*tan(c/2 + (d*x)/2)^4)/24 - (693*tan(c/2 + (d*x)/2)^5)/
320 + (791*tan(c/2 + (d*x)/2)^6)/240 + (1207*tan(c/2 + (d*x)/2)^7)/192 + (
123*tan(c/2 + (d*x)/2)^8)/4 + (1207*tan(c/2 + (d*x)/2)^9)/192 + (791*tan(c
/2 + (d*x)/2)^10)/240 - (693*tan(c/2 + (d*x)/2)^11)/320 - (49*tan(c/2 + (d
*x)/2)^12)/24 + (7*tan(c/2 + (d*x)/2)^13)/192 + (7*tan(c/2 + (d*x)/2)^14)/
16 + (7*tan(c/2 + (d*x)/2)^15)/64)/(d*(36*a^2*tan(c/2 + (d*x)/2)^5 - 20*a^
2*tan(c/2 + (d*x)/2)^4 - 20*a^2*tan(c/2 + (d*x)/2)^3 + 64*a^2*tan(c/2 + (d
*x)/2)^6 - 20*a^2*tan(c/2 + (d*x)/2)^7 - 90*a^2*tan(c/2 + (d*x)/2)^8 - 20*
a^2*tan(c/2 + (d*x)/2)^9 + 64*a^2*tan(c/2 + (d*x)/2)^10 + 36*a^2*tan(c/2 +
 (d*x)/2)^11 - 20*a^2*tan(c/2 + (d*x)/2)^12 - 20*a^2*tan(c/2 + (d*x)/2)^13
 + 4*a^2*tan(c/2 + (d*x)/2)^15 + a^2*tan(c/2 + (d*x)/2)^16 + a^2 + 4*a^2*t
an(c/2 + (d*x)/2))) - (7*atanh(tan(c/2 + (d*x)/2)))/(64*a^2*d)
```

3.62 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.62.1 Optimal result

Integrand size = 21, antiderivative size = 146

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{5\text{arctanh}(\sin(c+dx))}{64a^2d} + \frac{1}{64d(a-a\sin(c+dx))^2} + \frac{a^2}{32d(a+a\sin(c+dx))^4} - \frac{7a}{48d(a+a\sin(c+dx))^3} + \frac{1}{4d(a+a\sin(c+dx))^2} - \frac{5}{64d\left(a^2-a^2\sin(c+dx)\right)} - \frac{5}{32d\left(a^2+a^2\sin(c+dx)\right)}$$

output
```
5/64*arctanh(sin(d*x+c))/a^2/d+1/64/d/(a-a*sin(d*x+c))^2+1/32*a^2/d/(a+a*s
in(d*x+c))^4-7/48*a/d/(a+a*sin(d*x+c))^3+1/4/d/(a+a*sin(d*x+c))^2-5/64/d/(
a^2-a^2*sin(d*x+c))-5/32/d/(a^2+a^2*sin(d*x+c))
```

3.62.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.62

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{15\text{arctanh}(\sin(c+dx)) + \frac{-16-47\sin(c+dx)-14\sin^2(c+dx)+74\sin^3(c+dx)+66\sin^4(c+dx)-15\sin^5(c+dx)}{(-1+\sin(c+dx))^2(1+\sin(c+dx))^4}}{192a^2d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

output
```
(15*ArcTanh[Sin[c + d*x]] + (-16 - 47*Sin[c + d*x] - 14*Sin[c + d*x]^2 + 7
4*Sin[c + d*x]^3 + 66*Sin[c + d*x]^4 - 15*Sin[c + d*x]^5)/((-1 + Sin[c + d
*x])^2*(1 + Sin[c + d*x])^4))/(192*a^2*d)
```

3.62.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^5}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3186}$$

$$\frac{\int \frac{a^5\sin^5(c+dx)}{(a-a\sin(c+dx))^3(\sin(c+dx)a+a)^5}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(-\frac{a^2}{8(\sin(c+dx)a+a)^5}+\frac{7a}{16(\sin(c+dx)a+a)^4}+\frac{1}{32(a-a\sin(c+dx))^3}-\frac{1}{2(\sin(c+dx)a+a)^3}+\frac{5}{64(a^2-a^2\sin^2(c+dx))a}-\frac{5}{64(a-a\sin(c+d$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{5\text{arctanh}(\sin(c+dx))}{64a^2}+\frac{a^2}{32(a\sin(c+dx)+a)^4}-\frac{7a}{48(a\sin(c+dx)+a)^3}+\frac{1}{64(a-a\sin(c+dx))^2}+\frac{1}{4(a\sin(c+dx)+a)^2}-\frac{5}{64a(a-a\sin(c+dx))}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

output
```
((5*ArcTanh[Sin[c + d*x]])/(64*a^2) + 1/(64*(a - a*Sin[c + d*x])^2) - 5/(6
4*a*(a - a*Sin[c + d*x])) + a^2/(32*(a + a*Sin[c + d*x])^4) - (7*a)/(48*(a
 + a*Sin[c + d*x])^3) + 1/(4*(a + a*Sin[c + d*x])^2) - 5/(32*a*(a + a*Sin[
c + d*x])))/d
```

3.62.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.62.4 Maple [A] (verified)

Time = 2.00 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{1}{64(\sin(dx+c)-1)^{2}}+\frac{5}{64(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{128}+\frac{1}{32(1+\sin(dx+c))^{4}}-\frac{7}{48(1+\sin(dx+c))^{3}}+\frac{1}{4(1+\sin(dx+c))^{2}}-\frac{5}{32(1+\sin(\cdots}}{d\,a^{2}}$
default	$\frac{\frac{1}{64(\sin(dx+c)-1)^{2}}+\frac{5}{64(\sin(dx+c)-1)}-\frac{5\ln(\sin(dx+c)-1)}{128}+\frac{1}{32(1+\sin(dx+c))^{4}}-\frac{7}{48(1+\sin(dx+c))^{3}}+\frac{1}{4(1+\sin(dx+c))^{2}}-\frac{5}{32(1+\sin(\cdots}}{d\,a^{2}}$
risch	$-\frac{i\left(416i\,\mathrm{e}^{8i(dx+c)}-221\,\mathrm{e}^{3i(dx+c)}-132i\,\mathrm{e}^{2i(dx+c)}-14\,\mathrm{e}^{5i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}-56i\,\mathrm{e}^{6i(dx+c)}+14\,\mathrm{e}^{7i(dx+c)}-132i\,\mathrm{e}^{10i(dx+c)}\cdots\right.}{96\left(\mathrm{e}^{i(dx+c)}+i\right)^{8}\left(-i+\mathrm{e}^{i(dx+c)}\right)^{4}d\,a^{2}}$

input
```
int(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/64/(sin(d*x+c)-1)^2+5/64/(sin(d*x+c)-1)-5/128*ln(sin(d*x+c)-1)+
1/32/(1+sin(d*x+c))^4-7/48/(1+sin(d*x+c))^3+1/4/(1+sin(d*x+c))^2-5/32/(1+s
in(d*x+c))+5/128*ln(1+sin(d*x+c)))
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.36

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{132\cos(dx+c)^4 - 236\cos(dx+c)^2 - 15\left(\cos(dx+c)^6 - 2\cos(dx+c)^4\sin(dx+c) - 2\cos(dx+c\right.}{}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/384*(132*cos(d*x + c)^4 - 236*cos(d*x + c)^2 - 15*(cos(d*x + c)^6 - 2*c
os(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(sin(d*x + c) + 1) + 15*
(cos(d*x + c)^6 - 2*cos(d*x + c)^4*sin(d*x + c) - 2*cos(d*x + c)^4)*log(-s
in(d*x + c) + 1) - 2*(15*cos(d*x + c)^4 + 44*cos(d*x + c)^2 - 12)*sin(d*x
+ c) + 72)/(a^2*d*cos(d*x + c)^6 - 2*a^2*d*cos(d*x + c)^4*sin(d*x + c) - 2
*a^2*d*cos(d*x + c)^4)
```

3.62.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx =$$

$$-\frac{\frac{2\left(15\sin(dx+c)^5-66\sin(dx+c)^4-74\sin(dx+c)^3+14\sin(dx+c)^2+47\sin(dx+c)+16\right)}{a^2\sin(dx+c)^6+2a^2\sin(dx+c)^5-a^2\sin(dx+c)^4-4a^2\sin(dx+c)^3-a^2\sin(dx+c)^2+2a^2\sin(dx+c)+a^2}-\frac{15\log(\sin(dx+c)+1)}{a^2}+\frac{15\log(\sin}{}}{384\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/384*(2*(15*sin(d*x + c)^5 - 66*sin(d*x + c)^4 - 74*sin(d*x + c)^3 + 14*
sin(d*x + c)^2 + 47*sin(d*x + c) + 16)/(a^2*sin(d*x + c)^6 + 2*a^2*sin(d*x
 + c)^5 - a^2*sin(d*x + c)^4 - 4*a^2*sin(d*x + c)^3 - a^2*sin(d*x + c)^2 +
 2*a^2*sin(d*x + c) + a^2) - 15*log(sin(d*x + c) + 1)/a^2 + 15*log(sin(d*x
 + c) - 1)/a^2)/d
```

3.62.8 Giac [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx$$

$$=\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^2}-\frac{60\log(|\sin(dx+c)-1|)}{a^2}+\frac{6\left(15\sin(dx+c)^2-10\sin(dx+c)-1\right)}{a^2(\sin(dx+c)-1)^2}-\frac{125\sin(dx+c)^4+740\sin(dx+c)^3+1086\sin(dx+}{a^2(\sin(dx+c)+1)^4}}{1536\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1536*(60*log(abs(sin(d*x + c) + 1))/a^2 - 60*log(abs(sin(d*x + c) - 1))/
a^2 + 6*(15*sin(d*x + c)^2 - 10*sin(d*x + c) - 1)/(a^2*(sin(d*x + c) - 1)^
2) - (125*sin(d*x + c)^4 + 740*sin(d*x + c)^3 + 1086*sin(d*x + c)^2 + 676*
sin(d*x + c) + 157)/(a^2*(sin(d*x + c) + 1)^4))/d
```

3.62.9 Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.47

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{32\,a^2\,d} - \frac{\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{32}+\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{8}+\frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{96}-\frac{5\tan\left(\frac{c}{2}+\right.}{3}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-12\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}-17\,a^2\tan\left(\frac{c}{2}+\right.\right.}$$

input

```
int(tan(c + d*x)^5/(a + a*sin(c + d*x))^2,x)
```

output

```
(5*atanh(tan(c/2 + (d*x)/2)))/(32*a^2*d) - ((5*tan(c/2 + (d*x)/2))/32 + (5
*tan(c/2 + (d*x)/2)^2)/8 + (35*tan(c/2 + (d*x)/2)^3)/96 - (5*tan(c/2 + (d*
x)/2)^4)/3 - (121*tan(c/2 + (d*x)/2)^5)/48 - (119*tan(c/2 + (d*x)/2)^6)/12
 - (121*tan(c/2 + (d*x)/2)^7)/48 - (5*tan(c/2 + (d*x)/2)^8)/3 + (35*tan(c/
2 + (d*x)/2)^9)/96 + (5*tan(c/2 + (d*x)/2)^10)/8 + (5*tan(c/2 + (d*x)/2)^1
1)/32)/(d*(2*a^2*tan(c/2 + (d*x)/2)^2 - 12*a^2*tan(c/2 + (d*x)/2)^3 - 17*a
^2*tan(c/2 + (d*x)/2)^4 + 8*a^2*tan(c/2 + (d*x)/2)^5 + 28*a^2*tan(c/2 + (d
*x)/2)^6 + 8*a^2*tan(c/2 + (d*x)/2)^7 - 17*a^2*tan(c/2 + (d*x)/2)^8 - 12*a
^2*tan(c/2 + (d*x)/2)^9 + 2*a^2*tan(c/2 + (d*x)/2)^10 + 4*a^2*tan(c/2 + (d
*x)/2)^11 + a^2*tan(c/2 + (d*x)/2)^12 + a^2 + 4*a^2*tan(c/2 + (d*x)/2)))
```

3.63 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.63.1 Optimal result

Integrand size = 21, antiderivative size = 104

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{8a^2d} + \frac{a}{12d(a+a\sin(c+dx))^3} - \frac{1}{4d(a+a\sin(c+dx))^2} + \frac{1}{16d\,(a^2-a^2\sin(c+dx))} + \frac{3}{16d\,(a^2+a^2\sin(c+dx))}$$

output
```
-1/8*arctanh(sin(d*x+c))/a^2/d+1/12*a/d/(a+a*sin(d*x+c))^3-1/4/d/(a+a*sin(
d*x+c))^2+1/16/d/(a^2-a^2*sin(d*x+c))+3/16/d/(a^2+a^2*sin(d*x+c))
```

3.63.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{6\operatorname{arctanh}(\sin(c+dx)) - \frac{3}{1-\sin(c+dx)} - \frac{4}{(1+\sin(c+dx))^3} + \frac{12}{(1+\sin(c+dx))^2} - \frac{9}{1+\sin(c+dx)}}{48a^2d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

output
```
-1/48*(6*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 4/(1 + Sin[c + d*x
])^3 + 12/(1 + Sin[c + d*x])^2 - 9/(1 + Sin[c + d*x]))/(a^2*d)
```

3.63.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a\sin(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^3\sin^3(c+dx)}{(a-a\sin(c+dx))^2(\sin(c+dx)a+a)^4}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{a}{4(\sin(c+dx)a+a)^4}+\frac{1}{2(\sin(c+dx)a+a)^3}-\frac{1}{8(a^2-a^2\sin^2(c+dx))a}+\frac{1}{16(a-a\sin(c+dx))^2a}-\frac{3}{16(\sin(c+dx)a+a)^2a}\right)d(a\sin(c+d}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\text{arctanh}(\sin(c+dx))}{8a^2}+\frac{a}{12(a\sin(c+dx)+a)^3}-\frac{1}{4(a\sin(c+dx)+a)^2}+\frac{1}{16a(a-a\sin(c+dx))}+\frac{3}{16a(a\sin(c+dx)+a)}}{d}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

output
```
(-1/8*ArcTanh[Sin[c + d*x]]/a^2 + 1/(16*a*(a - a*Sin[c + d*x])) + a/(12*(a
 + a*Sin[c + d*x])^3) - 1/(4*(a + a*Sin[c + d*x])^2) + 3/(16*a*(a + a*Sin[
c + d*x])))/d
```

3.63.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.63.4 Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{\frac{1}{12(1+\sin(dx+c))^3}-\frac{1}{4(1+\sin(dx+c))^2}+\frac{3}{16(1+\sin(dx+c))}-\frac{\ln(1+\sin(dx+c))}{16}-\frac{1}{16(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{16}}{d\,a^2}$
default	$\frac{\frac{1}{12(1+\sin(dx+c))^3}-\frac{1}{4(1+\sin(dx+c))^2}+\frac{3}{16(1+\sin(dx+c))}-\frac{\ln(1+\sin(dx+c))}{16}-\frac{1}{16(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{16}}{d\,a^2}$
risch	$\frac{i\left(-19\,\mathrm{e}^{3i(dx+c)}-12i\mathrm{e}^{2i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}+19\,\mathrm{e}^{5i(dx+c)}+40i\mathrm{e}^{4i(dx+c)}-12i\mathrm{e}^{6i(dx+c)}+3\,\mathrm{e}^{7i(dx+c)}\right)}{12\left(\mathrm{e}^{i(dx+c)}+i\right)^6\left(-i+\mathrm{e}^{i(dx+c)}\right)^2 d\,a^2}+\frac{\ln\left(-i+\mathrm{e}^{i(dx+c)}\right)}{8a^2d}$

input
```
int(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/12/(1+sin(d*x+c))^3-1/4/(1+sin(d*x+c))^2+3/16/(1+sin(d*x+c))-1/
16*ln(1+sin(d*x+c))-1/16/(sin(d*x+c)-1)+1/16*ln(sin(d*x+c)-1))
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.71

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$$
$$= \frac{12\cos(dx+c)^2 - 3\left(\cos(dx+c)^4 - 2\cos(dx+c)^2\sin(dx+c) - 2\cos(dx+c)^2\right)\log(\sin(dx+c)+1)}{48\left(a^2d\cos(dx+c)^4 - 2\right.}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/48*(12*cos(d*x + c)^2 - 3*(cos(d*x + c)^4 - 2*cos(d*x + c)^2*sin(d*x + c
) - 2*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + 3*(cos(d*x + c)^4 - 2*cos(d*
x + c)^2*sin(d*x + c) - 2*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(3*co
s(d*x + c)^2 + 4)*sin(d*x + c) - 16)/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d
*x + c)^2*sin(d*x + c) - 2*a^2*d*cos(d*x + c)^2)
```

3.63.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\tan^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{2\left(3\sin(dx+c)^3 - 6\sin(dx+c)^2 - 7\sin(dx+c) - 2\right)}{a^2\sin(dx+c)^4 + 2a^2\sin(dx+c)^3 - 2a^2\sin(dx+c) - a^2} - \frac{3\log(\sin(dx+c)+1)}{a^2} + \frac{3\log(\sin(dx+c)-1)}{a^2}}{48d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/48*(2*(3*sin(d*x + c)^3 - 6*sin(d*x + c)^2 - 7*sin(d*x + c) - 2)/(a^2*si
n(d*x + c)^4 + 2*a^2*sin(d*x + c)^3 - 2*a^2*sin(d*x + c) - a^2) - 3*log(si
n(d*x + c) + 1)/a^2 + 3*log(sin(d*x + c) - 1)/a^2)/d
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\frac{6\log(|\sin(dx+c)+1|)}{a^2} - \frac{6\log(|\sin(dx+c)-1|)}{a^2} + \frac{6\sin(dx+c)}{a^2(\sin(dx+c)-1)} - \frac{11\sin(dx+c)^3+51\sin(dx+c)^2+45\sin(dx+c)+13}{a^2(\sin(dx+c)+1)^3}}{96\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/96*(6*log(abs(sin(d*x + c) + 1))/a^2 - 6*log(abs(sin(d*x + c) - 1))/a^2
 + 6*sin(d*x + c)/(a^2*(sin(d*x + c) - 1)) - (11*sin(d*x + c)^3 + 51*sin(d
*x + c)^2 + 45*sin(d*x + c) + 13)/(a^2*(sin(d*x + c) + 1)^3))/d
```

3.63.9 Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.31

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$$

$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \frac{13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{12} + \frac{10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3} + \frac{13\tan\left(\frac{c}{2}+\right.}{12}}{d\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + 4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 - 4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 - 10\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right.}$$

$$- \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d}$$

input
```
int(tan(c + d*x)^3/(a + a*sin(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)/4 + tan(c/2 + (d*x)/2)^2 + (13*tan(c/2 + (d*x)/2)^3)/1
2 + (10*tan(c/2 + (d*x)/2)^4)/3 + (13*tan(c/2 + (d*x)/2)^5)/12 + tan(c/2 +
 (d*x)/2)^6 + tan(c/2 + (d*x)/2)^7/4)/(d*(4*a^2*tan(c/2 + (d*x)/2)^2 - 4*a
^2*tan(c/2 + (d*x)/2)^3 - 10*a^2*tan(c/2 + (d*x)/2)^4 - 4*a^2*tan(c/2 + (d
*x)/2)^5 + 4*a^2*tan(c/2 + (d*x)/2)^6 + 4*a^2*tan(c/2 + (d*x)/2)^7 + a^2*t
an(c/2 + (d*x)/2)^8 + a^2 + 4*a^2*tan(c/2 + (d*x)/2))) - atanh(tan(c/2 + (
d*x)/2))/(4*a^2*d)
```

3.63. $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.64 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.64.1 Optimal result

Integrand size = 19, antiderivative size = 60

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{4a^2d} + \frac{1}{4d(a+a\sin(c+dx))^2} - \frac{1}{4d\,(a^2+a^2\sin(c+dx))}$$

output
```
1/4*arctanh(sin(d*x+c))/a^2/d+1/4/d/(a+a*sin(d*x+c))^2-1/4/d/(a^2+a^2*sin(
d*x+c))
```

3.64.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.60

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx)) - \frac{\sin(c+dx)}{(1+\sin(c+dx))^2}}{4a^2d}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

output
```
(ArcTanh[Sin[c + d*x]] - Sin[c + d*x]/(1 + Sin[c + d*x])^2)/(4*a^2*d)
```

3.64.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a\sin(c+dx)}{(a-a\sin(c+dx))(\sin(c+dx)a+a)^3}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{1}{2(\sin(c+dx)a+a)^3}+\frac{1}{4(a^2-a^2\sin^2(c+dx))a}+\frac{1}{4(\sin(c+dx)a+a)^2a}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\text{arctanh}(\sin(c+dx))}{4a^2}-\frac{1}{4a(a\sin(c+dx)+a)}+\frac{1}{4(a\sin(c+dx)+a)^2}}{d}$$

input
```
Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

output
```
(ArcTanh[Sin[c + d*x]]/(4*a^2) + 1/(4*(a + a*Sin[c + d*x])^2) - 1/(4*a*(a
+ a*Sin[c + d*x])))/d
```

3.64.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.64.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{8}+\frac{1}{4(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8}}{d\,a^2}$	55
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{8}+\frac{1}{4(1+\sin(dx+c))^2}-\frac{1}{4(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{8}}{d\,a^2}$	55
risch	$-\frac{i(e^{3i(dx+c)}-e^{i(dx+c)})}{2d\,a^2(e^{i(dx+c)}+i)^4}-\frac{\ln(-i+e^{i(dx+c)})}{4a^2d}+\frac{\ln(e^{i(dx+c)}+i)}{4a^2d}$	88

input
```
int(tan(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/8*ln(sin(d*x+c)-1)+1/4/(1+sin(d*x+c))^2-1/4/(1+sin(d*x+c))+1/8
*ln(1+sin(d*x+c)))
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.73

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx$$
$$= \frac{\left(\cos\left(dx+c\right)^2-2\,\sin\left(dx+c\right)-2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(\cos\left(dx+c\right)^2-2\,\sin\left(dx+c\right)-2\right)\log\left(-\sin\right.}{8\left(a^2d\cos\left(dx+c\right)^2-2\,a^2d\sin\left(dx+c\right)-2\,a^2d\right)}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/8*((cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - (cos(d*
x + c)^2 - 2*sin(d*x + c) - 2)*log(-sin(d*x + c) + 1) + 2*sin(d*x + c))/(a
^2*d*cos(d*x + c)^2 - 2*a^2*d*sin(d*x + c) - 2*a^2*d)
```

3.64.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\tan(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\frac{2\,\sin(dx+c)}{a^2\sin(dx+c)^2+2\,a^2\sin(dx+c)+a^2}-\frac{\log(\sin(dx+c)+1)}{a^2}+\frac{\log(\sin(dx+c)-1)}{a^2}}{8\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/8*(2*sin(d*x + c)/(a^2*sin(d*x + c)^2 + 2*a^2*sin(d*x + c) + a^2) - log
(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c) - 1)/a^2)/d
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right|\right)}{a^2}-\frac{\log\left(\left|\frac{1}{\sin(dx+c)}+\sin(dx+c)-2\right|\right)}{a^2}-\frac{\frac{1}{\sin(dx+c)}+\sin(dx+c)+6}{a^2\left(\frac{1}{\sin(dx+c)}+\sin(dx+c)+2\right)}}{16\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/16*(log(abs(1/sin(d*x + c) + sin(d*x + c) + 2))/a^2 - log(abs(1/sin(d*x
+ c) + sin(d*x + c) - 2))/a^2 - (1/sin(d*x + c) + sin(d*x + c) + 6)/(a^2*(
1/sin(d*x + c) + sin(d*x + c) + 2)))/d
```

3.64.9 Mupad [B] (verification not implemented)

Time = 7.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.93

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,a^2\,d} - \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{2}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+6\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2\right)}$$

input
```
int(tan(c + d*x)/(a + a*sin(c + d*x))^2,x)
```

output
```
atanh(tan(c/2 + (d*x)/2))/(2*a^2*d) - (tan(c/2 + (d*x)/2)/2 + tan(c/2 + (d
*x)/2)^3/2)/(d*(6*a^2*tan(c/2 + (d*x)/2)^2 + 4*a^2*tan(c/2 + (d*x)/2)^3 +
a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a^2*tan(c/2 + (d*x)/2)))
```

3.65 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.65.1 Optimal result

Integrand size = 19, antiderivative size = 52

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\log(\sin(c+dx))}{a^2d} - \frac{\log(1+\sin(c+dx))}{a^2d} + \frac{1}{d\,(a^2+a^2\sin(c+dx))}$$

output
```
ln(sin(d*x+c))/a^2/d-ln(1+sin(d*x+c))/a^2/d+1/d/(a^2+a^2*sin(d*x+c))
```

3.65.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\log(\sin(c+dx)) - \log(1+\sin(c+dx)) + \frac{1}{1+\sin(c+dx)}}{a^2d}$$

input
```
Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

output
```
(Log[Sin[c + d*x]] - Log[1 + Sin[c + d*x]] + (1 + Sin[c + d*x])^(-1))/(a^2
*d)
```

3.65.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3186}$$

$$\frac{\int \frac{\csc(c+dx)}{a(\sin(c+dx)a+a)^2}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\int \left(\frac{\csc(c+dx)}{a^3}-\frac{1}{a^2(\sin(c+dx)a+a)}-\frac{1}{a(\sin(c+dx)a+a)^2}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\log(a\sin(c+dx))}{a^2}-\frac{\log(a\sin(c+dx)+a)}{a^2}+\frac{1}{a(a\sin(c+dx)+a)}}{d}$$

input
```
Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^2,x]
```

output
```
(Log[a*Sin[c + d*x]]/a^2 - Log[a + a*Sin[c + d*x]]/a^2 + 1/(a*(a + a*Sin[c
 + d*x])))/d
```

3.65.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.65.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\ln(\sin(dx+c))+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{d\,a^2}$	37
default	$\frac{\ln(\sin(dx+c))+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))}{d\,a^2}$	37
risch	$\frac{2i\mathrm{e}^{i(dx+c)}}{d\,a^2\left(\mathrm{e}^{i(dx+c)}+i\right)^2}-\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2d}+\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^2d}$	74

input
```
int(cot(d*x+c)/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(ln(sin(d*x+c))+1/(1+sin(d*x+c))-ln(1+sin(d*x+c)))
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{(\sin(dx+c)+1)\log\left(\frac{1}{2}\sin(dx+c)\right) - (\sin(dx+c)+1)\log(\sin(dx+c)+1)+1}{a^2d\sin(dx+c)+a^2d}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
((sin(d*x + c) + 1)*log(1/2*sin(d*x + c)) - (sin(d*x + c) + 1)*log(sin(d*x
 + c) + 1) + 1)/(a^2*d*sin(d*x + c) + a^2*d)
```

3.65.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\cot(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.88

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\frac{1}{a^2\sin(dx+c)+a^2} - \frac{\log(\sin(dx+c)+1)}{a^2} + \frac{\log(\sin(dx+c))}{a^2}}{d}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
(1/(a^2*sin(d*x + c) + a^2) - log(sin(d*x + c) + 1)/a^2 + log(sin(d*x + c)
)/a^2)/d
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{a\left(\frac{\log\left(\left|-\frac{a}{a\sin(dx+c)+a}+1\right|\right)}{a^3}+\frac{1}{(a\sin(dx+c)+a)a^2}\right)}{d}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
a*(log(abs(-a/(a*sin(d*x + c) + a) + 1))/a^3 + 1/((a*sin(d*x + c) + a)*a^2
))/d
```

3.65.9 Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.67

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d} - \frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^2\,d} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^2\right)}$$

input
```
int(cot(c + d*x)/(a + a*sin(c + d*x))^2,x)
```

output
```
log(tan(c/2 + (d*x)/2))/(a^2*d) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^2*d)
- (2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2 + 2*a^2*tan(c/
2 + (d*x)/2)))
```

3.66 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.66.1 Optimal result

Integrand size = 21, antiderivative size = 65

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{2\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2a^2d} + \frac{2\log(\sin(c+dx))}{a^2d} - \frac{2\log(1+\sin(c+dx))}{a^2d}$$

output
```
2*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a^2/d+2*ln(sin(d*x+c))/a^2/d-2*ln(1+si
n(d*x+c))/a^2/d
```

3.66.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{4\csc(c+dx) - \csc^2(c+dx) + 4\log(\sin(c+dx)) - 4\log(1+\sin(c+dx))}{2a^2d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

output
```
(4*Csc[c + d*x] - Csc[c + d*x]^2 + 4*Log[Sin[c + d*x]] - 4*Log[1 + Sin[c +
 d*x]])/(2*a^2*d)
```

3.66.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^3(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{a^3(\sin(c+dx)a+a)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{\csc^3(c+dx)}{a^3}-\frac{2\csc^2(c+dx)}{a^3}+\frac{2\csc(c+dx)}{a^3}-\frac{2}{a^2(\sin(c+dx)a+a)}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^2(c+dx)}{2a^2}+\frac{2\csc(c+dx)}{a^2}+\frac{2\log(a\sin(c+dx))}{a^2}-\frac{2\log(a\sin(c+dx)+a)}{a^2}}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^2,x]
```

output
```
((2*Csc[c + d*x])/a^2 - Csc[c + d*x]^2/(2*a^2) + (2*Log[a*Sin[c + d*x]])/a
^2 - (2*Log[a + a*Sin[c + d*x]])/a^2)/d
```

3.66.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.66.4 Maple [A] (verified)

Time = 2.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

method	result	size
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{2}{\sin(dx+c)}+2\ln(\sin(dx+c))-2\ln(1+\sin(dx+c))}{d\,a^2}$	49
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{2}{\sin(dx+c)}+2\ln(\sin(dx+c))-2\ln(1+\sin(dx+c))}{d\,a^2}$	49
risch	$\frac{2i\left(-i\,e^{2i(dx+c)}+2\,e^{3i(dx+c)}-2\,e^{i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}-\frac{4\ln\left(e^{i(dx+c)}+i\right)}{a^2d}+\frac{2\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}$	100

input
```
int(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/2/sin(d*x+c)^2+2/sin(d*x+c)+2*ln(sin(d*x+c))-2*ln(1+sin(d*x+c)
))
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx$$
$$= \frac{4\left(\cos\left(dx+c\right)^2-1\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-4\left(\cos\left(dx+c\right)^2-1\right)\log\left(\sin\left(dx+c\right)+1\right)-4\sin\left(dx+c\right)+}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(4*(cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 4*(cos(d*x + c)^2 - 1)
*log(sin(d*x + c) + 1) - 4*sin(d*x + c) + 1)/(a^2*d*cos(d*x + c)^2 - a^2*d
)
```

3.66.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**3/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.66.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Timed out
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.77

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx =$$
$$-\frac{\frac{32\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{16\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2}+\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^4}+\frac{24\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-8\tan\left(\frac{1}{2}\,dx+\right.}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{8\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/8*(32*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 16*log(abs(tan(1/2*d*x +
 1/2*c)))/a^2 + (a^2*tan(1/2*d*x + 1/2*c)^2 - 8*a^2*tan(1/2*d*x + 1/2*c))/
a^4 + (24*tan(1/2*d*x + 1/2*c)^2 - 8*tan(1/2*d*x + 1/2*c) + 1)/(a^2*tan(1/
2*d*x + 1/2*c)^2))/d
```

3.66.9 Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.58

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{2\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^2\,d}-\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^2\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2\,d}+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{8}\right)}{a^2\,d}$$

input
```
int(cot(c + d*x)^3/(a + a*sin(c + d*x))^2,x)
```

output
```
(2*log(tan(c/2 + (d*x)/2)))/(a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (4*
log(tan(c/2 + (d*x)/2) + 1))/(a^2*d) + tan(c/2 + (d*x)/2)/(a^2*d) + (cot(c
/2 + (d*x)/2)^2*(tan(c/2 + (d*x)/2) - 1/8))/(a^2*d)
```

3.67 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2} \, dx$

3.67.1 Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2} \, dx = -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{4a^2d}$$

output
```
-1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a^2/d
```

3.67.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2} \, dx = \frac{\csc^4(c+dx)(-6+3\cos(2(c+dx))+8\sin(c+dx))}{12a^2d}$$

input
```
Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

output
```
(Csc[c + d*x]^4*(-6 + 3*Cos[2*(c + d*x)] + 8*Sin[c + d*x]))/(12*a^2*d)
```

3.67.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3186, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^5(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{ 3186}$$

$$\frac{\int \frac{\csc^5(c+dx)(a-a\sin(c+dx))^2}{a^5}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{\int \left(\frac{\csc^5(c+dx)}{a^3}-\frac{2\csc^4(c+dx)}{a^3}+\frac{\csc^3(c+dx)}{a^3}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\csc^4(c+dx)}{4a^2}+\frac{2\csc^3(c+dx)}{3a^2}-\frac{\csc^2(c+dx)}{2a^2}}{d}$$

input
```
Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^2,x]
```

output
```
(-1/2*Csc[c + d*x]^2/a^2 + (2*Csc[c + d*x]^3)/(3*a^2) - Csc[c + d*x]^4/(4*
a^2))/d
```

3.67.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.67.4 Maple [A] (verified)

Time = 4.70 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{2}{3\sin(dx+c)^3}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}}{d\,a^2}$	39
default	$\frac{\frac{2}{3\sin(dx+c)^3}-\frac{1}{4\sin(dx+c)^4}-\frac{1}{2\sin(dx+c)^2}}{d\,a^2}$	39
risch	$\frac{2\,e^{6i(dx+c)}-8\,e^{4i(dx+c)}-\frac{16ie^{5i(dx+c)}}{3}+2\,e^{2i(dx+c)}+\frac{16ie^{3i(dx+c)}}{3}}{d\,a^2\left(e^{2i(dx+c)}-1\right)^4}$	80

input
```
int(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(2/3/sin(d*x+c)^3-1/4/sin(d*x+c)^4-1/2/sin(d*x+c)^2)
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{6\,\cos\left(dx+c\right)^2+8\,\sin\left(dx+c\right)-9}{12\,\left(a^2d\cos\left(dx+c\right)^4-2\,a^2d\cos\left(dx+c\right)^2+a^2d\right)}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/12*(6*cos(d*x + c)^2 + 8*sin(d*x + c) - 9)/(a^2*d*cos(d*x + c)^4 - 2*a^2
*d*cos(d*x + c)^2 + a^2*d)
```

3.67.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**5/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{6\,\sin\left(dx+c\right)^2-8\,\sin\left(dx+c\right)+3}{12\,a^2d\sin\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{6\,\sin\left(dx+c\right)^2 - 8\,\sin\left(dx+c\right) + 3}{12\,a^2 d\sin\left(dx+c\right)^4}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/12*(6*sin(d*x + c)^2 - 8*sin(d*x + c) + 3)/(a^2*d*sin(d*x + c)^4)
```

3.67.9 Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\frac{\sin(c+d\,x)^2}{2} - \frac{2\,\sin(c+d\,x)}{3} + \frac{1}{4}}{a^2\,d\sin\left(c+d\,x\right)^4}$$

input
```
int(cot(c + d*x)^5/(a + a*sin(c + d*x))^2,x)
```

output
```
-(sin(c + d*x)^2/2 - (2*sin(c + d*x))/3 + 1/4)/(a^2*d*sin(c + d*x)^4)
```

3.68 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.68.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d} - \frac{\csc^6(c+dx)}{6a^2d}$$

output
```
1/2*csc(d*x+c)^2/a^2/d-2/3*csc(d*x+c)^3/a^2/d+2/5*csc(d*x+c)^5/a^2/d-1/6*c
sc(d*x+c)^6/a^2/d
```

3.68.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\csc^2(c+dx)\left(15-20\csc(c+dx)+12\csc^3(c+dx)-5\csc^4(c+dx)\right)}{30a^2d}$$

input
```
Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

output
```
(Csc[c + d*x]^2*(15 - 20*Csc[c + d*x] + 12*Csc[c + d*x]^3 - 5*Csc[c + d*x]
^4))/(30*a^2*d)
```

3.68.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^7(c+dx)}{(a\sin(c+dx)+a)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(c+dx)^7(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \; 3186$$

$$\frac{\int \frac{\csc^7(c+dx)(a-a\sin(c+dx))^3(\sin(c+dx)a+a)}{a^7}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 84$$

$$\frac{\int \left(\frac{\csc^7(c+dx)}{a^3}-\frac{2\csc^6(c+dx)}{a^3}+\frac{2\csc^4(c+dx)}{a^3}-\frac{\csc^3(c+dx)}{a^3}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{\csc^6(c+dx)}{6a^2}+\frac{2\csc^5(c+dx)}{5a^2}-\frac{2\csc^3(c+dx)}{3a^2}+\frac{\csc^2(c+dx)}{2a^2}}{d}$$

input
```
Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^2,x]
```

output
```
(Csc[c + d*x]^2/(2*a^2) - (2*Csc[c + d*x]^3)/(3*a^2) + (2*Csc[c + d*x]^5)/
(5*a^2) - Csc[c + d*x]^6/(6*a^2))/d
```

3.68.3.1 Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.68.4 Maple [A] (verified)

Time = 9.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{-\frac{1}{6\sin(dx+c)^6}-\frac{2}{3\sin(dx+c)^3}+\frac{2}{5\sin(dx+c)^5}+\frac{1}{2\sin(dx+c)^2}}{d\,a^2}$
default	$\frac{-\frac{1}{6\sin(dx+c)^6}-\frac{2}{3\sin(dx+c)^3}+\frac{2}{5\sin(dx+c)^5}+\frac{1}{2\sin(dx+c)^2}}{d\,a^2}$
risch	$-\frac{2\left(15\,e^{10i(dx+c)}-60\,e^{8i(dx+c)}-40i e^{9i(dx+c)}+10\,e^{6i(dx+c)}+24i e^{7i(dx+c)}-60\,e^{4i(dx+c)}-24i e^{5i(dx+c)}+15\,e^{2i(dx+c)}+4\right.}{15 d\,a^2\left(e^{2i(dx+c)}-1\right)^6}$

input
```
int(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/6/sin(d*x+c)^6-2/3/sin(d*x+c)^3+2/5/sin(d*x+c)^5+1/2/sin(d*x+c
)^2)
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{15\cos(dx+c)^4-30\cos(dx+c)^2+4\left(5\cos(dx+c)^2-2\right)\sin(dx+c)+10}{30\left(a^2d\cos(dx+c)^6-3\,a^2d\cos(dx+c)^4+3\,a^2d\cos(dx+c)^2-a^2d\right)}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/30*(15*cos(d*x + c)^4 - 30*cos(d*x + c)^2 + 4*(5*cos(d*x + c)^2 - 2)*si
n(d*x + c) + 10)/(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*
cos(d*x + c)^2 - a^2*d)
```

3.68.6 Sympy [F]

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\cot^7(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**7/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{15\sin(dx+c)^4-20\sin(dx+c)^3+12\sin(dx+c)-5}{30\,a^2d\sin(dx+c)^6}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*
sin(d*x + c)^6)
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{15\sin{(dx+c)}^4 - 20\sin{(dx+c)}^3 + 12\sin{(dx+c)} - 5}{30\,a^2d\sin{(dx+c)}^6}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/30*(15*sin(d*x + c)^4 - 20*sin(d*x + c)^3 + 12*sin(d*x + c) - 5)/(a^2*d*
sin(d*x + c)^6)
```

3.68.9 Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{15\sin{(c+d\,x)}^4 - 20\sin{(c+d\,x)}^3 + 12\sin{(c+d\,x)} - 5}{30\,a^2\,d\sin{(c+d\,x)}^6}$$

input
```
int(cot(c + d*x)^7/(a + a*sin(c + d*x))^2,x)
```

output
```
(12*sin(c + d*x) - 20*sin(c + d*x)^3 + 15*sin(c + d*x)^4 - 5)/(30*a^2*d*si
n(c + d*x)^6)
```

3.69 $\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.69.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\csc^2(c+dx)}{2a^2d} + \frac{2\csc^3(c+dx)}{3a^2d} + \frac{\csc^4(c+dx)}{4a^2d} - \frac{4\csc^5(c+dx)}{5a^2d} + \frac{\csc^6(c+dx)}{6a^2d} + \frac{2\csc^7(c+dx)}{7a^2d} - \frac{\csc^8(c+dx)}{8a^2d}$$

output
```
-1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d+1/4*csc(d*x+c)^4/a^2/d-4/5*
csc(d*x+c)^5/a^2/d+1/6*csc(d*x+c)^6/a^2/d+2/7*csc(d*x+c)^7/a^2/d-1/8*csc(d
*x+c)^8/a^2/d
```

3.69.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\csc^2(c+dx)\left(-420+560\csc(c+dx)+210\csc^2(c+dx)-672\csc^3(c+dx)+140\csc^4(c+dx)+240\csc^5\right.}{840a^2d}$$

input
```
Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]
```

output
```
(Csc[c + d*x]^2*(-420 + 560*Csc[c + d*x] + 210*Csc[c + d*x]^2 - 672*Csc[c
+ d*x]^3 + 140*Csc[c + d*x]^4 + 240*Csc[c + d*x]^5 - 105*Csc[c + d*x]^6))/
(840*a^2*d)
```

3.69.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^9(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^9(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^9(c+dx)(a-a\sin(c+dx))^4(\sin(c+dx)a+a)^2}{a^9}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\csc^9(c+dx)}{a^3}-\frac{2\csc^8(c+dx)}{a^3}-\frac{\csc^7(c+dx)}{a^3}+\frac{4\csc^6(c+dx)}{a^3}-\frac{\csc^5(c+dx)}{a^3}-\frac{2\csc^4(c+dx)}{a^3}+\frac{\csc^3(c+dx)}{a^3}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^8(c+dx)}{8a^2}+\frac{2\csc^7(c+dx)}{7a^2}+\frac{\csc^6(c+dx)}{6a^2}-\frac{4\csc^5(c+dx)}{5a^2}+\frac{\csc^4(c+dx)}{4a^2}+\frac{2\csc^3(c+dx)}{3a^2}-\frac{\csc^2(c+dx)}{2a^2}}{d}$$

input
```
Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^2,x]
```

output
```
(-1/2*Csc[c + d*x]^2/a^2 + (2*Csc[c + d*x]^3)/(3*a^2) + Csc[c + d*x]^4/(4*
a^2) - (4*Csc[c + d*x]^5)/(5*a^2) + Csc[c + d*x]^6/(6*a^2) + (2*Csc[c + d*
x]^7)/(7*a^2) - Csc[c + d*x]^8/(8*a^2))/d
```

3.69.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.69.4 Maple [A] (verified)

Time = 17.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.62

method	result
derivativedivides	$\frac{\frac{1}{4\sin(dx+c)^4}+\frac{2}{3\sin(dx+c)^3}-\frac{1}{8\sin(dx+c)^8}-\frac{4}{5\sin(dx+c)^5}+\frac{2}{7\sin(dx+c)^7}-\frac{1}{2\sin(dx+c)^2}+\frac{1}{6\sin(dx+c)^6}}{d\,a^2}$
default	$\frac{\frac{1}{4\sin(dx+c)^4}+\frac{2}{3\sin(dx+c)^3}-\frac{1}{8\sin(dx+c)^8}-\frac{4}{5\sin(dx+c)^5}+\frac{2}{7\sin(dx+c)^7}-\frac{1}{2\sin(dx+c)^2}+\frac{1}{6\sin(dx+c)^6}}{d\,a^2}$
risch	$\frac{2\,\mathrm{e}^{14i(dx+c)}-8\,\mathrm{e}^{12i(dx+c)}-\frac{16i\mathrm{e}^{13i(dx+c)}}{3}+\frac{10\,\mathrm{e}^{10i(dx+c)}}{3}+\frac{16i\mathrm{e}^{11i(dx+c)}}{15}-\frac{80\,\mathrm{e}^{8i(dx+c)}}{3}-\frac{1376i\mathrm{e}^{9i(dx+c)}}{105}+\frac{10\,\mathrm{e}^{6i(dx+c)}}{3}+1}{d\,a^2\left(\mathrm{e}^{2i(dx+c)}-1\right)^8}$

input
```
int(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(1/4/sin(d*x+c)^4+2/3/sin(d*x+c)^3-1/8/sin(d*x+c)^8-4/5/sin(d*x+c)
^5+2/7/sin(d*x+c)^7-1/2/sin(d*x+c)^2+1/6/sin(d*x+c)^6)
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx$$
$$= \frac{420\cos(dx+c)^6 - 1050\cos(dx+c)^4 + 700\cos(dx+c)^2 + 16\left(35\cos(dx+c)^4 - 28\cos(dx+c)^2 + 8\right)}{840\left(a^2d\cos(dx+c)^8 - 4a^2d\cos(dx+c)^6 + 6a^2d\cos(dx+c)^4 - 4a^2d\cos(dx+c)^2 + a\right.}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/840*(420*cos(d*x + c)^6 - 1050*cos(d*x + c)^4 + 700*cos(d*x + c)^2 + 16*
(35*cos(d*x + c)^4 - 28*cos(d*x + c)^2 + 8)*sin(d*x + c) - 175)/(a^2*d*cos
(d*x + c)^8 - 4*a^2*d*cos(d*x + c)^6 + 6*a^2*d*cos(d*x + c)^4 - 4*a^2*d*co
s(d*x + c)^2 + a^2*d)
```

3.69.6 Sympy [F]

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\int \frac{\cot^9(c+dx)}{\sin^2(c+dx)+2\sin(c+dx)+1}\,dx}{a^2}$$

input
```
integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**9/(sin(c + d*x)**2 + 2*sin(c + d*x) + 1), x)/a**2
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx =$$
$$-\frac{420\sin(dx+c)^6 - 560\sin(dx+c)^5 - 210\sin(dx+c)^4 + 672\sin(dx+c)^3 - 140\sin(dx+c)^2 - 240}{840\,a^2d\sin(dx+c)^8}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672
*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(
d*x + c)^8)
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{420\,\sin{(dx+c)}^6 - 560\,\sin{(dx+c)}^5 - 210\,\sin{(dx+c)}^4 + 672\,\sin{(dx+c)}^3 - 140\,\sin{(dx+c)}^2 - 240}{840\,a^2 d \sin{(dx+c)}^8}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/840*(420*sin(d*x + c)^6 - 560*sin(d*x + c)^5 - 210*sin(d*x + c)^4 + 672
*sin(d*x + c)^3 - 140*sin(d*x + c)^2 - 240*sin(d*x + c) + 105)/(a^2*d*sin(
d*x + c)^8)
```

3.69.9 Mupad [B] (verification not implemented)

Time = 6.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.60

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{-420\sin{(c+d\,x)}^6 + 560\sin{(c+d\,x)}^5 + 210\sin{(c+d\,x)}^4 - 672\sin{(c+d\,x)}^3 + 140\sin{(c+d\,x)}^2 + 240}{840\,a^2\,d\sin{(c+d\,x)}^8}$$

input
```
int(cot(c + d*x)^9/(a + a*sin(c + d*x))^2,x)
```

output
```
(240*sin(c + d*x) + 140*sin(c + d*x)^2 - 672*sin(c + d*x)^3 + 210*sin(c +
d*x)^4 + 560*sin(c + d*x)^5 - 420*sin(c + d*x)^6 - 105)/(840*a^2*d*sin(c +
 d*x)^8)
```

3.70 $\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.70.1 Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\csc^2(c+dx)}{2a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} - \frac{\csc^4(c+dx)}{2a^2d} + \frac{6\csc^5(c+dx)}{5a^2d} - \frac{6\csc^7(c+dx)}{7a^2d} + \frac{\csc^8(c+dx)}{4a^2d} + \frac{2\csc^9(c+dx)}{9a^2d} - \frac{\csc^{10}(c+dx)}{10a^2d}$$

output
```
1/2*csc(d*x+c)^2/a^2/d-2/3*csc(d*x+c)^3/a^2/d-1/2*csc(d*x+c)^4/a^2/d+6/5*c
sc(d*x+c)^5/a^2/d-6/7*csc(d*x+c)^7/a^2/d+1/4*csc(d*x+c)^8/a^2/d+2/9*csc(d*
x+c)^9/a^2/d-1/10*csc(d*x+c)^10/a^2/d
```

3.70.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\csc^2(c+dx)\left(630 - 840\csc(c+dx) - 630\csc^2(c+dx) + 1512\csc^3(c+dx) - 1080\csc^5(c+dx) + 315\csc\right.}{1260a^2d}$$

input
```
Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]
```

output
```
(Csc[c + d*x]^2*(630 - 840*Csc[c + d*x] - 630*Csc[c + d*x]^2 + 1512*Csc[c
+ d*x]^3 - 1080*Csc[c + d*x]^5 + 315*Csc[c + d*x]^6 + 280*Csc[c + d*x]^7 -
 126*Csc[c + d*x]^8))/(1260*a^2*d)
```

3.70.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{11}(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^{11}(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^{11}(c+dx)(a-a\sin(c+dx))^5(\sin(c+dx)a+a)^3}{a^{11}}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\csc^{11}(c+dx)}{a^3}-\frac{2\csc^{10}(c+dx)}{a^3}-\frac{2\csc^9(c+dx)}{a^3}+\frac{6\csc^8(c+dx)}{a^3}-\frac{6\csc^6(c+dx)}{a^3}+\frac{2\csc^5(c+dx)}{a^3}+\frac{2\csc^4(c+dx)}{a^3}-\frac{\csc^3(c+dx)}{a^3}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^{10}(c+dx)}{10a^2}+\frac{2\csc^9(c+dx)}{9a^2}+\frac{\csc^8(c+dx)}{4a^2}-\frac{6\csc^7(c+dx)}{7a^2}+\frac{6\csc^5(c+dx)}{5a^2}-\frac{\csc^4(c+dx)}{2a^2}-\frac{2\csc^3(c+dx)}{3a^2}+\frac{\csc^2(c+dx)}{2a^2}}{d}$$

input
```
Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^2,x]
```

output
```
(Csc[c + d*x]^2/(2*a^2) - (2*Csc[c + d*x]^3)/(3*a^2) - Csc[c + d*x]^4/(2*a
^2) + (6*Csc[c + d*x]^5)/(5*a^2) - (6*Csc[c + d*x]^7)/(7*a^2) + Csc[c + d*
x]^8/(4*a^2) + (2*Csc[c + d*x]^9)/(9*a^2) - Csc[c + d*x]^10/(10*a^2))/d
```

3.70.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.70.4 Maple [A] (verified)

Time = 31.77 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^4}+\frac{1}{2\sin(dx+c)^2}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{10\sin(dx+c)^{10}}+\frac{1}{4\sin(dx+c)^8}-\frac{6}{7\sin(dx+c)^7}+\frac{2}{9\sin(dx+c)^9}+\frac{6}{5\sin(dx+c)^5}}{d\,a^2}$
default	$\frac{-\frac{1}{2\sin(dx+c)^4}+\frac{1}{2\sin(dx+c)^2}-\frac{2}{3\sin(dx+c)^3}-\frac{1}{10\sin(dx+c)^{10}}+\frac{1}{4\sin(dx+c)^8}-\frac{6}{7\sin(dx+c)^7}+\frac{2}{9\sin(dx+c)^9}+\frac{6}{5\sin(dx+c)^5}}{d\,a^2}$
risch	$-\frac{2\left(315\,e^{18i(dx+c)}-1260\,e^{16i(dx+c)}+840ie^{3i(dx+c)}+1260\,e^{14i(dx+c)}-840ie^{17i(dx+c)}-8820\,e^{12i(dx+c)}+168ie^{5i(dx+c)}+\right.}{}$

input
```
int(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-1/2/sin(d*x+c)^4+1/2/sin(d*x+c)^2-2/3/sin(d*x+c)^3-1/10/sin(d*x+
c)^10+1/4/sin(d*x+c)^8-6/7/sin(d*x+c)^7+2/9/sin(d*x+c)^9+6/5/sin(d*x+c)^5)
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.12

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{630\cos(dx+c)^8 - 1890\cos(dx+c)^6 + 1890\cos(dx+c)^4 - 945\cos(dx+c)^2 + 8\left(105\cos(dx+c)^6\right.}{1260\left(a^2d\cos(dx+c)^{10} - 5a^2d\cos(dx+c)^8 + 10a^2d\cos(dx+c)^6 - 10a^2d\,\mathrm{co}\right.}$$

input
```
integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/1260*(630*cos(d*x + c)^8 - 1890*cos(d*x + c)^6 + 1890*cos(d*x + c)^4 -
945*cos(d*x + c)^2 + 8*(105*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d
*x + c)^2 - 16)*sin(d*x + c) + 189)/(a^2*d*cos(d*x + c)^10 - 5*a^2*d*cos(d
*x + c)^8 + 10*a^2*d*cos(d*x + c)^6 - 10*a^2*d*cos(d*x + c)^4 + 5*a^2*d*co
s(d*x + c)^2 - a^2*d)
```

3.70.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{630\sin(dx+c)^8 - 840\sin(dx+c)^7 - 630\sin(dx+c)^6 + 1512\sin(dx+c)^5 - 1080\sin(dx+c)^3 + 315}{1260\,a^2d\sin(dx+c)^{10}}$$

input
```
integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 151
2*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x
+ c) - 126)/(a^2*d*sin(d*x + c)^10)
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{630\,\sin\left(dx+c\right)^8-840\,\sin\left(dx+c\right)^7-630\,\sin\left(dx+c\right)^6+1512\,\sin\left(dx+c\right)^5-1080\,\sin\left(dx+c\right)^3+315}{1260\,a^2d\sin\left(dx+c\right)^{10}}$$

input
```
integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/1260*(630*sin(d*x + c)^8 - 840*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 151
2*sin(d*x + c)^5 - 1080*sin(d*x + c)^3 + 315*sin(d*x + c)^2 + 280*sin(d*x
+ c) - 126)/(a^2*d*sin(d*x + c)^10)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{\frac{\sin(c+d\,x)^8}{2}-\frac{2\sin(c+d\,x)^7}{3}-\frac{\sin(c+d\,x)^6}{2}+\frac{6\sin(c+d\,x)^5}{5}-\frac{6\sin(c+d\,x)^3}{7}+\frac{\sin(c+d\,x)^2}{4}+\frac{2\,\sin(c+d\,x)}{9}-\frac{1}{10}}{a^2\,d\sin\left(c+d\,x\right)^{10}}$$

input
```
int(cot(c + d*x)^11/(a + a*sin(c + d*x))^2,x)
```

output
```
((2*sin(c + d*x))/9 + sin(c + d*x)^2/4 - (6*sin(c + d*x)^3)/7 + (6*sin(c +
 d*x)^5)/5 - sin(c + d*x)^6/2 - (2*sin(c + d*x)^7)/3 + sin(c + d*x)^8/2 -
1/10)/(a^2*d*sin(c + d*x)^10)
```

3.71 $\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx$

3.71.1 Optimal result

Integrand size = 21, antiderivative size = 199

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\csc^2(c+dx)}{2a^2d}+\frac{2\csc^3(c+dx)}{3a^2d}+\frac{3\csc^4(c+dx)}{4a^2d}-\frac{8\csc^5(c+dx)}{5a^2d}-\frac{\csc^6(c+dx)}{3a^2d}+\frac{12\csc^7(c+dx)}{7a^2d}-\frac{\csc^8(c+dx)}{4a^2d}-\frac{8\csc^9(c+dx)}{9a^2d}+\frac{3\csc^{10}(c+dx)}{10a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}-\frac{\csc^{12}(c+dx)}{12a^2d}$$

output
```
-1/2*csc(d*x+c)^2/a^2/d+2/3*csc(d*x+c)^3/a^2/d+3/4*csc(d*x+c)^4/a^2/d-8/5*
csc(d*x+c)^5/a^2/d-1/3*csc(d*x+c)^6/a^2/d+12/7*csc(d*x+c)^7/a^2/d-1/4*csc(
d*x+c)^8/a^2/d-8/9*csc(d*x+c)^9/a^2/d+3/10*csc(d*x+c)^10/a^2/d+2/11*csc(d*
x+c)^11/a^2/d-1/12*csc(d*x+c)^12/a^2/d
```

3.71.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{\csc^2(c+dx)\left(6930-9240\csc(c+dx)-10395\csc^2(c+dx)+22176\csc^3(c+dx)+4620\csc^4(c+dx)\right.}{}$$

input
```
Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]
```

output
```
-1/13860*(Csc[c + d*x]^2*(6930 - 9240*Csc[c + d*x] - 10395*Csc[c + d*x]^2
+ 22176*Csc[c + d*x]^3 + 4620*Csc[c + d*x]^4 - 23760*Csc[c + d*x]^5 + 3465
*Csc[c + d*x]^6 + 12320*Csc[c + d*x]^7 - 4158*Csc[c + d*x]^8 - 2520*Csc[c
+ d*x]^9 + 1155*Csc[c + d*x]^10))/(a^2*d)
```

3.71.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{13}(c+dx)}{(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^{13}(a\sin(c+dx)+a)^2}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^{13}(c+dx)(a-a\sin(c+dx))^6(\sin(c+dx)a+a)^4}{a^{13}}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\csc^{13}(c+dx)}{a^3}-\frac{2\csc^{12}(c+dx)}{a^3}-\frac{3\csc^{11}(c+dx)}{a^3}+\frac{8\csc^{10}(c+dx)}{a^3}+\frac{2\csc^9(c+dx)}{a^3}-\frac{12\csc^8(c+dx)}{a^3}+\frac{2\csc^7(c+dx)}{a^3}+\frac{8\csc^6(c+dx)}{a^3}-\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^{12}(c+dx)}{12a^2}+\frac{2\csc^{11}(c+dx)}{11a^2}+\frac{3\csc^{10}(c+dx)}{10a^2}-\frac{8\csc^9(c+dx)}{9a^2}-\frac{\csc^8(c+dx)}{4a^2}+\frac{12\csc^7(c+dx)}{7a^2}-\frac{\csc^6(c+dx)}{3a^2}-\frac{8\csc^5(c+dx)}{5a^2}+\frac{3\csc^4}{4}}{d}$$

input
```
Int[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^2,x]
```

output
```
(-1/2*Csc[c + d*x]^2/a^2 + (2*Csc[c + d*x]^3)/(3*a^2) + (3*Csc[c + d*x]^4)
/(4*a^2) - (8*Csc[c + d*x]^5)/(5*a^2) - Csc[c + d*x]^6/(3*a^2) + (12*Csc[c
 + d*x]^7)/(7*a^2) - Csc[c + d*x]^8/(4*a^2) - (8*Csc[c + d*x]^9)/(9*a^2) +
 (3*Csc[c + d*x]^10)/(10*a^2) + (2*Csc[c + d*x]^11)/(11*a^2) - Csc[c + d*x
]^12/(12*a^2))/d
```

3.71.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.71.4 Maple [A] (verified)

Time = 53.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.60

method	result
derivativedivides	$\frac{-\frac{8}{9\sin(dx+c)^9}-\frac{1}{4\sin(dx+c)^8}+\frac{12}{7\sin(dx+c)^7}+\frac{2}{11\sin(dx+c)^{11}}-\frac{8}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^6}-\frac{1}{12\sin(dx+c)^{12}}+\frac{3}{4\sin(dx+c)^4}+ 10}{d\,a^2}$
default	$\frac{-\frac{8}{9\sin(dx+c)^9}-\frac{1}{4\sin(dx+c)^8}+\frac{12}{7\sin(dx+c)^7}+\frac{2}{11\sin(dx+c)^{11}}-\frac{8}{5\sin(dx+c)^5}-\frac{1}{3\sin(dx+c)^6}-\frac{1}{12\sin(dx+c)^{12}}+\frac{3}{4\sin(dx+c)^4}+ 10}{d\,a^2}$
risch	$2\,e^{22i(dx+c)}-8\,e^{20i(dx+c)}+\frac{16ie^{3i(dx+c)}}{3}+\frac{46\,e^{18i(dx+c)}}{3}-\frac{1856ie^{17i(dx+c)}}{35}-96\,e^{16i(dx+c)}-\frac{16ie^{19i(dx+c)}}{5}+\frac{84\,e^{14i(dx+c)}}{5}+$

input
```
int(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^2*(-8/9/sin(d*x+c)^9-1/4/sin(d*x+c)^8+12/7/sin(d*x+c)^7+2/11/sin(d*x
+c)^11-8/5/sin(d*x+c)^5-1/3/sin(d*x+c)^6-1/12/sin(d*x+c)^12+3/4/sin(d*x+c)
^4+3/10/sin(d*x+c)^10-1/2/sin(d*x+c)^2+2/3/sin(d*x+c)^3)
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \frac{6930\cos(dx+c)^{10}-24255\cos(dx+c)^{8}+32340\cos(dx+c)^{6}-24255\cos(dx+c)^{4}+9702\cos(dx+}{13860\left(a^2d\cos(dx+c)^{12}-6a^2d\cos(dx+c)^{10}+15a^2d\cos(dx+}$$

input
```
integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/13860*(6930*cos(d*x + c)^10 - 24255*cos(d*x + c)^8 + 32340*cos(d*x + c)^
6 - 24255*cos(d*x + c)^4 + 9702*cos(d*x + c)^2 + 8*(1155*cos(d*x + c)^8 -
1848*cos(d*x + c)^6 + 1584*cos(d*x + c)^4 - 704*cos(d*x + c)^2 + 128)*sin(
d*x + c) - 1617)/(a^2*d*cos(d*x + c)^12 - 6*a^2*d*cos(d*x + c)^10 + 15*a^2
*d*cos(d*x + c)^8 - 20*a^2*d*cos(d*x + c)^6 + 15*a^2*d*cos(d*x + c)^4 - 6*
a^2*d*cos(d*x + c)^2 + a^2*d)
```

3.71.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**2,x)
```

output
```
Timed out
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{6930\sin(dx+c)^{10} - 9240\sin(dx+c)^9 - 10395\sin(dx+c)^8 + 22176\sin(dx+c)^7 + 4620\sin(dx+}{1386}$$

input
```
integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^
8 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 34
65*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(
d*x + c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx = -\frac{6930\sin(dx+c)^{10} - 9240\sin(dx+c)^9 - 10395\sin(dx+c)^8 + 22176\sin(dx+c)^7 + 4620\sin(dx+}{1386}$$

input
```
integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/13860*(6930*sin(d*x + c)^10 - 9240*sin(d*x + c)^9 - 10395*sin(d*x + c)^
8 + 22176*sin(d*x + c)^7 + 4620*sin(d*x + c)^6 - 23760*sin(d*x + c)^5 + 34
65*sin(d*x + c)^4 + 12320*sin(d*x + c)^3 - 4158*sin(d*x + c)^2 - 2520*sin(
d*x + c) + 1155)/(a^2*d*sin(d*x + c)^12)
```

3.71.9 Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.58

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^2}\,dx =$$
$$-\frac{\frac{\sin(c+d\,x)^{10}}{2}-\frac{2\sin(c+d\,x)^9}{3}-\frac{3\sin(c+d\,x)^8}{4}+\frac{8\sin(c+d\,x)^7}{5}+\frac{\sin(c+d\,x)^6}{3}-\frac{12\sin(c+d\,x)^5}{7}+\frac{\sin(c+d\,x)^4}{4}+\frac{8\sin(c+d\,x)^3}{9}-\cdots}{a^2\,d\sin\left(c+d\,x\right)^{12}}$$

input
```
int(cot(c + d*x)^13/(a + a*sin(c + d*x))^2,x)
```

output
```
-((8*sin(c + d*x)^3)/9 - (3*sin(c + d*x)^2)/10 - (2*sin(c + d*x))/11 + sin
(c + d*x)^4/4 - (12*sin(c + d*x)^5)/7 + sin(c + d*x)^6/3 + (8*sin(c + d*x)
^7)/5 - (3*sin(c + d*x)^8)/4 - (2*sin(c + d*x)^9)/3 + sin(c + d*x)^10/2 +
1/12)/(a^2*d*sin(c + d*x)^12)
```

3.72 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.72.1 Optimal result

Integrand size = 21, antiderivative size = 171

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{128a^3d} + \frac{1}{128ad(a-a\sin(c+dx))^2} + \frac{a^2}{40d(a+a\sin(c+dx))^5} - \frac{7a}{64d(a+a\sin(c+dx))^4} + \frac{1}{6d(a+a\sin(c+dx))^3} - \frac{5}{64ad(a+a\sin(c+dx))^2} - \frac{1}{32d\,(a^3-a^3\sin(c+dx))} - \frac{5}{128d\,(a^3+a^3\sin(c+dx))}$$

output
```
1/128*arctanh(sin(d*x+c))/a^3/d+1/128/a/d/(a-a*sin(d*x+c))^2+1/40*a^2/d/(a
+a*sin(d*x+c))^5-7/64*a/d/(a+a*sin(d*x+c))^4+1/6/d/(a+a*sin(d*x+c))^3-5/64
/a/d/(a+a*sin(d*x+c))^2-1/32/d/(a^3-a^3*sin(d*x+c))-5/128/d/(a^3+a^3*sin(d
*x+c))
```

3.72.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.60

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{15\text{arctanh}(\sin(c+dx))-\frac{112+351\sin(c+dx)+157\sin^2(c+dx)-540\sin^3(c+dx)-620\sin^4(c+dx)+45\sin^5(c+dx)+15\sin^6(c+dx)}{(-1+\sin(c+dx))^2(1+\sin(c+dx))^5}}{1920a^3d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

output
```
(15*ArcTanh[Sin[c + d*x]] - (112 + 351*Sin[c + d*x] + 157*Sin[c + d*x]^2 -
 540*Sin[c + d*x]^3 - 620*Sin[c + d*x]^4 + 45*Sin[c + d*x]^5 + 15*Sin[c +
d*x]^6)/((-1 + Sin[c + d*x])^2*(1 + Sin[c + d*x])^5))/(1920*a^3*d)
```

3.72.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a\sin(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^5}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^5\sin^5(c+dx)}{(a-a\sin(c+dx))^3(\sin(c+dx)a+a)^6}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{a^2}{8(\sin(c+dx)a+a)^6}+\frac{7a}{16(\sin(c+dx)a+a)^5}-\frac{1}{2(\sin(c+dx)a+a)^4}+\frac{1}{64(a-a\sin(c+dx))^3a}+\frac{5}{32(\sin(c+dx)a+a)^3a}+\frac{1}{128(a^2-a^2\sin^2(c\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\operatorname{arctanh}(\sin(c+dx))}{128a^3}+\frac{a^2}{40(a\sin(c+dx)+a)^5}-\frac{1}{32a^2(a-a\sin(c+dx))}-\frac{5}{128a^2(a\sin(c+dx)+a)}-\frac{7a}{64(a\sin(c+dx)+a)^4}+\frac{1}{6(a\sin(c+dx)+a)}}{d}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

output
```
(ArcTanh[Sin[c + d*x]]/(128*a^3) + 1/(128*a*(a - a*Sin[c + d*x])^2) - 1/(3
2*a^2*(a - a*Sin[c + d*x])) + a^2/(40*(a + a*Sin[c + d*x])^5) - (7*a)/(64*
(a + a*Sin[c + d*x])^4) + 1/(6*(a + a*Sin[c + d*x])^3) - 5/(64*a*(a + a*Si
n[c + d*x])^2) - 5/(128*a^2*(a + a*Sin[c + d*x])))/d
```

3.72.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.72.4 Maple [A] (verified)

Time = 3.86 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{\frac{1}{128(\sin(dx+c)-1)^2}+\frac{1}{32\sin(dx+c)-32}-\frac{\ln(\sin(dx+c)-1)}{256}+\frac{1}{40(1+\sin(dx+c))^5}-\frac{7}{64(1+\sin(dx+c))^4}+\frac{1}{6(1+\sin(dx+c))^3}-\frac{5}{64(1+\sin(\cdots}}{d\,a^3}$
default	$\frac{\frac{1}{128(\sin(dx+c)-1)^2}+\frac{1}{32\sin(dx+c)-32}-\frac{\ln(\sin(dx+c)-1)}{256}+\frac{1}{40(1+\sin(dx+c))^5}-\frac{7}{64(1+\sin(dx+c))^4}+\frac{1}{6(1+\sin(dx+c))^3}-\frac{5}{64(1+\sin(\cdots}}{d\,a^3}$
risch	$-\frac{i\left(-828i e^{8i(dx+c)}+15\,e^{13i(dx+c)}+2390\,e^{11i(dx+c)}-7183\,e^{9i(dx+c)}+2388\,e^{7i(dx+c)}+15\,e^{i(dx+c)}-7183\,e^{5i(dx+c)}+2390\cdots\right.}{960\left(-i+e^{i(dx+c)}\right)^4\left(e^{i(dx+c)}+i\right)\cdots}$

input
```
int(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/128/(sin(d*x+c)-1)^2+1/32/(sin(d*x+c)-1)-1/256*ln(sin(d*x+c)-1)
+1/40/(1+sin(d*x+c))^5-7/64/(1+sin(d*x+c))^4+1/6/(1+sin(d*x+c))^3-5/64/(1+
sin(d*x+c))^2-5/128/(1+sin(d*x+c))+1/256*ln(1+sin(d*x+c)))
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.45

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{30\cos(dx+c)^6+1150\cos(dx+c)^4-2076\cos(dx+c)^2-15\left(3\cos(dx+c)^6-4\cos(dx+c)^4+(\cdots\right.}{\cdots}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/3840*(30*cos(d*x + c)^6 + 1150*cos(d*x + c)^4 - 2076*cos(d*x + c)^2 - 1
5*(3*cos(d*x + c)^6 - 4*cos(d*x + c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^
4)*sin(d*x + c))*log(sin(d*x + c) + 1) + 15*(3*cos(d*x + c)^6 - 4*cos(d*x
+ c)^4 + (cos(d*x + c)^6 - 4*cos(d*x + c)^4)*sin(d*x + c))*log(-sin(d*x +
c) + 1) - 18*(5*cos(d*x + c)^4 + 50*cos(d*x + c)^2 - 16)*sin(d*x + c) + 67
2)/(3*a^3*d*cos(d*x + c)^6 - 4*a^3*d*cos(d*x + c)^4 + (a^3*d*cos(d*x + c)^
6 - 4*a^3*d*cos(d*x + c)^4)*sin(d*x + c))
```

3.72.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
d*x) + 1), x)/a**3
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.10

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\frac{2\left(15\sin(dx+c)^6+45\sin(dx+c)^5-620\sin(dx+c)^4-540\sin(dx+c)^3+157\sin(dx+c)^2+351\sin(dx+c)+112\right)}{a^3\sin(dx+c)^7+3a^3\sin(dx+c)^6+a^3\sin(dx+c)^5-5a^3\sin(dx+c)^4-5a^3\sin(dx+c)^3+a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3}-\frac{15\log(\sin(dx+c)\cdot}{a^3}}{3840\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/3840*(2*(15*sin(d*x + c)^6 + 45*sin(d*x + c)^5 - 620*sin(d*x + c)^4 - 5
40*sin(d*x + c)^3 + 157*sin(d*x + c)^2 + 351*sin(d*x + c) + 112)/(a^3*sin(
d*x + c)^7 + 3*a^3*sin(d*x + c)^6 + a^3*sin(d*x + c)^5 - 5*a^3*sin(d*x + c
)^4 - 5*a^3*sin(d*x + c)^3 + a^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3
) - 15*log(sin(d*x + c) + 1)/a^3 + 15*log(sin(d*x + c) - 1)/a^3)/d
```

3.72.8 Giac [A] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.80

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{60\log(|\sin(dx+c)+1|)}{a^3}-\frac{60\log(|\sin(dx+c)-1|)}{a^3}+\frac{30\left(3\sin(dx+c)^2+10\sin(dx+c)-9\right)}{a^3(\sin(dx+c)-1)^2}-\frac{137\sin(dx+c)^5+1285\sin(dx+c)^4+4970\sin(dx-}{a^3(\sin(dx+}}{15360\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/15360*(60*log(abs(sin(d*x + c) + 1))/a^3 - 60*log(abs(sin(d*x + c) - 1))
/a^3 + 30*(3*sin(d*x + c)^2 + 10*sin(d*x + c) - 9)/(a^3*(sin(d*x + c) - 1)
^2) - (137*sin(d*x + c)^5 + 1285*sin(d*x + c)^4 + 4970*sin(d*x + c)^3 + 60
10*sin(d*x + c)^2 + 3245*sin(d*x + c) + 673)/(a^3*(sin(d*x + c) + 1)^5))/d
```

3.72.9 Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.44

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{64}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{32}-\frac{17\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{96}+}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+6\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}+11\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-4\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}-39\,a^3\tan\left(\frac{c}{2}+\right.\right.}$$
$$+\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64\,a^3\,d}$$

input
```
int(tan(c + d*x)^5/(a + a*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^4/32 - (3*tan(c/2 + (d*x)/2)^2)/32 - (17*tan(c/2 + (d*
x)/2)^3)/96 - tan(c/2 + (d*x)/2)/64 + (527*tan(c/2 + (d*x)/2)^5)/960 + (90
1*tan(c/2 + (d*x)/2)^6)/80 + (711*tan(c/2 + (d*x)/2)^7)/80 + (901*tan(c/2
+ (d*x)/2)^8)/80 + (527*tan(c/2 + (d*x)/2)^9)/960 + tan(c/2 + (d*x)/2)^10/
32 - (17*tan(c/2 + (d*x)/2)^11)/96 - (3*tan(c/2 + (d*x)/2)^12)/32 - tan(c/
2 + (d*x)/2)^13/64)/(d*(11*a^3*tan(c/2 + (d*x)/2)^2 - 4*a^3*tan(c/2 + (d*x
)/2)^3 - 39*a^3*tan(c/2 + (d*x)/2)^4 - 38*a^3*tan(c/2 + (d*x)/2)^5 + 27*a^
3*tan(c/2 + (d*x)/2)^6 + 72*a^3*tan(c/2 + (d*x)/2)^7 + 27*a^3*tan(c/2 + (d
*x)/2)^8 - 38*a^3*tan(c/2 + (d*x)/2)^9 - 39*a^3*tan(c/2 + (d*x)/2)^10 - 4*
a^3*tan(c/2 + (d*x)/2)^11 + 11*a^3*tan(c/2 + (d*x)/2)^12 + 6*a^3*tan(c/2 +
 (d*x)/2)^13 + a^3*tan(c/2 + (d*x)/2)^14 + a^3 + 6*a^3*tan(c/2 + (d*x)/2))
) + atanh(tan(c/2 + (d*x)/2))/(64*a^3*d)
```

3.73 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.73.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\operatorname{arctanh}(\sin(c+dx))}{32a^3d} + \frac{a}{16d(a+a\sin(c+dx))^4} - \frac{1}{6d(a+a\sin(c+dx))^3} + \frac{3}{32ad(a+a\sin(c+dx))^2} + \frac{1}{32d\left(a^3-a^3\sin(c+dx)\right)} + \frac{1}{16d\left(a^3+a^3\sin(c+dx)\right)}$$

output
```
-1/32*arctanh(sin(d*x+c))/a^3/d+1/16*a/d/(a+a*sin(d*x+c))^4-1/6/d/(a+a*sin
(d*x+c))^3+3/32/a/d/(a+a*sin(d*x+c))^2+1/32/d/(a^3-a^3*sin(d*x+c))+1/16/d/
(a^3+a^3*sin(d*x+c))
```

3.73.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{3\operatorname{arctanh}(\sin(c+dx)) - \frac{3}{1-\sin(c+dx)} - \frac{6}{(1+\sin(c+dx))^4} + \frac{16}{(1+\sin(c+dx))^3} - \frac{9}{(1+\sin(c+dx))^2} - \frac{6}{1+\sin(c+dx)}}{96a^3d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/96*(3*ArcTanh[Sin[c + d*x]] - 3/(1 - Sin[c + d*x]) - 6/(1 + Sin[c + d*x
])^4 + 16/(1 + Sin[c + d*x])^3 - 9/(1 + Sin[c + d*x])^2 - 6/(1 + Sin[c + d
*x]))/(a^3*d)
```

3.73.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a\sin(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^3\sin^3(c+dx)}{(a-a\sin(c+dx))^2(\sin(c+dx)a+a)^5}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{a}{4(\sin(c+dx)a+a)^5}+\frac{1}{2(\sin(c+dx)a+a)^4}-\frac{3}{16(\sin(c+dx)a+a)^3a}-\frac{1}{32(a^2-a^2\sin^2(c+dx))a^2}+\frac{1}{32(a-a\sin(c+dx))^2a^2}-\frac{1}{16(\sin(c+d}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\text{arctanh}(\sin(c+dx))}{32a^3}+\frac{1}{32a^2(a-a\sin(c+dx))}+\frac{1}{16a^2(a\sin(c+dx)+a)}+\frac{a}{16(a\sin(c+dx)+a)^4}-\frac{1}{6(a\sin(c+dx)+a)^3}+\frac{3}{32a(a\sin(c+dx)+}}{d}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

output
```
(-1/32*ArcTanh[Sin[c + d*x]]/a^3 + 1/(32*a^2*(a - a*Sin[c + d*x])) + a/(16
*(a + a*Sin[c + d*x])^4) - 1/(6*(a + a*Sin[c + d*x])^3) + 3/(32*a*(a + a*S
in[c + d*x])^2) + 1/(16*a^2*(a + a*Sin[c + d*x])))/d
```

3.73.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.73.4 Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72

method	result
derivativedivides	$\frac{\frac{1}{16(1+\sin(dx+c))^4}-\frac{1}{6(1+\sin(dx+c))^3}+\frac{3}{32(1+\sin(dx+c))^2}+\frac{1}{16+16\sin(dx+c)}-\frac{\ln(1+\sin(dx+c))}{64}-\frac{1}{32(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)}{64}}{d\,a^3}$
default	$\frac{\frac{1}{16(1+\sin(dx+c))^4}-\frac{1}{6(1+\sin(dx+c))^3}+\frac{3}{32(1+\sin(dx+c))^2}+\frac{1}{16+16\sin(dx+c)}-\frac{\ln(1+\sin(dx+c))}{64}-\frac{1}{32(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)}{64}}{d\,a^3}$
risch	$\frac{i(-310\,e^{5i(dx+c)}-162ie^{4i(dx+c)}+88\,e^{3i(dx+c)}-18ie^{2i(dx+c)}+3\,e^{i(dx+c)}+88\,e^{7i(dx+c)}+162ie^{6i(dx+c)}+18ie^{8i(dx+c)}+3}{48\left(e^{i(dx+c)}+i\right)^8\left(-i+e^{i(dx+c)}\right)^2 d\,a^3}$

input
```
int(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/16/(1+sin(d*x+c))^4-1/6/(1+sin(d*x+c))^3+3/32/(1+sin(d*x+c))^2+
1/16/(1+sin(d*x+c))-1/64*ln(1+sin(d*x+c))-1/32/(sin(d*x+c)-1)+1/64*ln(sin(
d*x+c)-1))
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.79

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{6\,\cos\left(dx+c\right)^4+38\,\cos\left(dx+c\right)^2-3\left(3\,\cos\left(dx+c\right)^4-4\,\cos\left(dx+c\right)^2+\left(\cos\left(dx+c\right)^4-4\,\cos\left(dx+c\right.\right.\right.}{192\left(3\,a\right.}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/192*(6*cos(d*x + c)^4 + 38*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^4 - 4*cos(
d*x + c)^2 + (cos(d*x + c)^4 - 4*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x
 + c) + 1) + 3*(3*cos(d*x + c)^4 - 4*cos(d*x + c)^2 + (cos(d*x + c)^4 - 4*
cos(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 18*(cos(d*x + c)^2
+ 2)*sin(d*x + c) - 60)/(3*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 +
 (a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2)*sin(d*x + c))
```

3.73.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\tan^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
d*x) + 1), x)/a**3
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.16

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{2\left(3\,\sin(dx+c)^4+9\,\sin(dx+c)^3-25\,\sin(dx+c)^2-27\,\sin(dx+c)-8\right)}{a^3\sin(dx+c)^5+3\,a^3\sin(dx+c)^4+2\,a^3\sin(dx+c)^3-2\,a^3\sin(dx+c)^2-3\,a^3\sin(dx+c)-a^3}-\frac{3\,\log(\sin(dx+c)+1)}{a^3}+\frac{3\,\log(\sin(dx+c)-1)}{a^3}}{192\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/192*(2*(3*sin(d*x + c)^4 + 9*sin(d*x + c)^3 - 25*sin(d*x + c)^2 - 27*sin
(d*x + c) - 8)/(a^3*sin(d*x + c)^5 + 3*a^3*sin(d*x + c)^4 + 2*a^3*sin(d*x
+ c)^3 - 2*a^3*sin(d*x + c)^2 - 3*a^3*sin(d*x + c) - a^3) - 3*log(sin(d*x
+ c) + 1)/a^3 + 3*log(sin(d*x + c) - 1)/a^3)/d
```

3.73.8 Giac [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\frac{12\log(|\sin(dx+c)+1|)}{a^3} - \frac{12\log(|\sin(dx+c)-1|)}{a^3} + \frac{12(\sin(dx+c)+1)}{a^3(\sin(dx+c)-1)} - \frac{25\sin(dx+c)^4+148\sin(dx+c)^3+366\sin(dx+c)^2+260\sin(dx+c}{a^3(\sin(dx+c)+1)^4}}{768\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/768*(12*log(abs(sin(d*x + c) + 1))/a^3 - 12*log(abs(sin(d*x + c) - 1))/
a^3 + 12*(sin(d*x + c) + 1)/(a^3*(sin(d*x + c) - 1)) - (25*sin(d*x + c)^4
+ 148*sin(d*x + c)^3 + 366*sin(d*x + c)^2 + 260*sin(d*x + c) + 65)/(a^3*(s
in(d*x + c) + 1)^4))/d
```

3.73.9 Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.40

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9}{16} + \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{8} + \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{6} + \frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{8} + \frac{1}{}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} + 6a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 + 13a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 + 8a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7 - 14a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right.} - \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{16a^3d}$$

input
```
int(tan(c + d*x)^3/(a + a*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)/16 + (3*tan(c/2 + (d*x)/2)^2)/8 + (5*tan(c/2 + (d*x)/2
)^3)/6 + (37*tan(c/2 + (d*x)/2)^4)/8 + (101*tan(c/2 + (d*x)/2)^5)/24 + (37
*tan(c/2 + (d*x)/2)^6)/8 + (5*tan(c/2 + (d*x)/2)^7)/6 + (3*tan(c/2 + (d*x)
/2)^8)/8 + tan(c/2 + (d*x)/2)^9/16)/(d*(13*a^3*tan(c/2 + (d*x)/2)^2 + 8*a^
3*tan(c/2 + (d*x)/2)^3 - 14*a^3*tan(c/2 + (d*x)/2)^4 - 28*a^3*tan(c/2 + (d
*x)/2)^5 - 14*a^3*tan(c/2 + (d*x)/2)^6 + 8*a^3*tan(c/2 + (d*x)/2)^7 + 13*a
^3*tan(c/2 + (d*x)/2)^8 + 6*a^3*tan(c/2 + (d*x)/2)^9 + a^3*tan(c/2 + (d*x)
/2)^10 + a^3 + 6*a^3*tan(c/2 + (d*x)/2))) - atanh(tan(c/2 + (d*x)/2))/(16*
a^3*d)
```

3.73. $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.74 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.74.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{8a^3d} + \frac{1}{6d(a+a\sin(c+dx))^3} - \frac{1}{8ad(a+a\sin(c+dx))^2} - \frac{1}{8d\left(a^3+a^3\sin(c+dx)\right)}$$

output
```
1/8*arctanh(sin(d*x+c))/a^3/d+1/6/d/(a+a*sin(d*x+c))^3-1/8/a/d/(a+a*sin(d*
x+c))^2-1/8/d/(a^3+a^3*sin(d*x+c))
```

3.74.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.63

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{3\operatorname{arctanh}(\sin(c+dx)) - \frac{2+9\sin(c+dx)+3\sin^2(c+dx)}{(1+\sin(c+dx))^3}}{24a^3d}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

output
```
(3*ArcTanh[Sin[c + d*x]] - (2 + 9*Sin[c + d*x] + 3*Sin[c + d*x]^2)/(1 + Si
n[c + d*x])^3)/(24*a^3*d)
```

3.74.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a\sin(c+dx)+a)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \; 3186$$

$$\frac{\int \frac{a\sin(c+dx)}{(a-a\sin(c+dx))(\sin(c+dx)a+a)^4}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 86$$

$$\frac{\int \left(-\frac{1}{2(\sin(c+dx)a+a)^4}+\frac{1}{4(\sin(c+dx)a+a)^3a}+\frac{1}{8(a^2-a^2\sin^2(c+dx))a^2}+\frac{1}{8(\sin(c+dx)a+a)^2a^2}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\text{arctanh}(\sin(c+dx))}{8a^3}-\frac{1}{8a^2(a\sin(c+dx)+a)}-\frac{1}{8a(a\sin(c+dx)+a)^2}+\frac{1}{6(a\sin(c+dx)+a)^3}}{d}$$

input
```
Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

output
```
(ArcTanh[Sin[c + d*x]]/(8*a^3) + 1/(6*(a + a*Sin[c + d*x])^3) - 1/(8*a*(a
+ a*Sin[c + d*x])^2) - 1/(8*a^2*(a + a*Sin[c + d*x])))/d
```

3.74.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.74.4 Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.82

method	result	siz
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{16}+\frac{1}{6(1+\sin(dx+c))^3}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{8(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{16}}{d\,a^3}$	67
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{16}+\frac{1}{6(1+\sin(dx+c))^3}-\frac{1}{8(1+\sin(dx+c))^2}-\frac{1}{8(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{16}}{d\,a^3}$	67
risch	$-\frac{i\left(3\,e^{i(dx+c)}-14\,e^{3i(dx+c)}-18ie^{2i(dx+c)}+18ie^{4i(dx+c)}+3\,e^{5i(dx+c)}\right)}{12d\,a^3\left(e^{i(dx+c)}+i\right)^6}-\frac{\ln\left(-i+e^{i(dx+c)}\right)}{8a^3d}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{8a^3d}$	125

input
```
int(tan(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-1/16*ln(sin(d*x+c)-1)+1/6/(1+sin(d*x+c))^3-1/8/(1+sin(d*x+c))^2-
1/8/(1+sin(d*x+c))+1/16*ln(1+sin(d*x+c)))
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(74) = 148$.

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.88

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{6\cos(dx+c)^2 - 3\left(3\cos(dx+c)^2 + \left(\cos(dx+c)^2 - 4\right)\sin(dx+c) - 4\right)\log(\sin(dx+c)+1) + 3\,(3}{48\left(3\,a^3d\cos(dx+c)^2 - 4\,a^3d + (a^3d\right.}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/48*(6*cos(d*x + c)^2 - 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2 - 4)*sin(d
*x + c) - 4)*log(sin(d*x + c) + 1) + 3*(3*cos(d*x + c)^2 + (cos(d*x + c)^2
 - 4)*sin(d*x + c) - 4)*log(-sin(d*x + c) + 1) - 18*sin(d*x + c) - 10)/(3*
a^3*d*cos(d*x + c)^2 - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 4*a^3*d)*sin(d*x
+ c))
```

3.74.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\tan(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x
) + 1), x)/a**3
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.20

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$= -\frac{\frac{2\left(3\sin(dx+c)^2+9\sin(dx+c)+2\right)}{a^3\sin(dx+c)^3+3a^3\sin(dx+c)^2+3a^3\sin(dx+c)+a^3} - \frac{3\log(\sin(dx+c)+1)}{a^3} + \frac{3\log(\sin(dx+c)-1)}{a^3}}{48\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/48*(2*(3*sin(d*x + c)^2 + 9*sin(d*x + c) + 2)/(a^3*sin(d*x + c)^3 + 3*a
^3*sin(d*x + c)^2 + 3*a^3*sin(d*x + c) + a^3) - 3*log(sin(d*x + c) + 1)/a^
3 + 3*log(sin(d*x + c) - 1)/a^3)/d
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{6\log(|\sin(dx+c)+1|)}{a^3} - \frac{6\log(|\sin(dx+c)-1|)}{a^3} - \frac{11\sin(dx+c)^3+45\sin(dx+c)^2+69\sin(dx+c)+19}{a^3(\sin(dx+c)+1)^3}}{96\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/96*(6*log(abs(sin(d*x + c) + 1))/a^3 - 6*log(abs(sin(d*x + c) - 1))/a^3
- (11*sin(d*x + c)^3 + 45*sin(d*x + c)^2 + 69*sin(d*x + c) + 19)/(a^3*(sin
(d*x + c) + 1)^3))/d
```

3.74.9 Mupad [B] (verification not implemented)

Time = 8.50 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.27

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{4a^3d} + \frac{-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{4} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{6} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 6a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 15a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 20a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 15a^3\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(tan(c + d*x)/(a + a*sin(c + d*x))^3,x)
```

output
```
atanh(tan(c/2 + (d*x)/2))/(4*a^3*d) + (tan(c/2 + (d*x)/2)^2/2 - tan(c/2 +
(d*x)/2)/4 + tan(c/2 + (d*x)/2)^3/6 + tan(c/2 + (d*x)/2)^4/2 - tan(c/2 + (
d*x)/2)^5/4)/(d*(15*a^3*tan(c/2 + (d*x)/2)^2 + 20*a^3*tan(c/2 + (d*x)/2)^3
 + 15*a^3*tan(c/2 + (d*x)/2)^4 + 6*a^3*tan(c/2 + (d*x)/2)^5 + a^3*tan(c/2
+ (d*x)/2)^6 + a^3 + 6*a^3*tan(c/2 + (d*x)/2)))
```

3.75 $\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.75.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(1+\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+a\sin(c+dx))^2} + \frac{1}{d\,(a^3+a^3\sin(c+dx))}$$

output
```
ln(sin(d*x+c))/a^3/d-ln(1+sin(d*x+c))/a^3/d+1/2/a/d/(a+a*sin(d*x+c))^2+1/d
/(a^3+a^3*sin(d*x+c))
```

3.75.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.70

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{2\log(\sin(c+dx)) - 2\log(1+\sin(c+dx)) + \frac{3+2\sin(c+dx)}{(1+\sin(c+dx))^2}}{2a^3d}$$

input
```
Integrate[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

output
```
(2*Log[Sin[c + d*x]] - 2*Log[1 + Sin[c + d*x]] + (3 + 2*Sin[c + d*x])/(1 +
 Sin[c + d*x])^2)/(2*a^3*d)
```

3.75.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3186}$$

$$\frac{\int \frac{\csc(c+dx)}{a(\sin(c+dx)a+a)^3}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\int \left(\frac{\csc(c+dx)}{a^4}-\frac{1}{a^3(\sin(c+dx)a+a)}-\frac{1}{a^2(\sin(c+dx)a+a)^2}-\frac{1}{a(\sin(c+dx)a+a)^3}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\log(a\sin(c+dx))}{a^3}-\frac{\log(a\sin(c+dx)+a)}{a^3}+\frac{1}{a^2(a\sin(c+dx)+a)}+\frac{1}{2a(a\sin(c+dx)+a)^2}}{d}$$

input
```
Int[Cot[c + d*x]/(a + a*Sin[c + d*x])^3,x]
```

output
```
(Log[a*Sin[c + d*x]]/a^3 - Log[a + a*Sin[c + d*x]]/a^3 + 1/(2*a*(a + a*Sin
[c + d*x])^2) + 1/(a^2*(a + a*Sin[c + d*x])))/d
```

3.75.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.75.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.66

method	result	size
derivativedivides	$\frac{\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))+\ln(\sin(dx+c))}{d\,a^3}$	49
default	$\frac{\frac{1}{2(1+\sin(dx+c))^2}+\frac{1}{1+\sin(dx+c)}-\ln(1+\sin(dx+c))+\ln(\sin(dx+c))}{d\,a^3}$	49
risch	$\frac{2i\left(-e^{i(dx+c)}+3i e^{2i(dx+c)}+e^{3i(dx+c)}\right)}{d\,a^3\left(e^{i(dx+c)}+i\right)^4}-\frac{2\ln\left(e^{i(dx+c)}+i\right)}{a^3 d}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{a^3 d}$	98

input
```
int(cot(d*x+c)/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(1/2/(1+sin(d*x+c))^2+1/(1+sin(d*x+c))-ln(1+sin(d*x+c))+ln(sin(d*x
+c)))
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.41

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{2\left(\cos\left(dx+c\right)^2-2\sin\left(dx+c\right)-2\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)-2\left(\cos\left(dx+c\right)^2-2\sin\left(dx+c\right)-2\right)\log\left(\sin\right.}{2\left(a^3d\cos\left(dx+c\right)^2-2a^3d\sin\left(dx+c\right)-2a^3d\right)}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(2*(cos(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(1/2*sin(d*x + c)) - 2*(co
s(d*x + c)^2 - 2*sin(d*x + c) - 2)*log(sin(d*x + c) + 1) - 2*sin(d*x + c)
- 3)/(a^3*d*cos(d*x + c)^2 - 2*a^3*d*sin(d*x + c) - 2*a^3*d)
```

3.75.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\cot\left(c+dx\right)}{\sin^3\left(c+dx\right)+3\sin^2\left(c+dx\right)+3\sin\left(c+dx\right)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c + d*x
) + 1), x)/a**3
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{2\sin(dx+c)+3}{a^3\sin(dx+c)^2+2a^3\sin(dx+c)+a^3}-\frac{2\log(\sin(dx+c)+1)}{a^3}+\frac{2\log(\sin(dx+c))}{a^3}}{2d}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((2*sin(d*x + c) + 3)/(a^3*sin(d*x + c)^2 + 2*a^3*sin(d*x + c) + a^3)
- 2*log(sin(d*x + c) + 1)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\frac{2\log(|\sin(dx+c)+1|)}{a^3} - \frac{2\log(|\sin(dx+c)|)}{a^3} - \frac{2\sin(dx+c)+3}{a^3(\sin(dx+c)+1)^2}}{2d}$$

input
```
integrate(cot(d*x+c)/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(2*log(abs(sin(d*x + c) + 1))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 - (2
*sin(d*x + c) + 3)/(a^3*(sin(d*x + c) + 1)^2))/d
```

3.75.9 Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.00

$$\int \frac{\cot(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3 d} - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+6a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a^3\right)} - \frac{2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^3 d}$$

input
```
int(cot(c + d*x)/(a + a*sin(c + d*x))^3,x)
```

output
```
log(tan(c/2 + (d*x)/2))/(a^3*d) - (4*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x
)/2)^2 + 4*tan(c/2 + (d*x)/2)^3)/(d*(6*a^3*tan(c/2 + (d*x)/2)^2 + 4*a^3*ta
n(c/2 + (d*x)/2)^3 + a^3*tan(c/2 + (d*x)/2)^4 + a^3 + 4*a^3*tan(c/2 + (d*x
)/2))) - (2*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d)
```

3.76 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.76.1 Optimal result

Integrand size = 21, antiderivative size = 86

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{3\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^3d} + \frac{5\log(\sin(c+dx))}{a^3d} - \frac{5\log(1+\sin(c+dx))}{a^3d} + \frac{2}{d\,(a^3+a^3\sin(c+dx))}$$

output
```
3*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^3/d+5*ln(sin(d*x+c))/a^3/d-5*ln(1+si
n(d*x+c))/a^3/d+2/d/(a^3+a^3*sin(d*x+c))
```

3.76.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{6\csc(c+dx)-\csc^2(c+dx)+10\log(\sin(c+dx))-10\log(1+\sin(c+dx))+\frac{4}{1+\sin(c+dx)}}{2a^3d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

output
```
(6*Csc[c + d*x] - Csc[c + d*x]^2 + 10*Log[Sin[c + d*x]] - 10*Log[1 + Sin[c
 + d*x]] + 4/(1 + Sin[c + d*x]))/(2*a^3*d)
```

3.76.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^3(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{a^3(\sin(c+dx)a+a)^2}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{\csc^3(c+dx)}{a^4}-\frac{3\csc^2(c+dx)}{a^4}+\frac{5\csc(c+dx)}{a^4}-\frac{5}{a^3(\sin(c+dx)a+a)}-\frac{2}{a^2(\sin(c+dx)a+a)^2}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^2(c+dx)}{2a^3}+\frac{3\csc(c+dx)}{a^3}+\frac{5\log(a\sin(c+dx))}{a^3}-\frac{5\log(a\sin(c+dx)+a)}{a^3}+\frac{2}{a^2(a\sin(c+dx)+a)}}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^3,x]
```

output
```
((3*Csc[c + d*x])/a^3 - Csc[c + d*x]^2/(2*a^3) + (5*Log[a*Sin[c + d*x]])/a
^3 - (5*Log[a + a*Sin[c + d*x]])/a^3 + 2/(a^2*(a + a*Sin[c + d*x])))/d
```

3.76.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.76.4 Maple [A] (verified)

Time = 5.84 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.71

method	result	size
derivativedivides	$\frac{\frac{2}{1+\sin(dx+c)}-5\ln(1+\sin(dx+c))-\frac{1}{2\sin(dx+c)^2}+\frac{3}{\sin(dx+c)}+5\ln(\sin(dx+c))}{d\,a^3}$	61
default	$\frac{\frac{2}{1+\sin(dx+c)}-5\ln(1+\sin(dx+c))-\frac{1}{2\sin(dx+c)^2}+\frac{3}{\sin(dx+c)}+5\ln(\sin(dx+c))}{d\,a^3}$	61
risch	$\frac{2i\left(5i\,\mathrm{e}^{4i(dx+c)}+5\,\mathrm{e}^{5i(dx+c)}-5i\,\mathrm{e}^{2i(dx+c)}-8\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^2\left(\mathrm{e}^{i(dx+c)}+i\right)^2a^3d}-\frac{10\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^3d}+\frac{5\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{a^3d}$	137

input
```
int(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(2/(1+sin(d*x+c))-5*ln(1+sin(d*x+c))-1/2/sin(d*x+c)^2+3/sin(d*x+c)
+5*ln(sin(d*x+c)))
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.71

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{10\cos(dx+c)^2 + 10\left(\cos(dx+c)^2 + \left(\cos(dx+c)^2-1\right)\sin(dx+c)-1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 10\,(\cos}{2\left(a^3d\cos(dx+c)^2 - a^3d + \left(a^3d\cos(dx\right.\right.}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(10*cos(d*x + c)^2 + 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 - 1)*sin(d*x
 + c) - 1)*log(1/2*sin(d*x + c)) - 10*(cos(d*x + c)^2 + (cos(d*x + c)^2 -
1)*sin(d*x + c) - 1)*log(sin(d*x + c) + 1) - 5*sin(d*x + c) - 9)/(a^3*d*co
s(d*x + c)^2 - a^3*d + (a^3*d*cos(d*x + c)^2 - a^3*d)*sin(d*x + c))
```

3.76.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**3/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
d*x) + 1), x)/a**3
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\frac{10\sin(dx+c)^2+5\sin(dx+c)-1}{a^3\sin(dx+c)^3+a^3\sin(dx+c)^2} - \frac{10\log(\sin(dx+c)+1)}{a^3} + \frac{10\log(\sin(dx+c))}{a^3}}{2d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((10*sin(d*x + c)^2 + 5*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^3 + a^3*si
n(d*x + c)^2) - 10*log(sin(d*x + c) + 1)/a^3 + 10*log(sin(d*x + c))/a^3)/d
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.79

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{80\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{40\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{30\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+40\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+53\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+10\tan\left(\frac{1}{2}\,dx+\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)^2a^3}}{8\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/8*(80*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 40*log(abs(tan(1/2*d*x +
 1/2*c)))/a^3 - (30*tan(1/2*d*x + 1/2*c)^4 + 40*tan(1/2*d*x + 1/2*c)^3 + 5
3*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x + 1/2*c) - 1)/((tan(1/2*d*x + 1/
2*c)^2 + tan(1/2*d*x + 1/2*c))^2*a^3) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 12*a
^3*tan(1/2*d*x + 1/2*c))/a^6)/d
```

3.76.9 Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.97

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$

$$= \frac{5\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a^3\,d}-\frac{10\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{a^3\,d}$$

$$+\frac{-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{1}{2}}{d\,\left(4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+8\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^3\,d}$$

input
```
int(cot(c + d*x)^3/(a + a*sin(c + d*x))^3,x)
```

output
```
(5*log(tan(c/2 + (d*x)/2)))/(a^3*d) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) - (10
*log(tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (5*tan(c/2 + (d*x)/2) + (23*tan(c/
2 + (d*x)/2)^2)/2 - 10*tan(c/2 + (d*x)/2)^3 - 1/2)/(d*(4*a^3*tan(c/2 + (d*
x)/2)^2 + 8*a^3*tan(c/2 + (d*x)/2)^3 + 4*a^3*tan(c/2 + (d*x)/2)^4)) + (3*t
an(c/2 + (d*x)/2))/(2*a^3*d)
```

3.77 $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.77.1 Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{4\csc(c+dx)}{a^3d} - \frac{2\csc^2(c+dx)}{a^3d} + \frac{\csc^3(c+dx)}{a^3d} - \frac{\csc^4(c+dx)}{4a^3d} + \frac{4\log(\sin(c+dx))}{a^3d} - \frac{4\log(1+\sin(c+dx))}{a^3d}$$

output
```
4*csc(d*x+c)/a^3/d-2*csc(d*x+c)^2/a^3/d+csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^
4/a^3/d+4*ln(sin(d*x+c))/a^3/d-4*ln(1+sin(d*x+c))/a^3/d
```

3.77.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.72

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{16\csc(c+dx) - 8\csc^2(c+dx) + 4\csc^3(c+dx) - \csc^4(c+dx) + 16\log(\sin(c+dx)) - 16\log(1+\sin(c+dx))}{4a^3d}$$

input
```
Integrate[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

output
```
(16*Csc[c + d*x] - 8*Csc[c + d*x]^2 + 4*Csc[c + d*x]^3 - Csc[c + d*x]^4 +
16*Log[Sin[c + d*x]] - 16*Log[1 + Sin[c + d*x]])/(4*a^3*d)
```

3.77.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^5(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{ 3186}$$

$$\frac{\int \frac{\csc^5(c+dx)(a-a\sin(c+dx))^2}{a^5(\sin(c+dx)a+a)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(\frac{\csc^5(c+dx)}{a^4}-\frac{3\csc^4(c+dx)}{a^4}+\frac{4\csc^3(c+dx)}{a^4}-\frac{4\csc^2(c+dx)}{a^4}+\frac{4\csc(c+dx)}{a^4}-\frac{4}{a^3(\sin(c+dx)a+a)}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\csc^4(c+dx)}{4a^3}+\frac{\csc^3(c+dx)}{a^3}-\frac{2\csc^2(c+dx)}{a^3}+\frac{4\csc(c+dx)}{a^3}+\frac{4\log(a\sin(c+dx))}{a^3}-\frac{4\log(a\sin(c+dx)+a)}{a^3}}{d}$$

input
```
Int[Cot[c + d*x]^5/(a + a*Sin[c + d*x])^3,x]
```

output
```
((4*Csc[c + d*x])/a^3 - (2*Csc[c + d*x]^2)/a^3 + Csc[c + d*x]^3/a^3 - Csc[
c + d*x]^4/(4*a^3) + (4*Log[a*Sin[c + d*x]])/a^3 - (4*Log[a + a*Sin[c + d*
x]])/a^3)/d
```

3.77.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.77.4 Maple [A] (verified)

Time = 12.75 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.70

method	result
derivativedivides	$\frac{-4\ln(1+\sin(dx+c))-\frac{1}{4\sin(dx+c)^4}+\frac{1}{\sin(dx+c)^3}-\frac{2}{\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+4\ln(\sin(dx+c))}{d\,a^3}$
default	$\frac{-4\ln(1+\sin(dx+c))-\frac{1}{4\sin(dx+c)^4}+\frac{1}{\sin(dx+c)^3}-\frac{2}{\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+4\ln(\sin(dx+c))}{d\,a^3}$
risch	$\frac{4i\left(-2i e^{6i(dx+c)}+2\,e^{7i(dx+c)}+5i e^{4i(dx+c)}-8\,e^{5i(dx+c)}-2i e^{2i(dx+c)}+8\,e^{3i(dx+c)}-2\,e^{i(dx+c)}\right)}{d\,a^3\left(e^{2i(dx+c)}-1\right)^4}-\frac{8\ln\left(e^{i(dx+c)}+i\right)}{a^3 d}+$

input
```
int(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-4*ln(1+sin(d*x+c))-1/4/sin(d*x+c)^4+1/sin(d*x+c)^3-2/sin(d*x+c)^
2+4/sin(d*x+c)+4*ln(sin(d*x+c)))
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.36

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{8\cos(dx+c)^2 + 16\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)\log\left(\frac{1}{2}\sin(dx+c)\right) - 16\left(\cos(dx+c)^4 - 2\cos\right.}{4\left(a^3d\cos(dx+c)^4 - 2a^3d\cos(dx+\right.}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/4*(8*cos(d*x + c)^2 + 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(1/2
*sin(d*x + c)) - 16*(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*log(sin(d*x +
c) + 1) - 4*(4*cos(d*x + c)^2 - 5)*sin(d*x + c) - 9)/(a^3*d*cos(d*x + c)^4
 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

3.77.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\cot^5(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**5/(a+a*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**5/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
d*x) + 1), x)/a**3
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.78

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$= -\frac{\frac{16\log(\sin(dx+c)+1)}{a^3} - \frac{16\log(\sin(dx+c))}{a^3} - \frac{16\sin(dx+c)^3 - 8\sin(dx+c)^2 + 4\sin(dx+c) - 1}{a^3\sin(dx+c)^4}}{4d}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*(16*log(sin(d*x + c) + 1)/a^3 - 16*log(sin(d*x + c))/a^3 - (16*sin(d*
x + c)^3 - 8*sin(d*x + c)^2 + 4*sin(d*x + c) - 1)/(a^3*sin(d*x + c)^4))/d
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.81

$$\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{1536\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{768\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}+\frac{1600\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-456\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+108\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,\tan}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{192\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/192*(1536*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 768*log(abs(tan(1/2*
d*x + 1/2*c)))/a^3 + (1600*tan(1/2*d*x + 1/2*c)^4 - 456*tan(1/2*d*x + 1/2*
c)^3 + 108*tan(1/2*d*x + 1/2*c)^2 - 24*tan(1/2*d*x + 1/2*c) + 3)/(a^3*tan(
1/2*d*x + 1/2*c)^4) + 3*(a^9*tan(1/2*d*x + 1/2*c)^4 - 8*a^9*tan(1/2*d*x +
1/2*c)^3 + 36*a^9*tan(1/2*d*x + 1/2*c)^2 - 152*a^9*tan(1/2*d*x + 1/2*c))/a
^12)/d
```

3.77.9 Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.78

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3}\,dx &= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8\,a^3\,d}-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{16\,a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,a^3\,d}\\&+\frac{4\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d}-\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^3\,d}+\frac{19\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a^3\,d}\\&+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(38\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\frac{1}{4}\right)}{16\,a^3\,d}\end{aligned}$$

input
```
int(cot(c + d*x)^5/(a + a*sin(c + d*x))^3,x)
```

output
```
tan(c/2 + (d*x)/2)^3/(8*a^3*d) - (9*tan(c/2 + (d*x)/2)^2)/(16*a^3*d) - tan
(c/2 + (d*x)/2)^4/(64*a^3*d) + (4*log(tan(c/2 + (d*x)/2)))/(a^3*d) - (8*lo
g(tan(c/2 + (d*x)/2) + 1))/(a^3*d) + (19*tan(c/2 + (d*x)/2))/(8*a^3*d) + (
cot(c/2 + (d*x)/2)^4*(2*tan(c/2 + (d*x)/2) - 9*tan(c/2 + (d*x)/2)^2 + 38*t
an(c/2 + (d*x)/2)^3 - 1/4))/(16*a^3*d)
```

3.77. $\int \frac{\cot^5(c+dx)}{(a+a\sin(c+dx))^3} \, dx$

3.78 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.78.1 Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx=\frac{\csc^3(c+dx)}{3a^3d}-\frac{3\csc^4(c+dx)}{4a^3d}+\frac{3\csc^5(c+dx)}{5a^3d}-\frac{\csc^6(c+dx)}{6a^3d}$$

output
```
1/3*csc(d*x+c)^3/a^3/d-3/4*csc(d*x+c)^4/a^3/d+3/5*csc(d*x+c)^5/a^3/d-1/6*c
sc(d*x+c)^6/a^3/d
```

3.78.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.66

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\csc^3(c+dx)\,(20-45\csc(c+dx)+36\csc^2(c+dx)-10\csc^3(c+dx))}{60a^3d}$$

input
```
Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^3*(20 - 45*Csc[c + d*x] + 36*Csc[c + d*x]^2 - 10*Csc[c + d*x
]^3))/(60*a^3*d)
```

3.78.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^7(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^7(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^7(c+dx)(a-a\sin(c+dx))^3}{a^7}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{53}$$

$$\frac{\int \left(\frac{\csc^7(c+dx)}{a^4}-\frac{3\csc^6(c+dx)}{a^4}+\frac{3\csc^5(c+dx)}{a^4}-\frac{\csc^4(c+dx)}{a^4}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^6(c+dx)}{6a^3}+\frac{3\csc^5(c+dx)}{5a^3}-\frac{3\csc^4(c+dx)}{4a^3}+\frac{\csc^3(c+dx)}{3a^3}}{d}$$

input
```
Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^3/(3*a^3) - (3*Csc[c + d*x]^4)/(4*a^3) + (3*Csc[c + d*x]^5)/
(5*a^3) - Csc[c + d*x]^6/(6*a^3))/d
```

3.78.3.1 Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.78.4 Maple [A] (verified)

Time = 23.64 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.67

method	result	size
derivativedivides	$\frac{-\frac{3}{4\sin(dx+c)^4}+\frac{3}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}+\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$	49
default	$\frac{-\frac{3}{4\sin(dx+c)^4}+\frac{3}{5\sin(dx+c)^5}-\frac{1}{6\sin(dx+c)^6}+\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$	49
risch	$-\frac{4i\left(-45i\,e^{8i(dx+c)}+10\,e^{9i(dx+c)}+130i\,e^{6i(dx+c)}-102\,e^{7i(dx+c)}-45i\,e^{4i(dx+c)}+102\,e^{5i(dx+c)}-10\,e^{3i(dx+c)}\right)}{15d\,a^3\left(e^{2i(dx+c)}-1\right)^6}$	104

input
```
int(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-3/4/sin(d*x+c)^4+3/5/sin(d*x+c)^5-1/6/sin(d*x+c)^6+1/3/sin(d*x+c
)^3)
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{45\cos(dx+c)^2 - 4\left(5\cos(dx+c)^2 - 14\right)\sin(dx+c) - 55}{60\left(a^3d\cos(dx+c)^6 - 3a^3d\cos(dx+c)^4 + 3a^3d\cos(dx+c)^2 - a^3d\right)}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/60*(45*cos(d*x + c)^2 - 4*(5*cos(d*x + c)^2 - 14)*sin(d*x + c) - 55)/(a
^3*d*cos(d*x + c)^6 - 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^2 - a^
3*d)
```

3.78.6 Sympy [F]

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\cot^7(c+dx)}{\sin^3(c+dx)+3\sin^2(c+dx)+3\sin(c+dx)+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**7/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
d*x) + 1), x)/a**3
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{20\sin(dx+c)^3 - 45\sin(dx+c)^2 + 36\sin(dx+c) - 10}{60\,a^3d\sin(dx+c)^6}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d
*sin(d*x + c)^6)
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{20\,\sin\left(dx+c\right)^3-45\,\sin\left(dx+c\right)^2+36\,\sin\left(dx+c\right)-10}{60\,a^3d\sin\left(dx+c\right)^6}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/60*(20*sin(d*x + c)^3 - 45*sin(d*x + c)^2 + 36*sin(d*x + c) - 10)/(a^3*d
*sin(d*x + c)^6)
```

3.78.9 Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{20\sin\left(c+d\,x\right)^3-45\sin\left(c+d\,x\right)^2+36\,\sin\left(c+d\,x\right)-10}{60\,a^3\,d\sin\left(c+d\,x\right)^6}$$

input
```
int(cot(c + d*x)^7/(a + a*sin(c + d*x))^3,x)
```

output
```
(36*sin(c + d*x) - 45*sin(c + d*x)^2 + 20*sin(c + d*x)^3 - 10)/(60*a^3*d*s
in(c + d*x)^6)
```

3.79 $\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.79.1 Optimal result

Integrand size = 21, antiderivative size = 109

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{2\csc^5(c+dx)}{5a^3d}-\frac{\csc^6(c+dx)}{3a^3d}+\frac{3\csc^7(c+dx)}{7a^3d}-\frac{\csc^8(c+dx)}{8a^3d}$$

output
```
-1/3*csc(d*x+c)^3/a^3/d+3/4*csc(d*x+c)^4/a^3/d-2/5*csc(d*x+c)^5/a^3/d-1/3*
csc(d*x+c)^6/a^3/d+3/7*csc(d*x+c)^7/a^3/d-1/8*csc(d*x+c)^8/a^3/d
```

3.79.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\csc^3(c+dx)\left(280-630\csc(c+dx)+336\csc^2(c+dx)+280\csc^3(c+dx)-360\csc^4(c+dx)+105\csc\right.}{840a^3d}$$

input
```
Integrate[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]
```

output
```
-1/840*(Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 336*Csc[c + d*x]^2 + 280*
Csc[c + d*x]^3 - 360*Csc[c + d*x]^4 + 105*Csc[c + d*x]^5))/(a^3*d)
```

3.79.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 84, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^9(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^9(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^9(c+dx)(a-a\sin(c+dx))^4(\sin(c+dx)a+a)}{a^9}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{84}$$

$$\frac{\int \left(\frac{\csc^9(c+dx)}{a^4}-\frac{3\csc^8(c+dx)}{a^4}+\frac{2\csc^7(c+dx)}{a^4}+\frac{2\csc^6(c+dx)}{a^4}-\frac{3\csc^5(c+dx)}{a^4}+\frac{\csc^4(c+dx)}{a^4}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^8(c+dx)}{8a^3}+\frac{3\csc^7(c+dx)}{7a^3}-\frac{\csc^6(c+dx)}{3a^3}-\frac{2\csc^5(c+dx)}{5a^3}+\frac{3\csc^4(c+dx)}{4a^3}-\frac{\csc^3(c+dx)}{3a^3}}{d}$$

input
```
Int[Cot[c + d*x]^9/(a + a*Sin[c + d*x])^3,x]
```

output
```
(-1/3*Csc[c + d*x]^3/a^3 + (3*Csc[c + d*x]^4)/(4*a^3) - (2*Csc[c + d*x]^5)
/(5*a^3) - Csc[c + d*x]^6/(3*a^3) + (3*Csc[c + d*x]^7)/(7*a^3) - Csc[c + d
*x]^8/(8*a^3))/d
```

3.79.3.1 Defintions of rubi rules used

rule 84
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p + 2, 0
] && GtQ[n + 2*p, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.79.4 Maple [A] (verified)

Time = 41.73 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

method	result
derivativedivides	$\frac{-\frac{1}{8\sin(dx+c)^8}-\frac{1}{3\sin(dx+c)^6}+\frac{3}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}-\frac{2}{5\sin(dx+c)^5}+\frac{3}{7\sin(dx+c)^7}}{d\,a^3}$
default	$\frac{-\frac{1}{8\sin(dx+c)^8}-\frac{1}{3\sin(dx+c)^6}+\frac{3}{4\sin(dx+c)^4}-\frac{1}{3\sin(dx+c)^3}-\frac{2}{5\sin(dx+c)^5}+\frac{3}{7\sin(dx+c)^7}}{d\,a^3}$
risch	$\frac{4i\left(-315i e^{12i(dx+c)}+70\,e^{13i(dx+c)}+700i e^{10i(dx+c)}-686\,e^{11i(dx+c)}+70i e^{8i(dx+c)}+268\,e^{9i(dx+c)}+700i e^{6i(dx+c)}-268\,e^{7}\right.}{105d\,a^3\left(e^{2i(dx+c)}-1\right)^8}$

input
```
int(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-1/8/sin(d*x+c)^8-1/3/sin(d*x+c)^6+3/4/sin(d*x+c)^4-1/3/sin(d*x+c
)^3-2/5/sin(d*x+c)^5+3/7/sin(d*x+c)^7)
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{630\,\cos{(dx+c)}^4-980\,\cos{(dx+c)}^2-8\left(35\,\cos{(dx+c)}^4-112\,\cos{(dx+c)}^2+32\right)\sin{(dx+c)}+245}{840\left(a^3d\cos{(dx+c)}^8-4\,a^3d\cos{(dx+c)}^6+6\,a^3d\cos{(dx+c)}^4-4\,a^3d\cos{(dx+c)}^2+a^3d\right)}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/840*(630*cos(d*x + c)^4 - 980*cos(d*x + c)^2 - 8*(35*cos(d*x + c)^4 - 11
2*cos(d*x + c)^2 + 32)*sin(d*x + c) + 245)/(a^3*d*cos(d*x + c)^8 - 4*a^3*d
*cos(d*x + c)^6 + 6*a^3*d*cos(d*x + c)^4 - 4*a^3*d*cos(d*x + c)^2 + a^3*d)
```

3.79.6 Sympy [F]

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\int \frac{\cot^9{(c+dx)}}{\sin^3{(c+dx)}+3\sin^2{(c+dx)}+3\sin{(c+dx)}+1}\,dx}{a^3}$$

input
```
integrate(cot(d*x+c)**9/(a+a*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**9/(sin(c + d*x)**3 + 3*sin(c + d*x)**2 + 3*sin(c +
d*x) + 1), x)/a**3
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{280\,\sin{(dx+c)}^5-630\,\sin{(dx+c)}^4+336\,\sin{(dx+c)}^3+280\,\sin{(dx+c)}^2-360\,\sin{(dx+c)}+105}{840\,a^3d\sin{(dx+c)}^8}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280
*sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{280\,\sin\left(dx+c\right)^5 - 630\,\sin\left(dx+c\right)^4 + 336\,\sin\left(dx+c\right)^3 + 280\,\sin\left(dx+c\right)^2 - 360\,\sin\left(dx+c\right) + 105}{840\,a^3 d\sin\left(dx+c\right)^8}$$

input
```
integrate(cot(d*x+c)^9/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/840*(280*sin(d*x + c)^5 - 630*sin(d*x + c)^4 + 336*sin(d*x + c)^3 + 280
*sin(d*x + c)^2 - 360*sin(d*x + c) + 105)/(a^3*d*sin(d*x + c)^8)
```

3.79.9 Mupad [B] (verification not implemented)

Time = 6.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.61

$$\int \frac{\cot^9(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{280\sin\left(c+d\,x\right)^5 - 630\sin\left(c+d\,x\right)^4 + 336\sin\left(c+d\,x\right)^3 + 280\sin\left(c+d\,x\right)^2 - 360\,\sin\left(c+d\,x\right) + 105}{840\,a^3\,d\sin\left(c+d\,x\right)^8}$$

input
```
int(cot(c + d*x)^9/(a + a*sin(c + d*x))^3,x)
```

output
```
-(280*sin(c + d*x)^2 - 360*sin(c + d*x) + 336*sin(c + d*x)^3 - 630*sin(c +
 d*x)^4 + 280*sin(c + d*x)^5 + 105)/(840*a^3*d*sin(c + d*x)^8)
```

3.80 $\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.80.1 Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\csc^3(c+dx)}{3a^3d} - \frac{3\csc^4(c+dx)}{4a^3d} + \frac{\csc^5(c+dx)}{5a^3d} + \frac{5\csc^6(c+dx)}{6a^3d} - \frac{5\csc^7(c+dx)}{7a^3d} - \frac{\csc^8(c+dx)}{8a^3d} + \frac{\csc^9(c+dx)}{3a^3d} - \frac{\csc^{10}(c+dx)}{10a^3d}$$

output
```
1/3*csc(d*x+c)^3/a^3/d-3/4*csc(d*x+c)^4/a^3/d+1/5*csc(d*x+c)^5/a^3/d+5/6*c
sc(d*x+c)^6/a^3/d-5/7*csc(d*x+c)^7/a^3/d-1/8*csc(d*x+c)^8/a^3/d+1/3*csc(d*
x+c)^9/a^3/d-1/10*csc(d*x+c)^10/a^3/d
```

3.80.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\csc^3(c+dx)\,(280 - 630\csc(c+dx) + 168\csc^2(c+dx) + 700\csc^3(c+dx) - 600\csc^4(c+dx) - 105\csc^5(\ldots}{840a^3d}$$

input
```
Integrate[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^3*(280 - 630*Csc[c + d*x] + 168*Csc[c + d*x]^2 + 700*Csc[c +
 d*x]^3 - 600*Csc[c + d*x]^4 - 105*Csc[c + d*x]^5 + 280*Csc[c + d*x]^6 - 8
4*Csc[c + d*x]^7))/(840*a^3*d)
```

3.80.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{11}(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^{11}(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^{11}(c+dx)(a-a\sin(c+dx))^5(\sin(c+dx)a+a)^2}{a^{11}}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\csc^{11}(c+dx)}{a^4}-\frac{3\csc^{10}(c+dx)}{a^4}+\frac{\csc^9(c+dx)}{a^4}+\frac{5\csc^8(c+dx)}{a^4}-\frac{5\csc^7(c+dx)}{a^4}-\frac{\csc^6(c+dx)}{a^4}+\frac{3\csc^5(c+dx)}{a^4}-\frac{\csc^4(c+dx)}{a^4}\right)d(a\sin}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^{10}(c+dx)}{10a^3}+\frac{\csc^9(c+dx)}{3a^3}-\frac{\csc^8(c+dx)}{8a^3}-\frac{5\csc^7(c+dx)}{7a^3}+\frac{5\csc^6(c+dx)}{6a^3}+\frac{\csc^5(c+dx)}{5a^3}-\frac{3\csc^4(c+dx)}{4a^3}+\frac{\csc^3(c+dx)}{3a^3}}{d}$$

input
```
Int[Cot[c + d*x]^11/(a + a*Sin[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^3/(3*a^3) - (3*Csc[c + d*x]^4)/(4*a^3) + Csc[c + d*x]^5/(5*a
^3) + (5*Csc[c + d*x]^6)/(6*a^3) - (5*Csc[c + d*x]^7)/(7*a^3) - Csc[c + d*
x]^8/(8*a^3) + Csc[c + d*x]^9/(3*a^3) - Csc[c + d*x]^10/(10*a^3))/d
```

3.80.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.80.4 Maple [A] (verified)

Time = 69.79 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{-\frac{5}{7\sin(dx+c)^7}-\frac{1}{10\sin(dx+c)^{10}}+\frac{5}{6\sin(dx+c)^6}+\frac{1}{3\sin(dx+c)^9}+\frac{1}{5\sin(dx+c)^5}-\frac{3}{4\sin(dx+c)^4}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$
default	$\frac{-\frac{5}{7\sin(dx+c)^7}-\frac{1}{10\sin(dx+c)^{10}}+\frac{5}{6\sin(dx+c)^6}+\frac{1}{3\sin(dx+c)^9}+\frac{1}{5\sin(dx+c)^5}-\frac{3}{4\sin(dx+c)^4}-\frac{1}{8\sin(dx+c)^8}+\frac{1}{3\sin(dx+c)^3}}{d\,a^3}$
risch	$-\frac{4i\left(-315ie^{16i(dx+c)}+70\,e^{17i(dx+c)}+490ie^{14i(dx+c)}-658\,e^{15i(dx+c)}+35ie^{12i(dx+c)}-90\,e^{13i(dx+c)}+2268ie^{10i(dx+c)}-\right.}{105d\,a^3\left(e^{\right.}}$

input
```
int(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(-5/7/sin(d*x+c)^7-1/10/sin(d*x+c)^10+5/6/sin(d*x+c)^6+1/3/sin(d*x
+c)^9+1/5/sin(d*x+c)^5-3/4/sin(d*x+c)^4-1/8/sin(d*x+c)^8+1/3/sin(d*x+c)^3)
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.05

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{630\cos(dx+c)^6 - 1190\cos(dx+c)^4 + 595\cos(dx+c)^2 - 8\left(35\cos(dx+c)^6 - 126\cos(dx+c)^4 + \right.}{840\left(a^3d\cos(dx+c)^{10} - 5a^3d\cos(dx+c)^8 + 10a^3d\cos(dx+c)^6 - 10a^3d\cos(dx+c)^4\right.}$$

input
```
integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/840*(630*cos(d*x + c)^6 - 1190*cos(d*x + c)^4 + 595*cos(d*x + c)^2 - 8*
(35*cos(d*x + c)^6 - 126*cos(d*x + c)^4 + 72*cos(d*x + c)^2 - 16)*sin(d*x
+ c) - 119)/(a^3*d*cos(d*x + c)^10 - 5*a^3*d*cos(d*x + c)^8 + 10*a^3*d*cos
(d*x + c)^6 - 10*a^3*d*cos(d*x + c)^4 + 5*a^3*d*cos(d*x + c)^2 - a^3*d)
```

3.80.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**11/(a+a*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{280\sin(dx+c)^7 - 630\sin(dx+c)^6 + 168\sin(dx+c)^5 + 700\sin(dx+c)^4 - 600\sin(dx+c)^3 - 105\,\mathrm{si}}{840\,a^3d\sin(dx+c)^{10}}$$

input
```
integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*
sin(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c
) - 84)/(a^3*d*sin(d*x + c)^10)
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{280\,\sin\left(dx+c\right)^7 - 630\,\sin\left(dx+c\right)^6 + 168\,\sin\left(dx+c\right)^5 + 700\,\sin\left(dx+c\right)^4 - 600\,\sin\left(dx+c\right)^3 - 105\,\mathrm{si}}{840\,a^3 d\sin\left(dx+c\right)^{10}}$$

input
```
integrate(cot(d*x+c)^11/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/840*(280*sin(d*x + c)^7 - 630*sin(d*x + c)^6 + 168*sin(d*x + c)^5 + 700*
sin(d*x + c)^4 - 600*sin(d*x + c)^3 - 105*sin(d*x + c)^2 + 280*sin(d*x + c
) - 84)/(a^3*d*sin(d*x + c)^10)
```

3.80.9 Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{11}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{280\sin\left(c+d\,x\right)^7 - 630\sin\left(c+d\,x\right)^6 + 168\sin\left(c+d\,x\right)^5 + 700\sin\left(c+d\,x\right)^4 - 600\sin\left(c+d\,x\right)^3 - 105\,\mathrm{si}}{840\,a^3\,d\sin\left(c+d\,x\right)^{10}}$$

input
```
int(cot(c + d*x)^11/(a + a*sin(c + d*x))^3,x)
```

output
```
(280*sin(c + d*x) - 105*sin(c + d*x)^2 - 600*sin(c + d*x)^3 + 700*sin(c +
d*x)^4 + 168*sin(c + d*x)^5 - 630*sin(c + d*x)^6 + 280*sin(c + d*x)^7 - 84
)/(840*a^3*d*sin(c + d*x)^10)
```

3.81 $\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$

3.81.1 Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{\csc^3(c+dx)}{3a^3d}+\frac{3\csc^4(c+dx)}{4a^3d}-\frac{4\csc^6(c+dx)}{3a^3d}+\frac{6\csc^7(c+dx)}{7a^3d}+\frac{3\csc^8(c+dx)}{4a^3d}-\frac{8\csc^9(c+dx)}{9a^3d}+\frac{3\csc^{11}(c+dx)}{11a^3d}-\frac{\csc^{12}(c+dx)}{12a^3d}$$

output
```
-1/3*csc(d*x+c)^3/a^3/d+3/4*csc(d*x+c)^4/a^3/d-4/3*csc(d*x+c)^6/a^3/d+6/7*
csc(d*x+c)^7/a^3/d+3/4*csc(d*x+c)^8/a^3/d-8/9*csc(d*x+c)^9/a^3/d+3/11*csc(
d*x+c)^11/a^3/d-1/12*csc(d*x+c)^12/a^3/d
```

3.81.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.61

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{\csc^3(c+dx)\left(-924+2079\csc(c+dx)-3696\csc^3(c+dx)+2376\csc^4(c+dx)+2079\csc^5(c+dx)-24\right.}{2772a^3d}$$

input
```
Integrate[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]
```

output
```
(Csc[c + d*x]^3*(-924 + 2079*Csc[c + d*x] - 3696*Csc[c + d*x]^3 + 2376*Csc
[c + d*x]^4 + 2079*Csc[c + d*x]^5 - 2464*Csc[c + d*x]^6 + 756*Csc[c + d*x]
^8 - 231*Csc[c + d*x]^9))/(2772*a^3*d)
```

3.81.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^{13}(c+dx)}{(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^{13}(a\sin(c+dx)+a)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^{13}(c+dx)(a-a\sin(c+dx))^6(\sin(c+dx)a+a)^3}{a^{13}}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\csc^{13}(c+dx)}{a^4}-\frac{3\csc^{12}(c+dx)}{a^4}+\frac{8\csc^{10}(c+dx)}{a^4}-\frac{6\csc^9(c+dx)}{a^4}-\frac{6\csc^8(c+dx)}{a^4}+\frac{8\csc^7(c+dx)}{a^4}-\frac{3\csc^5(c+dx)}{a^4}+\frac{\csc^4(c+dx)}{a^4}\right)d(a}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^{12}(c+dx)}{12a^3}+\frac{3\csc^{11}(c+dx)}{11a^3}-\frac{8\csc^9(c+dx)}{9a^3}+\frac{3\csc^8(c+dx)}{4a^3}+\frac{6\csc^7(c+dx)}{7a^3}-\frac{4\csc^6(c+dx)}{3a^3}+\frac{3\csc^4(c+dx)}{4a^3}-\frac{\csc^3(c+dx)}{3a^3}}{d}$$

input
```
Int[Cot[c + d*x]^13/(a + a*Sin[c + d*x])^3,x]
```

output
```
(-1/3*Csc[c + d*x]^3/a^3 + (3*Csc[c + d*x]^4)/(4*a^3) - (4*Csc[c + d*x]^6)
/(3*a^3) + (6*Csc[c + d*x]^7)/(7*a^3) + (3*Csc[c + d*x]^8)/(4*a^3) - (8*Cs
c[c + d*x]^9)/(9*a^3) + (3*Csc[c + d*x]^11)/(11*a^3) - Csc[c + d*x]^12/(12
*a^3))/d
```

3.81.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.81.4 Maple [A] (verified)

Time = 117.45 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.61

method	result
derivativedivides	$\frac{\frac{3}{4\sin(dx+c)^4}-\frac{4}{3\sin(dx+c)^6}-\frac{1}{3\sin(dx+c)^3}-\frac{8}{9\sin(dx+c)^9}+\frac{3}{11\sin(dx+c)^{11}}-\frac{1}{12\sin(dx+c)^{12}}+\frac{3}{4\sin(dx+c)^8}+\frac{6}{7\sin(dx+c)^7}}{d\,a^3}$
default	$\frac{\frac{3}{4\sin(dx+c)^4}-\frac{4}{3\sin(dx+c)^6}-\frac{1}{3\sin(dx+c)^3}-\frac{8}{9\sin(dx+c)^9}+\frac{3}{11\sin(dx+c)^{11}}-\frac{1}{12\sin(dx+c)^{12}}+\frac{3}{4\sin(dx+c)^8}+\frac{6}{7\sin(dx+c)^7}}{d\,a^3}$
risch	$4i(-2772ie^{8i(dx+c)}+462\,e^{21i(dx+c)}+9702ie^{12i(dx+c)}-4158\,e^{19i(dx+c)}+1848ie^{6i(dx+c)}-2376\,e^{17i(dx+c)}+27720ie^{10i(dx}$

input
```
int(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^3*(3/4/sin(d*x+c)^4-4/3/sin(d*x+c)^6-1/3/sin(d*x+c)^3-8/9/sin(d*x+c)
^9+3/11/sin(d*x+c)^11-1/12/sin(d*x+c)^12+3/4/sin(d*x+c)^8+6/7/sin(d*x+c)^7
)
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.28

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx$$
$$= \frac{2079\cos(dx+c)^8 - 4620\cos(dx+c)^6 + 3465\cos(dx+c)^4 - 1386\cos(dx+c)^2 - 4\left(231\cos(dx+c)^8\right.}{2772\left(a^3d\cos(dx+c)^{12} - 6a^3d\cos(dx+c)^{10} + 15a^3d\cos(dx+c)^8 - 20a^3d\cos\right.}$$

input
```
integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2772*(2079*cos(d*x + c)^8 - 4620*cos(d*x + c)^6 + 3465*cos(d*x + c)^4 -
1386*cos(d*x + c)^2 - 4*(231*cos(d*x + c)^8 - 924*cos(d*x + c)^6 + 792*cos
(d*x + c)^4 - 352*cos(d*x + c)^2 + 64)*sin(d*x + c) + 231)/(a^3*d*cos(d*x
+ c)^12 - 6*a^3*d*cos(d*x + c)^10 + 15*a^3*d*cos(d*x + c)^8 - 20*a^3*d*cos
(d*x + c)^6 + 15*a^3*d*cos(d*x + c)^4 - 6*a^3*d*cos(d*x + c)^2 + a^3*d)
```

3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cot(d*x+c)**13/(a+a*sin(d*x+c))**3,x)
```

output
```
Timed out
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx =$$
$$-\frac{924\sin(dx+c)^9 - 2079\sin(dx+c)^8 + 3696\sin(dx+c)^6 - 2376\sin(dx+c)^5 - 2079\sin(dx+c)^4 -}{2772\,a^3d\sin(dx+c)^{12}}$$

input
```
integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 -
2376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(
d*x + c) + 231)/(a^3*d*sin(d*x + c)^12)
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = -\frac{924\,\sin(dx+c)^9 - 2079\,\sin(dx+c)^8 + 3696\,\sin(dx+c)^6 - 2376\,\sin(dx+c)^5 - 2079\,\sin(dx+c)^4 -}{2772\,a^3 d\sin(dx+c)^{12}}$$

input
```
integrate(cot(d*x+c)^13/(a+a*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2772*(924*sin(d*x + c)^9 - 2079*sin(d*x + c)^8 + 3696*sin(d*x + c)^6 -
2376*sin(d*x + c)^5 - 2079*sin(d*x + c)^4 + 2464*sin(d*x + c)^3 - 756*sin(
d*x + c) + 231)/(a^3*d*sin(d*x + c)^12)
```

3.81.9 Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.59

$$\int \frac{\cot^{13}(c+dx)}{(a+a\sin(c+dx))^3}\,dx = \frac{-\frac{\sin(c+d\,x)^9}{3}+\frac{3\sin(c+d\,x)^8}{4}-\frac{4\sin(c+d\,x)^6}{3}+\frac{6\sin(c+d\,x)^5}{7}+\frac{3\sin(c+d\,x)^4}{4}-\frac{8\sin(c+d\,x)^3}{9}+\frac{3\,\sin(c+d\,x)}{11}-\frac{1}{12}}{a^3\,d\sin(c+d\,x)^{12}}$$

input
```
int(cot(c + d*x)^13/(a + a*sin(c + d*x))^3,x)
```

output
```
((3*sin(c + d*x))/11 - (8*sin(c + d*x)^3)/9 + (3*sin(c + d*x)^4)/4 + (6*si
n(c + d*x)^5)/7 - (4*sin(c + d*x)^6)/3 + (3*sin(c + d*x)^8)/4 - sin(c + d*
x)^9/3 - 1/12)/(a^3*d*sin(c + d*x)^12)
```

3.82 $\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.82.1 Optimal result

Integrand size = 21, antiderivative size = 195

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = &-\frac{\operatorname{arctanh}(\sin(c+dx))}{128a^4d}+\frac{a^2}{48d(a+a\sin(c+dx))^6}\\ &-\frac{7a}{80d(a+a\sin(c+dx))^5}+\frac{1}{8d(a+a\sin(c+dx))^4}\\ &-\frac{5}{96ad(a+a\sin(c+dx))^3}+\frac{1}{256d\left(a^2-a^2\sin(c+dx)\right)^2}\\ &-\frac{5}{256d\left(a^2+a^2\sin(c+dx)\right)^2}\\ &-\frac{3}{256d\left(a^4-a^4\sin(c+dx)\right)}-\frac{1}{256d\left(a^4+a^4\sin(c+dx)\right)}\end{aligned}$$

output
```
-1/128*arctanh(sin(d*x+c))/a^4/d+1/48*a^2/d/(a+a*sin(d*x+c))^6-7/80*a/d/(a
+a*sin(d*x+c))^5+1/8/d/(a+a*sin(d*x+c))^4-5/96/a/d/(a+a*sin(d*x+c))^3+1/25
6/d/(a^2-a^2*sin(d*x+c))^2-5/256/d/(a^2+a^2*sin(d*x+c))^2-3/256/d/(a^4-a^4
*sin(d*x+c))-1/256/d/(a^4+a^4*sin(d*x+c))
```

3.82.2 Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.57

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx =$$
$$-\frac{30\text{arctanh}(\sin(c+dx)) - \frac{2(-48-177\sin(c+dx)-132\sin^2(c+dx)+257\sin^3(c+dx)+440\sin^4(c+dx)+65\sin^5(c+dx)+60\sin^6(c+dx)}{(-1+\sin(c+dx))^2(1+\sin(c+dx))^6}}{3840a^4d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

output
```
-1/3840*(30*ArcTanh[Sin[c + d*x]] - (2*(-48 - 177*Sin[c + d*x] - 132*Sin[c
 + d*x]^2 + 257*Sin[c + d*x]^3 + 440*Sin[c + d*x]^4 + 65*Sin[c + d*x]^5 +
60*Sin[c + d*x]^6 + 15*Sin[c + d*x]^7))/((-1 + Sin[c + d*x])^2*(1 + Sin[c
+ d*x])^6))/(a^4*d)
```

3.82.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a\sin(c+dx)+a)^4}dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(c+dx)^5}{(a\sin(c+dx)+a)^4}dx$$
$$\Big\downarrow \; 3186$$
$$\frac{\int \frac{a^5\sin^5(c+dx)}{(a-a\sin(c+dx))^3(\sin(c+dx)a+a)^7}d(a\sin(c+dx))}{d}$$
$$\Big\downarrow \; 99$$
$$\frac{\int \Big(-\frac{a^2}{8(\sin(c+dx)a+a)^7}+\frac{7a}{16(\sin(c+dx)a+a)^6}-\frac{1}{2(\sin(c+dx)a+a)^5}+\frac{5}{32(\sin(c+dx)a+a)^4a}+\frac{1}{128(a-a\sin(c+dx))^3a^2}+\frac{5}{128(\sin(c+dx)a}}{d}$$

$\downarrow$ 2009

$$\frac{-\frac{\text{arctanh}(\sin(c+dx))}{128a^4}-\frac{3}{256a^3(a-a\sin(c+dx))}-\frac{1}{256a^3(a\sin(c+dx)+a)}+\frac{a^2}{48(a\sin(c+dx)+a)^6}+\frac{1}{256a^2(a-a\sin(c+dx))^2}-\frac{}{256a^2(a\sin}}{d}$$

input
```
Int[Tan[c + d*x]^5/(a + a*Sin[c + d*x])^4,x]
```

output
```
(-1/128*ArcTanh[Sin[c + d*x]]/a^4 + 1/(256*a^2*(a - a*Sin[c + d*x])^2) - 3
/(256*a^3*(a - a*Sin[c + d*x])) + a^2/(48*(a + a*Sin[c + d*x])^6) - (7*a)/
(80*(a + a*Sin[c + d*x])^5) + 1/(8*(a + a*Sin[c + d*x])^4) - 5/(96*a*(a +
a*Sin[c + d*x])^3) - 5/(256*a^2*(a + a*Sin[c + d*x])^2) - 1/(256*a^3*(a +
a*Sin[c + d*x])))/d
```

3.82.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.82.4 Maple [A] (verified)

Time = 8.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.65

method	result
derivativedivides	$\frac{\frac{1}{48(1+\sin(dx+c))^6}-\frac{7}{80(1+\sin(dx+c))^5}+\frac{1}{8(1+\sin(dx+c))^4}-\frac{5}{96(1+\sin(dx+c))^3}-\frac{5}{256(1+\sin(dx+c))^2}-\frac{1}{256(1+\sin(dx+c))}-\frac{\ln(1+\mathrm{si}}{2}}{d\,a^4}$
default	$\frac{\frac{1}{48(1+\sin(dx+c))^6}-\frac{7}{80(1+\sin(dx+c))^5}+\frac{1}{8(1+\sin(dx+c))^4}-\frac{5}{96(1+\sin(dx+c))^3}-\frac{5}{256(1+\sin(dx+c))^2}-\frac{1}{256(1+\sin(dx+c))}-\frac{\ln(1+\mathrm{si}}{2}}{d\,a^4}$
risch	$\frac{i\left(120i e^{2i(dx+c)}-5727\,e^{5i(dx+c)}-365\,e^{13i(dx+c)}-15\,e^{i(dx+c)}+5727\,e^{11i(dx+c)}+4133\,e^{7i(dx+c)}+365\,e^{3i(dx+c)}-4133\,e^{9i(}\right.}{960\left(-i+e^{i(dx+c}\right.}$

input
```
int(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(1/48/(1+sin(d*x+c))^6-7/80/(1+sin(d*x+c))^5+1/8/(1+sin(d*x+c))^4-
5/96/(1+sin(d*x+c))^3-5/256/(1+sin(d*x+c))^2-1/256/(1+sin(d*x+c))-1/256*ln
(1+sin(d*x+c))+1/256/(sin(d*x+c)-1)^2+3/256/(sin(d*x+c)-1)+1/256*ln(sin(d*
x+c)-1))
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.49

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{120\,\cos\left(dx+c\right)^6-1240\,\cos\left(dx+c\right)^4+1856\,\cos\left(dx+c\right)^2+15\left(\cos\left(dx+c\right)^8-8\,\cos\left(dx+c\right)^6+8\,\mathrm{c}\right.}{}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/3840*(120*cos(d*x + c)^6 - 1240*cos(d*x + c)^4 + 1856*cos(d*x + c)^2 +
15*(cos(d*x + c)^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)
^6 - 2*cos(d*x + c)^4)*sin(d*x + c))*log(sin(d*x + c) + 1) - 15*(cos(d*x +
 c)^8 - 8*cos(d*x + c)^6 + 8*cos(d*x + c)^4 - 4*(cos(d*x + c)^6 - 2*cos(d*
x + c)^4)*sin(d*x + c))*log(-sin(d*x + c) + 1) + 2*(15*cos(d*x + c)^6 - 11
0*cos(d*x + c)^4 + 432*cos(d*x + c)^2 - 160)*sin(d*x + c) - 640)/(a^4*d*co
s(d*x + c)^8 - 8*a^4*d*cos(d*x + c)^6 + 8*a^4*d*cos(d*x + c)^4 - 4*(a^4*d*
cos(d*x + c)^6 - 2*a^4*d*cos(d*x + c)^4)*sin(d*x + c))
```

3.82.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\tan^5(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(tan(d*x+c)**5/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(tan(c + d*x)**5/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c +
d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.09

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\frac{2\left(15\sin(dx+c)^7+60\sin(dx+c)^6+65\sin(dx+c)^5+440\sin(dx+c)^4+257\sin(dx+c)^3-132\sin(dx+c)^2-177\sin(dx+c)-48\right)}{a^4\sin(dx+c)^8+4a^4\sin(dx+c)^7+4a^4\sin(dx+c)^6-4a^4\sin(dx+c)^5-10a^4\sin(dx+c)^4-4a^4\sin(dx+c)^3+4a^4\sin(dx+c)^2+4a^4\sin(dx+c)+a^4} - \cdots}{3840\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/3840*(2*(15*sin(d*x + c)^7 + 60*sin(d*x + c)^6 + 65*sin(d*x + c)^5 + 440
*sin(d*x + c)^4 + 257*sin(d*x + c)^3 - 132*sin(d*x + c)^2 - 177*sin(d*x +
c) - 48)/(a^4*sin(d*x + c)^8 + 4*a^4*sin(d*x + c)^7 + 4*a^4*sin(d*x + c)^6
 - 4*a^4*sin(d*x + c)^5 - 10*a^4*sin(d*x + c)^4 - 4*a^4*sin(d*x + c)^3 + 4
*a^4*sin(d*x + c)^2 + 4*a^4*sin(d*x + c) + a^4) - 15*log(sin(d*x + c) + 1)
/a^4 + 15*log(sin(d*x + c) - 1)/a^4)/d
```

3.82.8 Giac [A] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.75

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{60\log(|\sin(dx+c)+1|)}{a^4} - \frac{60\log(|\sin(dx+c)-1|)}{a^4} + \frac{30\left(3\sin(dx+c)^2-12\sin(dx+c)+7\right)}{a^4(\sin(dx+c)-1)^2} - \frac{147\sin(dx+c)^6+822\sin(dx+c)^5+1605\sin(d\cdots}{\cdots}}{15360\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/15360*(60*log(abs(sin(d*x + c) + 1))/a^4 - 60*log(abs(sin(d*x + c) - 1)
)/a^4 + 30*(3*sin(d*x + c)^2 - 12*sin(d*x + c) + 7)/(a^4*(sin(d*x + c) - 1
)^2) - (147*sin(d*x + c)^6 + 822*sin(d*x + c)^5 + 1605*sin(d*x + c)^4 + 34
0*sin(d*x + c)^3 - 675*sin(d*x + c)^2 - 522*sin(d*x + c) - 117)/(a^4*(sin(
d*x + c) + 1)^6))/d
```

3.82.9 Mupad [B] (verification not implemented)

Time = 12.14 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.44

$$\int \frac{\tan^5(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$

$$= \frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{8}+\frac{73\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{1}}{192}}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+8a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}+24a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+24a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}-36a^4\tan\left(\frac{c}{2}+\right.\right.}$$

$$-\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{64a^4d}$$

input
```
int(tan(c + d*x)^5/(a + a*sin(c + d*x))^4,x)
```

output
```
(tan(c/2 + (d*x)/2)/64 + tan(c/2 + (d*x)/2)^2/8 + (73*tan(c/2 + (d*x)/2)^3
)/192 + (5*tan(c/2 + (d*x)/2)^4)/12 - (139*tan(c/2 + (d*x)/2)^5)/320 + (10
73*tan(c/2 + (d*x)/2)^6)/120 + (10277*tan(c/2 + (d*x)/2)^7)/960 + (237*tan
(c/2 + (d*x)/2)^8)/10 + (10277*tan(c/2 + (d*x)/2)^9)/960 + (1073*tan(c/2 +
 (d*x)/2)^10)/120 - (139*tan(c/2 + (d*x)/2)^11)/320 + (5*tan(c/2 + (d*x)/2
)^12)/12 + (73*tan(c/2 + (d*x)/2)^13)/192 + tan(c/2 + (d*x)/2)^14/8 + tan(
c/2 + (d*x)/2)^15/64)/(d*(24*a^4*tan(c/2 + (d*x)/2)^2 + 24*a^4*tan(c/2 + (
d*x)/2)^3 - 36*a^4*tan(c/2 + (d*x)/2)^4 - 120*a^4*tan(c/2 + (d*x)/2)^5 - 8
8*a^4*tan(c/2 + (d*x)/2)^6 + 88*a^4*tan(c/2 + (d*x)/2)^7 + 198*a^4*tan(c/2
 + (d*x)/2)^8 + 88*a^4*tan(c/2 + (d*x)/2)^9 - 88*a^4*tan(c/2 + (d*x)/2)^10
 - 120*a^4*tan(c/2 + (d*x)/2)^11 - 36*a^4*tan(c/2 + (d*x)/2)^12 + 24*a^4*t
an(c/2 + (d*x)/2)^13 + 24*a^4*tan(c/2 + (d*x)/2)^14 + 8*a^4*tan(c/2 + (d*x
)/2)^15 + a^4*tan(c/2 + (d*x)/2)^16 + a^4 + 8*a^4*tan(c/2 + (d*x)/2))) - a
tanh(tan(c/2 + (d*x)/2))/(64*a^4*d)
```

3.83 $\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.83.1 Optimal result

Integrand size = 21, antiderivative size = 132

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{a}{20d(a+a\sin(c+dx))^5} - \frac{1}{8d(a+a\sin(c+dx))^4} + \frac{1}{16ad(a+a\sin(c+dx))^3} + \frac{1}{32d\left(a^2+a^2\sin(c+dx)\right)^2} + \frac{1}{64d\left(a^4-a^4\sin(c+dx)\right)} + \frac{1}{64d\left(a^4+a^4\sin(c+dx)\right)}$$

output
```
1/20*a/d/(a+a*sin(d*x+c))^5-1/8/d/(a+a*sin(d*x+c))^4+1/16/a/d/(a+a*sin(d*x
+c))^3+1/32/d/(a^2+a^2*sin(d*x+c))^2+1/64/d/(a^4-a^4*sin(d*x+c))+1/64/d/(a
^4+a^4*sin(d*x+c))
```

3.83.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.38

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{1+4\sin(c+dx)+5\sin^2(c+dx)}{20a^4d(-1+\sin(c+dx))(1+\sin(c+dx))^5}$$

input
```
Integrate[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

output
```
-1/20*(1 + 4*Sin[c + d*x] + 5*Sin[c + d*x]^2)/(a^4*d*(-1 + Sin[c + d*x])*(
1 + Sin[c + d*x])^5)
```

3.83.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a\sin(c+dx)+a)^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^3}{(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \; 3186$$

$$\frac{\int \frac{a^3\sin^3(c+dx)}{(a-a\sin(c+dx))^2(\sin(c+dx)a+a)^6}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(-\frac{a}{4(\sin(c+dx)a+a)^6}+\frac{1}{2(\sin(c+dx)a+a)^5}-\frac{3}{16(\sin(c+dx)a+a)^4a}-\frac{1}{16(\sin(c+dx)a+a)^3a^2}+\frac{1}{64(a-a\sin(c+dx))^2a^3}-\frac{1}{64(\sin(c+dx)a}\right.}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{64a^3(a-a\sin(c+dx))}+\frac{1}{64a^3(a\sin(c+dx)+a)}+\frac{1}{32a^2(a\sin(c+dx)+a)^2}+\frac{a}{20(a\sin(c+dx)+a)^5}-\frac{1}{8(a\sin(c+dx)+a)^4}+\frac{1}{16a(a\sin(c+dx)+a}}{d}$$

input
```
Int[Tan[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

output
```
(1/(64*a^3*(a - a*Sin[c + d*x])) + a/(20*(a + a*Sin[c + d*x])^5) - 1/(8*(a
 + a*Sin[c + d*x])^4) + 1/(16*a*(a + a*Sin[c + d*x])^3) + 1/(32*a^2*(a + a
*Sin[c + d*x])^2) + 1/(64*a^3*(a + a*Sin[c + d*x])))/d
```

3.83.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.83.4 Maple [A] (verified)

Time = 5.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

method	result	size
derivativedivides	$\frac{\frac{1}{20(1+\sin(dx+c))^5}-\frac{1}{8(1+\sin(dx+c))^4}+\frac{1}{16(1+\sin(dx+c))^3}+\frac{1}{32(1+\sin(dx+c))^2}+\frac{1}{64+64\sin(dx+c)}-\frac{1}{64(\sin(dx+c)-1)}}{d\,a^4}$	81
default	$\frac{\frac{1}{20(1+\sin(dx+c))^5}-\frac{1}{8(1+\sin(dx+c))^4}+\frac{1}{16(1+\sin(dx+c))^3}+\frac{1}{32(1+\sin(dx+c))^2}+\frac{1}{64+64\sin(dx+c)}-\frac{1}{64(\sin(dx+c)-1)}}{d\,a^4}$	81
risch	$-\frac{4\left(8i\,e^{7i(dx+c)}+5\,e^{8i(dx+c)}-8i\,e^{5i(dx+c)}-14\,e^{6i(dx+c)}+5\,e^{4i(dx+c)}\right)}{5\left(e^{i(dx+c)}+i\right)^{10}\left(-i+e^{i(dx+c)}\right)^2 d\,a^4}$	95

input
```
int(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(1/20/(1+sin(d*x+c))^5-1/8/(1+sin(d*x+c))^4+1/16/(1+sin(d*x+c))^3+
1/32/(1+sin(d*x+c))^2+1/64/(1+sin(d*x+c))-1/64/(sin(d*x+c)-1))
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{5\cos(dx+c)^2 - 4\sin(dx+c) - 6}{20\left(a^4d\cos(dx+c)^6 - 8a^4d\cos(dx+c)^4 + 8a^4d\cos(dx+c)^2 - 4\left(a^4d\cos(dx+c)^4 - 2a^4d\cos(dx+\right.\right.}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/20*(5*cos(d*x + c)^2 - 4*sin(d*x + c) - 6)/(a^4*d*cos(d*x + c)^6 - 8*a^
4*d*cos(d*x + c)^4 + 8*a^4*d*cos(d*x + c)^2 - 4*(a^4*d*cos(d*x + c)^4 - 2*
a^4*d*cos(d*x + c)^2)*sin(d*x + c))
```

3.83.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\tan^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(tan(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(tan(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c +
d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{5\sin(dx+c)^2 + 4\sin(dx+c) + 1}{20\left(a^4\sin(dx+c)^6 + 4a^4\sin(dx+c)^5 + 5a^4\sin(dx+c)^4 - 5a^4\sin(dx+c)^2 - 4a^4\sin(dx+c) - a^4\right)}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/20*(5*sin(d*x + c)^2 + 4*sin(d*x + c) + 1)/((a^4*sin(d*x + c)^6 + 4*a^4
*sin(d*x + c)^5 + 5*a^4*sin(d*x + c)^4 - 5*a^4*sin(d*x + c)^2 - 4*a^4*sin(
d*x + c) - a^4)*d)
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{5}{a^4(\sin(dx+c)-1)} - \frac{5\sin(dx+c)^4+30\sin(dx+c)^3+80\sin(dx+c)^2+50\sin(dx+c)+11}{a^4(\sin(dx+c)+1)^5}}{320\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/320*(5/(a^4*(sin(d*x + c) - 1)) - (5*sin(d*x + c)^4 + 30*sin(d*x + c)^3
 + 80*sin(d*x + c)^2 + 50*sin(d*x + c) + 11)/(a^4*(sin(d*x + c) + 1)^5))/d
```

3.83.9 Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.30

$$\int \frac{\tan^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + \frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5} + \frac{56\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{5} + \frac{32\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{5}}{a^4\,d\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^2\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)^{10}}$$

input
```
int(tan(c + d*x)^3/(a + a*sin(c + d*x))^4,x)
```

output
```
(4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8 + (32*cos(c/2 + (d*x)/2)^5*si
n(c/2 + (d*x)/2)^7)/5 + (56*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^6)/5 +
 (32*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)/5 + 4*cos(c/2 + (d*x)/2)^8
*sin(c/2 + (d*x)/2)^4)/(a^4*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))^2*
(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^10)
```

3.84 $\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.84.1 Optimal result

Integrand size = 19, antiderivative size = 105

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\operatorname{arctanh}(\sin(c+dx))}{16a^4d} + \frac{1}{8d(a+a\sin(c+dx))^4} - \frac{1}{12ad(a+a\sin(c+dx))^3} - \frac{1}{16d\left(a^2+a^2\sin(c+dx)\right)^2} - \frac{1}{16d\left(a^4+a^4\sin(c+dx)\right)}$$

output
```
1/16*arctanh(sin(d*x+c))/a^4/d+1/8/d/(a+a*sin(d*x+c))^4-1/12/a/d/(a+a*sin(
d*x+c))^3-1/16/d/(a^2+a^2*sin(d*x+c))^2-1/16/d/(a^4+a^4*sin(d*x+c))
```

3.84.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.59

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{3\operatorname{arctanh}(\sin(c+dx)) - \frac{4+19\sin(c+dx)+12\sin^2(c+dx)+3\sin^3(c+dx)}{(1+\sin(c+dx))^4}}{48a^4d}$$

input
```
Integrate[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

output
```
(3*ArcTanh[Sin[c + d*x]] - (4 + 19*Sin[c + d*x] + 12*Sin[c + d*x]^2 + 3*Si
n[c + d*x]^3)/(1 + Sin[c + d*x])^4)/(48*a^4*d)
```

3.84.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a\sin(c+dx)+a)^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)}{(a\sin(c+dx)+a)^4}dx$$
$$\Big\downarrow \text{3186}$$
$$\frac{\int \frac{a\sin(c+dx)}{(a-a\sin(c+dx))(\sin(c+dx)a+a)^5}d(a\sin(c+dx))}{d}$$
$$\Big\downarrow \text{86}$$
$$\frac{\int \left(-\frac{1}{2(\sin(c+dx)a+a)^5}+\frac{1}{4(\sin(c+dx)a+a)^4a}+\frac{1}{8(\sin(c+dx)a+a)^3a^2}+\frac{1}{16(a^2-a^2\sin^2(c+dx))a^3}+\frac{1}{16(\sin(c+dx)a+a)^2a^3}\right)d(a\sin(c+dx))}{d}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{\text{arctanh}(\sin(c+dx))}{16a^4}-\frac{1}{16a^3(a\sin(c+dx)+a)}-\frac{1}{16a^2(a\sin(c+dx)+a)^2}-\frac{1}{12a(a\sin(c+dx)+a)^3}+\frac{1}{8(a\sin(c+dx)+a)^4}}{d}$$

input
```
Int[Tan[c + d*x]/(a + a*Sin[c + d*x])^4,x]
```

output
```
(ArcTanh[Sin[c + d*x]]/(16*a^4) + 1/(8*(a + a*Sin[c + d*x])^4) - 1/(12*a*(
a + a*Sin[c + d*x])^3) - 1/(16*a^2*(a + a*Sin[c + d*x])^2) - 1/(16*a^3*(a
+ a*Sin[c + d*x])))/d
```

3.84.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.84.4 Maple [A] (verified)

Time = 5.98 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{32}+\frac{1}{8(1+\sin(dx+c))^4}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{1}{16(1+\sin(dx+c))^2}-\frac{1}{16(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{32}}{d\,a^4}$
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{32}+\frac{1}{8(1+\sin(dx+c))^4}-\frac{1}{12(1+\sin(dx+c))^3}-\frac{1}{16(1+\sin(dx+c))^2}-\frac{1}{16(1+\sin(dx+c))}+\frac{\ln(1+\sin(dx+c))}{32}}{d\,a^4}$
risch	$-\frac{i\left(-3\,\mathrm{e}^{i(dx+c)}+85\,\mathrm{e}^{3i(dx+c)}+24i\mathrm{e}^{2i(dx+c)}-80i\mathrm{e}^{4i(dx+c)}-85\,\mathrm{e}^{5i(dx+c)}+24i\mathrm{e}^{6i(dx+c)}+3\,\mathrm{e}^{7i(dx+c)}\right)}{24d\,a^4\left(\mathrm{e}^{i(dx+c)}+i\right)^8}-\frac{\ln\left(-i+\mathrm{e}^{i(dx+}\right.}{16a^4d}$

input
```
int(tan(d*x+c)/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(-1/32*ln(sin(d*x+c)-1)+1/8/(1+sin(d*x+c))^4-1/12/(1+sin(d*x+c))^3
-1/16/(1+sin(d*x+c))^2-1/16/(1+sin(d*x+c))+1/32*ln(1+sin(d*x+c)))
```

3.84.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(95) = 190$.

Time = 0.31 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.89

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{24\cos(dx+c)^2 + 3\left(\cos(dx+c)^4 - 8\cos(dx+c)^2 - 4\left(\cos(dx+c)^2-2\right)\sin(dx+c)+8\right)\log(\sin(dx}{96\left(a^4d\cos(dx+c)^4 -\right.}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/96*(24*cos(d*x + c)^2 + 3*(cos(d*x + c)^4 - 8*cos(d*x + c)^2 - 4*(cos(d*
x + c)^2 - 2)*sin(d*x + c) + 8)*log(sin(d*x + c) + 1) - 3*(cos(d*x + c)^4
- 8*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2)*sin(d*x + c) + 8)*log(-sin(d*x
 + c) + 1) + 2*(3*cos(d*x + c)^2 - 22)*sin(d*x + c) - 32)/(a^4*d*cos(d*x +
 c)^4 - 8*a^4*d*cos(d*x + c)^2 + 8*a^4*d - 4*(a^4*d*cos(d*x + c)^2 - 2*a^4
*d)*sin(d*x + c))
```

3.84.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\tan(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(tan(c + d*x)/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c + d*x
)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.15

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$
$$= -\frac{\frac{2\left(3\sin(dx+c)^3+12\sin(dx+c)^2+19\sin(dx+c)+4\right)}{a^4\sin(dx+c)^4+4a^4\sin(dx+c)^3+6a^4\sin(dx+c)^2+4a^4\sin(dx+c)+a^4}-\frac{3\log(\sin(dx+c)+1)}{a^4}+\frac{3\log(\sin(dx+c)-1)}{a^4}}{96\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/96*(2*(3*sin(d*x + c)^3 + 12*sin(d*x + c)^2 + 19*sin(d*x + c) + 4)/(a^4
*sin(d*x + c)^4 + 4*a^4*sin(d*x + c)^3 + 6*a^4*sin(d*x + c)^2 + 4*a^4*sin(
d*x + c) + a^4) - 3*log(sin(d*x + c) + 1)/a^4 + 3*log(sin(d*x + c) - 1)/a^
4)/d
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$
$$= \frac{\frac{12\log(|\sin(dx+c)+1|)}{a^4}-\frac{12\log(|\sin(dx+c)-1|)}{a^4}-\frac{25\sin(dx+c)^4+124\sin(dx+c)^3+246\sin(dx+c)^2+252\sin(dx+c)+57}{a^4(\sin(dx+c)+1)^4}}{384\,d}$$

input
```
integrate(tan(d*x+c)/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
1/384*(12*log(abs(sin(d*x + c) + 1))/a^4 - 12*log(abs(sin(d*x + c) - 1))/a
^4 - (25*sin(d*x + c)^4 + 124*sin(d*x + c)^3 + 246*sin(d*x + c)^2 + 252*si
n(d*x + c) + 57)/(a^4*(sin(d*x + c) + 1)^4))/d
```

3.84.9 Mupad [B] (verification not implemented)

Time = 10.51 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.29

$$\int \frac{\tan(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{8\,a^4\,d} + \frac{-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7}{8}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{43\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{24}+\frac{10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+\frac{43\,\mathrm{ta}}{}}{d\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+8\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+28\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+56\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+70\,a^4\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(tan(c + d*x)/(a + a*sin(c + d*x))^4,x)
```

output
```
atanh(tan(c/2 + (d*x)/2))/(8*a^4*d) + (tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d
*x)/2)/8 + (43*tan(c/2 + (d*x)/2)^3)/24 + (10*tan(c/2 + (d*x)/2)^4)/3 + (4
3*tan(c/2 + (d*x)/2)^5)/24 + tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^7/8
)/(d*(28*a^4*tan(c/2 + (d*x)/2)^2 + 56*a^4*tan(c/2 + (d*x)/2)^3 + 70*a^4*t
an(c/2 + (d*x)/2)^4 + 56*a^4*tan(c/2 + (d*x)/2)^5 + 28*a^4*tan(c/2 + (d*x)
/2)^6 + 8*a^4*tan(c/2 + (d*x)/2)^7 + a^4*tan(c/2 + (d*x)/2)^8 + a^4 + 8*a^
4*tan(c/2 + (d*x)/2)))
```

3.85 $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.85.1 Optimal result

Integrand size = 21, antiderivative size = 106

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{4\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^4d} + \frac{9\log(\sin(c+dx))}{a^4d} - \frac{9\log(1+\sin(c+dx))}{a^4d} + \frac{1}{d\,(a^2+a^2\sin(c+dx))^2} + \frac{5}{d\,(a^4+a^4\sin(c+dx))}$$

output
```
4*csc(d*x+c)/a^4/d-1/2*csc(d*x+c)^2/a^4/d+9*ln(sin(d*x+c))/a^4/d-9*ln(1+si
n(d*x+c))/a^4/d+1/d/(a^2+a^2*sin(d*x+c))^2+5/d/(a^4+a^4*sin(d*x+c))
```

3.85.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.69

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{8\csc(c+dx) - \csc^2(c+dx) + 18\log(\sin(c+dx)) - 18\log(1+\sin(c+dx)) + \frac{2}{(1+\sin(c+dx))^2} + \frac{10}{1+\sin(c+dx)}}{2a^4d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

output
```
(8*Csc[c + d*x] - Csc[c + d*x]^2 + 18*Log[Sin[c + d*x]] - 18*Log[1 + Sin[c
 + d*x]] + 2/(1 + Sin[c + d*x])^2 + 10/(1 + Sin[c + d*x]))/(2*a^4*d)
```

3.85.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3186, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^3(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^3(c+dx)(a-a\sin(c+dx))}{a^3(\sin(c+dx)a+a)^3}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(\frac{\csc^3(c+dx)}{a^5}-\frac{4\csc^2(c+dx)}{a^5}+\frac{9\csc(c+dx)}{a^5}-\frac{9}{a^4(\sin(c+dx)a+a)}-\frac{5}{a^3(\sin(c+dx)a+a)^2}-\frac{2}{a^2(\sin(c+dx)a+a)^3}\right)d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^2(c+dx)}{2a^4}+\frac{4\csc(c+dx)}{a^4}+\frac{9\log(a\sin(c+dx))}{a^4}-\frac{9\log(a\sin(c+dx)+a)}{a^4}+\frac{5}{a^3(a\sin(c+dx)+a)}+\frac{1}{a^2(a\sin(c+dx)+a)^2}}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + a*Sin[c + d*x])^4,x]
```

output
```
((4*Csc[c + d*x])/a^4 - Csc[c + d*x]^2/(2*a^4) + (9*Log[a*Sin[c + d*x]])/a
^4 - (9*Log[a + a*Sin[c + d*x]])/a^4 + 1/(a^2*(a + a*Sin[c + d*x])^2) + 5/
(a^3*(a + a*Sin[c + d*x])))/d
```

3.85.3.1 Defintions of rubi rules used

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.85.4 Maple [A] (verified)

Time = 15.89 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+9\ln(\sin(dx+c))+\frac{1}{(1+\sin(dx+c))^2}+\frac{5}{1+\sin(dx+c)}-9\ln(1+\sin(dx+c))}{d\,a^4}$
default	$\frac{-\frac{1}{2\sin(dx+c)^2}+\frac{4}{\sin(dx+c)}+9\ln(\sin(dx+c))+\frac{1}{(1+\sin(dx+c))^2}+\frac{5}{1+\sin(dx+c)}-9\ln(1+\sin(dx+c))}{d\,a^4}$
risch	$\frac{2i\left(27i\,\mathrm{e}^{6i(dx+c)}+9\,\mathrm{e}^{7i(dx+c)}-50i\,\mathrm{e}^{4i(dx+c)}-39\,\mathrm{e}^{5i(dx+c)}+27i\,\mathrm{e}^{2i(dx+c)}+39\,\mathrm{e}^{3i(dx+c)}-9\,\mathrm{e}^{i(dx+c)}\right)}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^2\left(\mathrm{e}^{i(dx+c)}+i\right)^4a^4d}-\frac{18\ln(\mathrm{e}^{i(dx+c)}+i)}{a^4d}$

input
```
int(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(-1/2/sin(d*x+c)^2+4/sin(d*x+c)+9*ln(sin(d*x+c))+1/(1+sin(d*x+c))^
2+5/(1+sin(d*x+c))-9*ln(1+sin(d*x+c)))
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.85

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{27\cos(dx+c)^2 - 18\left(\cos(dx+c)^4 - 3\cos(dx+c)^2 - 2\left(\cos(dx+c)^2 - 1\right)\sin(dx+c) + 2\right)\log\left(\frac{1}{2}\,\mathrm{s}\right.}{2\left(a^4d\cos(dx+c)^4\right.}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/2*(27*cos(d*x + c)^2 - 18*(cos(d*x + c)^4 - 3*cos(d*x + c)^2 - 2*(cos(d
*x + c)^2 - 1)*sin(d*x + c) + 2)*log(1/2*sin(d*x + c)) + 18*(cos(d*x + c)^
4 - 3*cos(d*x + c)^2 - 2*(cos(d*x + c)^2 - 1)*sin(d*x + c) + 2)*log(sin(d*
x + c) + 1) + 6*(3*cos(d*x + c)^2 - 4)*sin(d*x + c) - 26)/(a^4*d*cos(d*x +
 c)^4 - 3*a^4*d*cos(d*x + c)^2 + 2*a^4*d - 2*(a^4*d*cos(d*x + c)^2 - a^4*d
)*sin(d*x + c))
```

3.85.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\cot^3(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(cot(d*x+c)**3/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(cot(c + d*x)**3/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c +
d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.97

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\frac{18\sin(dx+c)^3+27\sin(dx+c)^2+6\sin(dx+c)-1}{a^4\sin(dx+c)^4+2a^4\sin(dx+c)^3+a^4\sin(dx+c)^2} - \frac{18\log(\sin(dx+c)+1)}{a^4} + \frac{18\log(\sin(dx+c))}{a^4}}{2d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/2*((18*sin(d*x + c)^3 + 27*sin(d*x + c)^2 + 6*sin(d*x + c) - 1)/(a^4*sin
(d*x + c)^4 + 2*a^4*sin(d*x + c)^3 + a^4*sin(d*x + c)^2) - 18*log(sin(d*x
+ c) + 1)/a^4 + 18*log(sin(d*x + c))/a^4)/d
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.75

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{144\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4} - \frac{72\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4} + \frac{108\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-16\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2} + \frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-16a^4\tan \cdots}{a^8}}{8d}$$

input
```
integrate(cot(d*x+c)^3/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/8*(144*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 72*log(abs(tan(1/2*d*x
+ 1/2*c)))/a^4 + (108*tan(1/2*d*x + 1/2*c)^2 - 16*tan(1/2*d*x + 1/2*c) + 1
)/(a^4*tan(1/2*d*x + 1/2*c)^2) + (a^4*tan(1/2*d*x + 1/2*c)^2 - 16*a^4*tan(
1/2*d*x + 1/2*c))/a^8 - 4*(75*tan(1/2*d*x + 1/2*c)^4 + 272*tan(1/2*d*x + 1
/2*c)^3 + 402*tan(1/2*d*x + 1/2*c)^2 + 272*tan(1/2*d*x + 1/2*c) + 75)/(a^4
*(tan(1/2*d*x + 1/2*c) + 1)^4))/d
```

3.85.9 Mupad [B] (verification not implemented)

Time = 6.44 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.15

$$\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{9\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4 d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8a^4 d} - \frac{48\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + \frac{129\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2} + 10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 29\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - 6\tan\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{1}{2} \cdots}{d\left(4a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + 16a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 + 24a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 + 16a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 4a^4\tan\left(\frac{c}{2}+ \cdots\right.\right.} - \frac{18\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^4 d} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^4 d}$$

input
```
int(cot(c + d*x)^3/(a + a*sin(c + d*x))^4,x)
```

output
```
(9*log(tan(c/2 + (d*x)/2)))/(a^4*d) - tan(c/2 + (d*x)/2)^2/(8*a^4*d) - (10
*tan(c/2 + (d*x)/2)^3 - 29*tan(c/2 + (d*x)/2)^2 - 6*tan(c/2 + (d*x)/2) + (
129*tan(c/2 + (d*x)/2)^4)/2 + 48*tan(c/2 + (d*x)/2)^5 + 1/2)/(d*(4*a^4*tan
(c/2 + (d*x)/2)^2 + 16*a^4*tan(c/2 + (d*x)/2)^3 + 24*a^4*tan(c/2 + (d*x)/2
)^4 + 16*a^4*tan(c/2 + (d*x)/2)^5 + 4*a^4*tan(c/2 + (d*x)/2)^6)) - (18*log
(tan(c/2 + (d*x)/2) + 1))/(a^4*d) + (2*tan(c/2 + (d*x)/2))/(a^4*d)
```

3.85. $\int \frac{\cot^3(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.86 $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.86.1 Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{8\csc(c+dx)}{a^4d} - \frac{4\csc^2(c+dx)}{a^4d} + \frac{8\csc^3(c+dx)}{3a^4d} - \frac{7\csc^4(c+dx)}{4a^4d} + \frac{4\csc^5(c+dx)}{5a^4d} - \frac{\csc^6(c+dx)}{6a^4d} + \frac{8\log(\sin(c+dx))}{a^4d} - \frac{8\log(1+\sin(c+dx))}{a^4d}$$

output
```
8*csc(d*x+c)/a^4/d-4*csc(d*x+c)^2/a^4/d+8/3*csc(d*x+c)^3/a^4/d-7/4*csc(d*x
+c)^4/a^4/d+4/5*csc(d*x+c)^5/a^4/d-1/6*csc(d*x+c)^6/a^4/d+8*ln(sin(d*x+c))
/a^4/d-8*ln(1+sin(d*x+c))/a^4/d
```

3.86.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{480\csc(c+dx) - 240\csc^2(c+dx) + 160\csc^3(c+dx) - 105\csc^4(c+dx) + 48\csc^5(c+dx) - 10\csc^6(c+}{60a^4d}$$

input
```
Integrate[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]
```

output
```
(480*Csc[c + d*x] - 240*Csc[c + d*x]^2 + 160*Csc[c + d*x]^3 - 105*Csc[c +
d*x]^4 + 48*Csc[c + d*x]^5 - 10*Csc[c + d*x]^6 + 480*Log[Sin[c + d*x]] - 4
80*Log[1 + Sin[c + d*x]])/(60*a^4*d)
```

3.86.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3186, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^7(c+dx)}{(a\sin(c+dx)+a)^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^7(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^7(c+dx)(a-a\sin(c+dx))^3}{a^7(\sin(c+dx)a+a)}d(a\sin(c+dx))}{d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{\csc^7(c+dx)}{a^5}-\frac{4\csc^6(c+dx)}{a^5}+\frac{7\csc^5(c+dx)}{a^5}-\frac{8\csc^4(c+dx)}{a^5}+\frac{8\csc^3(c+dx)}{a^5}-\frac{8\csc^2(c+dx)}{a^5}+\frac{8\csc(c+dx)}{a^5}-\frac{8}{a^4(\sin(c+dx)a+a)}\right) d}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\csc^6(c+dx)}{6a^4}+\frac{4\csc^5(c+dx)}{5a^4}-\frac{7\csc^4(c+dx)}{4a^4}+\frac{8\csc^3(c+dx)}{3a^4}-\frac{4\csc^2(c+dx)}{a^4}+\frac{8\csc(c+dx)}{a^4}+\frac{8\log(a\sin(c+dx))}{a^4}-\frac{8\log(a\sin(c+dx)+}{a^4}}{d}$$

input
```
Int[Cot[c + d*x]^7/(a + a*Sin[c + d*x])^4,x]
```

output
```
((8*Csc[c + d*x])/a^4 - (4*Csc[c + d*x]^2)/a^4 + (8*Csc[c + d*x]^3)/(3*a^4
) - (7*Csc[c + d*x]^4)/(4*a^4) + (4*Csc[c + d*x]^5)/(5*a^4) - Csc[c + d*x]
^6/(6*a^4) + (8*Log[a*Sin[c + d*x]])/a^4 - (8*Log[a + a*Sin[c + d*x]])/a^4
)/d
```

3.86.3.1 Defintions of rubi rules used

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.86.4 Maple [A] (verified)

Time = 54.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

method	result
derivativedivides	$\frac{-8\ln(1+\sin(dx+c))-\frac{1}{6\sin(dx+c)^6}+\frac{4}{5\sin(dx+c)^5}-\frac{7}{4\sin(dx+c)^4}+\frac{8}{3\sin(dx+c)^3}-\frac{4}{\sin(dx+c)^2}+\frac{8}{\sin(dx+c)}+8\ln(\sin(dx+c))}{d\,a^4}$
default	$\frac{-8\ln(1+\sin(dx+c))-\frac{1}{6\sin(dx+c)^6}+\frac{4}{5\sin(dx+c)^5}-\frac{7}{4\sin(dx+c)^4}+\frac{8}{3\sin(dx+c)^3}-\frac{4}{\sin(dx+c)^2}+\frac{8}{\sin(dx+c)}+8\ln(\sin(dx+c))}{d\,a^4}$
risch	$\frac{4i(-60i\mathrm{e}^{10i(dx+c)}+60\,\mathrm{e}^{11i(dx+c)}+345i\mathrm{e}^{8i(dx+c)}-380\,\mathrm{e}^{9i(dx+c)}-610i\mathrm{e}^{6i(dx+c)}+936\,\mathrm{e}^{7i(dx+c)}+345i\mathrm{e}^{4i(dx+c)}-936\,\mathrm{e}^{5i(}}{15d\,a^4\left(\mathrm{e}^{2i(dx+c)}-1\right)^6}$

input
```
int(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d/a^4*(-8*ln(1+sin(d*x+c))-1/6/sin(d*x+c)^6+4/5/sin(d*x+c)^5-7/4/sin(d*x
+c)^4+8/3/sin(d*x+c)^3-4/sin(d*x+c)^2+8/sin(d*x+c)+8*ln(sin(d*x+c)))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.38

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{240\cos(dx+c)^4 - 585\cos(dx+c)^2 + 480\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - 1\right)\log\left(\frac{1}{2}\right.}{60\left(a^4\right.}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/60*(240*cos(d*x + c)^4 - 585*cos(d*x + c)^2 + 480*(cos(d*x + c)^6 - 3*co
s(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(1/2*sin(d*x + c)) - 480*(cos(d*x
+ c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*log(sin(d*x + c) + 1) -
16*(30*cos(d*x + c)^4 - 70*cos(d*x + c)^2 + 43)*sin(d*x + c) + 355)/(a^4*d
*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)
```

3.86.6 Sympy [F]

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\cot^7(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(cot(d*x+c)**7/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(cot(c + d*x)**7/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c +
d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{480\log(\sin(dx+c)+1)}{a^4} - \frac{480\log(\sin(dx+c))}{a^4} - \frac{480\sin(dx+c)^5-240\sin(dx+c)^4+160\sin(dx+c)^3-105\sin(dx+c)^2+48\sin(dx+c)-10}{a^4\sin(dx+c)^6}}{60\,d}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/60*(480*log(sin(d*x + c) + 1)/a^4 - 480*log(sin(d*x + c))/a^4 - (480*si
n(d*x + c)^5 - 240*sin(d*x + c)^4 + 160*sin(d*x + c)^3 - 105*sin(d*x + c)^
2 + 48*sin(d*x + c) - 10)/(a^4*sin(d*x + c)^6))/d
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.72

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx =$$
$$-\frac{\frac{30720\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^4}-\frac{15360\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}+\frac{37632\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-10080\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+2835\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4\cdots}{a^4\tan\left(\frac{1}{2}dx\cdots\right.}}{\cdots}$$

input
```
integrate(cot(d*x+c)^7/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/1920*(30720*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 15360*log(abs(tan(
1/2*d*x + 1/2*c)))/a^4 + (37632*tan(1/2*d*x + 1/2*c)^6 - 10080*tan(1/2*d*x
 + 1/2*c)^5 + 2835*tan(1/2*d*x + 1/2*c)^4 - 880*tan(1/2*d*x + 1/2*c)^3 + 2
40*tan(1/2*d*x + 1/2*c)^2 - 48*tan(1/2*d*x + 1/2*c) + 5)/(a^4*tan(1/2*d*x
+ 1/2*c)^6) + (5*a^20*tan(1/2*d*x + 1/2*c)^6 - 48*a^20*tan(1/2*d*x + 1/2*c
)^5 + 240*a^20*tan(1/2*d*x + 1/2*c)^4 - 880*a^20*tan(1/2*d*x + 1/2*c)^3 +
2835*a^20*tan(1/2*d*x + 1/2*c)^2 - 10080*a^20*tan(1/2*d*x + 1/2*c))/a^24)/
d
```

3.86.9 Mupad [B] (verification not implemented)

Time = 6.53 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.74

$$\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$
$$= \frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^4\,d}-\frac{189\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{128\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{8\,a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{40\,a^4\,d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{384\,a^4\,d}+\frac{8\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d}-\frac{16\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{a^4\,d}+\frac{21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\,a^4\,d}$$
$$+\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(336\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-\frac{189\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{2}+\frac{88\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{5}-\frac{1}{6}\right)}{64\,a^4\,d}$$

input
```
int(cot(c + d*x)^7/(a + a*sin(c + d*x))^4,x)
```

output
```
(11*tan(c/2 + (d*x)/2)^3)/(24*a^4*d) - (189*tan(c/2 + (d*x)/2)^2)/(128*a^4
*d) - tan(c/2 + (d*x)/2)^4/(8*a^4*d) + tan(c/2 + (d*x)/2)^5/(40*a^4*d) - t
an(c/2 + (d*x)/2)^6/(384*a^4*d) + (8*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (1
6*log(tan(c/2 + (d*x)/2) + 1))/(a^4*d) + (21*tan(c/2 + (d*x)/2))/(4*a^4*d)
 + (cot(c/2 + (d*x)/2)^6*((8*tan(c/2 + (d*x)/2))/5 - 8*tan(c/2 + (d*x)/2)^
2 + (88*tan(c/2 + (d*x)/2)^3)/3 - (189*tan(c/2 + (d*x)/2)^4)/2 + 336*tan(c
/2 + (d*x)/2)^5 - 1/6))/(64*a^4*d)
```

3.86. $\int \frac{\cot^7(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.87 $\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.87.1 Optimal result

Integrand size = 21, antiderivative size = 127

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{4\sec^5(c+dx)}{5a^4d}+\frac{12\sec^7(c+dx)}{7a^4d}-\frac{8\sec^9(c+dx)}{9a^4d}+\frac{\tan^3(c+dx)}{3a^4d}+\frac{9\tan^5(c+dx)}{5a^4d}+\frac{16\tan^7(c+dx)}{7a^4d}+\frac{8\tan^9(c+dx)}{9a^4d}$$

output
```
-4/5*sec(d*x+c)^5/a^4/d+12/7*sec(d*x+c)^7/a^4/d-8/9*sec(d*x+c)^9/a^4/d+1/3
*tan(d*x+c)^3/a^4/d+9/5*tan(d*x+c)^5/a^4/d+16/7*tan(d*x+c)^7/a^4/d+8/9*tan
(d*x+c)^9/a^4/d
```

3.87.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.98

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\sec(c+dx)(16128+1554\cos(c+dx)-16896\cos(2(c+dx))-999\cos(3(c+dx))+2816\cos(4(c+dx))}{80}$$

input
```
Integrate[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

output
```
(Sec[c + d*x]*(16128 + 1554*Cos[c + d*x] - 16896*Cos[2*(c + d*x)] - 999*Co
s[3*(c + d*x)] + 2816*Cos[4*(c + d*x)] + 37*Cos[5*(c + d*x)] + 34944*Sin[c
 + d*x] + 1776*Sin[2*(c + d*x)] - 9504*Sin[3*(c + d*x)] - 296*Sin[4*(c + d
*x)] + 352*Sin[5*(c + d*x)]))/(80640*a^4*d*(1 + Sin[c + d*x])^4)
```

3.87.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3190, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2}{(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3190}$$

$$\frac{\int \left(a^4\tan^2(c+dx)\sec^8(c+dx)-4a^4\tan^3(c+dx)\sec^7(c+dx)+6a^4\tan^4(c+dx)\sec^6(c+dx)-4a^4\tan^5(c+dx\right.}{a^8}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{8a^4\tan^9(c+dx)}{9d}+\frac{16a^4\tan^7(c+dx)}{7d}+\frac{9a^4\tan^5(c+dx)}{5d}+\frac{a^4\tan^3(c+dx)}{3d}-\frac{8a^4\sec^9(c+dx)}{9d}+\frac{12a^4\sec^7(c+dx)}{7d}-\frac{4a^4\sec^5(c+dx)}{5d}}{a^8}$$

input
```
Int[Tan[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

output
```
((-4*a^4*Sec[c + d*x]^5)/(5*d) + (12*a^4*Sec[c + d*x]^7)/(7*d) - (8*a^4*Se
c[c + d*x]^9)/(9*d) + (a^4*Tan[c + d*x]^3)/(3*d) + (9*a^4*Tan[c + d*x]^5)/
(5*d) + (16*a^4*Tan[c + d*x]^7)/(7*d) + (8*a^4*Tan[c + d*x]^9)/(9*d))/a^8
```

3.87.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3190
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Simp[a^(2*m)   Int[ExpandIntegrand[(g*Tan[e + f*x]
)^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; F
reeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

3.87.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 5.72 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{4i\left(504i\,\mathrm{e}^{5i(dx+c)}+315\,\mathrm{e}^{6i(dx+c)}-528i\,\mathrm{e}^{3i(dx+c)}-777\,\mathrm{e}^{4i(dx+c)}+88i\,\mathrm{e}^{i(dx+c)}+297\,\mathrm{e}^{2i(dx+c)}-11\right)}{315\left(-i+\mathrm{e}^{i(dx+c)}\right)\left(\mathrm{e}^{i(dx+c)}+i\right)^{9} d\,a^{4}}$
derivativedivides	$\frac{-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{116}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{62}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{83}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{a^{4}d}$
default	$\frac{-\frac{1}{16\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{16}{9\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{9}}+\frac{8}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{8}}-\frac{116}{7\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{7}}+\frac{62}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{6}}-\frac{83}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{a^{4}d}$

input
```
int(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/315*I*(504*I*exp(5*I*(d*x+c))+315*exp(6*I*(d*x+c))-528*I*exp(3*I*(d*x+c
))-777*exp(4*I*(d*x+c))+88*I*exp(I*(d*x+c))+297*exp(2*I*(d*x+c))-11)/(-I+e
xp(I*(d*x+c)))/(exp(I*(d*x+c))+I)^9/d/a^4
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$
$$= \frac{88\cos(dx+c)^4 - 220\cos(dx+c)^2 + \left(22\cos(dx+c)^4 - 165\cos(dx+c)^2 + 175\right)\sin(dx+c)}{315\left(a^4d\cos(dx+c)^5 - 8a^4d\cos(dx+c)^3 + 8a^4d\cos(dx+c) - 4\left(a^4d\cos(dx+c)^3 - 2a^4d\cos(dx+c\right.\right.}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/315*(88*cos(d*x + c)^4 - 220*cos(d*x + c)^2 + (22*cos(d*x + c)^4 - 165*c
os(d*x + c)^2 + 175)*sin(d*x + c) + 140)/(a^4*d*cos(d*x + c)^5 - 8*a^4*d*c
os(d*x + c)^3 + 8*a^4*d*cos(d*x + c) - 4*(a^4*d*cos(d*x + c)^3 - 2*a^4*d*c
os(d*x + c))*sin(d*x + c))
```

3.87.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\tan^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(tan(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(tan(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c +
d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.87.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(113) = 226.

Time = 0.20 (sec) , antiderivative size = 356, normalized size of antiderivative = 2.80

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$
$$= \frac{8\left(\frac{16\sin(dx+c)}{\cos(dx+c)+1} + \frac{54\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{201\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{294\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{378\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{210\sin(dx\cdots}{(\cos(dx+c)\cdots}\right.}{315\left(a^4 + \frac{8a^4\sin(dx+c)}{\cos(dx+c)+1} + \frac{27a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{48a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{42a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{42a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{48a^4\sin(dx+c)^7}{(\cos(dx+c)+1)^7} - \frac{27\cdots}{(\cdots}\right.}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
8/315*(16*sin(d*x + c)/(cos(d*x + c) + 1) + 54*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 201*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 294*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + 378*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 210*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 105*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 2
)/((a^4 + 8*a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 27*a^4*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 48*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 42*a^4*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 - 42*a^4*sin(d*x + c)^6/(cos(d*x + c) +
 1)^6 - 48*a^4*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 27*a^4*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 - 8*a^4*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^4*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10)*d)
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.15

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{315}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)} - \frac{315\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8+3150\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1050\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+630\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-8064\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)^9}}{5040\,d}$$

input
```
integrate(tan(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/5040*(315/(a^4*(tan(1/2*d*x + 1/2*c) - 1)) - (315*tan(1/2*d*x + 1/2*c)^
8 + 3150*tan(1/2*d*x + 1/2*c)^7 + 1050*tan(1/2*d*x + 1/2*c)^6 + 630*tan(1/
2*d*x + 1/2*c)^5 - 8064*tan(1/2*d*x + 1/2*c)^4 - 6006*tan(1/2*d*x + 1/2*c)
^3 - 5274*tan(1/2*d*x + 1/2*c)^2 - 846*tan(1/2*d*x + 1/2*c) - 59)/(a^4*(ta
n(1/2*d*x + 1/2*c) + 1)^9))/d
```

3.87.9 Mupad [B] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.82

$$\int \frac{\tan^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\frac{16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{315}+\frac{128\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{315}+\frac{48\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}{35}+\frac{536\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{105}+\frac{112\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{}}{a^4\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\cos\left(\right.\right.}$$

input
```
int(tan(c + d*x)^2/(a + a*sin(c + d*x))^4,x)
```

output
```
((16*cos(c/2 + (d*x)/2)^10)/315 + (128*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x
)/2))/315 + (8*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^7)/3 + (16*cos(c/2
+ (d*x)/2)^4*sin(c/2 + (d*x)/2)^6)/3 + (48*cos(c/2 + (d*x)/2)^5*sin(c/2 +
(d*x)/2)^5)/5 + (112*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^4)/15 + (536*
cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^3)/105 + (48*cos(c/2 + (d*x)/2)^8*
sin(c/2 + (d*x)/2)^2)/35)/(a^4*d*(cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2))
*(cos(c/2 + (d*x)/2) + sin(c/2 + (d*x)/2))^9)
```

3.88 $\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.88.1 Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{4\text{arctanh}(\cos(c+dx))}{a^4d} - \frac{\cot(c+dx)}{a^4d} - \frac{2\cot(c+dx)}{5a^4d(1+\csc(c+dx))^3} + \frac{31\cot(c+dx)}{15a^4d(1+\csc(c+dx))^2} - \frac{104\cot(c+dx)}{15a^4d(1+\csc(c+dx))}$$

output
```
4*arctanh(cos(d*x+c))/a^4/d-94/15*cot(d*x+c)/a^4/d+2/5*cot(d*x+c)/a^4/d/(1
+sin(d*x+c))^3+13/15*cot(d*x+c)/a^4/d/(1+sin(d*x+c))^2+4*cot(d*x+c)/a^4/d/
(1+sin(d*x+c))
```

3.88.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(108) = 216.

Time = 0.74 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.92

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3\left(24\sin\left(\frac{1}{2}(c+dx)\right)-12\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+76\,\text{si}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(24*Sin[(c + d*x)/2] - 12*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]) + 76*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2])^2 - 38*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + 316*Sin[
(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 15*Cot[(c + d*x)/2]
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5 + 120*Log[Cos[(c + d*x)/2]]*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2])^5 - 120*Log[Sin[(c + d*x)/2]]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^5 + 15*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*
Tan[(c + d*x)/2]))/(30*d*(a + a*Sin[c + d*x])^4)
```

3.88.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^2(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{\int \left(\frac{\csc^2(c+dx)}{a^2}-\frac{4\csc(c+dx)}{a^2}-\frac{16}{a^2(\csc(c+dx)+1)}+\frac{9}{a^2}+\frac{9}{a^2(\csc(c+dx)+1)^2}-\frac{2}{a^2(\csc(c+dx)+1)^3}\right)dx}{a^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{4\text{arctanh}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{a^2d}-\frac{104\cot(c+dx)}{15a^2d(\csc(c+dx)+1)}+\frac{31\cot(c+dx)}{15a^2d(\csc(c+dx)+1)^2}-\frac{2\cot(c+dx)}{5a^2d(\csc(c+dx)+1)^3}}{a^2}$$

input
```
Int[Cot[c + d*x]^2/(a + a*Sin[c + d*x])^4,x]
```

output
```
((4*ArcTanh[Cos[c + d*x]])/(a^2*d) - Cot[c + d*x]/(a^2*d) - (2*Cot[c + d*x
])/(5*a^2*d*(1 + Csc[c + d*x])^3) + (31*Cot[c + d*x])/(15*a^2*d*(1 + Csc[c
 + d*x])^2) - (104*Cot[c + d*x])/(15*a^2*d*(1 + Csc[c + d*x])))/a^2
```

3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.88.4 Maple [A] (verified)

Time = 11.11 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{\left(\tan\left(\frac{dx}{2}\right.\right.}}{2d\,a^4}$
default	$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-8\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{32}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}+\frac{16}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}-\frac{88}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{\left(\tan\left(\frac{dx}{2}\right.\right.}}{2d\,a^4}$
risch	$-\frac{4\left(-320\,e^{4i(dx+c)}+150ie^{5i(dx+c)}+367\,e^{2i(dx+c)}-385ie^{3i(dx+c)}-47+205ie^{i(dx+c)}+30\,e^{6i(dx+c)}\right)}{15\left(e^{2i(dx+c)}-1\right)\left(e^{i(dx+c)}+i\right)^5a^4d}-\frac{4\ln\left(e^{i(dx+c)}-\right.}{a^4d}$

input
```
int(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/2/d/a^4*(tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)-8*ln(tan(1/2*d*x+1/2*c)
)-32/5/(tan(1/2*d*x+1/2*c)+1)^5+16/(tan(1/2*d*x+1/2*c)+1)^4-88/3/(tan(1/2*
d*x+1/2*c)+1)^3+28/(tan(1/2*d*x+1/2*c)+1)^2-36/(tan(1/2*d*x+1/2*c)+1))
```

3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(102) = 204$.

Time = 0.29 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.42

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$
$$= \frac{94\cos(dx+c)^4 + 222\cos(dx+c)^3 - 115\cos(dx+c)^2 + 30\left(\cos(dx+c)^4 - 2\cos(dx+c)^3 - 5\cos(dx\right.}{}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/15*(94*cos(d*x + c)^4 + 222*cos(d*x + c)^3 - 115*cos(d*x + c)^2 + 30*(co
s(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - (cos(d*x + c)^3 + 3*c
os(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d*x + c) + 2*cos(d*x + c) + 4)*log
(1/2*cos(d*x + c) + 1/2) - 30*(cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 5*cos(d
*x + c)^2 - (cos(d*x + c)^3 + 3*cos(d*x + c)^2 - 2*cos(d*x + c) - 4)*sin(d
*x + c) + 2*cos(d*x + c) + 4)*log(-1/2*cos(d*x + c) + 1/2) + (94*cos(d*x +
 c)^3 - 128*cos(d*x + c)^2 - 243*cos(d*x + c) - 6)*sin(d*x + c) - 237*cos(
d*x + c) + 6)/(a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 5*a^4*d*cos
(d*x + c)^2 + 2*a^4*d*cos(d*x + c) + 4*a^4*d - (a^4*d*cos(d*x + c)^3 + 3*a
^4*d*cos(d*x + c)^2 - 2*a^4*d*cos(d*x + c) - 4*a^4*d)*sin(d*x + c))
```

3.88.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\cot^2(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(cot(d*x+c)**2/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(cot(c + d*x)**2/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c +
d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(102) = 204$.

Time = 0.20 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.67

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{\frac{491\sin(dx+c)}{\cos(dx+c)+1}+\frac{1690\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2570\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{1815\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{555\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+15}{\frac{a^4\sin(dx+c)}{\cos(dx+c)+1}+\frac{5a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{10a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{120\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}-\frac{15\sin(dx}{a^4(\cos(dx+}}{30\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
-1/30*((491*sin(d*x + c)/(cos(d*x + c) + 1) + 1690*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 2570*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 1815*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 555*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15)/(
a^4*sin(d*x + c)/(cos(d*x + c) + 1) + 5*a^4*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 10*a^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 10*a^4*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 5*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*si
n(d*x + c)^6/(cos(d*x + c) + 1)^6) + 120*log(sin(d*x + c)/(cos(d*x + c) +
1))/a^4 - 15*sin(d*x + c)/(a^4*(cos(d*x + c) + 1)))/d
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{120\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4}-\frac{15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^4}-\frac{15\left(8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\frac{4\left(135\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+435\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+605\tan\left(\frac{1}{2}\right.\right.}{a^4\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right.}}{30\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/30*(120*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - 15*tan(1/2*d*x + 1/2*c)/a^
4 - 15*(8*tan(1/2*d*x + 1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)) + 4*(135*ta
n(1/2*d*x + 1/2*c)^4 + 435*tan(1/2*d*x + 1/2*c)^3 + 605*tan(1/2*d*x + 1/2*
c)^2 + 385*tan(1/2*d*x + 1/2*c) + 104)/(a^4*(tan(1/2*d*x + 1/2*c) + 1)^5))
/d
```

3.88.9 Mupad [B] (verification not implemented)

Time = 11.47 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.88

$$\int \frac{\cot^2(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^4\,d}$$
$$-\frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+121\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\frac{514\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{3}+\frac{338\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{3}+\frac{491\tan\left(\frac{c}{2}+\right.}{15}}{d\left(2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+10\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+20\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+20\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+10\,a^4\tan\left(\frac{c}{2}\right.\right.}$$
$$-\frac{4\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d}$$

input
```
int(cot(c + d*x)^2/(a + a*sin(c + d*x))^4,x)
```

output
```
tan(c/2 + (d*x)/2)/(2*a^4*d) - ((491*tan(c/2 + (d*x)/2))/15 + (338*tan(c/2
 + (d*x)/2)^2)/3 + (514*tan(c/2 + (d*x)/2)^3)/3 + 121*tan(c/2 + (d*x)/2)^4
 + 37*tan(c/2 + (d*x)/2)^5 + 1)/(d*(10*a^4*tan(c/2 + (d*x)/2)^2 + 20*a^4*t
an(c/2 + (d*x)/2)^3 + 20*a^4*tan(c/2 + (d*x)/2)^4 + 10*a^4*tan(c/2 + (d*x)
/2)^5 + 2*a^4*tan(c/2 + (d*x)/2)^6 + 2*a^4*tan(c/2 + (d*x)/2))) - (4*log(t
an(c/2 + (d*x)/2)))/(a^4*d)
```

3.89 $\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.89.1 Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{14\text{arctanh}(\cos(c+dx))}{a^4d} - \frac{9\cot(c+dx)}{a^4d} - \frac{\cot^3(c+dx)}{3a^4d} + \frac{2\cot(c+dx)\csc(c+dx)}{a^4d} + \frac{4\cot(c+dx)}{3a^4d(1+\csc(c+dx))^2} - \frac{44\cot(c+dx)}{3a^4d(1+\csc(c+dx))}$$

output
```
14*arctanh(cos(d*x+c))/a^4/d-33*cot(d*x+c)/a^4/d-11*cot(d*x+c)^3/a^4/d+14*
cot(d*x+c)*csc(d*x+c)/a^4/d+4/3*cot(d*x+c)*csc(d*x+c)^2/a^4/d/(1+sin(d*x+c
))^2+28/3*cot(d*x+c)*csc(d*x+c)^2/a^4/d/(1+sin(d*x+c))
```

3.89.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 359 vs. 2(120) = 240.

Time = 5.06 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.99

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^5\left(-\cos\left(\frac{1}{2}(c+dx)\right)\left(1+\cot\left(\frac{1}{2}(c+dx)\right)\right)^3+64\sin\left(\frac{1}{2}(c+dx)\right)+12\cdots\right.}{\cdots}$$

input
```
Integrate[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

output
```
((Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(-(Cos[(c + d*x)/2]*(1 + Cot[(c +
 d*x)/2])^3) + 64*Sin[(c + d*x)/2] + 12*(1 + Cot[(c + d*x)/2])^3*Sin[(c +
d*x)/2] - 32*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]) + 640*Sin[(c + d*x)/2]*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 - 104*Cot[(c + d*x)/2]*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^3 + 336*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^3 - 336*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^3 + 104*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*Tan[(c + d*
x)/2] - 12*Cos[(c + d*x)/2]*(1 + Tan[(c + d*x)/2])^3 + Sin[(c + d*x)/2]*(1
 + Tan[(c + d*x)/2])^3))/(24*a^4*d*(1 + Sin[c + d*x])^4)
```

3.89.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^4(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{ 3188}$$

$$\frac{\int \left(\csc^4(c+dx)-4\csc^3(c+dx)+8\csc^2(c+dx)-12\csc(c+dx)-\frac{20}{\csc(c+dx)+1}+\frac{4}{(\csc(c+dx)+1)^2}+16\right)dx}{a^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{14\text{arctanh}(\cos(c+dx))}{d}-\frac{\cot^3(c+dx)}{3d}-\frac{9\cot(c+dx)}{d}+\frac{2\cot(c+dx)\csc(c+dx)}{d}-\frac{44\cot(c+dx)}{3d(\csc(c+dx)+1)}+\frac{4\cot(c+dx)}{3d(\csc(c+dx)+1)^2}}{a^4}$$

input
```
Int[Cot[c + d*x]^4/(a + a*Sin[c + d*x])^4,x]
```

output
```
((14*ArcTanh[Cos[c + d*x]])/d - (9*Cot[c + d*x])/d - Cot[c + d*x]^3/(3*d)
+ (2*Cot[c + d*x]*Csc[c + d*x])/d + (4*Cot[c + d*x])/(3*d*(1 + Csc[c + d*x
])^2) - (44*Cot[c + d*x])/(3*d*(1 + Csc[c + d*x])))/a^4
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.89.4 Maple [A] (verified)

Time = 23.64 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{128}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{64}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{256}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{3\tan\left(\frac{d}{}\right.}}{8d\,a^4}$
default	$\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+35\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{128}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{64}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{256}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}-\frac{1}{3\tan\left(\frac{d}{}\right.}}{8d\,a^4}$
risch	$-\frac{4\left(-119\,e^{6i(dx+c)}+63ie^{7i(dx+c)}+204\,e^{4i(dx+c)}-192ie^{5i(dx+c)}+21\,e^{8i(dx+c)}-135\,e^{2i(dx+c)}+211ie^{3i(dx+c)}+33-78ie\right.}{3\left(e^{2i(dx+c)}-1\right)^3\left(e^{i(dx+c)}+i\right)^3a^4d}$

input
```
int(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8/d/a^4*(1/3*tan(1/2*d*x+1/2*c)^3-4*tan(1/2*d*x+1/2*c)^2+35*tan(1/2*d*x+
1/2*c)-128/3/(tan(1/2*d*x+1/2*c)+1)^3+64/(tan(1/2*d*x+1/2*c)+1)^2-256/(tan
(1/2*d*x+1/2*c)+1)-1/3/tan(1/2*d*x+1/2*c)^3+4/tan(1/2*d*x+1/2*c)^2-35/tan(
1/2*d*x+1/2*c)-112*ln(tan(1/2*d*x+1/2*c)))
```

3.89.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(130) = 260$.

Time = 0.31 (sec) , antiderivative size = 445, normalized size of antiderivative = 3.71

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{66\,\cos\left(dx+c\right)^5 - 24\,\cos\left(dx+c\right)^4 - 147\,\cos\left(dx+c\right)^3 + 29\,\cos\left(dx+c\right)^2 - 21\,\left(\cos\left(dx+c\right)^5 + 2\,\cos\left(\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/3*(66*cos(d*x + c)^5 - 24*cos(d*x + c)^4 - 147*cos(d*x + c)^3 + 29*cos(
d*x + c)^2 - 21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*
cos(d*x + c)^2 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos
(d*x + c) + 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(1/2*cos(d*x + c) + 1/2
) + 21*(cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 4*cos(d*x +
 c)^2 + (cos(d*x + c)^4 - cos(d*x + c)^3 - 3*cos(d*x + c)^2 + cos(d*x + c)
 + 2)*sin(d*x + c) + cos(d*x + c) + 2)*log(-1/2*cos(d*x + c) + 1/2) - (66*
cos(d*x + c)^4 + 90*cos(d*x + c)^3 - 57*cos(d*x + c)^2 - 86*cos(d*x + c) -
 4)*sin(d*x + c) + 82*cos(d*x + c) - 4)/(a^4*d*cos(d*x + c)^5 + 2*a^4*d*co
s(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 - 4*a^4*d*cos(d*x + c)^2 + a^4*d*cos
(d*x + c) + 2*a^4*d + (a^4*d*cos(d*x + c)^4 - a^4*d*cos(d*x + c)^3 - 3*a^4
*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c) + 2*a^4*d)*sin(d*x + c))
```

3.89.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\cot^4(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(cot(d*x+c)**4/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(cot(c + d*x)**4/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c +
d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(130) = 260$.

Time = 0.20 (sec) , antiderivative size = 285, normalized size of antiderivative = 2.38

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1} - \frac{72\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{984\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{1647\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{873\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - 1}{\frac{a^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{3a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{3a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}} + \frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1} - \frac{12\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^4} - \frac{336\,l\cdots}{\cdots}}{24\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/24*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 72*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 - 984*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 1647*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 873*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 3*a^4*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d
*x + c) + 1)^6) + (105*sin(d*x + c)/(cos(d*x + c) + 1) - 12*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^4 - 336*log
(sin(d*x + c)/(cos(d*x + c) + 1))/a^4)/d
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.49

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{336\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right|\right)}{a^4} - \frac{308\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6 + 51\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 723\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4 - 676\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 72\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + 9\cdots}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)^3 a^4} \cdots}{24\,d}$$

input
```
integrate(cot(d*x+c)^4/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/24*(336*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 - (308*tan(1/2*d*x + 1/2*c)^
6 + 51*tan(1/2*d*x + 1/2*c)^5 - 723*tan(1/2*d*x + 1/2*c)^4 - 676*tan(1/2*d
*x + 1/2*c)^3 - 72*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 1)/((
tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x + 1/2*c))^3*a^4) - (a^8*tan(1/2*d*x +
 1/2*c)^3 - 12*a^8*tan(1/2*d*x + 1/2*c)^2 + 105*a^8*tan(1/2*d*x + 1/2*c))/
a^12)/d
```

3.89.9 Mupad [B] (verification not implemented)

Time = 7.75 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.42

$$\int \frac{\cot^4(c+dx)}{(a+a\sin(c+dx))^4}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^4\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2\,a^4\,d} - \frac{14\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^4\,d} + \frac{35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a^4\,d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{291\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{8} + \frac{549\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{8} + 41\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 + 3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 - \frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8} + \frac{1}{24}\right)}{a^4\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)^3}$$

input
```
int(cot(c + d*x)^4/(a + a*sin(c + d*x))^4,x)
```

output
```
tan(c/2 + (d*x)/2)^3/(24*a^4*d) - tan(c/2 + (d*x)/2)^2/(2*a^4*d) - (14*log
(tan(c/2 + (d*x)/2)))/(a^4*d) + (35*tan(c/2 + (d*x)/2))/(8*a^4*d) - (cot(c
/2 + (d*x)/2)^3*(3*tan(c/2 + (d*x)/2)^2 - (3*tan(c/2 + (d*x)/2))/8 + 41*ta
n(c/2 + (d*x)/2)^3 + (549*tan(c/2 + (d*x)/2)^4)/8 + (291*tan(c/2 + (d*x)/2
)^5)/8 + 1/24))/(a^4*d*(tan(c/2 + (d*x)/2) + 1)^3)
```

3.90 $\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx$

3.90.1 Optimal result

Integrand size = 21, antiderivative size = 133

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx &= \frac{27\text{arctanh}(\cos(c+dx))}{2a^4d} - \frac{16\cot(c+dx)}{a^4d} - \frac{3\cot^3(c+dx)}{a^4d} \\ &\quad - \frac{\cot^5(c+dx)}{5a^4d} + \frac{11\cot(c+dx)\csc(c+dx)}{2a^4d} \\ &\quad + \frac{\cot(c+dx)\csc^3(c+dx)}{a^4d} - \frac{8\cot(c+dx)}{a^4d(1+\csc(c+dx))}\end{aligned}$$

output
```
27/2*arctanh(cos(d*x+c))/a^4/d-40*cot(d*x+c)/a^4/d-27*cot(d*x+c)^3/a^4/d-4
1/5*cot(d*x+c)^5/a^4/d+27/2*cot(d*x+c)*csc(d*x+c)/a^4/d+9*cot(d*x+c)*csc(d
*x+c)^3/a^4/d+8*cot(d*x+c)*csc(d*x+c)^4/a^4/d/(1+sin(d*x+c))
```

3.90.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 733 vs. 2(133) = 266.

Time = 6.64 (sec) , antiderivative size = 733, normalized size of antiderivative = 5.51

$$
\begin{aligned}
\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx
&= \frac{16\sin\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^7}{d(a+a\sin(c+dx))^4} \\
&- \frac{33\cot\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{5d(a+a\sin(c+dx))^4} \\
&+ \frac{11\csc^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{8d(a+a\sin(c+dx))^4} \\
&- \frac{53\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{160d(a+a\sin(c+dx))^4} \\
&+ \frac{\csc^4\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{16d(a+a\sin(c+dx))^4} \\
&- \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{160d(a+a\sin(c+dx))^4} \\
&+ \frac{27\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{2d(a+a\sin(c+dx))^4} \\
&- \frac{27\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{2d(a+a\sin(c+dx))^4} \\
&- \frac{11\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{8d(a+a\sin(c+dx))^4} \\
&- \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8}{16d(a+a\sin(c+dx))^4} \\
&+ \frac{33\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8\tan\left(\frac{1}{2}(c+dx)\right)}{5d(a+a\sin(c+dx))^4} \\
&+ \frac{53\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8\tan\left(\frac{1}{2}(c+dx)\right)}{160d(a+a\sin(c+dx))^4} \\
&+ \frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^8\tan\left(\frac{1}{2}(c+dx)\right)}{160d(a+a\sin(c+dx))^4}
\end{aligned}
$$

input
```
Integrate[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```

output
```
(16*Sin[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^7)/(d*(a + a*Si
n[c + d*x])^4) - (33*Cot[(c + d*x)/2]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)^8)/(5*d*(a + a*Sin[c + d*x])^4) + (11*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/
2] + Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) - (53*Cot[(c + d*x)
/2]*Csc[(c + d*x)/2]^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a
+ a*Sin[c + d*x])^4) + (Csc[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^8)/(16*d*(a + a*Sin[c + d*x])^4) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/
2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(160*d*(a + a*Sin[c + d*x])^
4) + (27*Log[Cos[(c + d*x)/2]]*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(2
*d*(a + a*Sin[c + d*x])^4) - (27*Log[Sin[(c + d*x)/2]]*(Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2])^8)/(2*d*(a + a*Sin[c + d*x])^4) - (11*Sec[(c + d*x)/2]^
2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(8*d*(a + a*Sin[c + d*x])^4) -
(Sec[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8)/(16*d*(a + a*
Sin[c + d*x])^4) + (33*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*
x)/2])/(5*d*(a + a*Sin[c + d*x])^4) + (53*Sec[(c + d*x)/2]^2*(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d*x)/2])/(160*d*(a + a*Sin[c + d*x])^4
) + (Sec[(c + d*x)/2]^4*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^8*Tan[(c + d
*x)/2])/(160*d*(a + a*Sin[c + d*x])^4)
```

3.90.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3188, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{(a\sin(c+dx)+a)^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^6(a\sin(c+dx)+a)^4}dx$$

$$\Big\downarrow \text{3188}$$

$$\frac{\int \left(a^2\csc^6(c+dx)-4a^2\csc^5(c+dx)+7a^2\csc^4(c+dx)-8a^2\csc^3(c+dx)+8a^2\csc^2(c+dx)-8a^2\csc(c+dx)\right.}{a^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{27a^2\operatorname{arctanh}(\cos(c+dx))}{2d} - \frac{a^2\cot^5(c+dx)}{5d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{16a^2\cot(c+dx)}{d} + \frac{a^2\cot(c+dx)\csc^3(c+dx)}{d} + \frac{11a^2\cot(c+dx)\csc(c+dx)}{2d}}{a^6}$$

input
```
Int[Cot[c + d*x]^6/(a + a*Sin[c + d*x])^4,x]
```

output
```
((27*a^2*ArcTanh[Cos[c + d*x]])/(2*d) - (16*a^2*Cot[c + d*x])/d - (3*a^2*C
ot[c + d*x]^3)/d - (a^2*Cot[c + d*x]^5)/(5*d) + (11*a^2*Cot[c + d*x]*Csc[c
 + d*x])/(2*d) + (a^2*Cot[c + d*x]*Csc[c + d*x]^3)/d - (8*a^2*Cot[c + d*x]
)/(d*(1 + Csc[c + d*x])))/a^6
```

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3188
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[a^p   Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e
 + f*x])^(m - p/2)/(a - b*Sin[e + f*x])^(p/2)), x], x], x] /; FreeQ[{a, b,
e, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m
- p/2, 0])
```

3.90.4 Maple [A] (verified)

Time = 40.61 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-48\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+222\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{}{\tan\left(\right.}}{32d\,a^4}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-48\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+222\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^5}+\frac{}{\tan\left(\right.}}{32d\,a^4}$
risch	$-\frac{135ie^{9i(dx+c)}-630\,e^{8i(dx+c)}+135\,e^{10i(dx+c)}-610ie^{7i(dx+c)}+1260\,e^{6i(dx+c)}+860ie^{5i(dx+c)}-1510\,e^{4i(dx+c)}-430ie^{3}}{5\left(e^{2i(dx+c)}-1\right)^5\left(e^{i(dx+c)}+i\right)a^4d}$

input
```
int(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/32/d/a^4*(1/5*tan(1/2*d*x+1/2*c)^5-2*tan(1/2*d*x+1/2*c)^4+11*tan(1/2*d*x
+1/2*c)^3-48*tan(1/2*d*x+1/2*c)^2+222*tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1
/2*c)^5+2/tan(1/2*d*x+1/2*c)^4-11/tan(1/2*d*x+1/2*c)^3+48/tan(1/2*d*x+1/2*
c)^2-222/tan(1/2*d*x+1/2*c)-432*ln(tan(1/2*d*x+1/2*c))-512/(tan(1/2*d*x+1/
2*c)+1))
```

3.90.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(136) = 272$.

Time = 0.31 (sec) , antiderivative size = 439, normalized size of antiderivative = 3.30

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{424\cos(dx+c)^6 + 154\cos(dx+c)^5 - 1060\cos(dx+c)^4 - 340\cos(dx+c)^3 + 800\cos(dx+c)^2 + 135}{}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/20*(424*cos(d*x + c)^6 + 154*cos(d*x + c)^5 - 1060*cos(d*x + c)^4 - 340*
cos(d*x + c)^3 + 800*cos(d*x + c)^2 + 135*(cos(d*x + c)^6 - 3*cos(d*x + c)
^4 + 3*cos(d*x + c)^2 - (cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^
3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*sin(d*x + c) - 1)*log(1/2*cos(d*x
 + c) + 1/2) - 135*(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 -
 (cos(d*x + c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 +
cos(d*x + c) + 1)*sin(d*x + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*(212*
cos(d*x + c)^5 + 135*cos(d*x + c)^4 - 395*cos(d*x + c)^3 - 225*cos(d*x + c
)^2 + 175*cos(d*x + c) + 80)*sin(d*x + c) + 190*cos(d*x + c) - 160)/(a^4*d
*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d
- (a^4*d*cos(d*x + c)^5 + a^4*d*cos(d*x + c)^4 - 2*a^4*d*cos(d*x + c)^3 -
2*a^4*d*cos(d*x + c)^2 + a^4*d*cos(d*x + c) + a^4*d)*sin(d*x + c))
```

3.90.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\int \frac{\cot^6(c+dx)}{\sin^4(c+dx)+4\sin^3(c+dx)+6\sin^2(c+dx)+4\sin(c+dx)+1}\,dx}{a^4}$$

input
```
integrate(cot(d*x+c)**6/(a+a*sin(d*x+c))**4,x)
```

output
```
Integral(cot(c + d*x)**6/(sin(c + d*x)**4 + 4*sin(c + d*x)**3 + 6*sin(c +
d*x)**2 + 4*sin(c + d*x) + 1), x)/a**4
```

3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(136) = 272.

Time = 0.19 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.10

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{45\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{185\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{870\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3670\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-1}{\frac{a^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{1110\sin(dx+c)}{\cos(dx+c)+1}-\frac{240\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{55\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{10\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^4}-\frac{2160\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}}{160\,d}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/160*((9*sin(d*x + c)/(cos(d*x + c) + 1) - 45*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 185*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 870*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 3670*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 1)/(a^4*si
n(d*x + c)^5/(cos(d*x + c) + 1)^5 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6) + (1110*sin(d*x + c)/(cos(d*x + c) + 1) - 240*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 55*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 10*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^4 - 2160*log(s
in(d*x + c)/(cos(d*x + c) + 1))/a^4)/d
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.53

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx = -\frac{\frac{2160\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{2560}{a^4\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}-\frac{4932\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1110\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+240\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-55\,\tan\left(\frac{1}{2}\,dx\right.}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5}}{}$$

input
```
integrate(cot(d*x+c)^6/(a+a*sin(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/160*(2160*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 2560/(a^4*(tan(1/2*d*x +
 1/2*c) + 1)) - (4932*tan(1/2*d*x + 1/2*c)^5 - 1110*tan(1/2*d*x + 1/2*c)^4
 + 240*tan(1/2*d*x + 1/2*c)^3 - 55*tan(1/2*d*x + 1/2*c)^2 + 10*tan(1/2*d*x
 + 1/2*c) - 1)/(a^4*tan(1/2*d*x + 1/2*c)^5) - (a^16*tan(1/2*d*x + 1/2*c)^5
 - 10*a^16*tan(1/2*d*x + 1/2*c)^4 + 55*a^16*tan(1/2*d*x + 1/2*c)^3 - 240*a
^16*tan(1/2*d*x + 1/2*c)^2 + 1110*a^16*tan(1/2*d*x + 1/2*c))/a^20)/d
```

3.90.9 Mupad [B] (verification not implemented)

Time = 6.88 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.57

$$\int \frac{\cot^6(c+dx)}{(a+a\sin(c+dx))^4}\,dx = \frac{11\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32\,a^4\,d}-\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{2\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16\,a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,a^4\,d}-\frac{27\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{2\,a^4\,d}+\frac{111\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{16\,a^4\,d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{367\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{16}+\frac{87\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{16}-\frac{37\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{32}+\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{32}-\frac{9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{160}+\frac{1}{160}\right)}{a^4\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}$$

input
```
int(cot(c + d*x)^6/(a + a*sin(c + d*x))^4,x)
```

output
```
(11*tan(c/2 + (d*x)/2)^3)/(32*a^4*d) - (3*tan(c/2 + (d*x)/2)^2)/(2*a^4*d)
- tan(c/2 + (d*x)/2)^4/(16*a^4*d) + tan(c/2 + (d*x)/2)^5/(160*a^4*d) - (27
*log(tan(c/2 + (d*x)/2)))/(2*a^4*d) + (111*tan(c/2 + (d*x)/2))/(16*a^4*d)
- (cot(c/2 + (d*x)/2)^5*((9*tan(c/2 + (d*x)/2)^2)/32 - (9*tan(c/2 + (d*x)/
2))/160 - (37*tan(c/2 + (d*x)/2)^3)/32 + (87*tan(c/2 + (d*x)/2)^4)/16 + (3
67*tan(c/2 + (d*x)/2)^5)/16 + 1/160))/(a^4*d*(tan(c/2 + (d*x)/2) + 1))
```

3.91 $\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

3.91.1 Optimal result

Integrand size = 23, antiderivative size = 162

$$\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \frac{11\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{2}f} - \frac{27\sec(e+fx)\sqrt{a(1+\sin(e+fx))}}{8f} - \frac{\sec^3(e+fx)\sqrt{a(1+\sin(e+fx))}}{12f} + \frac{29\sqrt{a+a\sin(e+fx)}\tan(e+fx)}{12f} + \frac{5\sqrt{a(1+\sin(e+fx))}\tan^3(e+fx)}{12f}$$

output
```
11/16*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/
2)/f*2^(1/2)-27/8*sec(f*x+e)*(a*(1+sin(f*x+e)))^(1/2)/f-1/12*sec(f*x+e)^3*
(a*(1+sin(f*x+e)))^(1/2)/f+29/12*(a+a*sin(f*x+e))^(1/2)*tan(f*x+e)/f+5/12*
(a*(1+sin(f*x+e)))^(1/2)*tan(f*x+e)^3/f
```

3.91.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.59 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.43

$$\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$$
$$= \frac{\left(\frac{6\sin\left(\frac{fx}{2}\right)}{\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)} - \frac{3\left(\cos\left(\frac{e}{2}\right)-\sin\left(\frac{e}{2}\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}{\cos\left(\frac{e}{2}\right)+\sin\left(\frac{e}{2}\right)} + (33+33i)(-1)^{3/4}\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\right.\right.\right.}{}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

output
```
(((6*Sin[(f*x)/2])/(Cos[e/2] + Sin[e/2]) - (3*(Cos[e/2] - Sin[e/2])*(Cos[(
e + f*x)/2] + Sin[(e + f*x)/2]))/(Cos[e/2] + Sin[e/2]) + (33 + 33*I)*(-1)^
(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(f*x)/4]*(Cos[(2*e + f*x)/4] - Si
n[(2*e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 48*Cos[(f*x)/
2]*(Cos[e/2] - Sin[e/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 48*(Cos
[e/2] + Sin[e/2])*Sin[(f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (
4*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2] - Sin[(e + f*
x)/2])^3 - (36*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/2]
- Sin[(e + f*x)/2]))*Sqrt[a*(1 + Sin[e + f*x])])/(24*f*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^3)
```

3.91.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3193, 3042, 3125, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\sqrt{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3193}$$

$$\int \sqrt{\sin(e+fx)a+a}dx - \int \sec^4(e+fx)\sqrt{\sin(e+fx)a+a}\left(1-2\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(e+fx)a+a}dx - \int \frac{\sqrt{\sin(e+fx)a+a}(1-2\sin(e+fx)^2)}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{3125}$$

$$-\int \frac{\sqrt{\sin(e+fx)a+a}(1-2\sin(e+fx)^2)}{\cos(e+fx)^4}dx - \frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{4901}$$

$$-\int \left(\sec^4(e+fx)\sqrt{a(\sin(e+fx)+1)} - 2\sec^2(e+fx)\sqrt{a(\sin(e+fx)+1)}\tan^2(e+fx)\right)dx - \frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{11a^2\cos(e+fx)}{8f(a\sin(e+fx)+a)^{3/2}} + \frac{11\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{8\sqrt{2}f} - \frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}} + \frac{4\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3af} - \frac{7\sec^3(e+fx)\sqrt{a\sin(e+fx)+a}}{3f} - \frac{11a\sec(e+fx)}{6f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^4,x]
```

output
```
(11*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x
]])])/(8*Sqrt[2]*f) + (11*a^2*Cos[e + f*x])/(8*f*(a + a*Sin[e + f*x])^(3/2
)) - (2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (11*a*Sec[e + f*x])
/(6*f*Sqrt[a + a*Sin[e + f*x]]) - (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x
]])/(3*f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2))/(3*a*f)
```

3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3193
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.91.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.09

method	result
default	$-\frac{96a^{\frac{5}{2}}\left(\cos^{2}(fx+e)\right)\sin(fx+e)+33(a-a\sin(fx+e))^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sin(fx+e)a-162a^{\frac{5}{2}}\left(\cos^{2}(fx+e)\right)+33(a-}{48a^{\frac{3}{2}}(\sin(fx+e)-1)\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/48/a^(3/2)*(96*a^(5/2)*cos(f*x+e)^2*sin(f*x+e)+33*(a-a*sin(f*x+e))^(3/2
)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a
-162*a^(5/2)*cos(f*x+e)^2+33*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a
-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a+20*a^(5/2)*sin(f*x+e)-4*a^(5/2))/(
sin(f*x+e)-1)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.23

$$\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$$
$$= \frac{33\sqrt{2}\sqrt{a}\cos(fx+e)^3\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{a\sin(fx+e)+a}\left(\sqrt{2}\cos(fx+e)-\sqrt{2}\sin(fx+e)+\sqrt{2}\right)\sqrt{a}+3a\cos(fx+e)-(a\cos(fx+}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{96f\cos}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/96*(33*sqrt(2)*sqrt(a)*cos(f*x + e)^3*log(-(a*cos(f*x + e)^2 + 2*sqrt(a*
sin(f*x + e) + a)*(sqrt(2)*cos(f*x + e) - sqrt(2)*sin(f*x + e) + sqrt(2))*
sqrt(a) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(c
os(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(
81*cos(f*x + e)^2 - 2*(24*cos(f*x + e)^2 + 5)*sin(f*x + e) + 2)*sqrt(a*sin
(f*x + e) + a))/(f*cos(f*x + e)^3)
```

3.91.6 Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{a(\sin(e+fx)+1)}\tan^4(e+fx)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**4,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*tan(e + f*x)**4, x)
```

3.91.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt{a\sin(fx+e)+a}\tan(fx+e)^4\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^4, x)
```

3.91.8 Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument
Value
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/2), x)
```

3.92 $\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$

3.92.1 Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}f} + \frac{5\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f} - \frac{2\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{af}$$

output
```
-2*sec(f*x+e)*(a+a*sin(f*x+e))^(3/2)/a/f-1/2*arctanh(1/2*cos(f*x+e)*a^(1/2
)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f*2^(1/2)+5*sec(f*x+e)*(a+a*sin(
f*x+e))^(1/2)/f
```

3.92.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.13

$$\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$$
$$= \frac{\sec(e+fx)\left(3+(1-i)\sqrt[4]{-1}\operatorname{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\sec\left(\frac{fx}{4}\right)\left(\cos\left(\frac{1}{4}(2e+fx)\right)-\sin\left(\frac{1}{4}(2e+fx)\right)\right)\right)(c\ldots}{f}$$

input
```
Integrate[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

output
```
(Sec[e + f*x]*(3 + (1 - I)*(-1)^(1/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*Sec[(
f*x)/4]*(Cos[(2*e + f*x)/4] - Sin[(2*e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin
[(e + f*x)/2]) - 2*Sin[e + f*x])*Sqrt[a*(1 + Sin[e + f*x])])/f
```

3.92.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3192, 27, 3042, 3334, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\sqrt{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3192}$$

$$\frac{2\int \frac{1}{2}\sec^2(e+fx)\sqrt{\sin(e+fx)a+a}(2\sin(e+fx)a+3a)dx}{a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec^2(e+fx)\sqrt{\sin(e+fx)a+a}(2\sin(e+fx)a+3a)dx}{a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(2\sin(e+fx)a+3a)}{\cos(e+fx)^2}dx}{a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{1}{2}a^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{5a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}}{a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{2}a^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\frac{5a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}}{a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\frac{5a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{a^2\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}f}}{a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{af}$$

input
```
Int[Sqrt[a + a*Sin[e + f*x]]*Tan[e + f*x]^2,x]
```

output
```
(-2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(a*f) + (-((a^(3/2)*ArcTanh[(
Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*f)) +
(5*a*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f)/a
```

3.92.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3192
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Simp[1/(b*m)   Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos
[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&
!IntegerQ[m] &&  !LtQ[m, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

3.92.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{(\sin(fx+e)+1)\left(\sqrt{a}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a-a\sin(fx+e)}+4a\sin(fx+e)-6a\right)}{2\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	89

input
```
int((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(sin(f*x+e)+1)*(a^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^
(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(1/2)+4*a*sin(f*x+e)-6*a)/cos(f*x+e)/(a+a*
sin(f*x+e))^(1/2)/f
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.67

$$\int\sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$$
$$=\frac{\sqrt{2}\sqrt{a}\cos(fx+e)\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+1)+3a\cos(fx+e)-(a\cos(fx+e)-2a)\sin(}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(fx+e)-\cos(fx+e)-2}\right.}{4f\cos(fx+e)}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*sqrt(a)*cos(f*x + e)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(
a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*
x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos
(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*sqrt(a*sin(f*x + e) +
 a)*(2*sin(f*x + e) - 3))/(f*cos(f*x + e))
```

3.92.6 Sympy [F]

$$\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\tan^2\left(e+fx\right)dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/2)*tan(f*x+e)**2,x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*tan(e + f*x)**2, x)
```

3.92.7 Maxima [F]

$$\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt{a\sin\left(fx+e\right)+a}\tan\left(fx+e\right)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e) + a)*tan(f*x + e)^2, x)
```

3.92.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(88) = 176$.

Time = 0.75 (sec) , antiderivative size = 428, normalized size of antiderivative = 4.24

$$\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$$
$$= \frac{\sqrt{2}\left(\log\left(\frac{2\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^2+2\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)+1\right)}{\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^2+1}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8*pi + 1/4
*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 - log(2*(tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*p
i + 1/4*f*x + 1/4*e)^3 - sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi +
 1/4*f*x + 1/4*e)^4 + log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8
*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e) - log(2*(tan(
-1/8*pi + 1/4*f*x + 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(
-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan
(-1/8*pi + 1/4*f*x + 1/4*e) - 18*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-
1/8*pi + 1/4*f*x + 1/4*e)^2 - sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)
/((tan(-1/8*pi + 1/4*f*x + 1/4*e)^3 + tan(-1/8*pi + 1/4*f*x + 1/4*e))*f)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\sin(e+fx)}\tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/2), x)
```

3.93 $\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

3.93.1 Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = -\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{3a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{f}$$

output
```
-arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f+3*a*cos(f*x+
e)/f/(a+a*sin(f*x+e))^(1/2)-cot(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.93.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(89) = 178.

Time = 0.93 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.31

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \frac{\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(-4\cos\left(\frac{1}{2}(e+fx)\right)+2\cos\left(\frac{3}{2}(e+fx)\right)+4\sin\left(\frac{1}{2}(e+fx)\right)-\log\right.}{f\left(1+\cot\left(\frac{1}{2}(e+fx)\right)\right)\left(\csc\left(\frac{1}{4}(e+\right.\right.}$$

input
```
Integrate[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
(Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(-4*Cos[(e + f*x)/2] + 2*Co
s[(3*(e + f*x))/2] + 4*Sin[(e + f*x)/2] - Log[1 + Cos[(e + f*x)/2] - Sin[(
e + f*x)/2]]*Sin[e + f*x] + Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*S
in[e + f*x] + 2*Sin[(3*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e +
 f*x)/4] - Sec[(e + f*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

3.93.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3195, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a\sin(e+fx)+a}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3195}$$

$$\frac{\int \frac{1}{2}\csc(e+fx)(a-3a\sin(e+fx))\sqrt{\sin(e+fx)a+a}dx}{a}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \csc(e+fx)(a-3a\sin(e+fx))\sqrt{\sin(e+fx)a+a}dx}{2a}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a-3a\sin(e+fx))\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx}{2a}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}$$

$$\Big\downarrow \text{3460}$$

$$\frac{a\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx+\frac{6a^2\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{2a}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{6a^2\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{2a}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\frac{6a^2\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{2a^2\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{2a}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{6a^2\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}$$

input
```
Int[Cot[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
-((Cot[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f) + ((-2*a^(3/2)*ArcTanh[(Sqrt[
a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (6*a^2*Cos[e + f*x])/(f*Sq
rt[a + a*Sin[e + f*x]]))/(2*a)
```

3.93.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3195
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Simp[1/a
  Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
 x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1
/2] &&  !LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.93.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.45

method	result
default	$\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(2\sqrt{a-a\sin(fx+e)}\sin(fx+e)a^{\frac{3}{2}}-\sqrt{a-a\sin(fx+e)}\,a^{\frac{3}{2}}-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{a}}\right)a^{2}\sin(fx\cdots\right.}{\sin(fx+e)a^{\frac{3}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(2*(a-a*sin(f*x+e))^(1/2)*sin(f*x
+e)*a^(3/2)-(a-a*sin(f*x+e))^(1/2)*a^(3/2)-arctanh((a-a*sin(f*x+e))^(1/2)/
a^(1/2))*a^2*sin(f*x+e))/sin(f*x+e)/a^(3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1
/2)/f
```

3.93.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(79) = 158.

Time = 0.31 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.13

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$$

$$= \frac{\left(\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7\,a\cos(fx+e)^2-4\left(\cos(fx+e)^2+(\cos(fx+e\cdots\right.}{\cos(\cdots}\right.}{\cdots}$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a
*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) +
 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) -
9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e)
- a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e)
- cos(f*x + e) - 1)) - 4*(2*cos(f*x + e)^2 + (2*cos(f*x + e) + 3)*sin(f*x
+ e) - cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*
cos(f*x + e) + f)*sin(f*x + e) - f)
```

3.93.6 Sympy [F]

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\cot^2\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*cot(e + f*x)**2, x)
```

3.93.7 Maxima [F]

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \sqrt{a\sin\left(fx+e\right)+a}\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^2, x)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.67

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx =$$

$$-\frac{\sqrt{2}\left(\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+8\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right.\right.\right.}{4f}$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e
))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e)) + 8*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*
f*x + 1/2*e) + 4*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1))*sqrt(a)/f
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \cot(e+f\,x)^2\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/2), x)
```

3.94 $\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

3.94.1 Optimal result

Integrand size = 23, antiderivative size = 163

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \frac{11\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{2a\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} + \frac{11a\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}} - \frac{a\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}$$

output
```
11/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))*a^(1/2)/f-2*a*cos(
f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+11/8*a*cot(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
-1/12*a*cot(f*x+e)*csc(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-1/3*cot(f*x+e)*csc(
f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/f
```

3.94.2 Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.90

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$$
$$= \frac{\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(252\cos\left(\frac{1}{2}(e+fx)\right)-250\cos\left(\frac{3}{2}(e+fx)\right)-114\cos\left(\frac{5}{2}(e+fx)\right)\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
(Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(252*Cos[(e + f*x)/2] - 25
0*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/2] + 48*Cos[(7*(e + f*x))/2
] - 252*Sin[(e + f*x)/2] + 99*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]
*Sin[e + f*x] - 99*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*
x] - 250*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] - 33*Log[1 + Cos[
(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 33*Log[1 - Cos[(e + f*
x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] + 48*Sin[(7*(e + f*x))/2]))/(24
*f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

3.94.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3197, 3042, 3125, 3523, 27, 3042, 3459, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\sqrt{a\sin(e+fx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sqrt{a\sin(e+fx)+a}}{\tan(e+fx)^4}dx$$
$$\Big\downarrow \text{3197}$$
$$\int \sqrt{\sin(e+fx)a+a}dx+\int \csc^4(e+fx)\sqrt{\sin(e+fx)a+a}\left(1-2\sin^2(e+fx)\right)dx$$
$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin(e+fx)a+a}dx+\int \frac{\sqrt{\sin(e+fx)a+a}(1-2\sin(e+fx)^2)}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{3125}$$

$$\int \frac{\sqrt{\sin(e+fx)a+a}(1-2\sin(e+fx)^2)}{\sin(e+fx)^4}dx-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3523}$$

$$\frac{\int \frac{1}{2}\csc^3(e+fx)(a-9a\sin(e+fx))\sqrt{\sin(e+fx)a+a}dx}{3a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \csc^3(e+fx)(a-9a\sin(e+fx))\sqrt{\sin(e+fx)a+a}dx}{6a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(a-9a\sin(e+fx))\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)^3}dx}{6a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3459}$$

$$\frac{-\frac{33}{4}a\int \csc^2(e+fx)\sqrt{\sin(e+fx)a+a}dx-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{33}{4}a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)^2}dx-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3251}$$

3.94. $\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

$$\frac{-\frac{33}{4}a\left(\frac{1}{2}\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{33}{4}a\left(\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3252}$$

$$\frac{-\frac{33}{4}a\left(-\frac{a\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}-\frac{33}{4}a\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{6a}-\frac{2a\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

input
```
Int[Cot[e + f*x]^4*Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
(-2*a*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (Cot[e + f*x]*Csc[e + f
*x]^2*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (-1/2*(a^2*Cot[e + f*x]*Csc[e + f*
x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (33*a*(-((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[
e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f) - (a*Cot[e + f*x])/(f*Sqrt[a + a*S
in[e + f*x]])))/4)/(6*a)
```

3.94. $\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$

3.94.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3197
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

rule 3523
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*d*(n + 1)*(c^2 -
d^2))   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a
*d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*
(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

3.94.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04

method	result
default	$-\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(48\sqrt{-a(\sin(fx+e)-1)}\,a^{\frac{7}{2}}\left(\sin^{3}(fx+e)\right)-15\sqrt{-a(\sin(fx+e)-1)}\,a^{\frac{7}{2}}+56(-a(\sin(fx+e)-1)\right.}{24a^{\frac{7}{2}}\sin(fx+e)^{3}\cos(fx+e)\sqrt{a+a\sin(fx+e}}$

input
```
int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(48*(-a*(sin(f*x+e)-1))^(1/
2)*a^(7/2)*sin(f*x+e)^3-15*(-a*(sin(f*x+e)-1))^(1/2)*a^(7/2)+56*(-a*(sin(f
*x+e)-1))^(3/2)*a^(5/2)-33*(-a*(sin(f*x+e)-1))^(5/2)*a^(3/2)-33*arctanh((-
a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*a^4*sin(f*x+e)^3)/a^(7/2)/sin(f*x+e)^3/co
s(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.94.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(141) = 282$.

Time = 0.31 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.33

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$$

$$= \frac{33\left(\cos\left(fx+e\right)^4 - 2\,\cos\left(fx+e\right)^2 - \left(\cos\left(fx+e\right)^3 + \cos\left(fx+e\right)^2 - \cos\left(fx+e\right) - 1\right)\sin\left(fx+e\right) + 1}{}$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/96*(33*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x +
e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 -
 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e)
- 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e)
+ (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e
)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) -
1)) + 4*(48*cos(f*x + e)^4 - 33*cos(f*x + e)^3 - 139*cos(f*x + e)^2 + (48*
cos(f*x + e)^3 + 81*cos(f*x + e)^2 - 58*cos(f*x + e) - 83)*sin(f*x + e) +
25*cos(f*x + e) + 83)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*co
s(f*x + e)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*
sin(f*x + e) + f)
```

3.94.6 Sympy [F]

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \sqrt{a\left(\sin\left(e+fx\right)+1\right)}\cot^4\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(a*(sin(e + f*x) + 1))*cot(e + f*x)**4, x)
```

3.94.7 Maxima [F]

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \sqrt{a\sin(fx+e)+a}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*sin(f*x + e) + a)*cot(f*x + e)^4, x)
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.29

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx$$
$$= \frac{\sqrt{2}\left(33\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)+192\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\right.\right.\right.}{}$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/96*sqrt(2)*(33*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/
2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*pi +
 1/2*f*x + 1/2*e)) + 192*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi +
 1/2*f*x + 1/2*e) + 4*(132*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^5 - 112*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e)^3 + 15*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*p
i + 1/2*f*x + 1/2*e))/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3)*sqrt(a)/
f
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt{a+a\sin(e+fx)}\,dx = \int \cot(e+f\,x)^4\,\sqrt{a+a\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/2), x)
```

3.95 $\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx$

3.95.1 Optimal result

Integrand size = 23, antiderivative size = 167

$$\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx = -\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{2\sqrt{2}f} + \frac{2a^3\cos^3(e+fx)}{3f(a+a\sin(e+fx))^{3/2}} - \frac{4a^2\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{7a\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{2f} + \frac{\sec^3(e+fx)(a+a\sin(e+fx))^{3/2}}{3f}$$

output
```
2/3*a^3*cos(f*x+e)^3/f/(a+a*sin(f*x+e))^(3/2)+1/3*sec(f*x+e)^3*(a+a*sin(f*
x+e))^(3/2)/f-1/4*a^(3/2)*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(
f*x+e))^(1/2))/f*2^(1/2)-4*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-7/2*a*s
ec(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.95.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.77 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

$$\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{a\sec^3(e+fx)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\sqrt{a(1+\sin(e+fx))}\left(-45+6\cos(2(e+fx\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

output
```
(a*Sec[e + f*x]^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2*Sqrt[a*(1 + Sin[
e + f*x])]*(-45 + 6*Cos[2*(e + f*x)] + (3 + 3*I)*(-1)^(3/4)*ArcTanh[(1/2 +
 I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(e + f*x
)/2])^3 + 54*Sin[e + f*x] + Sin[3*(e + f*x)]))/(6*f)
```

3.95.3 Rubi [A] (verified)

Time = 1.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 3193, 3042, 3126, 3042, 3125, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(a\sin(e+fx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(a\sin(e+fx)+a)^{3/2}dx$$

$$\Big\downarrow \text{3193}$$

$$\int (\sin(e+fx)a+a)^{3/2}dx-\int \sec^4(e+fx)(\sin(e+fx)a+a)^{3/2}\left(1-2\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(e+fx)a+a)^{3/2}dx-\int \frac{(\sin(e+fx)a+a)^{3/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\int \frac{(\sin(e+fx)a+a)^{3/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\int \frac{(\sin(e+fx)a+a)^{3/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3125}$$

$$-\int \frac{(\sin(e+fx)a+a)^{3/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{4901}$$

$$-\int \left(\sec^4(e+fx)(a(\sin(e+fx)+1))^{3/2}-2\sec^2(e+fx)(a(\sin(e+fx)+1))^{3/2}\tan^2(e+fx)\right)dx-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}f}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}+\frac{4\sec^3(e+fx)(a\sin(e+fx)+a)^{5/2}}{af}-\frac{23\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}+\frac{a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^4,x]
```

output
```
-1/2*(a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f
*x]])])/(Sqrt[2]*f) - (8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]])
- (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) + (a*Sec[e + f*x]*Sqrt
[a + a*Sin[e + f*x]])/(2*f) - (23*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(3/2
))/(3*f) + (4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(5/2))/(a*f)
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3193
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.95.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.83

method	result
default	$\frac{(\sin(fx+e)+1)\left(3a^{\frac{3}{2}}\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)(a-a\sin(fx+e))^{\frac{3}{2}}-8a^{3}\left(\cos^{2}(fx+e)\right)\sin(fx+e)-24a^{3}\left(\cos^{2}(fx+e)\right)-106\sin}{12a(\sin(fx+e)-1)\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
1/12*(sin(f*x+e)+1)/a/(sin(f*x+e)-1)*(3*a^(3/2)*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(3/2)-8*a^3*cos(f*x+e)^
2*sin(f*x+e)-24*a^3*cos(f*x+e)^2-106*sin(f*x+e)*a^3+102*a^3)/cos(f*x+e)/(a
+a*sin(f*x+e))^(1/2)/f
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.43

$$\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx = \frac{3\left(\sqrt{2}a\cos(fx+e)\sin(fx+e)-\sqrt{2}a\cos(fx+e)\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{a\sin(fx+e)+a}\left(\sqrt{2}\cos(f\right.}{\cos(fx+e}\right.}{}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
1/24*(3*(sqrt(2)*a*cos(f*x + e)*sin(f*x + e) - sqrt(2)*a*cos(f*x + e))*sqr
t(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(a*sin(f*x + e) + a)*(sqrt(2)*cos(f*x
+ e) - sqrt(2)*sin(f*x + e) + sqrt(2))*sqrt(a) + 3*a*cos(f*x + e) - (a*cos
(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*
sin(f*x + e) - cos(f*x + e) - 2)) - 4*(12*a*cos(f*x + e)^2 + (4*a*cos(f*x
+ e)^2 + 53*a)*sin(f*x + e) - 51*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x +
 e)*sin(f*x + e) - f*cos(f*x + e))
```

3.95.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**4,x)
```

output
```
Timed out
```

3.95.7 Maxima [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{3/2}\tan^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
Timed out
```

3.95.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 917 vs. $2(142) = 284$.

Time = 1.30 (sec) , antiderivative size = 917, normalized size of antiderivative = 5.49

$$\int (a + a\sin(e + fx))^{3/2} \tan^4(e + fx) \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
-1/96*sqrt(2)*(a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
 + 1/4*e)^12 - 12*a*log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8*p
i + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 + 12*a*log(2*
(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/
(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 - 78*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^10 - 36*a*log(2*(tan(-1/8*pi + 1/4*f*x
+ 1/4*e)^2 + 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x
+ 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
 + 1/4*e)^7 + 36*a*log(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 - 2*tan(-1/8*pi
 + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 - 1089*a*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^8 - 36*a*log
(2*(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) +
1)/(tan(-1/8*pi + 1/4*f*x + 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 + 36*a*log(2*(tan(-1/8*pi + 1/4*f*x
+ 1/4*e)^2 - 2*tan(-1/8*pi + 1/4*f*x + 1/4*e) + 1)/(tan(-1/8*pi + 1/4*f*x
+ 1/4*e)^2 + 1))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
 + 1/4*e)^5 - 996*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1...
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{3/2} \tan^4(e + fx) \, dx = \int \tan(e + f\,x)^4 \, (a + a\,\sin(e + f\,x))^{3/2} \, dx$$

input
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(3/2), x)
```

3.96 $\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx$

3.96.1 Optimal result

Integrand size = 23, antiderivative size = 88

$$\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{7\sec(e+fx)(a+a\sin(e+fx))^{3/2}}{3f} - \frac{2\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{3af}$$

output
```
7/3*sec(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-2/3*sec(f*x+e)*(a+a*sin(f*x+e))^(5
/2)/a/f+11/3*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 3.98 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{a\sec(e+fx)(15+\cos(2(e+fx))-8\sin(e+fx))\sqrt{a(1+\sin(e+fx))}}{3f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(a*Sec[e + f*x]*(15 + Cos[2*(e + f*x)] - 8*Sin[e + f*x])*Sqrt[a*(1 + Sin[e
 + f*x])])/(3*f)
```

3.96.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 3192, 27, 3042, 3334, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(a\sin(e+fx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(e+fx)^2(a\sin(e+fx)+a)^{3/2}dx$$

$$\Big\downarrow \text{ 3192}$$

$$\frac{2\int \frac{1}{2}\sec^2(e+fx)(\sin(e+fx)a+a)^{3/2}(2\sin(e+fx)a+5a)dx}{3a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \sec^2(e+fx)(\sin(e+fx)a+a)^{3/2}(2\sin(e+fx)a+5a)dx}{3a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{3/2}(2\sin(e+fx)a+5a)}{\cos(e+fx)^2}dx}{3a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}$$

$$\Big\downarrow \text{ 3334}$$

$$\frac{\frac{7a\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}-\frac{11}{2}a^2\int \sqrt{\sin(e+fx)a+a}dx}{3a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{7a\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}-\frac{11}{2}a^2\int \sqrt{\sin(e+fx)a+a}dx}{3a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}$$

$$\Big\downarrow \text{ 3125}$$

$$\frac{\frac{11a^3\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}+\frac{7a\sec(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}}{3a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{3af}$$

input
```
Int[(a + a*Sin[e + f*x])^(3/2)*Tan[e + f*x]^2,x]
```

output
```
(-2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/(3*a*f) + ((11*a^3*Cos[e + f*
x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (7*a*Sec[e + f*x]*(a + a*Sin[e + f*x])^
(3/2))/f)/(3*a)
```

3.96.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3192
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Simp[1/(b*m)   Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos
[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&
!IntegerQ[m] &&  !LtQ[m, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

3.96.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.62

method	result	size
default	$-\frac{2a^{2}(\sin(fx+e)+1)(\sin^{2}(fx+e)+4\sin(fx+e)-8)}{3\cos(fx+e)\sqrt{a+a\sin(fx+e)}\, f}$	55

input
```
int((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*a^2*(sin(f*x+e)+1)*(sin(f*x+e)^2+4*sin(f*x+e)-8)/cos(f*x+e)/(a+a*sin(
f*x+e))^(1/2)/f
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.55

$$\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx = \frac{2\left(a\cos\left(fx+e\right)^{2}-4\,a\sin\left(fx+e\right)+7\,a\right)\sqrt{a\sin\left(fx+e\right)+a}}{3\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
2/3*(a*cos(f*x + e)^2 - 4*a*sin(f*x + e) + 7*a)*sqrt(a*sin(f*x + e) + a)/(
f*cos(f*x + e))
```

3.96.6 Sympy [F]

$$\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx = \int (a(\sin(e+fx)+1))^{\frac{3}{2}}\tan^2(e+fx)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(3/2)*tan(f*x+e)**2,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*tan(e + f*x)**2, x)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.65

$$\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{8\left(2a^{\frac{3}{2}}-\frac{2a^{\frac{3}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{3a^{\frac{3}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{2a^{\frac{3}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{2a^{\frac{3}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}\right)}{3f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{3}{2}}}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
-8/3*(2*a^(3/2) - 2*a^(3/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 3*a^(3/2)*si
n(f*x + e)^2/(cos(f*x + e) + 1)^2 - 2*a^(3/2)*sin(f*x + e)^3/(cos(f*x + e)
 + 1)^3 + 2*a^(3/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4)/(f*(sin(f*x + e)/
(cos(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(3/2))
```

3.96.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. 2(76) = 152.

Time = 0.57 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.38

$$\int (a+a\sin(e+fx))^{3/2}\tan^2(e+fx)\,dx = -\frac{\sqrt{2}\left(3a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx+\frac{1}{4}e\right)^8+60a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\tan\left(-\right.\right.}{6\left(\tan\left(-\frac{1}{8}\pi+\frac{1}{4}fx\right.\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*(3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*
x + 1/4*e)^8 + 60*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*
f*x + 1/4*e)^6 + 50*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/
4*f*x + 1/4*e)^4 + 60*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi +
1/4*f*x + 1/4*e)^2 + 3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)))*sqrt(a)/((ta
n(-1/8*pi + 1/4*f*x + 1/4*e)^7 + 3*tan(-1/8*pi + 1/4*f*x + 1/4*e)^5 + 3*ta
n(-1/8*pi + 1/4*f*x + 1/4*e)^3 + tan(-1/8*pi + 1/4*f*x + 1/4*e))*f)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^{3/2} \tan^2(e + fx) \, dx = \int \tan(e + f\,x)^2\,(a + a\,\sin(e + f\,x))^{3/2}\, dx$$

input
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(3/2), x)
```

3.97 $\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

3.97.1 Optimal result

Integrand size = 23, antiderivative size = 121

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = -\frac{3a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f}+\frac{11a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} +\frac{5a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f}-\frac{\cot(e+fx)(a+a\sin(e+fx))^{3/2}}{f}$$

output
```
-3*a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-cot(f*x+e)
*(a+a*sin(f*x+e))^(3/2)/f+11/3*a^2*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+5/3
*a*cos(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.97.2 Mathematica [A] (verified)

Time = 5.86 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.93

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = -\frac{a\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(14\cos\left(\frac{1}{2}(e+fx)\right)-9\cos\left(\frac{3}{2}(e+fx)\right)+\cos\left(\frac{5}{2}(e+fx)\right)-14\,\mathrm{si}\right.}{3f\left(1+\cot\left(\frac{1}{2}(\right.\right.}$$

input
```
Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/3*(a*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(14*Cos[(e + f*x)/2]
 - 9*Cos[(3*(e + f*x))/2] + Cos[(5*(e + f*x))/2] - 14*Sin[(e + f*x)/2] + 9
*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] - 9*Log[1 - Cos
[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 9*Sin[(3*(e + f*x))/2] -
Sin[(5*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(
e + f*x)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

3.97.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 3195, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(a\sin(e+fx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3195}$$

$$\frac{\int \frac{1}{2}\csc(e+fx)(3a-5a\sin(e+fx))(\sin(e+fx)a+a)^{3/2}dx}{a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \csc(e+fx)(3a-5a\sin(e+fx))(\sin(e+fx)a+a)^{3/2}dx}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(3a-5a\sin(e+fx))(\sin(e+fx)a+a)^{3/2}}{\sin(e+fx)}dx}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{3455}$$

$$\frac{\frac{2}{3}\int \frac{1}{2}\csc(e+fx)\sqrt{\sin(e+fx)a+a}\left(9a^2-11a^2\sin(e+fx)\right)dx+\frac{10a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{1}{3}\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}\left(9a^2-11a^2\sin(e+fx)\right)dx+\frac{10a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\int \frac{\sqrt{\sin(e+fx)a+a}\left(9a^2-11a^2\sin(e+fx)\right)}{\sin(e+fx)}dx+\frac{10a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{3460}$$

$$\frac{\frac{1}{3}\left(9a^2\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx+\frac{22a^3\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{10a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{1}{3}\left(9a^2\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{22a^3\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{10a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\frac{1}{3}\left(\frac{22a^3\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{18a^3\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}\right)+\frac{10a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{10a^2\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}+\frac{1}{3}\left(\frac{22a^3\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{18a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}\right)}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{3/2}}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(3/2),x]
```

3.97. $\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

output
```
-((Cot[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/f) + ((10*a^2*Cos[e + f*x]*Sqr
t[a + a*Sin[e + f*x]])/(3*f) + ((-18*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x]
)/Sqrt[a + a*Sin[e + f*x]]])/f + (22*a^3*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e
 + f*x]]))/3)/(2*a)
```

3.97.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3195
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Simp[1/a
  Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
 x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1
/2] &&  !LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.97.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.27

method	result
default	$\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(-2(a-a\sin(fx+e))^{\frac{3}{2}}\sin(fx+e)\sqrt{a}+12\sqrt{a-a\sin(fx+e)}\sin(fx+e)a^{\frac{3}{2}}-9\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin}}{\sqrt{a}}\right.\right.}{3\sin(fx+e)\sqrt{a}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(-2*(a-a*sin(f*x+e))^(3/2)*si
n(f*x+e)*a^(1/2)+12*(a-a*sin(f*x+e))^(1/2)*sin(f*x+e)*a^(3/2)-9*arctanh((a
-a*sin(f*x+e))^(1/2)/a^(1/2))*a^2*sin(f*x+e)-3*(a-a*sin(f*x+e))^(1/2)*a^(3
/2))/sin(f*x+e)/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(105) = 210$.

Time = 0.32 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.60

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \frac{9\left(a\cos\left(fx+e\right)^2-\left(a\cos\left(fx+e\right)+a\right)\sin\left(fx+e\right)-a\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7\,a\cos(fx+e)^2-4\,(c\cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/12*(9*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a)*sin(f*x + e) - a)*sqrt(a)
*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x
 + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqr
t(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*
x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*
x + e) - cos(f*x + e) - 1)) + 4*(2*a*cos(f*x + e)^3 - 8*a*cos(f*x + e)^2 +
 a*cos(f*x + e) - (2*a*cos(f*x + e)^2 + 10*a*cos(f*x + e) + 11*a)*sin(f*x
+ e) + 11*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e)
 + f)*sin(f*x + e) - f)
```

3.97.6 Sympy [F]

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \int (a(\sin(e+fx)+1))^{\frac{3}{2}}\cot^2(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(3/2)*cot(e + f*x)**2, x)
```

3.97.7 Maxima [F]

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \int (a\sin(fx+e)+a)^{\frac{3}{2}}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^2, x)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.51

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \frac{\sqrt{2}\left(16\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 9\sqrt{2}a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi\cdots\right.\right.}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi\cdots\right.\right.}\right.\right.}{\cdots}$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/12*sqrt(2)*(16*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f
*x + 1/2*e)^3 - 9*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x +
 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e)) - 48*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e) - 12*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1))*sqrt(a)/f
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \int \cot(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^{3/2}\,dx$$

input
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(3/2), x)
```

3.98 $\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

3.98.1 Optimal result

Integrand size = 23, antiderivative size = 197

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \frac{37a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} - \frac{8a^2\cos(e+fx)}{3f\sqrt{a+a\sin(e+fx)}} + \frac{29a^2\cot(e+fx)}{24f\sqrt{a+a\sin(e+fx)}} - \frac{2a\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc(e+fx)\sqrt{a+a\sin(e+fx)}}{4f} - \frac{\cot(e+fx)\csc^2(e+fx)(a+a\sin(e+fx))^{3/2}}{3f}$$

output
```
37/8*a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-1/3*cot(
f*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(3/2)/f-8/3*a^2*cos(f*x+e)/f/(a+a*sin
(f*x+e))^(1/2)+29/24*a^2*cot(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-2/3*a*cos(f*x
+e)*(a+a*sin(f*x+e))^(1/2)/f-1/4*a*cot(f*x+e)*csc(f*x+e)*(a+a*sin(f*x+e))^
(1/2)/f
```

3.98.2 Mathematica [A] (verified)

Time = 1.32 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.70

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx =$$
$$-\frac{a\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(-276\cos\left(\frac{1}{2}(e+fx)\right)+326\cos\left(\frac{3}{2}(e+fx)\right)+78\cos\left(\frac{5}{2}(e+fx\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/24*(a*Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(-276*Cos[(e + f*x
)/2] + 326*Cos[(3*(e + f*x))/2] + 78*Cos[(5*(e + f*x))/2] - 72*Cos[(7*(e +
 f*x))/2] + 8*Cos[(9*(e + f*x))/2] + 276*Sin[(e + f*x)/2] - 333*Log[1 + Co
s[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 333*Log[1 - Cos[(e + f*x
)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 326*Sin[(3*(e + f*x))/2] - 78*Sin[
(5*(e + f*x))/2] + 111*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*
(e + f*x)] - 111*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f
*x)] - 72*Sin[(7*(e + f*x))/2] - 8*Sin[(9*(e + f*x))/2]))/(f*(1 + Cot[(e +
 f*x)/2])*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

3.98.3 Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.696$, Rules used = {3042, 3197, 3042, 3126, 3042, 3125, 3523, 27, 3042, 3454, 27, 3042, 3459, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(a\sin(e+fx)+a)^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^{3/2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \; 3197$$

$$\int (\sin(e+fx)a+a)^{3/2}dx+\int \csc^4(e+fx)(\sin(e+fx)a+a)^{3/2}\left(1-2\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(e+fx)a+a)^{3/2}dx+\int \frac{(\sin(e+fx)a+a)^{3/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{3126}$$

$$\int \frac{(\sin(e+fx)a+a)^{3/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx+\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx+\int \frac{(\sin(e+fx)a+a)^{3/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3125}$$

$$\int \frac{(\sin(e+fx)a+a)^{3/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}$$

$$\Big\downarrow \text{3523}$$

$$\frac{\int \frac{1}{2}\csc^3(e+fx)(3a-11a\sin(e+fx))(\sin(e+fx)a+a)^{3/2}dx}{3a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \csc^3(e+fx)(3a-11a\sin(e+fx))(\sin(e+fx)a+a)^{3/2}dx}{6a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(3a-11a\sin(e+fx))(\sin(e+fx)a+a)^{3/2}}{\sin(e+fx)^3}dx}{6a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{3454}$$

3.98. $\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

$$\frac{\frac{1}{2}\int -\frac{1}{2}\csc^2(e+fx)\sqrt{\sin(e+fx)a+a}\left(41\sin(e+fx)a^2+29a^2\right)dx-\frac{3a^2\cot(e+fx)\csc(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}}{6a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{1}{4}\int \csc^2(e+fx)\sqrt{\sin(e+fx)a+a}\left(41\sin(e+fx)a^2+29a^2\right)dx-\frac{3a^2\cot(e+fx)\csc(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}}{6a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{1}{4}\int \frac{\sqrt{\sin(e+fx)a+a}\left(41\sin(e+fx)a^2+29a^2\right)}{\sin(e+fx)^2}dx-\frac{3a^2\cot(e+fx)\csc(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}}{6a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{ 3459}$$

$$\frac{\frac{1}{4}\left(\frac{29a^3\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{111}{2}a^2\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx\right)-\frac{3a^2\cot(e+fx)\csc(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}}{6a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{1}{4}\left(\frac{29a^3\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{111}{2}a^2\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx\right)-\frac{3a^2\cot(e+fx)\csc(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}}{6a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{ 3252}$$

3.98. $\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

$$\frac{\frac{1}{4}\left(\frac{111a^3\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}+\frac{29a^3\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{3a^2\cot(e+fx)\csc(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}}{6a}-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

$$\Big\downarrow \text{219}$$

$$-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}+\frac{\frac{1}{4}\left(\frac{111a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{29a^3\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{3a^2\cot(e+fx)\csc(e+fx)\sqrt{a\sin(e+fx)+a}}{2f}}{6a}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

input
```
Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*S
qrt[a + a*Sin[e + f*x]])/(3*f) - (Cot[e + f*x]*Csc[e + f*x]^2*(a + a*Sin[e
 + f*x])^(3/2))/(3*f) + ((-3*a^2*Cot[e + f*x]*Csc[e + f*x]*Sqrt[a + a*Sin[
e + f*x]])/(2*f) + ((111*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a
*Sin[e + f*x]]])/f + (29*a^3*Cot[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/4
)/(6*a)
```

3.98.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3197
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

3.98. $\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx$

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

rule 3523
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*d*(n + 1)*(c^2 -
d^2))   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a
*d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*
(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

3.98.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.99

method	result
default	$\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(16(-a(\sin(fx+e)-1))^{\frac{3}{2}}(\sin^3(fx+e))a^{\frac{3}{2}}-96a^{\frac{5}{2}}\sqrt{-a(\sin(fx+e)-1)}(\sin^3(fx+e))+111\ \mathrm{arcta}\right.}{24a^{\frac{3}{2}}\sin(fx+e)^3\cos(fx+}$

input
```
int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)/a^(3/2)*(16*(-a*(sin(f*x+e)-
1))^(3/2)*sin(f*x+e)^3*a^(3/2)-96*a^(5/2)*(-a*(sin(f*x+e)-1))^(1/2)*sin(f*
x+e)^3+111*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+15*
(-a*(sin(f*x+e)-1))^(5/2)*a^(1/2)-8*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)-15*(
-a*(sin(f*x+e)-1))^(1/2)*a^(5/2))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))
^(1/2)/f
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(169) = 338$.

Time = 0.31 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.15

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \frac{111\left(a\cos\left(fx+e\right)^4-2\,a\cos\left(fx+e\right)^2-\left(a\cos\left(fx+e\right)^3+a\cos\left(fx+e\right)^2-a\cos\left(fx+e\right)\right.}{}$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/96*(111*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e)^3 + a*c
os(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*sqrt(a)*log((a*cos(f
*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*si
n(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*co
s(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(
cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(
f*x + e) - 1)) - 4*(16*a*cos(f*x + e)^5 - 64*a*cos(f*x + e)^4 - 17*a*cos(f
*x + e)^3 + 165*a*cos(f*x + e)^2 + 9*a*cos(f*x + e) - (16*a*cos(f*x + e)^4
 + 80*a*cos(f*x + e)^3 + 63*a*cos(f*x + e)^2 - 102*a*cos(f*x + e) - 93*a)*
sin(f*x + e) - 93*a)*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^4 - 2*f*cos
(f*x + e)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*cos(f*x + e) - f)*s
in(f*x + e) + f)
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Timed out
```

3.98.7 Maxima [F]

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx = \int (a\sin(fx+e)+a)^{\frac{3}{2}}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(3/2)*cot(f*x + e)^4, x)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.25

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx =$$

$$-\frac{\sqrt{2}\left(128\,a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3 - 111\sqrt{2}a\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right.\right.}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right.}\right.\right.}{}$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-1/96*sqrt(2)*(128*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2
*f*x + 1/2*e)^3 - 111*sqrt(2)*a*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f
*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e)) - 384*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(
-1/4*pi + 1/2*f*x + 1/2*e) - 4*(60*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*s
in(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 16*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 15*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1
)^3)*sqrt(a)/f
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{3/2}\,dx=\int \cot(e+f\,x)^4\,(a+a\,\sin(e+f\,x))^{3/2}\,dx$$

input
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(3/2), x)
```

3.99 $\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx$

3.99.1 Optimal result

Integrand size = 23, antiderivative size = 151

$$\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx = -\frac{2a^5\cos^5(e+fx)}{5f(a+a\sin(e+fx))^{5/2}} + \frac{8a^4\cos^3(e+fx)}{3f(a+a\sin(e+fx))^{3/2}} - \frac{12a^3\cos(e+fx)}{f\sqrt{a+a\sin(e+fx)}} - \frac{8a^2\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{f} + \frac{2a\sec^3(e+fx)(a+a\sin(e+fx))^{3/2}}{3f}$$

output
```
-2/5*a^5*cos(f*x+e)^5/f/(a+a*sin(f*x+e))^(5/2)+8/3*a^4*cos(f*x+e)^3/f/(a+a
*sin(f*x+e))^(3/2)+2/3*a*sec(f*x+e)^3*(a+a*sin(f*x+e))^(3/2)/f-12*a^3*cos(
f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-8*a^2*sec(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.99.2 Mathematica [A] (verified)

Time = 5.72 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.74

$$\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx = \frac{a^2\sqrt{a(1+\sin(e+fx))}(-1225+204\cos(2(e+fx))-3\cos(4(e+fx))+1488\sin(e+fx)+16}{60f\left(\cos\left(\frac{1}{2}(e+fx)\right)-\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

output
```
(a^2*Sqrt[a*(1 + Sin[e + f*x])]*(-1225 + 204*Cos[2*(e + f*x)] - 3*Cos[4*(e
 + f*x)] + 1488*Sin[e + f*x] + 16*Sin[3*(e + f*x)]))/(60*f*(Cos[(e + f*x)/
2] - Sin[(e + f*x)/2])^3*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]))
```

3.99.3 Rubi [A] (verified)

Time = 1.57 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.40, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3193, 3042, 3126, 3042, 3126, 3042, 3125, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(a\sin(e+fx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(a\sin(e+fx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3193}$$

$$\int (\sin(e+fx)a+a)^{5/2}dx-\int \sec^4(e+fx)(\sin(e+fx)a+a)^{5/2}\left(1-2\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(e+fx)a+a)^{5/2}dx-\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\int (\sin(e+fx)a+a)^{3/2}dx-\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int (\sin(e+fx)a+a)^{3/2}dx-\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3126}$$

$$-\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx+$$
$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-$$
$$\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-$$
$$\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3125}$$

$$-\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\cos(e+fx)^4}dx+$$
$$\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-$$
$$\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{4901}$$

$$-\int \left(\sec^4(e+fx)(a(\sin(e+fx)+1))^{5/2}-2\sec^2(e+fx)(a(\sin(e+fx)+1))^{5/2}\tan^2(e+fx)\right)dx+$$
$$\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-$$
$$\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{2009}$$

$$\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-$$
$$\frac{46a^2\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-$$
$$\frac{4\sec^3(e+fx)(a\sin(e+fx)+a)^{7/2}}{af}+\frac{26\sec^3(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}-$$
$$\frac{2a\sec^3(e+fx)(a\sin(e+fx)+a)^{3/2}}{3f}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^4,x]
```

3.99. $\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx$

output
```
(-46*a^2*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f) - (2*a*Cos[e + f*x]*
(a + a*Sin[e + f*x])^(3/2))/(5*f) - (2*a*Sec[e + f*x]^3*(a + a*Sin[e + f*x
])^(3/2))/(3*f) + (26*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(5/2))/(3*f) - (
4*Sec[e + f*x]^3*(a + a*Sin[e + f*x])^(7/2))/(a*f) + (8*a*((-8*a^2*Cos[e +
 f*x])/(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e
 + f*x]])/(3*f)))/5
```

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3193
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.99.4 Maple [A] (verified)

Time = 12.80 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2a^3(\sin(fx+e)+1)(3(\sin^4(fx+e))+8(\sin^3(fx+e))+48(\sin^2(fx+e))-192\sin(fx+e)+128)}{15(\sin(fx+e)-1)\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	87

input
```
int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x,method=_RETURNVERBOSE)
```

output
```
2/15*a^3*(sin(f*x+e)+1)/(sin(f*x+e)-1)*(3*sin(f*x+e)^4+8*sin(f*x+e)^3+48*s
in(f*x+e)^2-192*sin(f*x+e)+128)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.65

$$\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx = \frac{2\left(3a^2\cos(fx+e)^4-54a^2\cos(fx+e)^2+179a^2-8\left(a^2\cos(fx+e)^2+23a^2\right)\sin(fx+e)\right)\sqrt{}}{15\left(f\cos(fx+e)\sin(fx+e)-f\cos(fx+e)\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
2/15*(3*a^2*cos(f*x + e)^4 - 54*a^2*cos(f*x + e)^2 + 179*a^2 - 8*(a^2*cos(
f*x + e)^2 + 23*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e
)*sin(f*x + e) - f*cos(f*x + e))
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**4,x)
```

output
```
Timed out
```

3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(135) = 270$.

Time = 0.29 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.83

$$\int (a + a\sin(e + fx))^{5/2}\tan^4(e + fx)\,dx = \frac{32\left(8\,a^{\frac{5}{2}} - \frac{24\,a^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1} + \frac{44\,a^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2} - \frac{68\,a^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3} + \frac{75\,a^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4} - \frac{68\,a^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5} + \frac{44\,a^{\frac{5}{2}}}{(\cos(}\cdots\right.}{15\,f\left(\frac{3\sin(fx+e)}{\cos(fx+e)+1} - \frac{3\sin(fx+e)^2}{(\cos(fx+e)+1)^2} + \frac{\sin(fx+e)^3}{(\cos(fx+e)+1)^3} - 1\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1}\cdots\right.}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
32/15*(8*a^(5/2) - 24*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 44*a^(5/2)
*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 68*a^(5/2)*sin(f*x + e)^3/(cos(f*x
+ e) + 1)^3 + 75*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 68*a^(5/2)*
sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 44*a^(5/2)*sin(f*x + e)^6/(cos(f*x +
 e) + 1)^6 - 24*a^(5/2)*sin(f*x + e)^7/(cos(f*x + e) + 1)^7 + 8*a^(5/2)*si
n(f*x + e)^8/(cos(f*x + e) + 1)^8)/(f*(3*sin(f*x + e)/(cos(f*x + e) + 1) -
 3*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + sin(f*x + e)^3/(cos(f*x + e) + 1)
^3 - 1)*(sin(f*x + e)^2/(cos(f*x + e) + 1)^2 + 1)^(5/2))
```

3.99.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1485 vs. $2(135) = 270$.

Time = 7.65 (sec) , antiderivative size = 1485, normalized size of antiderivative = 9.83

$$\int (a + a\sin(e + fx))^{5/2}\tan^4(e + fx)\,dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
-1/122880*sqrt(2)*(2560*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*p
i + 1/4*f*x + 1/4*e)^16 - 79560*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi +
1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^13
 - 79560*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*
f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^13 - 9945*(pi - 2*f*x - 2*e)*
a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^13
+ 79560*a^2*arctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2
*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^13 - 225280*a^2*sgn(cos(-1/4
*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^14 - 397800*pi*a^2*
floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 - 397800*pi*a^2*sgn(cos(-1/4*pi + 1/2*
f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x +
1/4*e)^11 - 49725*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 + 397800*a^2*arctan(1/tan(-1/8*pi + 1
/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x
 + 1/4*e)^11 - 4352000*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi
 + 1/4*f*x + 1/4*e)^12 - 795600*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi +
1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9
- 795600*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*
f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 - 99450*(pi - 2*f*x - 2*...
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}\tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

input
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(5/2), x)
```

3.100 $\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx$

3.100.1 Optimal result

Integrand size = 23, antiderivative size = 118

$$\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx = \frac{124a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{9\sec(e+fx)(a+a\sin(e+fx))^{5/2}}{5f} - \frac{2\sec(e+fx)(a+a\sin(e+fx))^{7/2}}{5af}$$

output
```
9/5*sec(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f-2/5*sec(f*x+e)*(a+a*sin(f*x+e))^(7
/2)/a/f+124/15*a^3*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+31/15*a^2*cos(f*x+e
)*(a+a*sin(f*x+e))^(1/2)/f
```

3.100.2 Mathematica [A] (verified)

Time = 5.67 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

$$\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx = \frac{a^2\sec(e+fx)\sqrt{a(1+\sin(e+fx))}(330+22\cos(2(e+fx))-185\sin(e+fx)+3\sin(3(e+fx)}{30f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]
```

output
```
(a^2*Sec[e + f*x]*Sqrt[a*(1 + Sin[e + f*x])]*(330 + 22*Cos[2*(e + f*x)] -
185*Sin[e + f*x] + 3*Sin[3*(e + f*x)]))/(30*f)
```

3.100.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3192, 27, 3042, 3334, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(a\sin(e+fx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2(a\sin(e+fx)+a)^{5/2}dx$$

$$\Big\downarrow \text{3192}$$

$$\frac{2\int \frac{1}{2}\sec^2(e+fx)(\sin(e+fx)a+a)^{5/2}(2\sin(e+fx)a+7a)dx}{5a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec^2(e+fx)(\sin(e+fx)a+a)^{5/2}(2\sin(e+fx)a+7a)dx}{5a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^{5/2}(2\sin(e+fx)a+7a)}{\cos(e+fx)^2}dx}{5a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{9a\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}-\frac{31}{2}a^2\int(\sin(e+fx)a+a)^{3/2}dx}{5a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9a\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}-\frac{31}{2}a^2\int(\sin(e+fx)a+a)^{3/2}dx}{5a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}$$

$$\Big\downarrow \text{3126}$$

$$\frac{\frac{9a\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}-\frac{31}{2}a^2\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)}{5a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{9a\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}-\frac{31}{2}a^2\left(\frac{4}{3}a\int\sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)}{5a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}$$

$$\Big\downarrow \text{3125}$$

$$\frac{\frac{9a\sec(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}-\frac{31}{2}a^2\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)}{5a}-\frac{2\sec(e+fx)(a\sin(e+fx)+a)^{7/2}}{5af}$$

input
```
Int[(a + a*Sin[e + f*x])^(5/2)*Tan[e + f*x]^2,x]
```

output
```
(-2*Sec[e + f*x]*(a + a*Sin[e + f*x])^(7/2))/(5*a*f) + ((9*a*Sec[e + f*x]*
(a + a*Sin[e + f*x])^(5/2))/f - (31*a^2*((-8*a^2*Cos[e + f*x])/(3*f*Sqrt[a
 + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/(3*f)))/
2)/(5*a)
```

3.100.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3192
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Simp[1/(b*m)   Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos
[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&
!IntegerQ[m] &&  !LtQ[m, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

3.100.4 Maple [A] (verified)

Time = 2.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.57

method	result	size
default	$-\frac{2a^3(\sin(fx+e)+1)(3(\sin^3(fx+e))+11(\sin^2(fx+e))+44\sin(fx+e)-88)}{15\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	67

input
```
int((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*a^3*(sin(f*x+e)+1)*(3*sin(f*x+e)^3+11*sin(f*x+e)^2+44*sin(f*x+e)-88)
/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.59

$$\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx = \frac{2\left(11\,a^2\cos\left(fx+e\right)^2+77\,a^2+\left(3\,a^2\cos\left(fx+e\right)^2-47\,a^2\right)\sin\left(fx+e\right)\right)\sqrt{a\sin\left(fx+e\right)+a}}{15\,f\cos\left(fx+e\right)}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
2/15*(11*a^2*cos(f*x + e)^2 + 77*a^2 + (3*a^2*cos(f*x + e)^2 - 47*a^2)*sin
(f*x + e))*sqrt(a*sin(f*x + e) + a)/(f*cos(f*x + e))
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx = \text{Timed out}$$

input
```
integrate((a+a*sin(f*x+e))**(5/2)*tan(f*x+e)**2,x)
```

output
```
Timed out
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.62

$$\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx = -\frac{8\left(22\,a^{\frac{5}{2}}-\frac{22\,a^{\frac{5}{2}}\sin(fx+e)}{\cos(fx+e)+1}+\frac{55\,a^{\frac{5}{2}}\sin(fx+e)^2}{(\cos(fx+e)+1)^2}-\frac{50\,a^{\frac{5}{2}}\sin(fx+e)^3}{(\cos(fx+e)+1)^3}+\frac{55\,a^{\frac{5}{2}}\sin(fx+e)^4}{(\cos(fx+e)+1)^4}-\frac{22\,a^{\frac{5}{2}}\sin(fx+e)^5}{(\cos(fx+e)+1)^5}+\frac{22\,a^{\frac{5}{2}}\sin(fx+e)^6}{(\cos(fx+e)+1)^6}\right.}{15\,f\left(\frac{\sin(fx+e)}{\cos(fx+e)+1}-1\right)\left(\frac{\sin(fx+e)^2}{(\cos(fx+e)+1)^2}+1\right)^{\frac{5}{2}}}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
-8/15*(22*a^(5/2) - 22*a^(5/2)*sin(f*x + e)/(cos(f*x + e) + 1) + 55*a^(5/2
)*sin(f*x + e)^2/(cos(f*x + e) + 1)^2 - 50*a^(5/2)*sin(f*x + e)^3/(cos(f*x
 + e) + 1)^3 + 55*a^(5/2)*sin(f*x + e)^4/(cos(f*x + e) + 1)^4 - 22*a^(5/2)
*sin(f*x + e)^5/(cos(f*x + e) + 1)^5 + 22*a^(5/2)*sin(f*x + e)^6/(cos(f*x
+ e) + 1)^6)/(f*(sin(f*x + e)/(cos(f*x + e) + 1) - 1)*(sin(f*x + e)^2/(cos
(f*x + e) + 1)^2 + 1)^(5/2))
```

3.100.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1411 vs. $2(102) = 204$.

Time = 1.85 (sec) , antiderivative size = 1411, normalized size of antiderivative = 11.96

$$\int (a + a\sin(e + fx))^{5/2} \tan^2(e + fx) \, dx = \text{Too large to display}$$

input
```
integrate((a+a*sin(f*x+e))^(5/2)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
-1/3840*sqrt(2)*(1080*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(c
os(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 + 1080*pi
*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e
))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 + 135*(pi - 2*f*x - 2*e)*a^2*sgn(cos(
-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 - 1080*a^2*a
rctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)*tan(-1/8*pi + 1/4*f*x + 1/4*e)^11 + 3840*a^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^12 + 5400*pi*a^2*floor(-1/8*(pi -
2*f*x - 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1
/4*f*x + 1/4*e)^9 + 5400*pi*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sgn(ta
n(-1/8*pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 + 675*(pi -
 2*f*x - 2*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*
x + 1/4*e)^9 - 5400*a^2*arctan(1/tan(-1/8*pi + 1/4*f*x + 1/4*e))*sgn(cos(-
1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^9 + 99840*a^2*sg
n(cos(-1/4*pi + 1/2*f*x + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^10 + 1080
0*pi*a^2*floor(-1/8*(pi - 2*f*x - 2*e)/pi + 1/2)*sgn(cos(-1/4*pi + 1/2*f*x
 + 1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 + 10800*pi*a^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sgn(tan(-1/8*pi + 1/4*f*x + 1/4*e))*tan(-1/8*pi + 1/4
*f*x + 1/4*e)^7 + 1350*(pi - 2*f*x - 2*e)*a^2*sgn(cos(-1/4*pi + 1/2*f*x +
1/2*e))*tan(-1/8*pi + 1/4*f*x + 1/4*e)^7 - 10800*a^2*arctan(1/tan(-1/8*...
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^{5/2}\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,(a+a\,\sin\left(e+f\,x\right))^{5/2}\,dx$$

input
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(5/2), x)
```

3.101 $\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

3.101.1 Optimal result

Integrand size = 23, antiderivative size = 151

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = -\frac{5a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{f} + \frac{49a^3\cos(e+fx)}{15f\sqrt{a+a\sin(e+fx)}} + \frac{31a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} + \frac{7a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} - \frac{\cot(e+fx)(a+a\sin(e+fx))^{5/2}}{f}$$

output
```
-5*a^(5/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f+7/5*a*cos(
f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-cot(f*x+e)*(a+a*sin(f*x+e))^(5/2)/f+49/15*
a^3*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+31/15*a^2*cos(f*x+e)*(a+a*sin(f*x+
e))^(1/2)/f
```

3.101.2 Mathematica [A] (verified)

Time = 6.06 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.73

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{a^2\csc^4\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(125\cos\left(\frac{1}{2}(e+fx)\right)-93\cos\left(\frac{3}{2}(e+fx)\right)+25\cos\left(\frac{5}{2}(e+fx)\right)\right. -}{}$$

input
```
Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
-1/30*(a^2*Csc[(e + f*x)/2]^4*Sqrt[a*(1 + Sin[e + f*x])]*(125*Cos[(e + f*x
)/2] - 93*Cos[(3*(e + f*x))/2] + 25*Cos[(5*(e + f*x))/2] + 3*Cos[(7*(e + f
*x))/2] - 125*Sin[(e + f*x)/2] + 150*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f
*x)/2]]*Sin[e + f*x] - 150*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Si
n[e + f*x] - 93*Sin[(3*(e + f*x))/2] - 25*Sin[(5*(e + f*x))/2] + 3*Sin[(7*
(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2])*(Csc[(e + f*x)/4] - Sec[(e + f*x
)/4])*(Csc[(e + f*x)/4] + Sec[(e + f*x)/4]))
```

3.101.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.609$, Rules used = {3042, 3195, 27, 3042, 3455, 27, 3042, 3455, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(a\sin(e+fx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3195}$$

$$\frac{\int \frac{1}{2}\csc(e+fx)(5a-7a\sin(e+fx))(\sin(e+fx)a+a)^{5/2}dx}{a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \csc(e+fx)(5a-7a\sin(e+fx))(\sin(e+fx)a+a)^{5/2}dx}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(5a-7a\sin(e+fx))(\sin(e+fx)a+a)^{5/2}}{\sin(e+fx)}dx}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \text{3455}$$

$$\frac{\frac{2}{5}\int \frac{1}{2}\csc(e+fx)(\sin(e+fx)a+a)^{3/2}\left(25a^2-31a^2\sin(e+fx)\right)dx+\frac{14a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\int \csc(e+fx)(\sin(e+fx)a+a)^{3/2}\left(25a^2-31a^2\sin(e+fx)\right)dx+\frac{14a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}\int \frac{(\sin(e+fx)a+a)^{3/2}\left(25a^2-31a^2\sin(e+fx)\right)}{\sin(e+fx)}dx+\frac{14a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \; 3455$$

$$\frac{\frac{1}{5}\left(\frac{2}{3}\int \frac{1}{2}\csc(e+fx)\sqrt{\sin(e+fx)a+a}\left(75a^3-49a^3\sin(e+fx)\right)dx+\frac{62a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)+\frac{14a^2\cos(e+fx}{}}{2a}$$

$$\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}\left(75a^3-49a^3\sin(e+fx)\right)dx+\frac{62a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)+\frac{14a^2\cos(e+fx)(}{5}}{2a}$$

$$\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\int \frac{\sqrt{\sin(e+fx)a+a}\left(75a^3-49a^3\sin(e+fx)\right)}{\sin(e+fx)}dx+\frac{62a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)+\frac{14a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{2a}-\frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$$\Big\downarrow \; 3460$$

3.101. $\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(75a^3\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx+\frac{98a^4\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{62a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)+\frac{14a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{2a} - \frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$\downarrow$ 3042

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(75a^3\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{98a^4\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{62a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)+\frac{14a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{2a} - \frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$\downarrow$ 3252

$$\frac{\frac{1}{5}\left(\frac{1}{3}\left(\frac{98a^4\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{150a^4\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}\right)+\frac{62a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)+\frac{14a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}}{2a} - \frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

$\downarrow$ 219

$$\frac{\frac{14a^2\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}+\frac{1}{5}\left(\frac{62a^3\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}+\frac{1}{3}\left(\frac{98a^4\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-\frac{150a^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}\right)\right)}{2a} - \frac{\cot(e+fx)(a\sin(e+fx)+a)^{5/2}}{f}$$

input
```
Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
-((Cot[e + f*x]*(a + a*Sin[e + f*x])^(5/2))/f) + ((14*a^2*Cos[e + f*x]*(a
+ a*Sin[e + f*x])^(3/2))/(5*f) + ((62*a^3*Cos[e + f*x]*Sqrt[a + a*Sin[e +
f*x]])/(3*f) + ((-150*a^(7/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Si
n[e + f*x]]])/f + (98*a^4*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/3)/5
)/(2*a)
```

3.101. $\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

3.101.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3195
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Simp[1/a
  Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
 x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1
/2] &&  !LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3455
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Simp[1/(d*(m + n + 1))   Int[(a + b*Sin[e +
f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1
) + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e +
f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1
] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

3.101.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.07

method	result
default	$\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)\left(6(a-a\sin(fx+e))^{\frac{5}{2}}\sqrt{a}-40(a-a\sin(fx+e))^{\frac{3}{2}}a^{\frac{3}{2}}+90\sqrt{a-a\sin(fx+e)}\,a^{\frac{5}{2}}-75\text{ arc}\right.\right.}{15\sin(fx+e)\sqrt{a}\,\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*(6*(a-a*sin(f*x+
e))^(5/2)*a^(1/2)-40*(a-a*sin(f*x+e))^(3/2)*a^(3/2)+90*(a-a*sin(f*x+e))^(1
/2)*a^(5/2)-75*arctanh((a-a*sin(f*x+e))^(1/2)/a^(1/2))*a^3)-15*(a-a*sin(f*
x+e))^(1/2)*a^(5/2))/sin(f*x+e)/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/
f
```

3.101.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(131) = 262$.

Time = 0.30 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.40

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \frac{75\left(a^2\cos(fx+e)^2-a^2-\left(a^2\cos(fx+e)+a^2\right)\sin(fx+e)\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7\,a\cos(fx+e)}{\cdots}\right.}{\cdots}$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/60*(75*(a^2*cos(f*x + e)^2 - a^2 - (a^2*cos(f*x + e) + a^2)*sin(f*x + e)
)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 +
 (cos(f*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e)
 + a)*sqrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) -
a)*sin(f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 -
1)*sin(f*x + e) - cos(f*x + e) - 1)) + 4*(6*a^2*cos(f*x + e)^4 + 28*a^2*co
s(f*x + e)^3 - 40*a^2*cos(f*x + e)^2 - 13*a^2*cos(f*x + e) + 49*a^2 + (6*a
^2*cos(f*x + e)^3 - 22*a^2*cos(f*x + e)^2 - 62*a^2*cos(f*x + e) - 49*a^2)*
sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f*x + e)^2 - (f*cos(f*x + e
) + f)*sin(f*x + e) - f)
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.101.7 Maxima [F]

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \int (a\sin(fx+e)+a)^{\frac{5}{2}}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(5/2)*cot(f*x + e)^2, x)
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.48

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx =$$

$$-\frac{\sqrt{2}\Big(96\,a^2\mathrm{sgn}\big(\cos\big(-\tfrac{1}{4}\,\pi+\tfrac{1}{2}\,fx+\tfrac{1}{2}\,e\big)\big)\sin\big(-\tfrac{1}{4}\,\pi+\tfrac{1}{2}\,fx+\tfrac{1}{2}\,e\big)^5-320\,a^2\mathrm{sgn}\big(\cos\big(-\tfrac{1}{4}\,\pi+\tfrac{1}{2}\,fx+\tfrac{1}{2}\,e\big)\big)\,\mathrm{si}}{}$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(2)*(96*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/
2*f*x + 1/2*e)^5 - 320*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi
 + 1/2*f*x + 1/2*e)^3 + 75*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi
+ 1/2*f*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 360*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) + 60*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1
/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)/(2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2
- 1))*sqrt(a)/f
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \int \cot(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^{5/2}\,dx$$

input
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(5/2), x)
```

3.102 $\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

3.102.1 Optimal result

Integrand size = 23, antiderivative size = 227

$$\begin{aligned}\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx &= \frac{55a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8f} \\ &- \frac{9a^3\cos(e+fx)}{40f\sqrt{a+a\sin(e+fx)}} - \frac{16a^2\cos(e+fx)\sqrt{a+a\sin(e+fx)}}{15f} \\ &+ \frac{17a^2\cot(e+fx)\sqrt{a+a\sin(e+fx)}}{24f} - \frac{2a\cos(e+fx)(a+a\sin(e+fx))^{3/2}}{5f} \\ &- \frac{5a\cot(e+fx)\csc(e+fx)(a+a\sin(e+fx))^{3/2}}{12f} \\ &- \frac{\cot(e+fx)\csc^2(e+fx)(a+a\sin(e+fx))^{5/2}}{3f}\end{aligned}$$

output
```
55/8*a^(5/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f-2/5*a*co
s(f*x+e)*(a+a*sin(f*x+e))^(3/2)/f-5/12*a*cot(f*x+e)*csc(f*x+e)*(a+a*sin(f*
x+e))^(3/2)/f-1/3*cot(f*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(5/2)/f-9/40*a^
3*cos(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)-16/15*a^2*cos(f*x+e)*(a+a*sin(f*x+e)
)^(1/2)/f+17/24*a^2*cot(f*x+e)*(a+a*sin(f*x+e))^(1/2)/f
```

3.102.2 Mathematica [A] (verified)

Time = 6.66 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.59

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx =$$
$$-\frac{a^2\csc^{10}\left(\frac{1}{2}(e+fx)\right)\sqrt{a(1+\sin(e+fx))}\left(108\cos\left(\frac{1}{2}(e+fx)\right)+706\cos\left(\frac{3}{2}(e+fx)\right)-450\cos\left(\frac{5}{2}(e+fx\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
-1/120*(a^2*Csc[(e + f*x)/2]^10*Sqrt[a*(1 + Sin[e + f*x])]*(108*Cos[(e + f
*x)/2] + 706*Cos[(3*(e + f*x))/2] - 450*Cos[(5*(e + f*x))/2] - 156*Cos[(7*
(e + f*x))/2] + 100*Cos[(9*(e + f*x))/2] + 12*Cos[(11*(e + f*x))/2] - 108*
Sin[(e + f*x)/2] - 2475*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e
 + f*x] + 2475*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] +
 706*Sin[(3*(e + f*x))/2] + 450*Sin[(5*(e + f*x))/2] + 825*Log[1 + Cos[(e
+ f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 825*Log[1 - Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 156*Sin[(7*(e + f*x))/2] - 100*
Sin[(9*(e + f*x))/2] + 12*Sin[(11*(e + f*x))/2]))/(f*(1 + Cot[(e + f*x)/2]
)*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3)
```

3.102.3 Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.21, number of steps used = 22, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.913$, Rules used = {3042, 3197, 3042, 3126, 3042, 3126, 3042, 3125, 3523, 27, 3042, 3454, 27, 3042, 3454, 27, 3042, 3460, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(a\sin(e+fx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^{5/2}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{3197}$$

$$\int (\sin(e+fx)a+a)^{5/2}dx+\int \csc^4(e+fx)(\sin(e+fx)a+a)^{5/2}\left(1-2\sin^2(e+fx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int (\sin(e+fx)a+a)^{5/2}dx+\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx$$

$$\Big\downarrow \text{3126}$$

$$\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx+\frac{8}{5}a\int (\sin(e+fx)a+a)^{3/2}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{8}{5}a\int (\sin(e+fx)a+a)^{3/2}dx+\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3126}$$

$$\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx+\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx+\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\sin(e+fx)a+a}dx-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

$$\Big\downarrow \text{3125}$$

$$\int \frac{(\sin(e+fx)a+a)^{5/2}\left(1-2\sin(e+fx)^2\right)}{\sin(e+fx)^4}dx+\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}$$

3.102. $\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

$$\Big\downarrow \text{3523}$$

$$\frac{\int \frac{1}{2}\csc^3(e+fx)(5a-13a\sin(e+fx))(\sin(e+fx)a+a)^{5/2}dx}{3a}+\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \csc^3(e+fx)(5a-13a\sin(e+fx))(\sin(e+fx)a+a)^{5/2}dx}{6a}+\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(5a-13a\sin(e+fx))(\sin(e+fx)a+a)^{5/2}}{\sin(e+fx)^3}dx}{6a}+\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$$\Big\downarrow \text{3454}$$

$$\frac{\frac{1}{2}\int -\frac{1}{2}\csc^2(e+fx)(\sin(e+fx)a+a)^{3/2}\left(57\sin(e+fx)a^2+17a^2\right)dx-\frac{5a^2\cot(e+fx)\csc(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f}}{6a}+\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{4}\int \csc^2(e+fx)(\sin(e+fx)a+a)^{3/2}\left(57\sin(e+fx)a^2+17a^2\right)dx-\frac{5a^2\cot(e+fx)\csc(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f}}{6a}+\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

3.102. $\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{1}{4}\int \frac{(\sin(e+fx)a+a)^{3/2}(57\sin(e+fx)a^2+17a^2)}{\sin(e+fx)^2}dx-\frac{5a^2\cot(e+fx)\csc(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f}}{6a}+\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$$\Big\downarrow \; 3454$$

$$\frac{\frac{1}{4}\left(\frac{17a^3\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\int \frac{1}{2}\csc(e+fx)\sqrt{\sin(e+fx)a+a}(97\sin(e+fx)a^3+165a^3)\,dx\right)-\frac{5a^2\cot(e+fx)\,c\cdots}{\cdots}}{6a}\cdots\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{4}\left(\frac{17a^3\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{1}{2}\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}(97\sin(e+fx)a^3+165a^3)\,dx\right)-\frac{5a^2\cot(e+fx)\,c\cdots}{\cdots}}{6a}\cdots\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{1}{4}\left(\frac{17a^3\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}-\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)a+a}(97\sin(e+fx)a^3+165a^3)}{\sin(e+fx)}dx\right)-\frac{5a^2\cot(e+fx)\csc(e+fx)(a\sin(e+fx)+a)^{3/2}}{2f}}{6a}\cdots\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$$\Big\downarrow \; 3460$$

3.102. $\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{194a^4\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-165a^3\int\csc(e+fx)\sqrt{\sin(e+fx)a+a}dx\right)+\frac{17a^3\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}\right)-\frac{5a^2\cot(e+fx)\csc}{}}{6a}$$
$$\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-$$
$$\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$\downarrow$ 3042

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{194a^4\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}-165a^3\int\frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx\right)+\frac{17a^3\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}\right)-\frac{5a^2\cot(e+fx)\csc(e+fx)(a\sin(e+fx)-}{2f}}{6a}$$
$$\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-$$
$$\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$\downarrow$ 3252

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(\frac{330a^4\int\frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}+\frac{194a^4\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)+\frac{17a^3\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}\right)-\frac{5a^2\cot(e+fx)\csc(e+fx}{2f}}{6a}$$
$$\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)-$$
$$\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

$\downarrow$ 219

$$\frac{8}{5}a\left(-\frac{8a^2\cos(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}-\frac{2a\cos(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}\right)+$$
$$\frac{\frac{1}{4}\left(\frac{17a^3\cot(e+fx)\sqrt{a\sin(e+fx)+a}}{f}+\frac{1}{2}\left(\frac{330a^{7/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}+\frac{194a^4\cos(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)\right)-\frac{5a^2\cot(e+fx)\csc(e+fx)(a\sin}{2f}}{6a}$$
$$\frac{2a\cos(e+fx)(a\sin(e+fx)+a)^{3/2}}{5f}-\frac{\cot(e+fx)\csc^2(e+fx)(a\sin(e+fx)+a)^{5/2}}{3f}$$

input
```
Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(5/2),x]
```

3.102. $\int\cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

output
```
(-2*a*Cos[e + f*x]*(a + a*Sin[e + f*x])^(3/2))/(5*f) - (Cot[e + f*x]*Csc[e
 + f*x]^2*(a + a*Sin[e + f*x])^(5/2))/(3*f) + (8*a*((-8*a^2*Cos[e + f*x])/
(3*f*Sqrt[a + a*Sin[e + f*x]]) - (2*a*Cos[e + f*x]*Sqrt[a + a*Sin[e + f*x]
])/(3*f)))/5 + ((-5*a^2*Cot[e + f*x]*Csc[e + f*x]*(a + a*Sin[e + f*x])^(3/
2))/(2*f) + ((17*a^3*Cot[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f + ((330*a^(7
/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (194*a^4
*Cos[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/2)/4)/(6*a)
```

3.102.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3197
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3454
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Simp[b/(d*(n + 1)*(b*c +
 a*d))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp
[a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B
*(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0
])
```

rule 3460
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[-2*b*B*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(2*n + 3)*Sqrt[a +
b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*c - 2*a*d*(n + 1)))/(b
*d*(2*n + 3))   Int[Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])^n, x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0] &&  !LtQ[n, -1]
```

rule 3523
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*d*(n + 1)*(c^2 -
d^2))   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a
*d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*
(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

3.102. $\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx$

3.102.4 Maple [A] (verified)

Time = 6.97 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.98

method	result
default	$-\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(48(-a(\sin(fx+e)-1))^{\frac{5}{2}}(\sin^3(fx+e))\sqrt{a}-320(-a(\sin(fx+e)-1))^{\frac{3}{2}}(\sin^3(fx+e))a^{\frac{3}{2}}+480a\right.}{}$

input
```
int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/120*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(48*(-a*(sin(f*x+e)-1))^(5
/2)*sin(f*x+e)^3*a^(1/2)-320*(-a*(sin(f*x+e)-1))^(3/2)*sin(f*x+e)^3*a^(3/2
)+480*a^(5/2)*(-a*(sin(f*x+e)-1))^(1/2)*sin(f*x+e)^3-825*arctanh((-a*(sin(
f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+135*(-a*(sin(f*x+e)-1))^(5/2)*a
^(1/2)-440*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)+345*(-a*(sin(f*x+e)-1))^(1/2)
*a^(5/2))/sin(f*x+e)^3/a^(1/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. 2(195) = 390.

Time = 0.31 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.14

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \frac{825\left(a^2\cos(fx+e)^4-2\,a^2\cos(fx+e)^2+a^2-\left(a^2\cos(fx+e)^3+a^2\cos(fx+e)^2-a^2\cos\right.\right.}{}$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/480*(825*(a^2*cos(f*x + e)^4 - 2*a^2*cos(f*x + e)^2 + a^2 - (a^2*cos(f*x
 + e)^3 + a^2*cos(f*x + e)^2 - a^2*cos(f*x + e) - a^2)*sin(f*x + e))*sqrt(
a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f
*x + e) + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*s
qrt(a) - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(
f*x + e) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(
f*x + e) - cos(f*x + e) - 1)) - 4*(48*a^2*cos(f*x + e)^6 + 224*a^2*cos(f*x
 + e)^5 - 128*a^2*cos(f*x + e)^4 - 583*a^2*cos(f*x + e)^3 + 147*a^2*cos(f*
x + e)^2 + 399*a^2*cos(f*x + e) - 27*a^2 + (48*a^2*cos(f*x + e)^5 - 176*a^
2*cos(f*x + e)^4 - 304*a^2*cos(f*x + e)^3 + 279*a^2*cos(f*x + e)^2 + 426*a
^2*cos(f*x + e) + 27*a^2)*sin(f*x + e))*sqrt(a*sin(f*x + e) + a))/(f*cos(f
*x + e)^4 - 2*f*cos(f*x + e)^2 - (f*cos(f*x + e)^3 + f*cos(f*x + e)^2 - f*
cos(f*x + e) - f)*sin(f*x + e) + f)
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Timed out
```

3.102.7 Maxima [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.28

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \frac{\sqrt{2}\left(768\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)\right)\sin\left(-\frac{1}{4}\,\pi+\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^5-2560\,a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\,\pi+\right.\right.\right.}{}$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/480*sqrt(2)*(768*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1
/2*f*x + 1/2*e)^5 - 2560*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*
pi + 1/2*f*x + 1/2*e)^3 + 825*sqrt(2)*a^2*log(abs(-2*sqrt(2) + 4*sin(-1/4*
pi + 1/2*f*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))*
sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)) + 1920*a^2*sgn(cos(-1/4*pi + 1/2*f*x +
 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e) - 20*(108*a^2*sgn(cos(-1/4*pi + 1/
2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 176*a^2*sgn(cos(-1/4*pi
 + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 69*a^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))*sin(-1/4*pi + 1/2*f*x + 1/2*e))/(2*sin(-1/4*pi +
1/2*f*x + 1/2*e)^2 - 1)^3)*sqrt(a)/f
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^{5/2}\,dx = \int \cot(e+f\,x)^4\,\left(a+a\,\sin\left(e+f\,x\right)\right)^{5/2}\,dx$$

input
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(5/2), x)
```

3.103 $\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

3.103.1 Optimal result

Integrand size = 23, antiderivative size = 150

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{67\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{64\sqrt{2}\sqrt{a}f} - \frac{\sec(e+fx)(53+127\sin(e+fx))}{192f\sqrt{a+a\sin(e+fx)}} + \frac{a\sin(e+fx)\tan(e+fx)}{24f(a+a\sin(e+fx))^{3/2}} + \frac{\tan^3(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}$$

output

```
-67/128*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/f*2
^(1/2)/a^(1/2)-1/192*sec(f*x+e)*(53+127*sin(f*x+e))/f/(a+a*sin(f*x+e))^(1/
2)+1/24*a*sin(f*x+e)*tan(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)+1/3*tan(f*x+e)^3/
f/(a+a*sin(f*x+e))^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{(804+804i)(-1)^{3/4}\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx\right.\right.}{768f\sqrt{a(1+\sin(e+fx}$$

input
```
Integrate[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
((804 + 804*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*
x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - Sec[e + f*x]^3*(90 + 122*C
os[2*(e + f*x)] - 41*Sin[e + f*x] + 183*Sin[3*(e + f*x)]))/(768*f*Sqrt[a*(
1 + Sin[e + f*x])])
```

3.103.3 Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.61, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 3193, 3042, 3128, 219, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3193}$$

$$\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3128}$$

$$-\frac{2\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx$$

$$\Big\downarrow \text{219}$$

$$-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}$$

$$\Big\downarrow \text{4901}$$

$$-\int \left(\frac{\sec^4(e+fx)}{\sqrt{a(\sin(e+fx)+1)}}-\frac{2\sec^2(e+fx)\tan^2(e+fx)}{\sqrt{a(\sin(e+fx)+1)}}\right)dx-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}+\frac{61\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{64\sqrt{2}\sqrt{a}f}+\frac{61a\cos(e+fx)}{64f(a\sin(e+fx)+a)^{3/2}}+\frac{7\sec^3(e+fx)\sqrt{a\sin(e+fx)+a}}{12af}-\frac{5\sec^3(e+fx)}{6f\sqrt{a\sin(e+fx)+a}}-\frac{61\sec(e+fx)}{48f\sqrt{a\sin(e+fx)+a}}+\frac{7a\sec(e+fx)}{24f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[Tan[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
(61*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(6
4*Sqrt[2]*Sqrt[a]*f) - (Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sq
rt[a + a*Sin[e + f*x]])])/(Sqrt[a]*f) + (61*a*Cos[e + f*x])/(64*f*(a + a*S
in[e + f*x])^(3/2)) + (7*a*Sec[e + f*x])/(24*f*(a + a*Sin[e + f*x])^(3/2))
 - (61*Sec[e + f*x])/(48*f*Sqrt[a + a*Sin[e + f*x]]) - (5*Sec[e + f*x]^3)/
(6*f*Sqrt[a + a*Sin[e + f*x]]) + (7*Sec[e + f*x]^3*Sqrt[a + a*Sin[e + f*x]
])/(12*a*f)
```

3.103.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.103. $\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

rule 3193
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.103.4 Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.61

method	result
default	$\frac{366(\cos^2(fx+e))\sin(fx+e)a^{\frac{7}{2}}-201(\cos^2(fx+e))(a-a\sin(fx+e))^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}a^2+122(\cos^2(fx+e))a^{\frac{7}{2}}+40}{384a^{\frac{7}{2}}(\sin(fx+e)-1)(}$

input
```
int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/384*(366*cos(f*x+e)^2*sin(f*x+e)*a^(7/2)-201*cos(f*x+e)^2*(a-a*sin(f*x+e
))^(3/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^2+1
22*cos(f*x+e)^2*a^(7/2)+402*sin(f*x+e)*(a-a*sin(f*x+e))^(3/2)*arctanh(1/2*
(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^2-112*sin(f*x+e)*a^(7/2)
+402*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*sin(
f*x+e))^(3/2)*a^2-16*a^(7/2))/a^(7/2)/(sin(f*x+e)-1)/(sin(f*x+e)+1)/cos(f*
x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{201\sqrt{2}\left(\cos(fx+e)^3\sin(fx+e)+\cos(fx+e)^3\right)\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2-2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx}{\cos(fx+e)^2-(\cos(fx+e)+2}\right.}{768\left(af\cos(\right.}$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/768*(201*sqrt(2)*(cos(f*x + e)^3*sin(f*x + e) + cos(f*x + e)^3)*sqrt(a)*
log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f
*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*si
n(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(
f*x + e) - 2)) - 4*(61*cos(f*x + e)^2 + (183*cos(f*x + e)^2 - 56)*sin(f*x
+ e) - 8)*sqrt(a*sin(f*x + e) + a))/(a*f*cos(f*x + e)^3*sin(f*x + e) + a*f
*cos(f*x + e)^3)
```

3.103.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{\tan^4{(e+fx)}}{\sqrt{a\,(\sin{(e+fx)}+1)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(tan(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.103.7 Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx=\int \frac{\tan{(fx+e)}^4}{\sqrt{a\sin{(fx+e)}+a}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/sqrt(a*sin(f*x + e) + a), x)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.41

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\frac{201\sqrt{2}\log\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{201\sqrt{2}\log\left(-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{6\sqrt{2}\left(21\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-19\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right.\right.}}{768\,f}$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
-1/768*(201*sqrt(2)*log(sin(3/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(a)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) - 201*sqrt(2)*log(-sin(3/4*pi + 1/2*f*x + 1
/2*e) + 1)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 6*sqrt(2)*(21*s
in(3/4*pi + 1/2*f*x + 1/2*e)^3 - 19*sin(3/4*pi + 1/2*f*x + 1/2*e))/((sin(3
/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^2*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*
e))) + 16*sqrt(2)*(15*sin(3/4*pi + 1/2*f*x + 1/2*e)^2 - 1)/(sqrt(a)*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi + 1/2*f*x + 1/2*e)^3))/f
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/2), x)
```

3.104 $\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

3.104.1 Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{5\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4\sqrt{2}\sqrt{a}f} - \frac{\sec(e+fx)}{2f\sqrt{a+a\sin(e+fx)}} + \frac{3\sec(e+fx)\sqrt{a+a\sin(e+fx)}}{4af}$$

output
```
5/8*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/f*2^(1/
2)/a^(1/2)-1/2*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+3/4*sec(f*x+e)*(a+a*sin
(f*x+e))^(1/2)/a/f
```

3.104.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.10

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{\sec(e+fx)\left(-1+(5+5i)(-1)^{3/4}\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)\right.}{4f\sqrt{a(1+\sin(e+fx))}}$$

input
```
Integrate[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
-1/4*(Sec[e + f*x]*(-1 + (5 + 5*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/
4)*(-1 + Tan[(e + f*x)/4]]*(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])*(Cos[(e
+ f*x)/2] + Sin[(e + f*x)/2])^2 - 3*Sin[e + f*x]))/(f*Sqrt[a*(1 + Sin[e +
f*x])])
```

3.104.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 3191, 27, 3042, 3334, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3191}$$

$$\frac{\int -\frac{1}{2}\sec^2(e+fx)(a-4a\sin(e+fx))\sqrt{\sin(e+fx)a+a}dx}{2a^2}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \sec^2(e+fx)(a-4a\sin(e+fx))\sqrt{\sin(e+fx)a+a}dx}{4a^2}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{(a-4a\sin(e+fx))\sqrt{\sin(e+fx)a+a}}{\cos(e+fx)^2}dx}{4a^2}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3334}$$

$$-\frac{\frac{5}{2}a^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{3a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}}{4a^2}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5}{2}a^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx-\frac{3a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}}{4a^2}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{-\frac{5a^2\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{3a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}}{4a^2}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{5a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{2}f}-\frac{3a\sec(e+fx)\sqrt{a\sin(e+fx)+a}}{f}}{4a^2}-\frac{\sec(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Tan[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
-1/2*Sec[e + f*x]/(f*Sqrt[a + a*Sin[e + f*x]]) - ((-5*a^(3/2)*ArcTanh[(Sqr
t[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(Sqrt[2]*f) - (3*a
*Sec[e + f*x]*Sqrt[a + a*Sin[e + f*x]])/f)/(4*a^2)
```

3.104.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

3.104. $\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

rule 3191
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[b*((a + b*Sin[e + f*x])^m/(a*f*(2*m - 1)*Cos[e + f*x])),
x] - Simp[1/(a^2*(2*m - 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*((a*m - b*(2
*m - 1)*Sin[e + f*x])/Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

3.104.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.25

method	result
default	$\frac{5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a-a\sin(fx+e)}\,a\sin(fx+e)+5\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{a-a\sin(fx+e)}\,a+6a^{\frac{3}{2}}\sin(}{8a^{\frac{3}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*(a-a*si
n(f*x+e))^(1/2)*a*sin(f*x+e)+5*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*
2^(1/2)/a^(1/2))*(a-a*sin(f*x+e))^(1/2)*a+6*a^(3/2)*sin(f*x+e)+2*a^(3/2))/
a^(3/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(88) = 176.

Time = 0.31 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.87

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{5\sqrt{2}(\cos(fx+e)\sin(fx+e)+\cos(fx+e))\sqrt{a}\log\left(-\frac{a\cos(fx+e)^2+2\sqrt{2}\sqrt{a\sin(fx+e)+a}\sqrt{a}(\cos(fx+e)-\sin(fx+e)+}{\cos(fx+e)^2-(\cos(fx+e)+2)\sin(}\right.}{16\,(af\cos(fx+e)\sin(fx+e)+a}$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/16*(5*sqrt(2)*(cos(f*x + e)*sin(f*x + e) + cos(f*x + e))*sqrt(a)*log(-(a
*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e)
 - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x +
 e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e
) - 2)) + 4*sqrt(a*sin(f*x + e) + a)*(3*sin(f*x + e) + 1))/(a*f*cos(f*x +
e)*sin(f*x + e) + a*f*cos(f*x + e))
```

3.104.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(tan(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.104.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.46

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\frac{5\sqrt{2}\log\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{5\sqrt{2}\log\left(-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{2\sqrt{2}\left(3\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-2\right)}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\dots\right.\right.}}{16f}$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/16*(5*sqrt(2)*log(sin(3/4*pi + 1/2*f*x + 1/2*e) + 1)/(sqrt(a)*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))) - 5*sqrt(2)*log(-sin(3/4*pi + 1/2*f*x + 1/2*e)
+ 1)/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*sqrt(2)*(3*sin(3/4*
pi + 1/2*f*x + 1/2*e)^2 - 2)/((sin(3/4*pi + 1/2*f*x + 1/2*e)^3 - sin(3/4*p
i + 1/2*f*x + 1/2*e))*sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/2), x)
```

3.105 $\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

3.105.1 Optimal result

Integrand size = 23, antiderivative size = 62

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{a}f} - \frac{\cot(e+fx)}{f\sqrt{a+a\sin(e+fx)}}$$

output
```
arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f/a^(1/2)-cot(f*x+e)/f/
(a+a*sin(f*x+e))^(1/2)
```

3.105.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 138 vs. 2(62) = 124.

Time = 0.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.23

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\csc\left(\frac{1}{4}(e+fx)\right)\sec\left(\frac{1}{4}(e+fx)\right)\left(-2\cos\left(\frac{1}{2}(e+fx)\right)+2\sin\left(\frac{1}{2}(e+fx)\right)+\left(\log\left(1+\cos\left(\frac{1}{2}(e+fx)\right)-\text{si}\right.\right.\right.}{8f\sqrt{a(1+\sin(}}$$

input
```
Integrate[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
(Csc[(e + f*x)/4]*Sec[(e + f*x)/4]*(-2*Cos[(e + f*x)/2] + 2*Sin[(e + f*x)/
2] + (Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - Log[1 - Cos[(e + f*x)
/2] + Sin[(e + f*x)/2]])*Sin[e + f*x])*(1 + Tan[(e + f*x)/2]))/(8*f*Sqrt[a
*(1 + Sin[e + f*x])])
```

3.105.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3195, 27, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3195}$$

$$\frac{\int -\frac{1}{2}\csc(e+fx)\sqrt{\sin(e+fx)a+a}dx}{a}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx}{2a}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx}{2a}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3252}$$

$$\frac{\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Cot[e + f*x]^2/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]/(Sqrt[a]*f) - Cot
[e + f*x]/(f*Sqrt[a + a*Sin[e + f*x]])
```

3.105.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3195
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)/tan[(e_.) + (f_.)*(x_)]^2,
 x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^m/(f*Tan[e + f*x]), x] + Simp[1/a
  Int[(a + b*Sin[e + f*x])^m*((b*m - a*(m + 1)*Sin[e + f*x])/Sin[e + f*x]),
 x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m - 1
/2] &&  !LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.105.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.66

method	result	size
default	$-\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(-\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{a}}\right)a\sin(fx+e)+\sqrt{a-a\sin(fx+e)}\,\sqrt{a}\right)}{a^{\frac{3}{2}}\sin(fx+e)\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$	103

input
```
int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(-arctanh((a-a*sin(f*x+e))^(1/2)
/a^(1/2))*a*sin(f*x+e)+(a-a*sin(f*x+e))^(1/2)*a^(1/2))/a^(3/2)/sin(f*x+e)/
cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(54) = 108.

Time = 0.31 (sec) , antiderivative size = 263, normalized size of antiderivative = 4.24

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$
$$= \frac{\left(\cos(fx+e)^2-(\cos(fx+e)+1)\sin(fx+e)-1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a\cos(fx+e)^2+4\left(\cos(fx+e)^2+(\cos(fx+e\right.}{\cos(}\right.}{4\left(af\cos(\right.}$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/4*((cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log((a
*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) +
 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) -
9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e)
- a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e)
- cos(f*x + e) - 1)) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x
+ e) + 1))/(a*f*cos(f*x + e)^2 - a*f - (a*f*cos(f*x + e) + a*f)*sin(f*x +
e))
```

3.105.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(cot(e + f*x)**2/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.105.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\cot(fx+e)^2}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/sqrt(a*sin(f*x + e) + a), x)
```

3.105.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(54) = 108$.

Time = 0.35 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.13

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)}{a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)a\,\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{4f}$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(a)*(sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x
+ 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a*sgn(cos(-1
/4*pi + 1/2*f*x + 1/2*e))) - 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)/((2*sin(-1/4
*pi + 1/2*f*x + 1/2*e)^2 - 1)*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^2}{\sqrt{a+a\,\sin(e+f\,x)}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/2), x)
```

3.106 $\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

3.106.1 Optimal result

Integrand size = 23, antiderivative size = 135

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = -\frac{7\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8\sqrt{a}f}+\frac{9\cot(e+fx)}{8f\sqrt{a+a\sin(e+fx)}}+\frac{\cot(e+fx)\csc(e+fx)}{12f\sqrt{a+a\sin(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a+a\sin(e+fx)}}$$

output
```
-7/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/f/a^(1/2)+9/8*cot(
f*x+e)/f/(a+a*sin(f*x+e))^(1/2)+1/12*cot(f*x+e)*csc(f*x+e)/f/(a+a*sin(f*x+
e))^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/f/(a+a*sin(f*x+e))^(1/2)
```

3.106.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(135) = 270.

Time = 0.69 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.16

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)\left(36\cos\left(\frac{1}{2}(e+fx)\right)-46\cos\left(\frac{3}{2}(e+fx)\right)-54\cos\left(\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

output
```
(Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])*(36*Cos[(e + f*x
)/2] - 46*Cos[(3*(e + f*x))/2] - 54*Cos[(5*(e + f*x))/2] - 36*Sin[(e + f*x
)/2] - 63*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 63*L
og[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] - 46*Sin[(3*(e +
f*x))/2] + 54*Sin[(5*(e + f*x))/2] + 21*Log[1 + Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]]*Sin[3*(e + f*x)] - 21*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/
2]]*Sin[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*S
qrt[a*(1 + Sin[e + f*x])])
```

3.106.3 Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.87, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.870$, Rules used = {3042, 3197, 3042, 3128, 219, 3523, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\sqrt{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3197}$$

$$\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\int \frac{\csc^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx+\int \frac{1-2\sin(e+fx)^2}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx$$

$$\Big\downarrow \text{3128}$$

$$\int \frac{1-2\sin(e+fx)^2}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx-\frac{2\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\int \frac{1-2\sin(e+fx)^2}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}$$

$$\Big\downarrow \text{3523}$$

$$\frac{\int -\frac{\csc^3(e+fx)(7\sin(e+fx)a+a)}{2\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\csc^3(e+fx)(7\sin(e+fx)a+a)}{\sqrt{\sin(e+fx)a+a}}dx}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{7\sin(e+fx)a+a}{\sin(e+fx)^3\sqrt{\sin(e+fx)a+a}}dx}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3463}$$

$$-\frac{\frac{\int \frac{3\csc^2(e+fx)\left(\sin(e+fx)a^2+9a^2\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3\int \frac{\csc^2(e+fx)\left(\sin(e+fx)a^2+9a^2\right)}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{3\int \frac{\sin(e+fx)a^2+9a^2}{\sin(e+fx)^2\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3463}$$

3.106. $\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

$$-\frac{\frac{3\left(\frac{\int -\frac{\csc(e+fx)\left(7a^3-9a^3\sin(e+fx)\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\frac{3\left(-\frac{\int \frac{\csc(e+fx)\left(7a^3-9a^3\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{3\left(-\frac{\int \frac{7a^3-9a^3\sin(e+fx)}{\sin(e+fx)\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3464}$$

$$-\frac{\frac{3\left(-\frac{7a^2\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx-16a^3\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{3\left(-\frac{7a^2\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx-16a^3\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \textbf{3128}$$

3.106. $\displaystyle\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

$$-\frac{\frac{3\left(-\frac{7a^2\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{32a^3\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{2a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{3\left(-\frac{7a^2\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{16\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 3252}$$

$$-\frac{\frac{3\left(-\frac{\frac{16\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^3\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{2a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{3\left(-\frac{\frac{16\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{9a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{\sqrt{a}f}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Cot[e + f*x]^4/Sqrt[a + a*Sin[e + f*x]],x]
```

3.106. $\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$

output
```
-((Sqrt[2]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]
])])/(Sqrt[a]*f)) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*f*Sqrt[a + a*Sin[e +
f*x]]) - (-1/2*(a*Cot[e + f*x]*Csc[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])
+ (3*(-1/2*((-14*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e +
 f*x]]])/f + (16*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*S
qrt[a + a*Sin[e + f*x]])])/f)/a - (9*a^2*Cot[e + f*x])/(f*Sqrt[a + a*Sin[e
 + f*x]])))/(4*a))/(6*a)
```

3.106.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3197
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] + Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Sin[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2] &&  !LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3523
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*d*(n + 1)*(c^2 -
d^2))   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a
*d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*
(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

3.106.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

method	result
default	$\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(-21\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}}{\sqrt{a}}\right)(\sin^3(fx+e))a^3+27(-a(\sin(fx+e)-1))^{\frac{5}{2}}\sqrt{a}-56(-a(\sin(f\right.}{24\sin(fx+e)^3a^{\frac{7}{2}}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(-21*arctanh((-a*(sin(f*x+e)
-1))^(1/2)/a^(1/2))*sin(f*x+e)^3*a^3+27*(-a*(sin(f*x+e)-1))^(5/2)*a^(1/2)-
56*(-a*(sin(f*x+e)-1))^(3/2)*a^(3/2)+21*(-a*(sin(f*x+e)-1))^(1/2)*a^(5/2))
/sin(f*x+e)^3/a^(7/2)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(115) = 230$.

Time = 0.30 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.73

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx$$

$$= \frac{21\left(\cos\left(fx+e\right)^4 - 2\cos\left(fx+e\right)^2 - \left(\cos\left(fx+e\right)^3 + \cos\left(fx+e\right)^2 - \cos\left(fx+e\right) - 1\right)\sin\left(fx+e\right) + 1}{}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
1/96*(21*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x +
e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 -
 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e)
- 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e)
+ (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e
)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) -
1)) - 4*(27*cos(f*x + e)^3 + 25*cos(f*x + e)^2 - (27*cos(f*x + e)^2 + 2*co
s(f*x + e) - 17)*sin(f*x + e) - 19*cos(f*x + e) - 17)*sqrt(a*sin(f*x + e)
+ a))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f - (a*f*cos(f*x + e)
^3 + a*f*cos(f*x + e)^2 - a*f*cos(f*x + e) - a*f)*sin(f*x + e))
```

3.106.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\cot^4\left(e+fx\right)}{\sqrt{a\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/2),x)
```

output
```
Integral(cot(e + f*x)**4/sqrt(a*(sin(e + f*x) + 1)), x)
```

3.106.7 Maxima [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\cot(fx+e)^4}{\sqrt{a\sin(fx+e)+a}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^4/sqrt(a*sin(f*x + e) + a), x)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.37

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \frac{\frac{21\log\left(\left|\frac{1}{2}\sqrt{2}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{21\log\left(\left|-\frac{1}{2}\sqrt{2}+\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|\right)}{\sqrt{a}\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{2\sqrt{2}\left(108\sqrt{a}\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^5-112\sqrt{a}\sin\left(-\frac{1}{4}\right.\right.}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3 as}}{48f}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
1/48*(21*log(abs(1/2*sqrt(2) + sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(sqrt(a)*s
gn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 21*log(abs(-1/2*sqrt(2) + sin(-1/4*p
i + 1/2*f*x + 1/2*e)))/(sqrt(a)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 2*s
qrt(2)*(108*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 112*sqrt(a)*sin(-1/
4*pi + 1/2*f*x + 1/2*e)^3 + 21*sqrt(a)*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2
*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*a*sgn(cos(-1/4*pi + 1/2*f*x + 1/2
*e))))/f
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+a\sin(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^4}{\sqrt{a+a\,\sin\left(e+f\,x\right)}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/2),x)
```

output
```
int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/2), x)
```

3.107 $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.107.1 Optimal result

Integrand size = 23, antiderivative size = 177

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{7\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{256\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a+a\sin(e+fx))^{3/2}} - \frac{\sec(e+fx)(65+87\sin(e+fx))}{192f(a+a\sin(e+fx))^{3/2}} + \frac{a\sin(e+fx)\tan(e+fx)}{12f(a+a\sin(e+fx))^{5/2}} + \frac{\tan^3(e+fx)}{3f(a+a\sin(e+fx))^{3/2}}$$

output
```
7/256*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/192*sec(f*x+e)*(65+87*sin(f*x+
e))/f/(a+a*sin(f*x+e))^(3/2)+7/512*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/
(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)+1/12*a*sin(f*x+e)*tan(f*x+e)/f/(
a+a*sin(f*x+e))^(5/2)+1/3*tan(f*x+e)^3/f/(a+a*sin(f*x+e))^(3/2)
```

3.107.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.89

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{124 + \frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - \frac{32}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} - \frac{248\sin\left(\frac{1}{2}(e+\right.}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin}}{}$$

input
```
Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(124 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 32/
(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (248*Sin[(e + f*x)/2])/(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]) + 342*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Si
n[(e + f*x)/2]) - 171*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (21 + 21*I
)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2])^3 + (32*(Cos[(e + f*x)/2] + Sin[(e + f*x)
/2])^3)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2])^3 - (192*(Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2])^3)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(768*f*(a*(1
+ Sin[e + f*x]))^(3/2))
```

3.107.3 Rubi [A] (verified)

Time = 1.68 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3193, 3042, 3129, 3042, 3128, 219, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3193}$$

$$\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx-\int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{(\sin(e+fx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{3/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{3/2}}dx-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a} - \int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{3/2}}dx - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\Big\downarrow$ 3128

$$-\frac{\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2af} - \int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{3/2}}dx - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\Big\downarrow$ 219

$$-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{3/2}}dx - \frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\Big\downarrow$ 4901

$$-\int \left(\frac{\sec^4(e+fx)}{(a(\sin(e+fx)+1))^{3/2}} - \frac{2\sec^2(e+fx)\tan^2(e+fx)}{(a(\sin(e+fx)+1))^{3/2}}\right)dx - \frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f} - \frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}$$

$\Big\downarrow$ 2009

$$\frac{7\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{256\sqrt{2}a^{3/2}f} + \frac{7\cos(e+fx)}{256f(a\sin(e+fx)+a)^{3/2}} + \frac{\sec^3(e+fx)}{4af\sqrt{a\sin(e+fx)+a}} - \frac{\sec^3(e+fx)}{6f(a\sin(e+fx)+a)^{3/2}} - \frac{45\sec(e+fx)}{64af\sqrt{a\sin(e+fx)+a}} + \frac{9\sec(e+fx)}{32f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(7*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/(25
6*Sqrt[2]*a^(3/2)*f) + (7*Cos[e + f*x])/(256*f*(a + a*Sin[e + f*x])^(3/2))
 + (9*Sec[e + f*x])/(32*f*(a + a*Sin[e + f*x])^(3/2)) - Sec[e + f*x]^3/(6*
f*(a + a*Sin[e + f*x])^(3/2)) - (45*Sec[e + f*x])/(64*a*f*Sqrt[a + a*Sin[e
 + f*x]]) + Sec[e + f*x]^3/(4*a*f*Sqrt[a + a*Sin[e + f*x]])
```

3.107. $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}dx$

3.107.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3193
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(150) = 300$.

Time = 0.76 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.76

method	result
default	$-\frac{42(\cos^4(fx+e))a^{\frac{9}{2}}-1080(\cos^2(fx+e))\sin(fx+e)a^{\frac{9}{2}}-21\sin(fx+e)(\cos^2(fx+e))(a-a\sin(fx+e))^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right.}{}$

input
```
int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1536/a^(11/2)*(42*cos(f*x+e)^4*a^(9/2)-1080*cos(f*x+e)^2*sin(f*x+e)*a^(
9/2)-21*sin(f*x+e)*cos(f*x+e)^2*(a-a*sin(f*x+e))^(3/2)*arctanh(1/2*(a-a*si
n(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^3-648*cos(f*x+e)^2*a^(9/2)-63*c
os(f*x+e)^2*(a-a*sin(f*x+e))^(3/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*2^(1/2)*a^3+384*sin(f*x+e)*a^(9/2)+84*sin(f*x+e)*(a-a*sin(f*x
+e))^(3/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^3
+128*a^(9/2)+84*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e)
)^(1/2)*2^(1/2)/a^(1/2))*a^3)/(sin(f*x+e)-1)/(sin(f*x+e)+1)^2/cos(f*x+e)/(
a+a*sin(f*x+e))^(1/2)/f
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.53

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{21\sqrt{2}\left(\cos\left(fx+e\right)^5-2\cos\left(fx+e\right)^3\sin\left(fx+e\right)-2\cos\left(fx+e\right)^3\right)\sqrt{a}\log}{}$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/3072*(21*sqrt(2)*(cos(f*x + e)^5 - 2*cos(f*x + e)^3*sin(f*x + e) - 2*cos
(f*x + e)^3)*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e
) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*c
os(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2
)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(21*cos(f*x + e)^4 - 324*cos(f*x +
 e)^2 - 12*(45*cos(f*x + e)^2 - 16)*sin(f*x + e) + 64)*sqrt(a*sin(f*x + e)
 + a))/(a^2*f*cos(f*x + e)^5 - 2*a^2*f*cos(f*x + e)^3*sin(f*x + e) - 2*a^2
*f*cos(f*x + e)^3)
```

3.107.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\tan^4{(e+fx)}}{(a(\sin{(e+fx)}+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.107.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.24

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\frac{21\sqrt{2}\log\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{21\sqrt{2}\log\left(-\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{2\sqrt{2}\left(21\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}\ldots\right.\right.}{}}{3}$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/3072*(21*sqrt(2)*log(sin(3/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^(3/2)*sgn(cos
(-1/4*pi + 1/2*f*x + 1/2*e))) - 21*sqrt(2)*log(-sin(3/4*pi + 1/2*f*x + 1/2
*e) + 1)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*sqrt(2)*(21*sqr
t(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^8 - 312*sqrt(a)*sin(3/4*pi + 1/2*f*x +
1/2*e)^6 + 507*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^4 - 240*sqrt(a)*sin(3
/4*pi + 1/2*f*x + 1/2*e)^2 + 16*sqrt(a))/((sin(3/4*pi + 1/2*f*x + 1/2*e)^3
 - sin(3/4*pi + 1/2*f*x + 1/2*e))^3*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e)
)))/f
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(3/2), x)
```

3.108 $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.108.1 Optimal result

Integrand size = 23, antiderivative size = 134

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{32\sqrt{2}a^{3/2}f}+\frac{\cos(e+fx)}{32f(a+a\sin(e+fx))^{3/2}}-\frac{\sec(e+fx)}{4f(a+a\sin(e+fx))^{3/2}}+\frac{5\sec(e+fx)}{8af\sqrt{a+a\sin(e+fx)}}$$

output
```
1/32*cos(f*x+e)/f/(a+a*sin(f*x+e))^(3/2)-1/4*sec(f*x+e)/f/(a+a*sin(f*x+e))
^(3/2)+1/64*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))
/a^(3/2)/f*2^(1/2)+5/8*sec(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

3.108.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=$$

$$-\frac{\sec(e+fx)\left(-25-\cos(2(e+fx))+(2+2i)(-1)^{3/4}\operatorname{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-1)^{3/4}\left(-1+\tan\left(\frac{1}{4}(e+fx)\right)\right)\right)\left(\right.}{64f(a(1+\sin(e+f}$$

input
```
Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/64*(Sec[e + f*x]*(-25 - Cos[2*(e + f*x)] + (2 + 2*I)*(-1)^(3/4)*ArcTanh
[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] - Sin[(
e + f*x)/2])*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 - 40*Sin[e + f*x]))/(
f*(a*(1 + Sin[e + f*x]))^(3/2))
```

3.108.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 3191, 27, 3042, 3334, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3191}$$

$$\frac{\int -\frac{\sec^2(e+fx)(3a-8a\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{4a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^2(e+fx)(3a-8a\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{8a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{3a-8a\sin(e+fx)}{\cos(e+fx)^2\sqrt{\sin(e+fx)a+a}}dx}{8a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3334}$$

$$-\frac{\frac{1}{2}a^2\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx-\frac{5a\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{8a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{1}{2}a^2\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx-\frac{5a\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{8a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \textbf{3129}$$

$$-\frac{\frac{1}{2}a^2\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)-\frac{5a\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{8a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\frac{1}{2}a^2\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)-\frac{5a\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{8a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \textbf{3128}$$

$$-\frac{\frac{1}{2}a^2\left(-\frac{\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2af}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)-\frac{5a\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{8a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \textbf{219}$$

$$-\frac{\frac{1}{2}a^2\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)-\frac{5a\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{8a^2}-\frac{\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/4*Sec[e + f*x]/(f*(a + a*Sin[e + f*x])^(3/2)) - ((-5*a*Sec[e + f*x])/(f
*Sqrt[a + a*Sin[e + f*x]]) + (a^2*(-1/2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sq
rt[2]*Sqrt[a + a*Sin[e + f*x]])]/(Sqrt[2]*a^(3/2)*f) - Cos[e + f*x]/(2*f*(
a + a*Sin[e + f*x])^(3/2))))/2)/(8*a^2)
```

3.108. $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3191
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[b*((a + b*Sin[e + f*x])^m/(a*f*(2*m - 1)*Cos[e + f*x])),
x] - Simp[1/(a^2*(2*m - 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*((a*m - b*(2
*m - 1)*Sin[e + f*x])/Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

3.108.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.58

method	result
default	$\frac{-\sqrt{a-a\sin(fx+e)}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^{2}\left(\cos^{2}(fx+e)\right)+2a^{\frac{5}{2}}\left(\cos^{2}(fx+e)\right)+2\sqrt{a-a\sin(fx+e)}\,\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\,\mathrm{si}}}{}\right.}{64a^{\frac{7}{2}}\left(\sin(fx+e)+1\right)\cos(fx+e)\sqrt{a+}}$

input
```
int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64/a^(7/2)*(-(a-a*sin(f*x+e))^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2
^(1/2)/a^(1/2))*2^(1/2)*a^2*cos(f*x+e)^2+2*a^(5/2)*cos(f*x+e)^2+2*(a-a*sin
(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))
*sin(f*x+e)*a^2+40*a^(5/2)*sin(f*x+e)+2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arc
tanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^2+24*a^(5/2))/(sin(f*x+
e)+1)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(111) = 222.

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.77

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\cos\left(fx+e\right)^{3}-2\,\cos\left(fx+e\right)\sin\left(fx+e\right)-2\,\cos\left(fx+e\right)\right)\sqrt{a}\log\left(-\frac{a}{}\right.}{1}$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/128*(sqrt(2)*(cos(f*x + e)^3 - 2*cos(f*x + e)*sin(f*x + e) - 2*cos(f*x +
 e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*s
qrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*cos(f*x + e) - (a*cos(f*x +
 e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*
x + e) - cos(f*x + e) - 2)) - 4*(cos(f*x + e)^2 + 20*sin(f*x + e) + 12)*sq
rt(a*sin(f*x + e) + a))/(a^2*f*cos(f*x + e)^3 - 2*a^2*f*cos(f*x + e)*sin(f
*x + e) - 2*a^2*f*cos(f*x + e))
```

3.108.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\tan^2(e+fx)}{(a(\sin(e+fx)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.108.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\tan(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(3/2), x)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.90

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\frac{8\sqrt{2}}{a^{\frac{3}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)} - \frac{\sqrt{2}\left(9\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-7\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}\right.}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^2a^2\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}\right.\right.}}{64f}$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/64*(8*sqrt(2)/(a^(3/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi +
1/2*f*x + 1/2*e)) - sqrt(2)*(9*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^3 - 7
*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e))/((sin(3/4*pi + 1/2*f*x + 1/2*e)^2
- 1)^2*a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{{\tan\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(3/2), x)
```

3.109 $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.109.1 Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{3\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{2\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{3/2}f} - \frac{\cot(e+fx)}{af\sqrt{a+a\sin(e+fx)}}$$

output
```
3*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f-2*arctanh(1
/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f*2^(1/2)-co
t(f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)
```

3.109.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.81 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.82

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left((16+16i)(-1)^{3/4}\text{arctanh}\left(\left(\frac{1}{2}+\frac{i}{2}\right)(-\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*((16 + 16*I)*(-1)^(3/4)*ArcTanh[(
1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - Cot[(e + f*x)/4] + 2*(3*L
og[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]] - 3*Log[1 - Cos[(e + f*x)/2] +
 Sin[(e + f*x)/2]] + Sec[(e + f*x)/2] + Csc[e + f*x]*Sin[(e + f*x)/4]^2 -
Csc[e + f*x]*Sin[(e + f*x)/4]*Sin[(3*(e + f*x))/4])))/(4*f*(a*(1 + Sin[e +
 f*x]))^(3/2))
```

3.109.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 3194, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3194}$$

$$\frac{\int -\frac{\csc(e+fx)(3a-a\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\csc(e+fx)(3a-a\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{3a-a\sin(e+fx)}{\sin(e+fx)\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3464}$$

$$-\frac{3\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx-4a\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx-4a\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{2a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3128

$$-\frac{\frac{8a\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}+3\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx}{2a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 219

$$-\frac{3\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{4\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 3252

$$-\frac{\frac{4\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{6a\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{2a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

$\Big\downarrow$ 219

$$-\frac{\frac{4\sqrt{2}\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{6\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a^2}-\frac{\cot(e+fx)}{af\sqrt{a\sin(e+fx)+a}}$$

input
```
Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
-1/2*((-6*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]]
)/f + (4*Sqrt[2]*Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a +
a*Sin[e + f*x]])])/f)/a^2 - Cot[e + f*x]/(a*f*Sqrt[a + a*Sin[e + f*x]])
```

3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3194
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Si
mp[1/b^2   Int[(a + b*Sin[e + f*x])^(m + 1)*((b*m - a*(m + 1)*Sin[e + f*x])
/Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[m - 1/2] && LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.109.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.19

method	result
default	$-\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(\sin(fx+e)a^2\left(2\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)-3\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}}{\sqrt{a}}\right)\right)+\sqrt{a-a}\right.}{a^{\frac{7}{2}}\sin(fx+e)\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

3.109. $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

output
```
-1/a^(7/2)*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(sin(f*x+e)*a^2*(2*2^(
1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))-3*arctanh((a-a*si
n(f*x+e))^(1/2)/a^(1/2)))+(a-a*sin(f*x+e))^(1/2)*a^(3/2))/sin(f*x+e)/cos(f
*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(96) = 192$.

Time = 0.33 (sec) , antiderivative size = 421, normalized size of antiderivative = 3.73

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{3\left(\cos\left(fx+e\right)^2-\left(\cos\left(fx+e\right)+1\right)\sin\left(fx+e\right)-1\right)\sqrt{a}\log\left(\frac{a\cos(fx+e)^3-7a}{}\right.}{}$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(3*(cos(f*x + e)^2 - (cos(f*x + e) + 1)*sin(f*x + e) - 1)*sqrt(a)*log(
(a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e)
 + 3)*sin(f*x + e) - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a)
- 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e
) - a)/(cos(f*x + e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e
) - cos(f*x + e) - 1)) + 4*sqrt(2)*(a*cos(f*x + e)^2 - (a*cos(f*x + e) + a
)*sin(f*x + e) - a)*log(-(cos(f*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e)
 - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sq
rt(a) + 3*cos(f*x + e) + 2)/(cos(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x +
 e) - cos(f*x + e) - 2))/sqrt(a) + 4*sqrt(a*sin(f*x + e) + a)*(cos(f*x + e
) - sin(f*x + e) + 1))/(a^2*f*cos(f*x + e)^2 - a^2*f - (a^2*f*cos(f*x + e)
 + a^2*f)*sin(f*x + e))
```

3.109.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\cot^2\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.109.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\cot(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(3/2), x)
```

3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(96) = 192.

Time = 0.44 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.81

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{4\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{4\log(-\mathrm{s}}{a^2\mathrm{sgn}(}\right.}{4f}$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*
x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^2*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) + 4*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)
/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*log(-sin(-1/4*pi + 1/2*f*x
+ 1/2*e) + 1)/(a^2*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*sin(-1/4*pi +
1/2*f*x + 1/2*e)/((2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)*a^2*sgn(cos(-1/
4*pi + 1/2*f*x + 1/2*e))))/f
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{3/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(3/2), x)
```

3.110 $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.110.1 Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{3/2}f} - \frac{\cot(e+fx)}{8af\sqrt{a+a\sin(e+fx)}} + \frac{11\cot(e+fx)\csc(e+fx)}{12af\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+a\sin(e+fx)}}{3a^2f}$$

output
```
-1/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(3/2)/f-1/8*cot(
f*x+e)/a/f/(a+a*sin(f*x+e))^(1/2)+11/12*cot(f*x+e)*csc(f*x+e)/a/f/(a+a*sin
(f*x+e))^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2*(a+a*sin(f*x+e))^(1/2)/a^2/f
```

3.110.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 294 vs. 2(144) = 288.

Time = 0.93 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.04

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{\csc^9\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(-132\cos\left(\frac{1}{2}(e+fx)\right)\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(Csc[(e + f*x)/2]^9*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(-132*Cos[(e +
 f*x)/2] + 62*Cos[(3*(e + f*x))/2] + 6*Cos[(5*(e + f*x))/2] + 132*Sin[(e +
 f*x)/2] - 9*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[e + f*x] + 9
*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e + f*x] + 62*Sin[(3*(e
+ f*x))/2] - 6*Sin[(5*(e + f*x))/2] + 3*Log[1 + Cos[(e + f*x)/2] - Sin[(e
+ f*x)/2]]*Sin[3*(e + f*x)] - 3*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2
]]*Sin[3*(e + f*x)]))/(24*f*(Csc[(e + f*x)/4]^2 - Sec[(e + f*x)/4]^2)^3*(a
*(1 + Sin[e + f*x]))^(3/2))
```

3.110.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.82, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.783$, Rules used = {3042, 3196, 3042, 3251, 3042, 3251, 3042, 3252, 219, 3523, 27, 3042, 3459, 3042, 3251, 3042, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a\sin(e+fx)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4(a\sin(e+fx)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3196}$$

$$\frac{\int \csc^4(e+fx)\sqrt{\sin(e+fx)a+a}\left(\sin^2(e+fx)+1\right)dx}{a^2}-\frac{2\int \csc^3(e+fx)\sqrt{\sin(e+fx)a+a}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(\sin(e+fx)^2+1)}{\sin(e+fx)^4}dx}{a^2}-\frac{2\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)^3}dx}{a^2}$$

$$\Big\downarrow \text{3251}$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(\sin(e+fx)^2+1)}{\sin(e+fx)^4}dx}{a^2}-\frac{2\left(\frac{3}{4}\int \csc^2(e+fx)\sqrt{\sin(e+fx)a+a}dx-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(\sin(e+fx)^2+1)}{\sin(e+fx)^4}dx}{a^2}-\frac{2\left(\frac{3}{4}\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)^2}dx-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3251$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(\sin(e+fx)^2+1)}{\sin(e+fx)^4}dx}{a^2}-\frac{2\left(\frac{3}{4}\left(\frac{1}{2}\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(\sin(e+fx)^2+1)}{\sin(e+fx)^4}dx}{a^2}-\frac{2\left(\frac{3}{4}\left(\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3252$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(\sin(e+fx)^2+1)}{\sin(e+fx)^4}dx}{a^2}-\frac{2\left(\frac{3}{4}\left(-\frac{a\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(\sin(e+fx)^2+1)}{\sin(e+fx)^4}dx}{a^2}-\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3523$$

$$\frac{\frac{\int \frac{1}{2}\csc^3(e+fx)\sqrt{\sin(e+fx)a+a}(9\sin(e+fx)a+a)dx}{3a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 27$$

3.110. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

$$\frac{\frac{\int \csc^3(e+fx)\sqrt{\sin(e+fx)a+a}(9\sin(e+fx)a+a)dx}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\int \frac{\sqrt{\sin(e+fx)a+a}(9\sin(e+fx)a+a)}{\sin(e+fx)^3}dx}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3459}$$

$$\frac{\frac{\frac{39}{4}a\int \csc^2(e+fx)\sqrt{\sin(e+fx)a+a}dx-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\frac{39}{4}a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)^2}dx-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3251}$$

$$\frac{\frac{\frac{39}{4}a\left(\frac{1}{2}\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

3.110. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

$$\frac{\frac{\frac{39}{4}a\left(\frac{1}{2}\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-$$
$$\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3252$$

$$\frac{\frac{\frac{39}{4}a\left(-\frac{a\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-$$
$$\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{39}{4}a\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a^2\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a\sin(e+fx)+a}}{3f}}{a^2}-$$
$$\frac{2\left(\frac{3}{4}\left(-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

input
```
Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(3/2),x]
```

output
```
(-2*(-1/2*(a*Cot[e + f*x]*Csc[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (3*
(-((Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f) -
 (a*Cot[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]])))/4))/a^2 + (-1/3*(Cot[e +
f*x]*Csc[e + f*x]^2*Sqrt[a + a*Sin[e + f*x]])/f + (-1/2*(a^2*Cot[e + f*x]*
Csc[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) + (39*a*(-((Sqrt[a]*ArcTanh[(Sq
rt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f) - (a*Cot[e + f*x])/(f*Sq
rt[a + a*Sin[e + f*x]])))/4)/(6*a))/a^2
```

3.110. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3196
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[-2/(a*b)   Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^
3, x], x] + Simp[1/a^2   Int[(a + b*Sin[e + f*x])^(m + 2)*((1 + Sin[e + f*x
]^2)/Sin[e + f*x]^4), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && IntegerQ[m - 1/2] && LtQ[m, -1]
```

rule 3251
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((c + d*Sin[e
+ f*x])^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] + Sim
p[(2*n + 3)*((b*c - a*d)/(2*b*(n + 1)*(c^2 - d^2)))   Int[Sqrt[a + b*Sin[e
+ f*x]]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1] && NeQ[2*n + 3, 0] && IntegerQ[2*n]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.110. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx$

rule 3459
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((A_.) + (B_.)*sin[(e_.) + (
f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp
[(-b^2)*(B*c - A*d)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)
*(b*c + a*d)*Sqrt[a + b*Sin[e + f*x]])), x] + Simp[(A*b*d*(2*n + 3) - B*(b*
c - 2*a*d*(n + 1)))/(2*d*(n + 1)*(b*c + a*d))   Int[Sqrt[a + b*Sin[e + f*x]
]*(c + d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, A, B}, x
] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n,
-1]
```

rule 3523
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*d*(n + 1)*(c^2 -
d^2))   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a
*d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*
(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

3.110.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

method	result
default	$-\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(3(-a(\sin(fx+e)-1))^{\frac{5}{2}}a^{\frac{3}{2}}+3\,\text{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}}{\sqrt{a}}\right)a^{4}(\sin^{3}(fx+e))+8(-a(\sin(fx+\right.}{24a^{\frac{11}{2}}\sin(fx+e)^{3}\cos(fx+e)\sqrt{a+a\sin(fx+e)}\,f}$

input
```
int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/a^(11/2)*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(3*(-a*(sin(f*x+e)
-1))^(5/2)*a^(3/2)+3*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*a^4*sin(f*
x+e)^3+8*(-a*(sin(f*x+e)-1))^(3/2)*a^(5/2)-3*(-a*(sin(f*x+e)-1))^(1/2)*a^(
7/2))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(124) = 248$.

Time = 0.31 (sec) , antiderivative size = 383, normalized size of antiderivative = 2.66

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \frac{3\left(\cos\left(fx+e\right)^4 - 2\,\cos\left(fx+e\right)^2 - \left(\cos\left(fx+e\right)^3 + \cos\left(fx+e\right)^2 - \cos\left(fx\right.\right.\right.}{}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
1/96*(3*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x + e
)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3 -
7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) -
 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) +
 (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)
^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1
)) + 4*(3*cos(f*x + e)^3 + 17*cos(f*x + e)^2 - (3*cos(f*x + e)^2 - 14*cos(
f*x + e) - 25)*sin(f*x + e) - 11*cos(f*x + e) - 25)*sqrt(a*sin(f*x + e) +
a))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f - (a^2*f*cos(f*
x + e)^3 + a^2*f*cos(f*x + e)^2 - a^2*f*cos(f*x + e) - a^2*f)*sin(f*x + e)
)
```

3.110.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \int \frac{\cot^4\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(3/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(3/2), x)
```

3.110.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{a}\left(\frac{3\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)}{a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}+\frac{4\left(12\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^5+16\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3-3\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}{\left(2\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a^2\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}\right)}{96\,f}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
-1/96*sqrt(2)*sqrt(a)*(3*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2*
f*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^2*sgn(
cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 4*(12*sin(-1/4*pi + 1/2*f*x + 1/2*e)^5
+ 16*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 - 3*sin(-1/4*pi + 1/2*f*x + 1/2*e))/
((2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*a^2*sgn(cos(-1/4*pi + 1/2*f*x
+ 1/2*e))))/f
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{3/2}}\,dx=\int \frac{{\cot\left(e+f\,x\right)}^4}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{3/2}} \,d x$$

input
```
int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(3/2),x)
```

output
```
int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(3/2), x)
```

3.111 $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.111.1 Optimal result

Integrand size = 23, antiderivative size = 207

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{317\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{4096\sqrt{2}a^{5/2}f} + \frac{317\cos(e+fx)}{3072f(a+a\sin(e+fx))^{5/2}} - \frac{\sec(e+fx)(115+129\sin(e+fx))}{384f(a+a\sin(e+fx))^{5/2}} + \frac{317\cos(e+fx)}{4096af(a+a\sin(e+fx))^{3/2}} + \frac{5a\sin(e+fx)\tan(e+fx)}{48f(a+a\sin(e+fx))^{7/2}} + \frac{\tan^3(e+fx)}{3f(a+a\sin(e+fx))^{5/2}}$$

output
```
317/3072*cos(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-1/384*sec(f*x+e)*(115+129*sin
(f*x+e))/f/(a+a*sin(f*x+e))^(5/2)+317/4096*cos(f*x+e)/a/f/(a+a*sin(f*x+e))
^(3/2)+317/8192*arctanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1
/2))/a^(5/2)/f*2^(1/2)+5/48*a*sin(f*x+e)*tan(f*x+e)/f/(a+a*sin(f*x+e))^(7/
2)+1/3*tan(f*x+e)^3/f/(a+a*sin(f*x+e))^(5/2)
```

3.111.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.90

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{1312 + \frac{768\sin\left(\frac{1}{2}(e+fx)\right)}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3} - \frac{384}{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2} - \frac{2624\sin\left(\frac{1}{2}(e\right.}{\cos\left(\frac{1}{2}(e+fx)\right)+\mathrm{si}}}{}$$

input
```
Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
(1312 + (768*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 3
84/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - (2624*Sin[(e + f*x)/2])/(Cos[
(e + f*x)/2] + Sin[(e + f*x)/2]) + 2584*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2]
 + Sin[(e + f*x)/2]) - 1292*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + 402*
Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3 - 201*(Cos[(e + f
*x)/2] + Sin[(e + f*x)/2])^4 - (951 + 951*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2
)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^5 + (256*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e + f*x)/2] - Si
n[(e + f*x)/2])^3 - (1152*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5)/(Cos[(e
 + f*x)/2] - Sin[(e + f*x)/2]))/(12288*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.111.3 Rubi [A] (verified)

Time = 2.08 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.66, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3193, 3042, 3129, 3042, 3129, 3042, 3128, 219, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3193}$$

$$\int \frac{1}{(\sin(e+fx)a+a)^{5/2}}dx-\int \frac{\sec^4(e+fx)\left(1-2\sin^2(e+fx)\right)}{(\sin(e+fx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(\sin(e+fx)a+a)^{5/2}}dx-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{5/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx}{8a}-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{5/2}}dx-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx}{8a}-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{5/2}}dx-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3129}$$

$$-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{5/2}}dx+\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{5/2}}dx-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3128}$$

$$\frac{3\left(-\frac{\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2af}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{5/2}}dx-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$-\int \frac{1-2\sin(e+fx)^2}{\cos(e+fx)^4(\sin(e+fx)a+a)^{5/2}}dx+\frac{3\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{4901}$$

$$-\int \left(\frac{\sec^4(e+fx)}{(a(\sin(e+fx)+1))^{5/2}}-\frac{2\sec^2(e+fx)\tan^2(e+fx)}{(a(\sin(e+fx)+1))^{5/2}}\right)dx+\frac{3\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

3.111. $\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$\frac{1085\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{4096\sqrt{2}a^{5/2}f}+\frac{3\left(-\frac{\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)}{8a}-\frac{31\sec^3(e+fx)}{192a^2f\sqrt{a\sin(e+fx)+a}}-\frac{1085\sec(e+fx)}{3072a^2f\sqrt{a\sin(e+fx)+a}}+\frac{1085\cos(e+fx)}{4096af(a\sin(e+fx)+a)^{3/2}}-\frac{\cos(e+fx)}{4f(a\sin(e+fx)+a)^{5/2}}+\frac{53\sec^3(e+fx)}{96af(a\sin(e+fx)+a)^{3/2}}-\frac{\sec^3(e+fx)}{8f(a\sin(e+fx)+a)^{5/2}}+\frac{217\sec(e+fx)}{1536af(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
(1085*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/
(4096*Sqrt[2]*a^(5/2)*f) - Cos[e + f*x]/(4*f*(a + a*Sin[e + f*x])^(5/2)) -
 Sec[e + f*x]^3/(8*f*(a + a*Sin[e + f*x])^(5/2)) + (1085*Cos[e + f*x])/(40
96*a*f*(a + a*Sin[e + f*x])^(3/2)) + (217*Sec[e + f*x])/(1536*a*f*(a + a*S
in[e + f*x])^(3/2)) + (53*Sec[e + f*x]^3)/(96*a*f*(a + a*Sin[e + f*x])^(3/
2)) - (1085*Sec[e + f*x])/(3072*a^2*f*Sqrt[a + a*Sin[e + f*x]]) - (31*Sec[
e + f*x]^3)/(192*a^2*f*Sqrt[a + a*Sin[e + f*x]]) + (3*(-1/2*ArcTanh[(Sqrt[
a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])]/(Sqrt[2]*a^(3/2)*f) -
 Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^(3/2))))/(8*a)
```

3.111.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3193
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m, x] - Int[(a + b*Sin[e + f*x])^m*((
1 - 2*Sin[e + f*x]^2)/Cos[e + f*x]^4), x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[m - 1/2]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(176) = 352$.

Time = 0.98 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.84

method	result
default	$-\frac{1902(\cos^4(fx+e))\sin(fx+e)a^{\frac{11}{2}}+4438(\cos^4(fx+e))a^{\frac{11}{2}}-13888(\cos^2(fx+e))\sin(fx+e)a^{\frac{11}{2}}+951(\cos^4(fx+e))(a-a\sin(fx+e}{}$

input
```
int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24576/a^(15/2)*(1902*cos(f*x+e)^4*sin(f*x+e)*a^(11/2)+4438*cos(f*x+e)^4
*a^(11/2)-13888*cos(f*x+e)^2*sin(f*x+e)*a^(11/2)+951*cos(f*x+e)^4*(a-a*sin
(f*x+e))^(3/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)
*a^4-9920*cos(f*x+e)^2*a^(11/2)-3804*cos(f*x+e)^2*sin(f*x+e)*(a-a*sin(f*x+
e))^(3/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^4+
5632*sin(f*x+e)*a^(11/2)-7608*cos(f*x+e)^2*(a-a*sin(f*x+e))^(3/2)*arctanh(
1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^4+2560*a^(11/2)+7608
*sin(f*x+e)*(a-a*sin(f*x+e))^(3/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1
/2)/a^(1/2))*2^(1/2)*a^4+7608*(a-a*sin(f*x+e))^(3/2)*2^(1/2)*arctanh(1/2*(
a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*a^4)/(sin(f*x+e)-1)/(sin(f*x+e)+1)^
3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.48

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{951\sqrt{2}\left(3\cos(fx+e)^5-4\cos(fx+e)^3+\left(\cos(fx+e)^5-4\cos(fx+e)^3\right.\right.}{}$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/49152*(951*sqrt(2)*(3*cos(f*x + e)^5 - 4*cos(f*x + e)^3 + (cos(f*x + e)^
5 - 4*cos(f*x + e)^3)*sin(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 + 2*sqr
t(2)*sqrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) +
3*a*cos(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e
)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) - 4*(2219*cos(f
*x + e)^4 - 4960*cos(f*x + e)^2 + (951*cos(f*x + e)^4 - 6944*cos(f*x + e)^
2 + 2816)*sin(f*x + e) + 1280)*sqrt(a*sin(f*x + e) + a))/(3*a^3*f*cos(f*x
+ e)^5 - 4*a^3*f*cos(f*x + e)^3 + (a^3*f*cos(f*x + e)^5 - 4*a^3*f*cos(f*x
+ e)^3)*sin(f*x + e))
```

3.111.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\tan^4(e+fx)}{(a(\sin(e+fx)+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.111.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.89

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{\frac{128\sqrt{2}\left(9\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a}\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^3} - \frac{\sqrt{2}\left(201\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^7-1249\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^5+1567\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}f\right.}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^4a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\right.\right.}}{24576\,f}$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
-1/24576*(128*sqrt(2)*(9*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^2 - sqrt(a)
)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi + 1/2*f*x + 1/2*e)^3
) - sqrt(2)*(201*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^7 - 1249*sqrt(a)*si
n(3/4*pi + 1/2*f*x + 1/2*e)^5 + 1567*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)
^3 - 567*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e))/((sin(3/4*pi + 1/2*f*x + 1
/2*e)^2 - 1)^4*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\tan(e+f\,x)^4}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(5/2), x)
```

3.112 $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.112.1 Optimal result

Integrand size = 23, antiderivative size = 167

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = -\frac{11\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{128\sqrt{2}a^{5/2}f} - \frac{\sec(e+fx)}{6f(a+a\sin(e+fx))^{5/2}} - \frac{11\cos(e+fx)}{128af(a+a\sin(e+fx))^{3/2}} + \frac{17\sec(e+fx)}{48af(a+a\sin(e+fx))^{3/2}} + \frac{11\sec(e+fx)}{96a^2f\sqrt{a+a\sin(e+fx)}}$$

output
```
-1/6*sec(f*x+e)/f/(a+a*sin(f*x+e))^(5/2)-11/128*cos(f*x+e)/a/f/(a+a*sin(f*
x+e))^(3/2)+17/48*sec(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-11/256*arctanh(1/2
*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)+11/9
6*sec(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.70

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{-32 + \frac{64\sin\left(\frac{1}{2}(e+fx)\right)}{\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)} - 104\sin\left(\frac{1}{2}(e+fx)\right)\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\right.\right.}{}$$

input
```
Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
(-32 + (64*Sin[(e + f*x)/2])/(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) - 104*S
in[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]) + 52*(Cos[(e + f*x)/
2] + Sin[(e + f*x)/2])^2 - 30*Sin[(e + f*x)/2]*(Cos[(e + f*x)/2] + Sin[(e
+ f*x)/2])^3 + 15*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^4 + (33 + 33*I)*(-
1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^5 + (48*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])
^5)/(Cos[(e + f*x)/2] - Sin[(e + f*x)/2]))/(384*f*(a*(1 + Sin[e + f*x]))^(
5/2))
```

3.112.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 3191, 27, 3042, 3338, 3042, 3166, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{(a\sin(e+fx)+a)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3191}$$

$$\frac{\int -\frac{\sec^2(e+fx)(5a-12a\sin(e+fx))}{2(\sin(e+fx)a+a)^{3/2}}dx}{6a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sec^2(e+fx)(5a-12a\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{5a-12a\sin(e+fx)}{\cos(e+fx)^2(\sin(e+fx)a+a)^{3/2}}dx}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$$\Big\downarrow \text{3338}$$

$$-\frac{-\frac{11}{8}\int \frac{\sec^2(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx-\frac{17a\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{11}{8}\int \frac{1}{\cos(e+fx)^2\sqrt{\sin(e+fx)a+a}}dx-\frac{17a\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3166

$$-\frac{-\frac{11}{8}\left(\frac{3}{2}a\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{17a\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{11}{8}\left(\frac{3}{2}a\int \frac{1}{(\sin(e+fx)a+a)^{3/2}}dx+\frac{\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{17a\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3129

$$-\frac{-\frac{11}{8}\left(\frac{3}{2}a\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)+\frac{\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{17a\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3042

$$-\frac{-\frac{11}{8}\left(\frac{3}{2}a\left(\frac{\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)+\frac{\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{17a\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 3128

$$-\frac{-\frac{11}{8}\left(\frac{3}{2}a\left(-\frac{\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{2af}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)+\frac{\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{17a\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

$\Big\downarrow$ 219

3.112. $\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$-\frac{-\frac{11}{8}\left(\frac{3}{2}a\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{2\sqrt{2}a^{3/2}f}-\frac{\cos(e+fx)}{2f(a\sin(e+fx)+a)^{3/2}}\right)+\frac{\sec(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)-\frac{17a\sec(e+fx)}{4f(a\sin(e+fx)+a)^{3/2}}}{12a^2}-\frac{\sec(e+fx)}{6f(a\sin(e+fx)+a)^{5/2}}$$

input
```
Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
-1/6*Sec[e + f*x]/(f*(a + a*Sin[e + f*x])^(5/2)) - ((-17*a*Sec[e + f*x])/(
4*f*(a + a*Sin[e + f*x])^(3/2)) - (11*(Sec[e + f*x]/(f*Sqrt[a + a*Sin[e +
f*x]]) + (3*a*(-1/2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin
[e + f*x]])]/(Sqrt[2]*a^(3/2)*f) - Cos[e + f*x]/(2*f*(a + a*Sin[e + f*x])^
(3/2))))/2))/8)/(12*a^2)
```

3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3166
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.
)*(x_)]], x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(a*f*g*(p + 1)*S
qrt[a + b*Sin[e + f*x]])), x] + Simp[a*((2*p + 1)/(2*g^2*(p + 1)))   Int[(g
*Cos[e + f*x])^(p + 2)/(a + b*Sin[e + f*x])^(3/2), x], x] /; FreeQ[{a, b, e
, f, g}, x] && EqQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

rule 3191
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[b*((a + b*Sin[e + f*x])^m/(a*f*(2*m - 1)*Cos[e + f*x])),
x] - Simp[1/(a^2*(2*m - 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*((a*m - b*(2
*m - 1)*Sin[e + f*x])/Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

rule 3338
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*c -
 a*d)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(2*m + p + 1)
)), x] + Simp[(a*d*m + b*c*(m + p + 1))/(a*b*(2*m + p + 1))   Int[(g*Cos[e
+ f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
g, m, p}, x] && EqQ[a^2 - b^2, 0] && (LtQ[m, -1] || ILtQ[Simplify[m + p], 0
]) && NeQ[2*m + p + 1, 0]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(140) = 280$.

Time = 0.80 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.72

method	result
default	$-\frac{66\left(\cos^{2}(fx+e)\right)\sin(fx+e)a^{\frac{7}{2}}-33\left(\cos^{2}(fx+e)\right)\sin(fx+e)\sqrt{a-a\sin(fx+e)}\,\operatorname{arctanh}\left(\frac{\sqrt{a-a\sin(fx+e)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,a^{3}+154\left(\cos^{2}(fx+\right.}{}$

input
```
int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/768/a^(11/2)*(66*cos(f*x+e)^2*sin(f*x+e)*a^(7/2)-33*cos(f*x+e)^2*sin(f*
x+e)*(a-a*sin(f*x+e))^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(
1/2))*2^(1/2)*a^3+154*cos(f*x+e)^2*a^(7/2)-99*cos(f*x+e)^2*(a-a*sin(f*x+e)
)^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*2^(1/2)*a^3-44
8*sin(f*x+e)*a^(7/2)+132*(a-a*sin(f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*s
in(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*sin(f*x+e)*a^3-320*a^(7/2)+132*(a-a*sin(
f*x+e))^(1/2)*2^(1/2)*arctanh(1/2*(a-a*sin(f*x+e))^(1/2)*2^(1/2)/a^(1/2))*
a^3)/(sin(f*x+e)+1)^2/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.67

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{33\sqrt{2}\left(3\cos\left(fx+e\right)^3+\left(\cos\left(fx+e\right)^3-4\cos\left(fx+e\right)\right)\sin\left(fx+e\right)-4\cos\ldots}{\ldots}$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/1536*(33*sqrt(2)*(3*cos(f*x + e)^3 + (cos(f*x + e)^3 - 4*cos(f*x + e))*s
in(f*x + e) - 4*cos(f*x + e))*sqrt(a)*log(-(a*cos(f*x + e)^2 - 2*sqrt(2)*s
qrt(a*sin(f*x + e) + a)*sqrt(a)*(cos(f*x + e) - sin(f*x + e) + 1) + 3*a*co
s(f*x + e) - (a*cos(f*x + e) - 2*a)*sin(f*x + e) + 2*a)/(cos(f*x + e)^2 -
(cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2)) + 4*(77*cos(f*x + e)^
2 + (33*cos(f*x + e)^2 - 224)*sin(f*x + e) - 160)*sqrt(a*sin(f*x + e) + a)
)/(3*a^3*f*cos(f*x + e)^3 - 4*a^3*f*cos(f*x + e) + (a^3*f*cos(f*x + e)^3 -
 4*a^3*f*cos(f*x + e))*sin(f*x + e))
```

3.112.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\tan^2\left(e+fx\right)}{\left(a\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.112.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\tan\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(5/2), x)
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\frac{48\sqrt{2}}{a^{\frac{5}{2}}\operatorname{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)} - \frac{\sqrt{2}\left(15\sqrt{a}\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^5-56\sqrt{a}\sin\left(\frac{3}{4}\pi+\right.\right.}{\left(\sin\left(\frac{3}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)^2-1\right)^3a^3\mathrm{sg}}}{768\,f}$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/768*(48*sqrt(2)/(a^(5/2)*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))*sin(3/4*pi
+ 1/2*f*x + 1/2*e)) - sqrt(2)*(15*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^5
- 56*sqrt(a)*sin(3/4*pi + 1/2*f*x + 1/2*e)^3 + 33*sqrt(a)*sin(3/4*pi + 1/2
*f*x + 1/2*e))/((sin(3/4*pi + 1/2*f*x + 1/2*e)^2 - 1)^3*a^3*sgn(cos(-1/4*p
i + 1/2*f*x + 1/2*e))))/f
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{{\mathrm{tan}\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{5/2}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(5/2), x)
```

3.113 $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.113.1 Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{5\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{7\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{\sqrt{2}a^{5/2}f} - \frac{2\cos(e+fx)}{af(a+a\sin(e+fx))^{3/2}} - \frac{\cot(e+fx)}{af(a+a\sin(e+fx))^{3/2}}$$

output
```
5*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f-2*cos(f*x+e
)/a/f/(a+a*sin(f*x+e))^(3/2)-cot(f*x+e)/a/f/(a+a*sin(f*x+e))^(3/2)-7/2*arc
tanh(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1
/2)
```

3.113.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 451, normalized size of antiderivative = 3.20

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^3\left(8\sin\left(\frac{1}{2}(e+fx)\right)-4\left(\cos\left(\frac{1}{2}(e+fx)\right.\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^3*(8*Sin[(e + f*x)/2] - 4*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2]) + 2*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 +
 (28 + 28*I)*(-1)^(3/4)*ArcTanh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)
/4])]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - Cot[(e + f*x)/4]*(Cos[(e +
 f*x)/2] + Sin[(e + f*x)/2])^2 + 10*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*
x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 - 10*Log[1 - Cos[(e + f*x)/
2] + Sin[(e + f*x)/2]]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2 + (2*Sin[(e
 + f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^2)/(Cos[(e + f*x)/4] - Si
n[(e + f*x)/4]) - (2*Sin[(e + f*x)/4]*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2]
)^2)/(Cos[(e + f*x)/4] + Sin[(e + f*x)/4]) - (Cos[(e + f*x)/2] + Sin[(e +
f*x)/2])^2*Tan[(e + f*x)/4]))/(4*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.113.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3042, 3194, 27, 3042, 3457, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(e+fx)^2(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{ 3194}$$

$$\frac{\int -\frac{\csc(e+fx)(5a-3a\sin(e+fx))}{2(\sin(e+fx)a+a)^{3/2}}dx}{a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{\csc(e+fx)(5a-3a\sin(e+fx))}{(\sin(e+fx)a+a)^{3/2}}dx}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{5a-3a\sin(e+fx)}{\sin(e+fx)(\sin(e+fx)a+a)^{3/2}}dx}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3457}$$

$$-\frac{\frac{\int \frac{2\csc(e+fx)\left(5a^2-2a^2\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{2a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{\int \frac{\csc(e+fx)\left(5a^2-2a^2\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{\int \frac{5a^2-2a^2\sin(e+fx)}{\sin(e+fx)\sqrt{\sin(e+fx)a+a}}dx}{a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3464}$$

$$-\frac{\frac{5a\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx-7a^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\frac{5a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx-7a^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{\frac{\frac{14a^2\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}+5a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx}{a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{5a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{7\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$$\Big\downarrow \text{3252}$$

3.113. $\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$-\frac{\frac{\frac{7\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{10a^2\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

$\downarrow$ 219

$$-\frac{\frac{\frac{7\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{10a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{a^2}+\frac{4a\cos(e+fx)}{f(a\sin(e+fx)+a)^{3/2}}}{2a^2}-\frac{\cot(e+fx)}{af(a\sin(e+fx)+a)^{3/2}}$$

input
```
Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
-(Cot[e + f*x]/(a*f*(a + a*Sin[e + f*x])^(3/2))) - (((-10*a^(3/2)*ArcTanh[
(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f + (7*Sqrt[2]*a^(3/2)*A
rcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*Sin[e + f*x]])])/f)/a^2
+ (4*a*Cos[e + f*x])/(f*(a + a*Sin[e + f*x])^(3/2)))/(2*a^2)
```

3.113.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3194
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(a*f*Tan[e + f*x]), x] + Si
mp[1/b^2   Int[(a + b*Sin[e + f*x])^(m + 1)*((b*m - a*(m + 1)*Sin[e + f*x])
/Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && In
tegerQ[m - 1/2] && LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3457
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Simp[1/(a*(2*m + 1)*(b*c - a*d))
  Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b
*d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ
[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
 &&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.113.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\left(7 \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{2}\,(\sin^2(fx+e))a+7\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}\,\sqrt{2}}{2\sqrt{a}}\right)a\sin(fx+e)-10\operatorname{arctanh}\left(\frac{\sqrt{-a(\sin\right.\right.}{2a^{\frac{7}{2}}\,\mathrm{si}}$

input
```
int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^(7/2)*(7*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2))*2^(
1/2)*sin(f*x+e)^2*a+7*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2
)/a^(1/2))*a*sin(f*x+e)-10*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(
f*x+e)^2*a-10*arctanh((-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*a*sin(f*x+e)+4*(-
a*(sin(f*x+e)-1))^(1/2)*a^(1/2)*sin(f*x+e)+2*(-a*(sin(f*x+e)-1))^(1/2)*a^(
1/2))*(-a*(sin(f*x+e)-1))^(1/2)/sin(f*x+e)/cos(f*x+e)/(a+a*sin(f*x+e))^(1/
2)/f
```

3.113.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(122) = 244$.

Time = 0.32 (sec) , antiderivative size = 539, normalized size of antiderivative = 3.82

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{5\left(\cos\left(fx+e\right)^3+2\,\cos\left(fx+e\right)^2+\left(\cos\left(fx+e\right)^2-\cos\left(fx+e\right)-2\right)\sin\left(\right.}{}$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/4*(5*(cos(f*x + e)^3 + 2*cos(f*x + e)^2 + (cos(f*x + e)^2 - cos(f*x + e)
 - 2)*sin(f*x + e) - cos(f*x + e) - 2)*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a
*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e) - 2*
cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e) + (a
*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x + e)^3
+ cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) - 1))
+ 7*sqrt(2)*(a*cos(f*x + e)^3 + 2*a*cos(f*x + e)^2 - a*cos(f*x + e) + (a*c
os(f*x + e)^2 - a*cos(f*x + e) - 2*a)*sin(f*x + e) - 2*a)*log(-(cos(f*x +
e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e) + a
)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(cos(f*x
 + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a) + 4
*(2*cos(f*x + e)^2 + (2*cos(f*x + e) + 1)*sin(f*x + e) + cos(f*x + e) - 1)
*sqrt(a*sin(f*x + e) + a))/(a^3*f*cos(f*x + e)^3 + 2*a^3*f*cos(f*x + e)^2
- a^3*f*cos(f*x + e) - 2*a^3*f + (a^3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e
) - 2*a^3*f)*sin(f*x + e))
```

3.113.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{\cot^2{(e+fx)}}{(a(\sin{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.113.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{\cot{(fx+e)}^2}{(a\sin{(fx+e)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(5/2), x)
```

3.113.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(122) = 244.

Time = 0.38 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.86

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\frac{\sqrt{a}\left(\frac{7\sqrt{2}\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{7\sqrt{2}\log\left(-\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)}-\frac{10\log\left(\left|\sqrt{2}+2\,\mathrm{s}\right.\right.}{a^3\mathrm{sgn}\left(\cos\left(\right.\right.}\right.}{}$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/4*sqrt(a)*(7*sqrt(2)*log(sin(-1/4*pi + 1/2*f*x + 1/2*e) + 1)/(a^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) - 7*sqrt(2)*log(-sin(-1/4*pi + 1/2*f*x + 1/
2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 10*log(abs(sqrt(2) +
 2*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e
))) + 10*log(abs(-sqrt(2) + 2*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3*sgn(co
s(-1/4*pi + 1/2*f*x + 1/2*e))) - 2*(4*sqrt(2)*sin(-1/4*pi + 1/2*f*x + 1/2*
e)^3 - 3*sqrt(2)*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2*sin(-1/4*pi + 1/2*f*x
 + 1/2*e)^4 - 3*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 + 1)*a^3*sgn(cos(-1/4*pi
+ 1/2*f*x + 1/2*e))))/f
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^2}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(5/2), x)
```

3.114 $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

3.114.1 Optimal result

Integrand size = 23, antiderivative size = 191

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{45\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+a\sin(e+fx)}}\right)}{8a^{5/2}f} - \frac{4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a+a\sin(e+fx)}}\right)}{a^{5/2}f} - \frac{19\cot(e+fx)}{8a^2f\sqrt{a+a\sin(e+fx)}} + \frac{13\cot(e+fx)\csc(e+fx)}{12a^2f\sqrt{a+a\sin(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3a^2f\sqrt{a+a\sin(e+fx)}}$$

output
```
45/8*arctanh(cos(f*x+e)*a^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f-4*arctan
h(1/2*cos(f*x+e)*a^(1/2)*2^(1/2)/(a+a*sin(f*x+e))^(1/2))/a^(5/2)/f*2^(1/2)
-19/8*cot(f*x+e)/a^2/f/(a+a*sin(f*x+e))^(1/2)+13/12*cot(f*x+e)*csc(f*x+e)/
a^2/f/(a+a*sin(f*x+e))^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/a^2/f/(a+a*sin(f*
x+e))^(1/2)
```

3.114.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.74

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^5\left((1536+1536i)(-1)^{3/4}\operatorname{arctanh}\left(\left(\frac{1}{2}+\right.\right.\right.}{}$$

input
```
Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

output
```
((Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^5*((1536 + 1536*I)*(-1)^(3/4)*ArcTa
nh[(1/2 + I/2)*(-1)^(3/4)*(-1 + Tan[(e + f*x)/4])] - (8*Csc[(e + f*x)/2]^9
*(396*Cos[(e + f*x)/2] - 218*Cos[(3*(e + f*x))/2] - 114*Cos[(5*(e + f*x))/
2] - 396*Sin[(e + f*x)/2] - 405*Log[1 + Cos[(e + f*x)/2] - Sin[(e + f*x)/2
]]*Sin[e + f*x] + 405*Log[1 - Cos[(e + f*x)/2] + Sin[(e + f*x)/2]]*Sin[e +
 f*x] - 218*Sin[(3*(e + f*x))/2] + 114*Sin[(5*(e + f*x))/2] + 135*Log[1 +
Cos[(e + f*x)/2] - Sin[(e + f*x)/2]]*Sin[3*(e + f*x)] - 135*Log[1 - Cos[(e
 + f*x)/2] + Sin[(e + f*x)/2]]*Sin[3*(e + f*x)]))/(Csc[(e + f*x)/4]^2 - Se
c[(e + f*x)/4]^2)^3))/(192*f*(a*(1 + Sin[e + f*x]))^(5/2))
```

3.114.3 Rubi [A] (verified)

Time = 2.81 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.97, number of steps used = 30, number of rules used = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 1.261$, Rules used = {3042, 3196, 3042, 3258, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219, 3523, 27, 3042, 3463, 27, 3042, 3463, 27, 3042, 3464, 3042, 3128, 219, 3252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4(a\sin(e+fx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3196}$$

$$\frac{\int \frac{\csc^4(e+fx)(\sin^2(e+fx)+1)}{\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\int \frac{\csc^3(e+fx)}{\sqrt{\sin(e+fx)a+a}}dx}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\int \frac{1}{\sin(e+fx)^3\sqrt{\sin(e+fx)a+a}}dx}{a^2}$$

$$\Big\downarrow \text{3258}$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{\int \frac{\csc^2(e+fx)(a-3a\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{\int \frac{a-3a\sin(e+fx)}{\sin(e+fx)^2\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3463}$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{\frac{\int -\frac{\csc(e+fx)\left(7a^2-a^2\sin(e+fx)\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{-\frac{\int \frac{\csc(e+fx)\left(7a^2-a^2\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{-\frac{\int \frac{7a^2-a^2\sin(e+fx)}{\sin(e+fx)\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \text{3464}$$

3.114. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{-\frac{7a\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx-8a^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{-\frac{7a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx-8a^2\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3128$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{-\frac{\frac{16a^2\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}+7a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{-\frac{7a\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3252$$

3.114. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^2\int \frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 219$$

$$\frac{\int \frac{\sin(e+fx)^2+1}{\sin(e+fx)^4\sqrt{\sin(e+fx)a+a}}dx}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3523$$

$$\frac{\frac{\int -\frac{\csc^3(e+fx)(a-11a\sin(e+fx))}{2\sqrt{\sin(e+fx)a+a}}dx}{3a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\int \frac{\csc^3(e+fx)(a-11a\sin(e+fx))}{\sqrt{\sin(e+fx)a+a}}dx}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3042$$

3.114. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$\frac{-\frac{\int \frac{a-11a\sin(e+fx)}{\sin(e+fx)^3\sqrt{\sin(e+fx)a+a}}dx}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$\Big\downarrow$ 3463

$$\frac{-\frac{\frac{\int -\frac{3\csc^2(e+fx)\left(15a^2-a^2\sin(e+fx)\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$\Big\downarrow$ 27

$$\frac{-\frac{-\frac{3\int \frac{\csc^2(e+fx)\left(15a^2-a^2\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{-\frac{3\int \frac{15a^2-a^2\sin(e+fx)}{\sin(e+fx)^2\sqrt{\sin(e+fx)a+a}}dx}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$\Big\downarrow$ 3463

3.114. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$-\frac{-\frac{-\frac{3\left(\frac{\int -\frac{\csc(e+fx)\left(17a^3-15a^3\sin(e+fx)\right)}{2\sqrt{\sin(e+fx)a+a}}dx}{a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{-\frac{3\left(-\frac{\int \frac{\csc(e+fx)\left(17a^3-15a^3\sin(e+fx)\right)}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{-\frac{-\frac{3\left(-\frac{\int \frac{17a^3-15a^3\sin(e+fx)}{\sin(e+fx)\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \; 3464$$

3.114. $\displaystyle\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$-\frac{-\frac{-\frac{3\left(-\frac{17a^2\int \csc(e+fx)\sqrt{\sin(e+fx)a+a}dx-32a^3\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{-\frac{-\frac{3\left(-\frac{17a^2\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx-32a^3\int \frac{1}{\sqrt{\sin(e+fx)a+a}}dx}{2a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \ 3128$$

$$-\frac{-\frac{-\frac{3\left(-\frac{17a^2\int \frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{64a^3\int \frac{1}{2a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{2a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}-\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$$\Big\downarrow \ 219$$

3.114. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

$$\frac{-\frac{-\frac{3\left(-\frac{17a^2\int\frac{\sqrt{\sin(e+fx)a+a}}{\sin(e+fx)}dx+\frac{32\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}$$

$$\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$\downarrow$ 3252

$$\frac{-\frac{-\frac{3\left(-\frac{\frac{32\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{34a^3\int\frac{1}{a-\frac{a^2\cos^2(e+fx)}{\sin(e+fx)a+a}}d\frac{a\cos(e+fx)}{\sqrt{\sin(e+fx)a+a}}}{f}}{2a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}$$

$$\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

$\downarrow$ 219

$$\frac{-\frac{-\frac{3\left(-\frac{\frac{32\sqrt{2}a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{34a^{5/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{15a^2\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}\right)}{4a}-\frac{a\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}}{6a}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\sin(e+fx)+a}}}{a^2}$$

$$\frac{2\left(-\frac{-\frac{\frac{8\sqrt{2}a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{2}\sqrt{a\sin(e+fx)+a}}\right)}{f}-\frac{14a^{3/2}\text{arctanh}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a\sin(e+fx)+a}}\right)}{f}}{2a}-\frac{a\cot(e+fx)}{f\sqrt{a\sin(e+fx)+a}}}{4a}-\frac{\cot(e+fx)\csc(e+fx)}{2f\sqrt{a\sin(e+fx)+a}}\right)}{a^2}$$

input

```
Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(5/2),x]
```

3.114. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

output
```
(-2*(-1/2*(Cot[e + f*x]*Csc[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (-1/2
*((-14*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]])/f
 + (8*Sqrt[2]*a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a + a*S
in[e + f*x]])])/f)/a - (a*Cot[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]))/(4*a
)))/a^2 + (-1/3*(Cot[e + f*x]*Csc[e + f*x]^2)/(f*Sqrt[a + a*Sin[e + f*x]])
 - (-1/2*(a*Cot[e + f*x]*Csc[e + f*x])/(f*Sqrt[a + a*Sin[e + f*x]]) - (3*(
-1/2*((-34*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + a*Sin[e + f*x]]
])/f + (32*Sqrt[2]*a^(5/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/(Sqrt[2]*Sqrt[a
+ a*Sin[e + f*x]])])/f)/a - (15*a^2*Cot[e + f*x])/(f*Sqrt[a + a*Sin[e + f*
x]])))/(4*a))/(6*a))/a^2
```

3.114.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3196
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[-2/(a*b)   Int[(a + b*Sin[e + f*x])^(m + 2)/Sin[e + f*x]^
3, x], x] + Simp[1/a^2   Int[(a + b*Sin[e + f*x])^(m + 2)*((1 + Sin[e + f*x
]^2)/Sin[e + f*x]^4), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
 && IntegerQ[m - 1/2] && LtQ[m, -1]
```

rule 3252
```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]]/((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)]), x_Symbol] :> Simp[-2*(b/f)   Subst[Int[1/(b*c + a*d - d*x^2),
x], x, b*(Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]])], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3258
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/Sqrt[(a_) + (b_.)*sin[(e_.
) + (f_.)*(x_)]], x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((c + d*Sin[e + f*x])
^(n + 1)/(f*(n + 1)*(c^2 - d^2)*Sqrt[a + b*Sin[e + f*x]])), x] - Simp[1/(2*
b*(n + 1)*(c^2 - d^2))   Int[(c + d*Sin[e + f*x])^(n + 1)*(Simp[a*d - 2*b*c
*(n + 1) + b*d*(2*n + 3)*Sin[e + f*x], x]/Sqrt[a + b*Sin[e + f*x]]), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3463
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n
+ 1)/(f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*(n + 1)*(c^2 - d^2))   Int[(a
 + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*(a*d*m + b*c*(n +
1)) - B*(a*c*m + b*d*(n + 1)) + b*(B*c - A*d)*(m + n + 2)*Sin[e + f*x], x],
 x], x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0] && Eq
Q[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n, -1] && (IntegerQ[n] || EqQ[m
 + 1/2, 0])
```

rule 3464
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A
*b - a*B)/(b*c - a*d)   Int[1/Sqrt[a + b*Sin[e + f*x]], x], x] + Simp[(B*c
- A*d)/(b*c - a*d)   Int[Sqrt[a + b*Sin[e + f*x]]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.114. $\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx$

rule 3523
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Simp[1/(b*d*(n + 1)*(c^2 -
d^2))   Int[(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(a
*d*m + b*c*(n + 1)) + c*C*(a*c*m + b*d*(n + 1)) - b*(A*d^2*(m + n + 2) + C*
(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d,
 e, f, A, C, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && (LtQ[n, -1] || EqQ[m + n + 2, 0])
```

3.114.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.95

method	result
default	$-\frac{(\sin(fx+e)+1)\sqrt{-a(\sin(fx+e)-1)}\left(-135a^5 \operatorname{arctanh}\left(\frac{\sqrt{-a(\sin(fx+e)-1)}}{\sqrt{a}}\right)(\sin^3(fx+e))+57a^{\frac{5}{2}}(-a(\sin(fx+e)-1))^{\frac{5}{2}}+96\sqrt{2}\ \text{arc}}{24a^{\frac{15}{2}}\sin(fx+e)^3\cos(fx+e)\sqrt{a+a}}$

input
```
int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24/a^(15/2)*(sin(f*x+e)+1)*(-a*(sin(f*x+e)-1))^(1/2)*(-135*a^5*arctanh(
(-a*(sin(f*x+e)-1))^(1/2)/a^(1/2))*sin(f*x+e)^3+57*a^(5/2)*(-a*(sin(f*x+e)
-1))^(5/2)+96*2^(1/2)*arctanh(1/2*(-a*(sin(f*x+e)-1))^(1/2)*2^(1/2)/a^(1/2
))*a^5*sin(f*x+e)^3-88*a^(7/2)*(-a*(sin(f*x+e)-1))^(3/2)+39*a^(9/2)*(-a*(s
in(f*x+e)-1))^(1/2))/sin(f*x+e)^3/cos(f*x+e)/(a+a*sin(f*x+e))^(1/2)/f
```

3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(162) = 324.

Time = 0.32 (sec) , antiderivative size = 564, normalized size of antiderivative = 2.95

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{135\left(\cos(fx+e)^4 - 2\cos(fx+e)^2 - \left(\cos(fx+e)^3 + \cos(fx+e)^2 - \cos(\right.\right.}{}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="fricas")
```

output
```
1/96*(135*(cos(f*x + e)^4 - 2*cos(f*x + e)^2 - (cos(f*x + e)^3 + cos(f*x +
 e)^2 - cos(f*x + e) - 1)*sin(f*x + e) + 1)*sqrt(a)*log((a*cos(f*x + e)^3
- 7*a*cos(f*x + e)^2 + 4*(cos(f*x + e)^2 + (cos(f*x + e) + 3)*sin(f*x + e)
 - 2*cos(f*x + e) - 3)*sqrt(a*sin(f*x + e) + a)*sqrt(a) - 9*a*cos(f*x + e)
 + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) - a)/(cos(f*x +
e)^3 + cos(f*x + e)^2 + (cos(f*x + e)^2 - 1)*sin(f*x + e) - cos(f*x + e) -
 1)) + 192*sqrt(2)*(a*cos(f*x + e)^4 - 2*a*cos(f*x + e)^2 - (a*cos(f*x + e
)^3 + a*cos(f*x + e)^2 - a*cos(f*x + e) - a)*sin(f*x + e) + a)*log(-(cos(f
*x + e)^2 - (cos(f*x + e) - 2)*sin(f*x + e) - 2*sqrt(2)*sqrt(a*sin(f*x + e
) + a)*(cos(f*x + e) - sin(f*x + e) + 1)/sqrt(a) + 3*cos(f*x + e) + 2)/(co
s(f*x + e)^2 - (cos(f*x + e) + 2)*sin(f*x + e) - cos(f*x + e) - 2))/sqrt(a
) + 4*(57*cos(f*x + e)^3 + 83*cos(f*x + e)^2 - (57*cos(f*x + e)^2 - 26*cos
(f*x + e) - 91)*sin(f*x + e) - 65*cos(f*x + e) - 91)*sqrt(a*sin(f*x + e) +
 a))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f - (a^3*f*cos(f
*x + e)^3 + a^3*f*cos(f*x + e)^2 - a^3*f*cos(f*x + e) - a^3*f)*sin(f*x + e
))
```

3.114.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\int \frac{\cot^4{(e+fx)}}{(a(\sin{(e+fx)}+1))^{\frac{5}{2}}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(5/2),x)
```

output
```
Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(5/2), x)
```

3.114.7 Maxima [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx=\text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
Timed out
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.26

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \frac{\sqrt{2}\sqrt{a}\left(\frac{135\sqrt{2}\log\left(\frac{\left|-2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}{\left|2\sqrt{2}+4\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right|}\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} + \frac{192\log\left(\sin\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)+1\right)}{a^3\mathrm{sgn}\left(\cos\left(-\frac{1}{4}\pi+\frac{1}{2}fx+\frac{1}{2}e\right)\right)} - \frac{192}{a}\right.}{}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
1/96*sqrt(2)*sqrt(a)*(135*sqrt(2)*log(abs(-2*sqrt(2) + 4*sin(-1/4*pi + 1/2
*f*x + 1/2*e))/abs(2*sqrt(2) + 4*sin(-1/4*pi + 1/2*f*x + 1/2*e)))/(a^3*sgn
(cos(-1/4*pi + 1/2*f*x + 1/2*e))) + 192*log(sin(-1/4*pi + 1/2*f*x + 1/2*e)
 + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 192*log(-sin(-1/4*pi + 1
/2*f*x + 1/2*e) + 1)/(a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))) - 4*(228*si
n(-1/4*pi + 1/2*f*x + 1/2*e)^5 - 176*sin(-1/4*pi + 1/2*f*x + 1/2*e)^3 + 39
*sin(-1/4*pi + 1/2*f*x + 1/2*e))/((2*sin(-1/4*pi + 1/2*f*x + 1/2*e)^2 - 1)
^3*a^3*sgn(cos(-1/4*pi + 1/2*f*x + 1/2*e))))/f
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{(a+a\sin(e+fx))^{5/2}}\,dx = \int \frac{\cot(e+f\,x)^4}{(a+a\,\sin(e+f\,x))^{5/2}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(5/2),x)
```

output
```
int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(5/2), x)
```

3.115 $\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

3.115.1 Optimal result

Integrand size = 23, antiderivative size = 982

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = -\frac{361\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{126f}$$
$$+\frac{361\sec(e+fx)(1-\sin(e+fx))\sqrt[3]{a+a\sin(e+fx)}}{63f}$$
$$-\frac{\sec(e+fx)\left(65a^2-142a^2\sin(e+fx)\right)}{42f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}}$$
$$+\frac{361\left(1+\sqrt{3}\right)\sec(e+fx)(1-\sin(e+fx))(a+a\sin(e+fx))^{2/3}}{63f\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sin(e+fx)}\right)}$$
$$-\frac{361\sqrt[3]{2}E\left(\arccos\left(\frac{\sqrt[3]{2}\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+a\sin(e+fx)}}{\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sin(e+fx)}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sec(e+fx)(a+a\sin(e+fx))^{2/}}{21\ 3^{3/4}a^{2/3}f\sqrt{-\frac{\sqrt[3]{a+a\sin(e+fx)}\left(\sqrt[3]{2}\right.}{\left(\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{\right.}}}}$$
$$-\frac{361\left(1-\sqrt{3}\right)\operatorname{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}\sqrt[3]{a}-\left(1-\sqrt{3}\right)\sqrt[3]{a+a\sin(e+fx)}}{\sqrt[3]{2}\sqrt[3]{a}-\left(1+\sqrt{3}\right)\sqrt[3]{a+a\sin(e+fx)}}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)\sec(e+fx)(a+a}{63\ 2^{2/3}\sqrt[4]{3}a^{2/3}f\sqrt{-\frac{\sqrt[3]{a+a\sin(e+}}{\left(\sqrt[3]{2}\sqrt[3]{a}-\right.}}}$$
$$+\frac{3a^2\sin(e+fx)\tan(e+fx)}{2f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}}$$
$$-\frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{2/3}}$$

output
```
-361/126*sec(f*x+e)*(a+a*sin(f*x+e))^(1/3)/f+361/63*sec(f*x+e)*(1-sin(f*x+
e))*(a+a*sin(f*x+e))^(1/3)/f-1/42*sec(f*x+e)*(65*a^2-142*a^2*sin(f*x+e))/f
/(a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(2/3)+361/63*sec(f*x+e)*(1-sin(f*x+e))*
(a+a*sin(f*x+e))^(2/3)*(1+3^(1/2))/f/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/
3)*(1+3^(1/2)))-361/63*2^(1/3)*((2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1
-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)
/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/3)*a^(1/3)-(a+
a*sin(f*x+e))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)*a^(1/3)-(a+a*sin(f*
x+e))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1
/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(f*x+e))^(2/3)*
(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))*((2^(2/3)*a^(2/3)+2^(1/3)*a^(1/3)
*(a+a*sin(f*x+e))^(1/3)+(a+a*sin(f*x+e))^(2/3))/(2^(1/3)*a^(1/3)-(a+a*sin(
f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)*3^(1/4)/a^(2/3)/f/(-(a+a*sin(f*x+e))^(
1/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))/(2^(1/3)*a^(1/3)-(a+a*sin(f*
x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)-361/378*((2^(1/3)*a^(1/3)-(a+a*sin(f*x+e
))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)
))^2)^(1/2)/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/3)*
a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)*a^(1/3)-
(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1
/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(...
```

3.115.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 3.33 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.24

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$$

$$= \frac{\sqrt[3]{a(1+\sin(e+fx))}\left(-722\sqrt[3]{2}\cos\left(\frac{1}{4}(2e+\pi+2fx)\right)+361\ 2^{5/6}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right.\right.}{}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(1/3)*(-722*2^(1/3)*Cos[(2*e + Pi + 2*f*x)/4] + 36
1*2^(5/6)*HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Sin[(2*e + Pi + 2*f*x)/4]
^2]*Sec[(2*e + Pi + 2*f*x)/4]*Sqrt[1 - Sin[e + f*x]] + 3*Sec[e + f*x]^3*(C
os[(e + f*x)/2] + Sin[(e + f*x)/2])^(2/3)*(40 + 43*Cos[2*(e + f*x)] - 19*S
in[e + f*x] - 43*Sin[3*(e + f*x)])*Sin[(2*e + Pi + 2*f*x)/4]^(1/3)))/(189*
f*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^(2/3)*Sin[(2*e + Pi + 2*f*x)/4]^(1
/3))
```

3.115.3 Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 988, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.652$, Rules used = {3042, 3198, 111, 27, 170, 27, 161, 61, 61, 73, 837, 25, 27, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)\sqrt[3]{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4\sqrt[3]{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3198}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{a^4\sin^4(e+fx)}{(a-a\sin(e+fx))^{5/2}(\sin(e+fx)a+a)^{13/6}}d(a\sin(e+fx))}{af}$$

$$\Big\downarrow \text{111}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(-3\int -\frac{a^3\sin^2(e+fx)(\sin(e+fx)a+9a)}{3(a-a\sin(e+fx))^{5/2}(\sin(e+fx)a+a)^{13/6}}d(a\sin(e+fx)) - \frac{\cdots}{(a\cdots}\right.}{af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\int \frac{a^2\sin^2(e+fx)(\sin(e+fx)a+9a)}{(a-a\sin(e+fx))^{5/2}(\sin(e+fx)a+a)^{13/6}}d(a\sin(e+fx)) - \frac{\cdots}{(a-a\sin\cdots}\right.}{af}$$

$$\Big\downarrow \text{170}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3}{2}\int -\frac{a^2\sin(e+fx)(6a-17a\sin(e+fx))}{3(a-a\sin(e+fx))^{5/2}(\sin(e+fx)a+a)^{13/6}}d(a\sin(e+fx))+\right.\right.}{af}$$

$\Big\downarrow$ 27

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\int\frac{a\sin(e+fx)(6a-1}{(a-a\sin(e+fx))^{5/2}(\sin}\right.\right.}{af}$$

$\Big\downarrow$ 161

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\left(\frac{361}{63}\int\frac{}{(a-a\sin(e+fx))}\right.\right.\right.}{af}$$

$\Big\downarrow$ 61

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\left(\frac{361}{63}\left(\frac{2\int\frac{}{\sqrt{a-a\sin(e+}}}{}\right.\right.\right.\right.$$

$\Big\downarrow$ 61

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\left(\frac{361}{63}\left(\frac{2\left(-\frac{\int}{\sqrt{a-a\sin}}\right.}{}\right.\right.\right.\right.$$

$\Big\downarrow$ 73

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\left(\frac{361}{63}\left(\frac{2\left(-\frac{6\int\frac{a^4 s}{}}{\sqrt{2a-}}\right.}{}\right.\right.\right.\right.$$

$\Big\downarrow$ 837

3.115. $\int\sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\left(\frac{361}{63}\left(\frac{2\left(-\frac{6\left(-\frac{(1-\sqrt{\ }}{}\right.}{}\right.}{}\right.\right.\right.\right.$$

$\downarrow$ 25

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\left(\frac{361}{63}\left(\frac{2\left(-\frac{6\left(\frac{1}{2}\int\frac{2^{2/}}{}\right.}{}\right.}{}\right.\right.\right.\right.$$

$\downarrow$ 27

3.115. $\int\sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\left(\frac{361}{63}\left(2\left(-\frac{6\left(\int\frac{\sqrt[3]{2}}{}\right)}{}\right.\right.\right.\right.\right.$$

$\downarrow$ 766

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{7/6}}-\frac{1}{2}a\left(\frac{361}{63}\left(2\left(-\frac{6\left(\int\frac{\sqrt[3]{2}}{}\right)}{}\right.\right.\right.\right.\right.$$

$\downarrow$ 2420

3.115. $\int\sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{\sin(e+fx)a+a}\left(a\left(\frac{3a^2\sin^2(e+fx)}{2(a-a\sin(e+fx))^{3/2}(\sin(e+fx)a+a)^{7/6}}-\frac{1}{2}a\left(\frac{65a-142a\,\mathrm{si}}{21(a-a\sin(e+fx))^{3/2}}\right.\right.\right.$$

3.115. $\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

input
```
Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^4,x]
```

output
```
(Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]*((-3*a^3*S
in[e + f*x]^3)/((a - a*Sin[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(7/6)) + a
*((3*a^2*Sin[e + f*x]^2)/(2*(a - a*Sin[e + f*x])^(3/2)*(a + a*Sin[e + f*x]
)^(7/6)) - (a*((65*a - 142*a*Sin[e + f*x])/(21*(a - a*Sin[e + f*x])^(3/2)*
(a + a*Sin[e + f*x])^(7/6)) + (361*(1/(a*Sqrt[a - a*Sin[e + f*x]]*(a + a*S
in[e + f*x])^(1/6)) + (2*((-3*Sqrt[a - a*Sin[e + f*x]])/(a*(a + a*Sin[e +
f*x])^(1/6)) - (6*(-1/2*((1 - Sqrt[3])*a^(4/3)*EllipticF[ArcCos[(2^(1/3)*a
^(1/3) - (1 - Sqrt[3])*a^2*Sin[e + f*x]^2)/(2^(1/3)*a^(1/3) - (1 + Sqrt[3]
)*a^2*Sin[e + f*x]^2)], (2 + Sqrt[3])/4]*Sin[e + f*x]*(2^(1/3)*a^(1/3) - a
^2*Sin[e + f*x]^2)*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(7/3)*Sin[e + f*x]^2
+ a^4*Sin[e + f*x]^4)/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*a^2*Sin[e + f*x]^2)
^2])/(2^(2/3)*3^(1/4)*Sqrt[-((a^2*Sin[e + f*x]^2*(2^(1/3)*a^(1/3) - a^2*Si
n[e + f*x]^2))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*a^2*Sin[e + f*x]^2)^2)]*Sq
rt[2*a - a^6*Sin[e + f*x]^6]) + (-((3^(1/4)*a^(4/3)*EllipticE[ArcCos[(2^(1
/3)*a^(1/3) - (1 - Sqrt[3])*a^2*Sin[e + f*x]^2)/(2^(1/3)*a^(1/3) - (1 + Sq
rt[3])*a^2*Sin[e + f*x]^2)], (2 + Sqrt[3])/4]*Sin[e + f*x]*(2^(1/3)*a^(1/3
) - a^2*Sin[e + f*x]^2)*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(7/3)*Sin[e + f*
x]^2 + a^4*Sin[e + f*x]^4)/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*a^2*Sin[e + f*
x]^2)^2])/(2^(1/3)*Sqrt[-((a^2*Sin[e + f*x]^2*(2^(1/3)*a^(1/3) - a^2*Sin[e
 + f*x]^2))/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*a^2*Sin[e + f*x]^2)^2)]*Sq...
```

3.115.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

3.115. $\int \sqrt[3]{a + a\sin(e + fx)}\tan^4(e + fx)\,dx$

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.115. $\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx$

3.115.4 Maple [F]

$$\int (a+a\sin(fx+e))^{\frac{1}{3}} \left(\tan^4(fx+e)\right) dx$$

input
```
int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

output
```
int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x)
```

3.115.5 Fricas [F]

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int (a\sin(fx+e)+a)^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

3.115.6 Sympy [F]

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int \sqrt[3]{a(\sin(e+fx)+1)}\tan^4(e+fx)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**4,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(1/3)*tan(e + f*x)**4, x)
```

3.115.7 Maxima [F]

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int (a\sin(fx+e)+a)^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

3.115.8 Giac [F]

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int (a\sin(fx+e)+a)^{\frac{1}{3}}\tan(fx+e)^4\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^4, x)
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^4(e+fx)\,dx = \int \tan(e+f\,x)^4\,(a+a\,\sin(e+f\,x))^{1/3}\,dx$$

input
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^(1/3), x)
```

3.116 $\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$

3.116.1 Optimal result

Integrand size = 23, antiderivative size = 123

$$\begin{aligned}\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx &= -\frac{5a\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sqrt[6]{1+\sin(e+fx)}}{3\sqrt[6]{2}f(a+a\sin(e+fx))^{2/3}}\\ &\quad+\frac{7\sec(e+fx)\sqrt[3]{a+a\sin(e+fx)}}{f}-\frac{3\sec(e+fx)(a+a\sin(e+fx))^{4/3}}{af}\end{aligned}$$

output
```
-5/6*a*cos(f*x+e)*hypergeom([1/2, 7/6],[3/2],1/2-1/2*sin(f*x+e))*(1+sin(f*
x+e))^(1/6)*2^(5/6)/f/(a+a*sin(f*x+e))^(2/3)+7*sec(f*x+e)*(a+a*sin(f*x+e))
^(1/3)/f-3*sec(f*x+e)*(a+a*sin(f*x+e))^(4/3)/a/f
```

3.116.2 Mathematica [A] (verified)

Time = 2.37 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.70

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$$

$$= \frac{\sqrt[3]{a(1+\sin(e+fx))}\left(\frac{5\sqrt[3]{2}\left(-2\cos\left(\frac{1}{4}(2e+\pi+2fx)\right){}_2F_1\left(-\frac{1}{2},-\frac{1}{6};\frac{5}{6};\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+\sqrt{\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}\left(2\cos\left(\frac{1}{4}(2e+\pi+2\right.\right.}{\sqrt{\cos^2\left(\frac{1}{4}(2e+\pi+2fx)\right)}\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^{2/3}\sqrt[3]{\sin\left(\frac{1}{4}(2e+\pi+2\right.}}\right.}{3f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

output
```
((a*(1 + Sin[e + f*x]))^(1/3)*((5*2^(1/3)*(-2*Cos[(2*e + Pi + 2*f*x)/4]*Hy
pergeometricPFQ[{-1/2, -1/6}, {5/6}, Sin[(2*e + Pi + 2*f*x)/4]^2] + Sqrt[C
os[(2*e + Pi + 2*f*x)/4]^2]*(2*Cos[(2*e + Pi + 2*f*x)/4] + 3*Sin[(2*e + Pi
 + 2*f*x)/4])))/(Sqrt[Cos[(2*e + Pi + 2*f*x)/4]^2]*(Cos[(e + f*x)/2] + Sin
[(e + f*x)/2])^(2/3)*Sin[(2*e + Pi + 2*f*x)/4]^(1/3)) - 3*(5 + Sec[e + f*x
] - 2*Tan[e + f*x])))/(3*f)
```

3.116.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 3192, 27, 3042, 3334, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)\sqrt[3]{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2\sqrt[3]{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3192}$$

$$\frac{3\int \frac{1}{3}\sec^2(e+fx)\sqrt[3]{\sin(e+fx)a+a}(3\sin(e+fx)a+4a)dx}{a}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sec^2(e+fx)\sqrt[3]{\sin(e+fx)a+a}(3\sin(e+fx)a+4a)dx}{a}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{\sqrt[3]{\sin(e+fx)a+a}(3\sin(e+fx)a+4a)}{\cos(e+fx)^2}dx}{a}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}$$

$$\Big\downarrow \text{3334}$$

$$\frac{\frac{5}{3}a^2\int \frac{1}{(\sin(e+fx)a+a)^{2/3}}dx+\frac{7a\sec(e+fx)\sqrt[3]{a\sin(e+fx)+a}}{f}}{a}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5}{3}a^2\int \frac{1}{(\sin(e+fx)a+a)^{2/3}}dx+\frac{7a\sec(e+fx)\sqrt[3]{a\sin(e+fx)+a}}{f}}{a}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}$$

$$\Big\downarrow \text{3131}$$

$$\frac{\frac{5a^2(\sin(e+fx)+1)^{2/3}\int \frac{1}{(\sin(e+fx)+1)^{2/3}}dx}{3(a\sin(e+fx)+a)^{2/3}}+\frac{7a\sec(e+fx)\sqrt[3]{a\sin(e+fx)+a}}{f}}{a}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{5a^2(\sin(e+fx)+1)^{2/3}\int \frac{1}{(\sin(e+fx)+1)^{2/3}}dx}{3(a\sin(e+fx)+a)^{2/3}}+\frac{7a\sec(e+fx)\sqrt[3]{a\sin(e+fx)+a}}{f}}{a}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}$$

$$\Big\downarrow \text{3130}$$

$$\frac{\frac{7a\sec(e+fx)\sqrt[3]{a\sin(e+fx)+a}}{f}-\frac{5a^2\sqrt[6]{\sin(e+fx)+1}\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{6},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{3\sqrt[6]{2}f(a\sin(e+fx)+a)^{2/3}}}{a}-\frac{3\sec(e+fx)(a\sin(e+fx)+a)^{4/3}}{af}$$

input
```
Int[(a + a*Sin[e + f*x])^(1/3)*Tan[e + f*x]^2,x]
```

output
```
(-3*Sec[e + f*x]*(a + a*Sin[e + f*x])^(4/3))/(a*f) + ((-5*a^2*Cos[e + f*x]
*Hypergeometric2F1[1/2, 7/6, 3/2, (1 - Sin[e + f*x])/2]*(1 + Sin[e + f*x])
^(1/6))/(3*2^(1/6)*f*(a + a*Sin[e + f*x])^(2/3)) + (7*a*Sec[e + f*x]*(a +
a*Sin[e + f*x])^(1/3))/f)/a
```

3.116. $\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$

3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3192
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Simp[1/(b*m)   Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos
[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&
!IntegerQ[m] &&  !LtQ[m, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

3.116. $\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx$

3.116.4 Maple [F]

$$\int (a+a\sin(fx+e))^{\frac{1}{3}} \left(\tan^2(fx+e)\right) dx$$

input
```
int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

output
```
int((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x)
```

3.116.5 Fricas [F]

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx = \int (a\sin(fx+e)+a)^{\frac{1}{3}}\tan(fx+e)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

3.116.6 Sympy [F]

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx = \int \sqrt[3]{a(\sin(e+fx)+1)}\tan^2(e+fx)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**(1/3)*tan(f*x+e)**2,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(1/3)*tan(e + f*x)**2, x)
```

3.116.7 Maxima [F]

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx=\int (a\sin{(fx+e)}+a)^{\frac{1}{3}}\tan{(fx+e)}^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

3.116.8 Giac [F]

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx=\int (a\sin{(fx+e)}+a)^{\frac{1}{3}}\tan{(fx+e)}^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^(1/3)*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^(1/3)*tan(f*x + e)^2, x)
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt[3]{a+a\sin(e+fx)}\tan^2(e+fx)\,dx=\int \tan(e+f\,x)^2\,(a+a\,\sin{(e+f\,x)})^{1/3}\,dx$$

input
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^(1/3), x)
```

3.117 $\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

3.117.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \frac{6\sqrt{2}\,\text{AppellF1}\left(\frac{11}{6},-\frac{1}{2},2,\frac{17}{6},\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}(a+a\sin$$
$$\overline{\qquad 11a^2f \qquad}$$

output
```
6/11*AppellF1(11/6,2,-1/2,17/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)
*(a+a*sin(f*x+e))^(7/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f
```

3.117.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 35.53 (sec) , antiderivative size = 2692, normalized size of antiderivative = 33.65

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
((15/2 + (15*I)/2)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e +
f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3)
)/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x
)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])] + (AppellF1[5/3, 1/3, 4/3, 8/3,
 (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])] +
 I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 -
 I/2)*(1 + Cot[(e + f*x)/2])])*(1 + Cot[(e + f*x)/2]))) + ((-4 - Cot[e + f
*x])*(a*(1 + Sin[e + f*x]))^(1/3))/f + ((5/2 + (5*I)/2)*AppellF1[2/3, 1/3,
 1/3, 5/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f
*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/
3, 5/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)
/2])] + (AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]),
(1/2 - I/2)*(1 + Tan[(e + f*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2
+ I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + T
an[(e + f*x)/2]))) + (Cos[(3*(e + f*x))/2]*Csc[(e + f*x)/2]*Sec[(e + f*x)/
2]*(a*(1 + Sin[e + f*x]))^(1/3)*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)
/2]^2])^(2/3)*(8 + (1 + I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 +
Cot[(e + f*x)/2]))^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, ((1 + I) + (1 -
I)*Tan[(e + f*x)/2])/(2 + 2*Tan[(e + f*x)/2])]*(I + Tan[(e + f*x)/2]) - Ap
pellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I...
```

3.117.3 Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3198, 149, 1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)\sqrt[3]{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{a\sin(e+fx)+a}}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \text{3198}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{\csc^2(e+fx)\sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{5/6}}{a^2}d(a\sin(e+fx))}{af}$$

$\downarrow$ 149

$$\frac{6\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int\frac{a^{10}\sin^{10}(e+fx)\sqrt{2a-a^6\sin^6(e+fx)}}{(a-a^6\sin^6(e+fx))^2}d\sqrt[6]{\sin(e+fx)a+a}}{af}$$

$\downarrow$ 1013

$$\frac{6\sqrt{2}\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\int\frac{a^{10}\sin^{10}(e+fx)\sqrt{2-a^5\sin^6(e+fx)}}{\sqrt{2}(a-a^6\sin^6(e+fx))^2}d\sqrt[6]{s}}{af\sqrt{2-a^5\sin^6(e+fx)}}$$

$\downarrow$ 27

$$\frac{6\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\int\frac{a^{10}\sin^{10}(e+fx)\sqrt{2-a^5\sin^6(e+fx)}}{(a-a^6\sin^6(e+fx))^2}d\sqrt[6]{\sin(}}{af\sqrt{2-a^5\sin^6(e+fx)}}$$

$\downarrow$ 1012

$$\frac{6\sqrt{2}a^8\sin^{10}(e+fx)\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\operatorname{AppellF1}\left(\frac{11}{6},2,-\right.}{11f\sqrt{2-a^5\sin^6(e+fx)}}$$

input
```
Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
(6*Sqrt[2]*a^8*AppellF1[11/6, 2, -1/2, 17/6, a^5*Sin[e + f*x]^6, (a^5*Sin[
e + f*x]^6)/2]*Sin[e + f*x]^10*Sqrt[a - a*Sin[e + f*x]]*Sqrt[a + a*Sin[e +
 f*x]]*Sqrt[2*a - a^6*Sin[e + f*x]^6]*Tan[e + f*x])/(11*f*Sqrt[2 - a^5*Sin
[e + f*x]^6])
```

3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.117.4 Maple [F]

$$\int \left(\cot^{2}\left(f x +e \right)\right) \left(a +a \sin \left(f x +e \right)\right)^{\frac{1}{3}}d x$$

input
```
int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)
```

output
```
int(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x)
```

3.117.5 Fricas [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.117.6 Sympy [F]

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\cot^2\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**(1/3),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(1/3)*cot(e + f*x)**2, x)
```

3.117.7 Maxima [F]

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \int \left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}\cot\left(fx+e\right)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)
```

3.117.8 Giac [F]

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \int (a\sin(fx+e)+a)^{\frac{1}{3}}\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^2, x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \int \cot(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^{1/3}\,dx$$

input
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/3),x)
```

output
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^(1/3), x)
```

3.118 $\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

3.118.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$$
$$= \frac{12\sqrt{2}\,\text{AppellF1}\left(\frac{17}{6},-\frac{3}{2},4,\frac{23}{6},\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}(a+a\,\text{si}}{17a^3f}$$

output
```
12/17*AppellF1(17/6,4,-3/2,23/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e
)*(a+a*sin(f*x+e))^(10/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f
```

3.118.2 Mathematica [C] (warning: unable to verify)

Result contains complex when optimal does not.

Time = 42.17 (sec) , antiderivative size = 2796, normalized size of antiderivative = 34.95

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \text{Result too large to show}$$

input
```
Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
((239/54 + (77*Cot[e + f*x])/54 - (Cot[e + f*x]*Csc[e + f*x])/18 - (Cot[e
+ f*x]*Csc[e + f*x]^2)/3)*(a*(1 + Sin[e + f*x]))^(1/3))/f - ((70/9 + (70*I
)/9)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2
 - I/2)*(1 + Cot[(e + f*x)/2])]*(a*(1 + Sin[e + f*x]))^(1/3)*(1 + Tan[(e +
 f*x)/2]))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3, (1/2 + I/2)*(1 + Cot
[(e + f*x)/2]), (1/2 - I/2)*(1 + Cot[(e + f*x)/2])]*Sec[(e + f*x)/2] + App
ellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*
(1 + Cot[(e + f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]) + I*AppellF1
[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)*(1 + Cot[(e + f*x)/2]), (1/2 - I/2)*(1 +
Cot[(e + f*x)/2])]*(Csc[(e + f*x)/2] + Sec[(e + f*x)/2]))*(Cos[(e + f*x)/2
] + Sin[(e + f*x)/2])) - ((355/108 + (355*I)/108)*AppellF1[2/3, 1/3, 1/3,
5/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2]
)]*(a*(1 + Sin[e + f*x]))^(1/3))/(f*((5 + 5*I)*AppellF1[2/3, 1/3, 1/3, 5/3
, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])]
+ (AppellF1[5/3, 1/3, 4/3, 8/3, (1/2 + I/2)*(1 + Tan[(e + f*x)/2]), (1/2 -
 I/2)*(1 + Tan[(e + f*x)/2])] + I*AppellF1[5/3, 4/3, 1/3, 8/3, (1/2 + I/2)
*(1 + Tan[(e + f*x)/2]), (1/2 - I/2)*(1 + Tan[(e + f*x)/2])])*(1 + Tan[(e
+ f*x)/2]))) - (239*Cos[(3*(e + f*x))/2]*Csc[e + f*x]*(a*(1 + Sin[e + f*x]
))^(1/3)*((1 + Tan[(e + f*x)/2])/Sqrt[Sec[(e + f*x)/2]^2])^(2/3)*(8 + (1 +
 I)*2^(2/3)*(((1 - I)*(I + Cot[(e + f*x)/2]))/(1 + Cot[(e + f*x)/2]))^(...
```

3.118.3 Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3198, 149, 1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)\sqrt[3]{a\sin(e+fx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt[3]{a\sin(e+fx)+a}}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{3198}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{\csc^4(e+fx)(a-a\sin(e+fx))^{3/2}(\sin(e+fx)a+a)^{11/6}}{a^4}d(a\sin(e+fx))}{af}$$

$\Big\downarrow$ 149

$$\frac{6\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{a^{16}\sin^{16}(e+fx)\left(2a-a^6\sin^6(e+fx)\right)^{3/2}}{\left(a-a^6\sin^6(e+fx)\right)^4}d\sqrt[6]{\sin(e+fx)a+a}}{af}$$

$\Big\downarrow$ 1013

$$\frac{12\sqrt{2}\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\int \frac{a^{16}\sin^{16}(e+fx)\left(2-a^5\sin^6(e+fx)\right)^{3/2}}{2\sqrt{2}\left(a-a^6\sin^6(e+fx)\right)^4}d}{f\sqrt{2-a^5\sin^6(e+fx)}}$$

$\Big\downarrow$ 27

$$\frac{6\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\int \frac{a^{16}\sin^{16}(e+fx)\left(2-a^5\sin^6(e+fx)\right)^{3/2}}{\left(a-a^6\sin^6(e+fx)\right)^4}d\sqrt[6]{\mathrm{si}}}{f\sqrt{2-a^5\sin^6(e+fx)}}$$

$\Big\downarrow$ 1012

$$\frac{12\sqrt{2}a^{13}\sin^{16}(e+fx)\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\,\mathrm{AppellF1}\left(\frac{17}{6},4,\right.}{17f\sqrt{2-a^5\sin^6(e+fx)}}$$

input
```
Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
(12*Sqrt[2]*a^13*AppellF1[17/6, 4, -3/2, 23/6, a^5*Sin[e + f*x]^6, (a^5*Si
n[e + f*x]^6)/2]*Sin[e + f*x]^16*Sqrt[a - a*Sin[e + f*x]]*Sqrt[a + a*Sin[e
 + f*x]]*Sqrt[2*a - a^6*Sin[e + f*x]^6]*Tan[e + f*x])/(17*f*Sqrt[2 - a^5*S
in[e + f*x]^6])
```

3.118. $\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx$

3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.118.4 Maple [F]

$$\int \left(\cot^4\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{\frac{1}{3}}dx$$

input
```
int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)
```

output
```
int(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x)
```

3.118.5 Fricas [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.118.6 Sympy [F]

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \int \sqrt[3]{a\left(\sin\left(e+fx\right)+1\right)}\cot^4\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**(1/3),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**(1/3)*cot(e + f*x)**4, x)
```

3.118.7 Maxima [F]

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \int (a\sin(fx+e)+a)^{\frac{1}{3}}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)
```

3.118.8 Giac [F]

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \int (a\sin(fx+e)+a)^{\frac{1}{3}}\cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^(1/3)*cot(f*x + e)^4, x)
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)\sqrt[3]{a+a\sin(e+fx)}\,dx = \int \cot(e+f\,x)^4\,(a+a\,\sin(e+f\,x))^{1/3}\,dx$$

input
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/3),x)
```

output
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^(1/3), x)
```

3.119 $\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

3.119.1 Optimal result

Integrand size = 23, antiderivative size = 551

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{973\sec(e+fx)}{396f\sqrt[3]{a+a\sin(e+fx)}} - \frac{973\sec(e+fx)(1-\sin(e+fx))}{495f\sqrt[3]{a+a\sin(e+fx)}} - \frac{\sec(e+fx)(95a+356a\sin(e+fx))}{132f(1-\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{973\,\text{EllipticF}\left(\arccos\left(\frac{\sqrt[3]{2}\sqrt[3]{a}-(1-\sqrt{3})\sqrt[3]{a+a\sin(e+fx)}}{\sqrt[3]{2}\sqrt[3]{a}-(1+\sqrt{3})\sqrt[3]{a+a\sin(e+fx)}}\right),\frac{1}{4}(2+\sqrt{3})\right)\sec(e+fx)(a+a\sin(e+f}{495\sqrt[3]{2}\sqrt[4]{3}a^{4/3}f\sqrt{-\frac{\sqrt[3]{a+a\sin(e+fx)}}{\left(\sqrt[3]{2}\sqrt[3]{a}-(1+\sqrt{}\right.}}} + \frac{3a^2\sin(e+fx)\tan(e+fx)}{4f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}} + \frac{3a^2\sin^2(e+fx)\tan(e+fx)}{f(a-a\sin(e+fx))(a+a\sin(e+fx))^{4/3}}$$

output
```
973/396*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/3)-973/495*sec(f*x+e)*(1-sin(f*x+
e))/f/(a+a*sin(f*x+e))^(1/3)-1/132*sec(f*x+e)*(95*a+356*a*sin(f*x+e))/f/(1
-sin(f*x+e))/(a+a*sin(f*x+e))^(4/3)+973/2970*((2^(1/3)*a^(1/3)-(a+a*sin(f*
x+e))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1
/2)))^2)^(1/2)/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))*(2^(1/
3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)*a^(1/
3)-(a+a*sin(f*x+e))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))
^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*sec(f*x+e)*(a+a*sin(
f*x+e))^(2/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))*((2^(2/3)*a^(2/3)+2
^(1/3)*a^(1/3)*(a+a*sin(f*x+e))^(1/3)+(a+a*sin(f*x+e))^(2/3))/(2^(1/3)*a^(
1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)*2^(2/3)*3^(3/4)/a^(4/3)/
f/(-(a+a*sin(f*x+e))^(1/3)*(2^(1/3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3))/(2^(1/
3)*a^(1/3)-(a+a*sin(f*x+e))^(1/3)*(1+3^(1/2)))^2)^(1/2)+3/4*a^2*sin(f*x+e)
*tan(f*x+e)/f/(a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(4/3)+3*a^2*sin(f*x+e)^2*t
an(f*x+e)/f/(a-a*sin(f*x+e))/(a+a*sin(f*x+e))^(4/3)
```

3.119.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.18 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.23

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$
$$= \frac{973\sqrt{2}\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+\sec^3(e+fx)\sqrt{1-\sin(e+fx}}{495f\sqrt{1-\sin(e+fx)}\sqrt[3]{a(1+\sin(e+f}}$$

input
```
Integrate[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
(973*Sqrt[2]*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi +
 2*f*x)/4]^2] + Sec[e + f*x]^3*Sqrt[1 - Sin[e + f*x]]*(-49 - 64*Cos[2*(e +
 f*x)] + 22*Sin[e + f*x] - 128*Sin[3*(e + f*x)]))/(495*f*Sqrt[1 - Sin[e +
f*x]]*(a*(1 + Sin[e + f*x]))^(1/3))
```

3.119.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 578, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3042, 3198, 111, 27, 170, 27, 161, 61, 61, 73, 766}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^4}{\sqrt[3]{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3198}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{a^4\sin^4(e+fx)}{(a-a\sin(e+fx))^{5/2}(\sin(e+fx)a+a)^{17/6}}d(a\sin(e+fx))}{af}$$

$$\Big\downarrow \text{111}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\Big(3\int -\frac{a^3\sin^2(e+fx)(9a-a\sin(e+fx))}{3(a-a\sin(e+fx))^{5/2}(\sin(e+fx)a+a)^{17/6}}d(a\sin(e+fx))+ \ldots}{af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\Big(\frac{3a^3\sin^3(e+fx)}{(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{11/6}}-a\int \frac{a^2\sin^2(e+fx)(9a-a\sin \ldots}{(a-a\sin(e+fx))^{5/2}(\sin(e+ \ldots} \ldots}{af}$$

$$\Big\downarrow \text{170}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\Big(\frac{3a^3\sin^3(e+fx)}{(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{11/6}}-a\Big(\frac{3}{4}\int \frac{a^2\sin(e+fx)(35\sin(\ldots}{3(a-a\sin(e+fx))^{5/2}(\sin \ldots} \ldots}{af}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\Big(\frac{3a^3\sin^3(e+fx)}{(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{11/6}}-a\Big(\frac{1}{4}a\int \frac{a\sin(e+fx)(35\sin \ldots}{(a-a\sin(e+fx))^{5/2}(\sin \ldots} \ldots}{af}$$

$$\Big\downarrow \text{161}$$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{3a^3\sin^3(e+fx)}{(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{11/6}}-a\left(\frac{1}{4}a\left(\frac{356a\sin(e+f}{33(a-a\sin(e+fx))^{3/2}(a}\right.\right.\right.$$

$\downarrow$ 61

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{3a^3\sin^3(e+fx)}{(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{11/6}}-a\left(\frac{1}{4}a\left(\frac{356a\sin(e+}{33(a-a\sin(e+fx))^{3/2}}\right.\right.\right.$$

$\downarrow$ 61

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{3a^3\sin^3(e+fx)}{(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{11/6}}-a\left(\frac{1}{4}a\left(\frac{356a\sin(e}{33(a-a\sin(e+fx))^{3/2}}\right.\right.\right.$$

$\downarrow$ 73

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{3a^3\sin^3(e+fx)}{(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{11/6}}-a\left(\frac{1}{4}a\left(\frac{356a\sin(e}{33(a-a\sin(e+fx))^{3/2}}\right.\right.\right.$$

$\downarrow$ 766

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{3a^3\sin^3(e+fx)}{(a-a\sin(e+fx))^{3/2}(a\sin(e+fx)+a)^{11/6}}-a\left(\frac{1}{4}a\left(\frac{356a\sin(e}{33(a-a\sin(e+fx))^{3/2}}\right.\right.\right.$$

3.119. $\displaystyle\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

input
```
Int[Tan[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
(Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]*((3*a^3*Si
n[e + f*x]^3)/((a - a*Sin[e + f*x])^(3/2)*(a + a*Sin[e + f*x])^(11/6)) - a
*((-3*a^2*Sin[e + f*x]^2)/(4*(a - a*Sin[e + f*x])^(3/2)*(a + a*Sin[e + f*x
])^(11/6)) + (a*((95*a + 356*a*Sin[e + f*x])/(33*(a - a*Sin[e + f*x])^(3/2
)*(a + a*Sin[e + f*x])^(11/6)) - (973*(1/(a*Sqrt[a - a*Sin[e + f*x]]*(a +
a*Sin[e + f*x])^(5/6)) + (4*((-3*Sqrt[a - a*Sin[e + f*x]])/(5*a*(a + a*Sin
[e + f*x])^(5/6)) + (3^(3/4)*EllipticF[ArcCos[(2^(1/3)*a^(1/3) - (1 - Sqrt
[3])*a^2*Sin[e + f*x]^2)/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*a^2*Sin[e + f*x]
^2)], (2 + Sqrt[3])/4]*Sin[e + f*x]*(2^(1/3)*a^(1/3) - a^2*Sin[e + f*x]^2)
*Sqrt[(2^(2/3)*a^(2/3) + 2^(1/3)*a^(7/3)*Sin[e + f*x]^2 + a^4*Sin[e + f*x]
^4)/(2^(1/3)*a^(1/3) - (1 + Sqrt[3])*a^2*Sin[e + f*x]^2)^2])/(5*2^(1/3)*a^
(1/3)*Sqrt[-((a^2*Sin[e + f*x]^2*(2^(1/3)*a^(1/3) - a^2*Sin[e + f*x]^2))/(
2^(1/3)*a^(1/3) - (1 + Sqrt[3])*a^2*Sin[e + f*x]^2)^2)]*Sqrt[2*a - a^6*Sin
[e + f*x]^6])))/(3*a)))/99))/4)))/(a*f)
```

3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 161
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^2*c*d*e*g*(n + 1) + a^2*c*d*f*h*(n +
 1) + a*b*(d^2*e*g*(m + 1) + c^2*f*h*(m + 1) - c*d*(f*g + e*h)*(m + n + 2))
 + (a^2*d^2*f*h*(n + 1) - a*b*d^2*(f*g + e*h)*(n + 1) + b^2*(c^2*f*h*(m + 1
) - c*d*(f*g + e*h)*(m + 1) + d^2*e*g*(m + n + 2)))*x)/(b*d*(b*c - a*d)^2*(
m + 1)*(n + 1)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] - Simp[(a^2*d^2*f*
h*(2 + 3*n + n^2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
 3)) + b^2*(c^2*f*h*(2 + 3*m + m^2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
 d^2*e*g*(6 + m^2 + 5*n + n^2 + m*(2*n + 5))))/(b*d*(b*c - a*d)^2*(m + 1)*(
n + 1))   Int[(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && LtQ[m, -1] && LtQ[n, -1]
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.119. $\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.119.4 Maple [F]

$$\int \frac{\tan^4(fx+e)}{(a+a\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

output
```
int(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

3.119.5 Fricas [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(fx+e)^4}{(a\sin(fx+e)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

3.119.6 Sympy [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt[3]{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(tan(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
```

output
```
Integral(tan(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

3.119.7 Maxima [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^4}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

3.119.8 Giac [F]

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan\left(fx+e\right)^4}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^4}{\left(a+a\,\sin\left(e+f\,x\right)\right)^{1/3}}\,dx$$

input
```
int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^4/(a + a*sin(e + f*x))^(1/3), x)
```

3.120 $\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

3.120.1 Optimal result

Integrand size = 23, antiderivative size = 126

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = -\frac{3\sec(e+fx)}{5f\sqrt[3]{a+a\sin(e+fx)}} + \frac{11\sqrt[6]{2}\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{15f\sqrt[6]{1+\sin(e+fx)}\sqrt[3]{a+a\sin(e+fx)}} + \frac{4\sec(e+fx)(a+a\sin(e+fx))^{2/3}}{5af}$$

output
```
-3/5*sec(f*x+e)/f/(a+a*sin(f*x+e))^(1/3)+11/15*2^(1/6)*cos(f*x+e)*hypergeo
m([1/2, 5/6],[3/2],1/2-1/2*sin(f*x+e))/f/(1+sin(f*x+e))^(1/6)/(a+a*sin(f*x
+e))^(1/3)+4/5*sec(f*x+e)*(a+a*sin(f*x+e))^(2/3)/a/f
```

3.120.2 Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.79

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{-22\cos(e+fx)\,\text{Hypergeometric2F1}\left(\frac{1}{6},\frac{1}{2},\frac{7}{6},\sin^2\left(\frac{1}{4}(2e+\pi+2fx)\right)\right)+\sqrt{2-2\sin(e+fx)}(\sec(e+fx)}{5f\sqrt{2-2\sin(e+fx)}\sqrt[3]{a(1+\sin(e+fx))}}$$

input
```
Integrate[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
(-22*Cos[e + f*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sin[(2*e + Pi + 2*f*x)/
4]^2] + Sqrt[2 - 2*Sin[e + f*x]]*(Sec[e + f*x] + 4*Tan[e + f*x]))/(5*f*Sqr
t[2 - 2*Sin[e + f*x]]*(a*(1 + Sin[e + f*x]))^(1/3))
```

3.120.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 3191, 27, 3042, 3334, 3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a\sin(e+fx)+a}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(e+fx)^2}{\sqrt[3]{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3191}$$

$$\frac{3\int -\frac{1}{3}\sec^2(e+fx)(a-5a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}dx}{5a^2}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \sec^2(e+fx)(a-5a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}dx}{5a^2}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{(a-5a\sin(e+fx))(\sin(e+fx)a+a)^{2/3}}{\cos(e+fx)^2}dx}{5a^2}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

$\downarrow$ 3334

$$-\frac{\frac{11}{3}a^2\int \frac{1}{\sqrt[3]{\sin(e+fx)a+a}}dx-\frac{4a\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{f}}{5a^2}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

$\downarrow$ 3042

$$-\frac{\frac{11}{3}a^2\int \frac{1}{\sqrt[3]{\sin(e+fx)a+a}}dx-\frac{4a\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{f}}{5a^2}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

$\downarrow$ 3131

$$-\frac{\frac{11a^2\sqrt[3]{\sin(e+fx)+1}\int \frac{1}{\sqrt[3]{\sin(e+fx)+1}}dx}{3\sqrt[3]{a\sin(e+fx)+a}}-\frac{4a\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{f}}{5a^2}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

$\downarrow$ 3042

$$-\frac{\frac{11a^2\sqrt[3]{\sin(e+fx)+1}\int \frac{1}{\sqrt[3]{\sin(e+fx)+1}}dx}{3\sqrt[3]{a\sin(e+fx)+a}}-\frac{4a\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{f}}{5a^2}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

$\downarrow$ 3130

$$-\frac{-\frac{11\sqrt[6]{2}a^2\cos(e+fx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{3}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{3f\sqrt[6]{\sin(e+fx)+1}\sqrt[3]{a\sin(e+fx)+a}}-\frac{4a\sec(e+fx)(a\sin(e+fx)+a)^{2/3}}{f}}{5a^2}-\frac{3\sec(e+fx)}{5f\sqrt[3]{a\sin(e+fx)+a}}$$

input
```
Int[Tan[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
(-3*Sec[e + f*x])/(5*f*(a + a*Sin[e + f*x])^(1/3)) - ((-11*2^(1/6)*a^2*Cos
[e + f*x]*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Sin[e + f*x])/2])/(3*f*(1
+ Sin[e + f*x])^(1/6)*(a + a*Sin[e + f*x])^(1/3)) - (4*a*Sec[e + f*x]*(a +
 a*Sin[e + f*x])^(2/3))/f)/(5*a^2)
```

3.120. $\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

rule 3191
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[b*((a + b*Sin[e + f*x])^m/(a*f*(2*m - 1)*Cos[e + f*x])),
x] - Simp[1/(a^2*(2*m - 1))   Int[(a + b*Sin[e + f*x])^(m + 1)*((a*m - b*(2
*m - 1)*Sin[e + f*x])/Cos[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && LtQ[m, 0]
```

rule 3334
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*
c + a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(a*f*g*(p + 1)))
, x] + Simp[b*((a*d*m + b*c*(m + p + 1))/(a*g^2*(p + 1)))   Int[(g*Cos[e +
f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, c, d, e,
 f, g}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, -1] && LtQ[p, -1]
```

3.120. $\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

3.120.4 Maple [F]

$$\int \frac{\tan^2(fx+e)}{(a+a\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

output
```
int(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

3.120.5 Fricas [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
integral(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

3.120.6 Sympy [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt[3]{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(tan(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)
```

output
```
Integral(tan(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)
```

3.120.7 Maxima [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

3.120.8 Giac [F]

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(fx+e)^2}{(a\sin(fx+e)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(tan(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(tan(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\tan^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\tan(e+f\,x)^2}{(a+a\,\sin(e+f\,x))^{1/3}}\,dx$$

input
```
int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/3),x)
```

output
```
int(tan(e + f*x)^2/(a + a*sin(e + f*x))^(1/3), x)
```

3.121 $\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

3.121.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{6\sqrt{2}\,\text{AppellF1}\left(\frac{7}{6},-\frac{1}{2},2,\frac{13}{6},\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}(a+a\sin(}{7a^2f}$$

output
```
6/7*AppellF1(7/6,2,-1/2,13/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*(
a+a*sin(f*x+e))^(5/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f
```

3.121.2 Mathematica [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$

input
```
Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
Integrate[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3), x]
```

3.121.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3198, 149, 1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^2\sqrt[3]{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3198}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{\csc^2(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt[6]{\sin(e+fx)a+a}}{a^2}d(a\sin(e+fx))}{af}$$

$$\Big\downarrow \text{149}$$

$$\frac{6\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{a^6\sin^6(e+fx)\sqrt{2a-a^6\sin^6(e+fx)}}{(a-a^6\sin^6(e+fx))^2}d\sqrt[6]{\sin(e+fx)a+a}}{af}$$

$$\Big\downarrow \text{1013}$$

$$\frac{6\sqrt{2}\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\int \frac{a^6\sin^6(e+fx)\sqrt{2-a^5\sin^6(e+fx)}}{\sqrt{2}(a-a^6\sin^6(e+fx))^2}d\sqrt[6]{\sin}}{af\sqrt{2-a^5\sin^6(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\int \frac{a^6\sin^6(e+fx)\sqrt{2-a^5\sin^6(e+fx)}}{(a-a^6\sin^6(e+fx))^2}d\sqrt[6]{\sin(e}}{af\sqrt{2-a^5\sin^6(e+fx)}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{6\sqrt{2}a^4\sin^6(e+fx)\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\operatorname{AppellF1}\left(\frac{7}{6},2,-\frac{1}{2},\right.}{7f\sqrt{2-a^5\sin^6(e+fx)}}$$

3.121. $\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}dx$

input
```
Int[Cot[e + f*x]^2/(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
(6*Sqrt[2]*a^4*AppellF1[7/6, 2, -1/2, 13/6, a^5*Sin[e + f*x]^6, (a^5*Sin[e
 + f*x]^6)/2]*Sin[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f
*x]]*Sqrt[2*a - a^6*Sin[e + f*x]^6]*Tan[e + f*x])/(7*f*Sqrt[2 - a^5*Sin[e
+ f*x]^6])
```

3.121.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.121.4 Maple [F]

$$\int \frac{\cot^2(fx+e)}{(a+a\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

output
```
int(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x)
```

3.121.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.121.6 Sympy [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt[3]{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(cot(f*x+e)**2/(a+a*sin(f*x+e))**(1/3),x)
```

output
```
Integral(cot(e + f*x)**2/(a*(sin(e + f*x) + 1))**(1/3), x)
```

3.121.7 Maxima [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

3.121.8 Giac [F]

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot\left(fx+e\right)^2}{\left(a\sin\left(fx+e\right)+a\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cot(f*x+e)^2/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^2/(a*sin(f*x + e) + a)^(1/3), x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^2(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{{\mathrm{cot}\left(e+f\,x\right)}^2}{{\left(a+a\,\sin\left(e+f\,x\right)\right)}^{1/3}}\,dx$$

input
```
int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/3),x)
```

output
```
int(cot(e + f*x)^2/(a + a*sin(e + f*x))^(1/3), x)
```

3.122 $\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$

3.122.1 Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \frac{12\sqrt{2}\,\text{AppellF1}\left(\frac{13}{6},-\frac{3}{2},4,\frac{19}{6},\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}(a+a\sin$$
$$13a^3f$$

output
```
12/13*AppellF1(13/6,4,-3/2,19/6,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e
)*(a+a*sin(f*x+e))^(8/3)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f
```

3.122.2 Mathematica [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx$$

input
```
Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
Integrate[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3), x]
```

3.122.3 Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.70, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 3198, 149, 1013, 27, 1012}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(e+fx)^4\sqrt[3]{a\sin(e+fx)+a}}dx$$

$$\Big\downarrow \text{3198}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{\csc^4(e+fx)(a-a\sin(e+fx))^{3/2}(\sin(e+fx)a+a)^{7/6}}{a^4}d(a\sin(e+fx))}{af}$$

$$\Big\downarrow \text{149}$$

$$\frac{6\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{a^{12}\sin^{12}(e+fx)\left(2a-a^6\sin^6(e+fx)\right)^{3/2}}{\left(a-a^6\sin^6(e+fx)\right)^4}d\sqrt[6]{\sin(e+fx)a+a}}{af}$$

$$\Big\downarrow \text{1013}$$

$$\frac{12\sqrt{2}\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\int \frac{a^{12}\sin^{12}(e+fx)\left(2-a^5\sin^6(e+fx)\right)^{3/2}}{2\sqrt{2}\left(a-a^6\sin^6(e+fx)\right)^4}a}{f\sqrt{2-a^5\sin^6(e+fx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{6\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\int \frac{a^{12}\sin^{12}(e+fx)\left(2-a^5\sin^6(e+fx)\right)^{3/2}}{\left(a-a^6\sin^6(e+fx)\right)^4}d\sqrt[6]{\mathrm{si}}}{f\sqrt{2-a^5\sin^6(e+fx)}}$$

$$\Big\downarrow \text{1012}$$

$$\frac{12\sqrt{2}a^9\sin^{12}(e+fx)\tan(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\sqrt{2a-a^6\sin^6(e+fx)}\,\text{AppellF1}\left(\frac{13}{6},4,-\right.}{13f\sqrt{2-a^5\sin^6(e+fx)}}$$

3.122. $\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}dx$

input
```
Int[Cot[e + f*x]^4/(a + a*Sin[e + f*x])^(1/3),x]
```

output
```
(12*Sqrt[2]*a^9*AppellF1[13/6, 4, -3/2, 19/6, a^5*Sin[e + f*x]^6, (a^5*Sin
[e + f*x]^6)/2]*Sin[e + f*x]^12*Sqrt[a - a*Sin[e + f*x]]*Sqrt[a + a*Sin[e
+ f*x]]*Sqrt[2*a - a^6*Sin[e + f*x]^6]*Tan[e + f*x])/(13*f*Sqrt[2 - a^5*Si
n[e + f*x]^6])
```

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 149
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_] :> With[{k = Denominator[m]}, Simp[k/b   Subst[Int[x^(k*(m + 1
) - 1)*(c - a*(d/b) + d*(x^k/b))^n*(e - a*(f/b) + f*(x^k/b))^p, x], x, (a +
 b*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && FractionQ[m] &&
IntegerQ[2*n] && IntegerQ[p]
```

rule 1012
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

rule 1013
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p])   Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
 FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &
& NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.122.4 Maple [F]

$$\int \frac{\cot^4(fx+e)}{(a+a\sin(fx+e))^{\frac{1}{3}}}dx$$

input
```
int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

output
```
int(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x)
```

3.122.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="fricas")
```

output
```
Timed out
```

3.122.6 Sympy [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot^4(e+fx)}{\sqrt[3]{a(\sin(e+fx)+1)}}\,dx$$

input
```
integrate(cot(f*x+e)**4/(a+a*sin(f*x+e))**(1/3),x)
```

output
```
Integral(cot(e + f*x)**4/(a*(sin(e + f*x) + 1))**(1/3), x)
```

3.122.7 Maxima [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot(fx+e)^4}{(a\sin(fx+e)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="maxima")
```

output
```
integrate(cot(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

3.122.8 Giac [F]

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot(fx+e)^4}{(a\sin(fx+e)+a)^{\frac{1}{3}}}\,dx$$

input
```
integrate(cot(f*x+e)^4/(a+a*sin(f*x+e))^(1/3),x, algorithm="giac")
```

output
```
integrate(cot(f*x + e)^4/(a*sin(f*x + e) + a)^(1/3), x)
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\cot^4(e+fx)}{\sqrt[3]{a+a\sin(e+fx)}}\,dx = \int \frac{\cot(e+f\,x)^4}{(a+a\,\sin(e+f\,x))^{1/3}}\,dx$$

input
```
int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/3),x)
```

output
```
int(cot(e + f*x)^4/(a + a*sin(e + f*x))^(1/3), x)
```

3.123 $\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx$

3.123.1 Optimal result

Integrand size = 23, antiderivative size = 269

$$\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx$$
$$= \frac{a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+p}{2},\frac{3+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}$$
$$+\frac{3a^3\cos^2(e+fx)^{\frac{1+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+p}{2},\frac{2+p}{2},\frac{4+p}{2},\sin^2(e+fx)\right)\sin(e+fx)(g\tan(e+fx))^{1+p}}{fg(2+p)}$$
$$+\frac{a^3\cos^2(e+fx)^{\frac{1+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+p}{2},\frac{4+p}{2},\frac{6+p}{2},\sin^2(e+fx)\right)\sin^3(e+fx)(g\tan(e+fx))^{1+p}}{fg(4+p)}$$
$$+\frac{3a^3\,\text{Hypergeometric2F1}\left(2,\frac{3+p}{2},\frac{5+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{3+p}}{fg^3(3+p)}$$

output
```
a^3*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(p+
1)/f/g/(p+1)+3*a^3*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*
p],[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(p+1)/f/g/(2+p)+a^3*(
cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([2+1/2*p, 1/2+1/2*p],[3+1/2*p],sin(f*x
+e)^2)*sin(f*x+e)^3*(g*tan(f*x+e))^(p+1)/f/g/(4+p)+3*a^3*hypergeom([2, 3/2
+1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/(3+p)
```

3.123.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 22.05 (sec) , antiderivative size = 4715, normalized size of antiderivative = 17.53

$$\int (a + a\sin(e + fx))^3 (g\tan(e + fx))^p \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

output
```
(4*(3 + p)*AppellF1[(1 + p)/2, p, 4, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Cos[(e + f*x)/2]^9*Sin[(e + f*x)/2]*(a + a*Sin[e + f*x])^3*
(g*Tan[e + f*x])^p)/(f*(1 + p)*(2*(4*AppellF1[(3 + p)/2, p, 5, (5 + p)/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - p*AppellF1[(3 + p)/2, 1 + p, 4,
 (5 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x])
+ (3 + p)*AppellF1[(1 + p)/2, p, 4, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2]*(1 + Cos[e + f*x]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6)
 + (24*(4 + p)*AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(
e + f*x)/2]^2]*Cos[(e + f*x)/2]^8*Sin[(e + f*x)/2]^2*(a + a*Sin[e + f*x])^
3*(g*Tan[e + f*x])^p)/(f*(2 + p)*(2*(4*AppellF1[2 + p/2, p, 5, 3 + p/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - p*AppellF1[2 + p/2, 1 + p, 4, 3 +
 p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*(-1 + Cos[e + f*x]) + (4 +
 p)*AppellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*(1 + Cos[e + f*x]))*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6) + (60*(Ap
pellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] - AppellF1[(1 + p)/2, p, 4, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x
)/2]^2])*Cos[(e + f*x)/2]^5*Sin[(e + f*x)/2]^3*(a + a*Sin[e + f*x])^3*(g*T
an[e + f*x])^p)/(f*(1 + p)*(2*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] - ((-2*(3*AppellF1[(3 + p)/2, p, 4, (5 +
 p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*AppellF1[(3 + p)/2,...
```

3.123.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^3(g\tan(e+fx))^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a\sin(e+fx)+a)^3(g\tan(e+fx))^p dx$$

$$\Big\downarrow \; 3189$$

$$\int \left(a^3(g\tan(e+fx))^p+a^3\sin^3(e+fx)(g\tan(e+fx))^p+3a^3\sin^2(e+fx)(g\tan(e+fx))^p+3a^3\sin(e+fx)(g\tan\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{3a^3(g\tan(e+fx))^{p+3}\,\text{Hypergeometric2F1}\left(2,\frac{p+3}{2},\frac{p+5}{2},-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^3(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,\frac{p+1}{2},\frac{p+3}{2},-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{3a^3\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{p+1}{2},\frac{p+2}{2},\frac{p+4}{2},\sin^2(e+fx)\right)}{fg(p+2)}+\frac{a^3\sin^3(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{p+1}{2},\frac{p+4}{2},\frac{p+6}{2},\sin^2(e+fx)\right)}{fg(p+4)}$$

input
```
Int[(a + a*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

output
```
(a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e
+ f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^3*(Cos[e + f*x]^2)^((1 + p)/2)*Hyper
geometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]
*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^3*(Cos[e + f*x]^2)^((1 + p)/
2)*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sin[
e + f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a^3*Hypergeometric
2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f
*g^3*(3 + p))
```

3.123. $\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx$

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.123.4 Maple [F]

$$\int (a + a\sin(fx + e))^3 (g\tan(fx + e))^p \, dx$$

input
```
int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

output
```
int((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

3.123.5 Fricas [F]

$$\int (a + a\sin(e + fx))^3 (g\tan(e + fx))^p \, dx = \int (a\sin(fx + e) + a)^3 (g\tan(fx + e))^p \, dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

output
```
integral(-(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x + e)^2 - 4*a^3)*sin
(f*x + e))*(g*tan(f*x + e))^p, x)
```

3.123.6 Sympy [F]

$$\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx = a^3\left(\int (g\tan{(e+fx)})^p\,dx + \int 3(g\tan{(e+fx)})^p\sin{(e+fx)}\,dx + \int 3(g\tan{(e+fx)})^p\sin^2{(e+fx)}\,dx + \int (g\tan{(e+fx)})^p\sin^3{(e+fx)}\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)
```

output
```
a**3*(Integral((g*tan(e + f*x))**p, x) + Integral(3*(g*tan(e + f*x))**p*si
n(e + f*x), x) + Integral(3*(g*tan(e + f*x))**p*sin(e + f*x)**2, x) + Inte
gral((g*tan(e + f*x))**p*sin(e + f*x)**3, x))
```

3.123.7 Maxima [F]

$$\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \int (a\sin{(fx+e)}+a)^3(g\tan{(fx+e)})^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

3.123.8 Giac [F]

$$\int (a+a\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \int (a\sin{(fx+e)}+a)^3(g\tan{(fx+e)})^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int (a + a\sin(e + fx))^3 (g\tan(e + fx))^p \,dx = \int (g\tan(e + f\,x))^p\,(a + a\,\sin(e + f\,x))^3\,dx$$

input
```
int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^3,x)
```

output
```
int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^3, x)
```

3.124 $\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx$

3.124.1 Optimal result

Integrand size = 23, antiderivative size = 187

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx$$
$$= \frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+p}{2},\frac{3+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)}$$
$$+ \frac{2a^2\cos^2(e+fx)^{\frac{1+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+p}{2},\frac{2+p}{2},\frac{4+p}{2},\sin^2(e+fx)\right)\sin(e+fx)(g\tan(e+fx))^{1+p}}{fg(2+p)}$$
$$+ \frac{a^2\,\text{Hypergeometric2F1}\left(2,\frac{3+p}{2},\frac{5+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{3+p}}{fg^3(3+p)}$$

output
```
a^2*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(p+
1)/f/g/(p+1)+2*a^2*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*
p],[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(p+1)/f/g/(2+p)+a^2*h
ypergeom([2, 3/2+1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/
g^3/(3+p)
```

3.124.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 17.28 (sec) , antiderivative size = 2054, normalized size of antiderivative = 10.98

$$\int (a + a\sin(e + fx))^2 (g\tan(e + fx))^p \, dx = \text{Result too large to show}$$

input
```
Integrate[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

output
```
(2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(a + a*Sin[e + f*x])^2*Tan[(e + f*x
)/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e
+ f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3
 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(1 + p)*AppellF1[1 +
 p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x
)/2])*(g*Tan[e + f*x])^p*(Cos[(e + f*x)/2]^4*Tan[e + f*x]^p + 4*Cos[(e + f
*x)/2]^3*Sin[(e + f*x)/2]*Tan[e + f*x]^p + 6*Cos[(e + f*x)/2]^2*Sin[(e + f
*x)/2]^2*Tan[e + f*x]^p + 4*Cos[(e + f*x)/2]*Sin[(e + f*x)/2]^3*Tan[e + f*
x]^p + Sin[(e + f*x)/2]^4*Tan[e + f*x]^p))/(f*(1 + p)*(2 + p)*(Cos[(e + f*
x)/2] + Sin[(e + f*x)/2])^4*((2*p*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Sec[
e + f*x]^2*Tan[(e + f*x)/2]*((2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2,
Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2 + p)*AppellF1[(1 + p)/2, p
, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - 4*(2 + p)*Appel
lF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 4*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2]*Tan[(e + f*x)/2])*Tan[e + f*x]^(-1 + p))/((1 + p)*(2 + p)) + (S
ec[(e + f*x)/2]^2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*((2 + p)*AppellF1[(1
 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*(2
+ p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e +...
```

3.124.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^2(g\tan(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a\sin(e+fx)+a)^2(g\tan(e+fx))^p dx$$

$$\Big\downarrow \text{3189}$$

$$\int \left(a^2(g\tan(e+fx))^p+a^2\sin^2(e+fx)(g\tan(e+fx))^p+2a^2\sin(e+fx)(g\tan(e+fx))^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(g\tan(e+fx))^{p+3}\,\text{Hypergeometric2F1}\left(2,\frac{p+3}{2},\frac{p+5}{2},-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{a^2(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,\frac{p+1}{2},\frac{p+3}{2},-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2a^2\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{p+1}{2},\frac{p+2}{2},\frac{p+4}{2},\sin^2(e+fx)\right)}{fg(p+2)}$$

input
```
Int[(a + a*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

output
```
(a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e
+ f*x])^(1 + p))/(f*g*(1 + p)) + (2*a^2*(Cos[e + f*x]^2)^((1 + p)/2)*Hyper
geometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]
*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (a^2*Hypergeometric2F1[2, (3 +
p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p)
)
```

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.124.4 Maple [F]

$$\int (a + a\sin(fx+e))^2 (g\tan(fx+e))^p\,dx$$

input
```
int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

output
```
int((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

3.124.5 Fricas [F]

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int (a\sin(fx+e)+a)^2(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

output
```
integral(-(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*a^2)*(g*tan(f*x + e
))^p, x)
```

3.124.6 Sympy [F]

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx = a^2\left(\int (g\tan(e+fx))^p\,dx + \int 2(g\tan(e+fx))^p\sin(e+fx)\,dx + \int (g\tan(e+fx))^p\sin^2(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)
```

output
```
a**2*(Integral((g*tan(e + f*x))**p, x) + Integral(2*(g*tan(e + f*x))**p*si
n(e + f*x), x) + Integral((g*tan(e + f*x))**p*sin(e + f*x)**2, x))
```

3.124.7 Maxima [F]

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx=\int (a\sin{(fx+e)}+a)^2(g\tan{(fx+e)})^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

3.124.8 Giac [F]

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx=\int (a\sin{(fx+e)}+a)^2(g\tan{(fx+e)})^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^2(g\tan(e+fx))^p\,dx=\int (g\tan(e+f\,x))^p\,(a+a\,\sin{(e+f\,x)})^2\,dx$$

input
```
int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^2,x)
```

output
```
int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^2, x)
```

3.125 $\int (a + a\sin(e + fx))(g\tan(e + fx))^p \, dx$

3.125.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int (a + a\sin(e + fx))(g\tan(e + fx))^p \, dx = \frac{a \operatorname{Hypergeometric2F1}\left(1, \frac{1+p}{2}, \frac{3+p}{2}, -\tan^2(e + fx)\right) (g\tan(e + fx))^{1+p}}{fg(1 + p)} + \frac{a\cos^2(e + fx)^{\frac{1+p}{2}} \operatorname{Hypergeometric2F1}\left(\frac{1+p}{2}, \frac{2+p}{2}, \frac{4+p}{2}, \sin^2(e + fx)\right) \sin(e + fx)(g\tan(e + fx))^{1+p}}{fg(2 + p)}$$

output
```
a*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(p+1)
/f/g/(p+1)+a*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p],[2+
1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(p+1)/f/g/(2+p)
```

3.125.2 Mathematica [F]

$$\int (a + a\sin(e + fx))(g\tan(e + fx))^p \, dx = \int (a + a\sin(e + fx))(g\tan(e + fx))^p \, dx$$

input
```
Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p, x]
```

3.125.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3189, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)(g\tan(e+fx))^p\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (a\sin(e+fx)+a)(g\tan(e+fx))^pdx$$

$$\Big\downarrow \ 3189$$

$$\int \left(a(g\tan(e+fx))^p+a\sin(e+fx)(g\tan(e+fx))^p\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,\frac{p+1}{2},\frac{p+3}{2},-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{a\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{p+1}{2},\frac{p+2}{2},\frac{p+4}{2},\sin^2(e+fx)\right)}{fg(p+2)}$$

input
```
Int[(a + a*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

output
```
(a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
f*x])^(1 + p))/(f*g*(1 + p)) + (a*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeomet
ric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Ta
n[e + f*x])^(1 + p))/(f*g*(2 + p))
```

3.125.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3189
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.125.4 Maple [F]

$$\int (a + a\sin(fx + e))(g\tan(fx + e))^p\,dx$$

input
```
int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

output
```
int((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

3.125.5 Fricas [F]

$$\int (a + a\sin(e + fx))(g\tan(e + fx))^p\,dx = \int (a\sin(fx + e) + a)(g\tan(fx + e))^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

3.125.6 Sympy [F]

$$\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx = a\left(\int (g\tan(e+fx))^p\,dx + \int (g\tan(e+fx))^p\sin(e+fx)\,dx\right)$$

input
```
integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))**p,x)
```

output
```
a*(Integral((g*tan(e + f*x))**p, x) + Integral((g*tan(e + f*x))**p*sin(e +
 f*x), x))
```

3.125.7 Maxima [F]

$$\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx = \int (a\sin(fx+e)+a)(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

3.125.8 Giac [F]

$$\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx = \int (a\sin(fx+e)+a)(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))(g\tan(e+fx))^p\,dx = \int (g\tan(e+f\,x))^p\,(a+a\,\sin(e+f\,x))\,dx$$

input
```
int((g*tan(e + f*x))^p*(a + a*sin(e + f*x)),x)
```

output
```
int((g*tan(e + f*x))^p*(a + a*sin(e + f*x)), x)
```

3.126 $\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx$

3.126.1 Optimal result

Integrand size = 23, antiderivative size = 108

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx=\frac{(g\tan(e+fx))^{1+p}}{afg(1+p)}-\frac{\cos^2(e+fx)^{\frac{3+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{2+p}{2},\frac{3+p}{2},\frac{4+p}{2},\sin^2(e+fx)\right)\sec(e+fx)(g\tan(e+fx))^{2+p}}{afg^2(2+p)}$$

output
```
(g*tan(f*x+e))^(p+1)/a/f/g/(p+1)-(cos(f*x+e)^2)^(3/2+1/2*p)*hypergeom([1+1
/2*p, 3/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)*(g*tan(f*x+e))^(2+p)/a
/f/g^2/(2+p)
```

3.126.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 232 vs. 2(108) = 216.

Time = 2.28 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.15

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx=\frac{2\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^p\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^2\tan\left(\frac{1}{2}(e+fx)\right)\left(\left(6+5p+p^2\right)\text{Hy}\right.}{}$$

input
```
Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]
```

output
```
(2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^2*Tan[(e + f*x)/2]*((6 + 5*p + p^2)*Hypergeometric2F1[(1 + p)/2, 2 + p,
 (3 + p)/2, Tan[(e + f*x)/2]^2] - (1 + p)*Tan[(e + f*x)/2]*(2*(3 + p)*Hype
rgeometric2F1[(2 + p)/2, 2 + p, (4 + p)/2, Tan[(e + f*x)/2]^2] - (2 + p)*H
ypergeometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e +
 f*x)/2]))*(g*Tan[e + f*x])^p)/(f*(1 + p)*(2 + p)*(3 + p)*(a + a*Sin[e + f
*x]))
```

3.126.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 3185, 3042, 3087, 17, 3097}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\tan(e+fx))^p}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(g\tan(e+fx))^p}{a\sin(e+fx)+a}dx$$

$$\Big\downarrow \text{3185}$$

$$\frac{\int \sec^2(e+fx)(g\tan(e+fx))^pdx}{a}-\frac{\int \sec(e+fx)(g\tan(e+fx))^{p+1}dx}{ag}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sec(e+fx)^2(g\tan(e+fx))^pdx}{a}-\frac{\int \sec(e+fx)(g\tan(e+fx))^{p+1}dx}{ag}$$

$$\Big\downarrow \text{3087}$$

$$\frac{\int (g\tan(e+fx))^pd\tan(e+fx)}{af}-\frac{\int \sec(e+fx)(g\tan(e+fx))^{p+1}dx}{ag}$$

$$\Big\downarrow \text{17}$$

$$\frac{(g\tan(e+fx))^{p+1}}{afg(p+1)}-\frac{\int \sec(e+fx)(g\tan(e+fx))^{p+1}dx}{ag}$$

$$\Big\downarrow \text{3097}$$

$$\frac{(g\tan(e+fx))^{p+1}}{afg(p+1)} - \frac{\sec(e+fx)\cos^2(e+fx)^{\frac{p+3}{2}}(g\tan(e+fx))^{p+2}\,\text{Hypergeometric2F1}\left(\frac{p+2}{2},\frac{p+3}{2},\frac{p+4}{2},\sin^2(e+fx)\right)}{afg^2(p+2)}$$

input
```
Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x]),x]
```

output
```
(g*Tan[e + f*x])^(1 + p)/(a*f*g*(1 + p)) - ((Cos[e + f*x]^2)^((3 + p)/2)*H
ypergeometric2F1[(2 + p)/2, (3 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sec[e +
f*x]*(g*Tan[e + f*x])^(2 + p))/(a*f*g^2*(2 + p))
```

3.126.3.1 Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3097
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n + 1)*((Cos[e
+ f*x]^2)^((m + n + 1)/2)/(b*f*(n + 1)))*Hypergeometric2F1[(n + 1)/2, (m +
n + 1)/2, (n + 3)/2, Sin[e + f*x]^2], x] /; FreeQ[{a, b, e, f, m, n}, x] &&
  !IntegerQ[(n - 1)/2] &&  !IntegerQ[m/2]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

3.126.4 Maple [F]

$$\int \frac{(g\tan(fx+e))^p}{a+a\sin(fx+e)}dx$$

input
```
int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)
```

output
```
int((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x)
```

3.126.5 Fricas [F]

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx = \int \frac{(g\tan(fx+e))^p}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

3.126.6 Sympy [F]

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx = \frac{\int \frac{(g\tan(e+fx))^p}{\sin(e+fx)+1}\,dx}{a}$$

input
```
integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e)),x)
```

output
```
Integral((g*tan(e + f*x))**p/(sin(e + f*x) + 1), x)/a
```

3.126.7 Maxima [F]

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx = \int \frac{(g\tan(fx+e))^p}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

3.126.8 Giac [F]

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx = \int \frac{(g\tan(fx+e))^p}{a\sin(fx+e)+a}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a), x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g\tan(e+fx))^p}{a+a\sin(e+fx)}\,dx = \int \frac{(g\tan(e+f\,x))^p}{a+a\,\sin(e+f\,x)}\,dx$$

input
```
int((g*tan(e + f*x))^p/(a + a*sin(e + f*x)),x)
```

output
```
int((g*tan(e + f*x))^p/(a + a*sin(e + f*x)), x)
```

3.127 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx$

3.127.1 Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \frac{(g\tan(e+fx))^{1+p}}{a^2fg(1+p)} - \frac{2\cos^2(e+fx)^{\frac{5+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{2+p}{2},\frac{5+p}{2},\frac{4+p}{2},\sin^2(e+fx)\right)\sec^3(e+fx)(g\tan(e+fx))^{2+p}}{a^2fg^2(2+p)} + \frac{2(g\tan(e+fx))^{3+p}}{a^2fg^3(3+p)}$$

output
```
(g*tan(f*x+e))^(p+1)/a^2/f/g/(p+1)-2*(cos(f*x+e)^2)^(5/2+1/2*p)*hypergeom(
[1+1/2*p, 5/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)^3*(g*tan(f*x+e))^(
2+p)/a^2/f/g^2/(2+p)+2*(g*tan(f*x+e))^(3+p)/a^2/f/g^3/(3+p)
```

3.127.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 626 vs. 2(138) = 276.

Time = 6.13 (sec) , antiderivative size = 626, normalized size of antiderivative = 4.54

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \frac{2\left(\cos(e+fx)\sec^2\left(\frac{1}{2}(e+fx)\right)\right)^p\left(\cos\left(\frac{1}{2}(e+fx)\right)+\sin\left(\frac{1}{2}(e+fx)\right)\right)^4\tan\left(\frac{1}{2}(e+fx)\right)\left(\frac{\text{Hypergeometric2F1}(}{}\right.}{}$$

input
```
Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

output
```
(2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2
])^4*Tan[(e + f*x)/2]*(Hypergeometric2F1[(1 + p)/2, 2 + p, (3 + p)/2, Tan[
(e + f*x)/2]^2]/(1 + p) - (2*Hypergeometric2F1[(1 + p)/2, 3 + p, (3 + p)/2
, Tan[(e + f*x)/2]^2])/(1 + p) + (2*Hypergeometric2F1[(1 + p)/2, 4 + p, (3
 + p)/2, Tan[(e + f*x)/2]^2])/(1 + p) - (2*Hypergeometric2F1[(2 + p)/2, 2
+ p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (6*Hyperge
ometric2F1[(2 + p)/2, 3 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/
2])/(2 + p) - (8*Hypergeometric2F1[(2 + p)/2, 4 + p, (4 + p)/2, Tan[(e + f
*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (Hypergeometric2F1[2 + p, (3 + p)/2,
 (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) - (6*Hypergeom
etric2F1[(3 + p)/2, 3 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]
^2)/(3 + p) + (12*Hypergeometric2F1[(3 + p)/2, 4 + p, (5 + p)/2, Tan[(e +
f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (2*Hypergeometric2F1[3 + p, (4 +
p)/2, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^3)/(4 + p) - (8*Hype
rgeometric2F1[(4 + p)/2, 4 + p, (6 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*
x)/2]^3)/(4 + p) + (2*Hypergeometric2F1[4 + p, (5 + p)/2, (7 + p)/2, Tan[(
e + f*x)/2]^2]*Tan[(e + f*x)/2]^4)/(5 + p))*(g*Tan[e + f*x])^p)/(f*(a + a*
Sin[e + f*x])^2)
```

3.127.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3190, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\tan(e+fx))^p}{(a\sin(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(g\tan(e+fx))^p}{(a\sin(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3190}$$

$$\frac{\int \left(a^2\sec^4(e+fx)(g\tan(e+fx))^p + a^2\sec^2(e+fx)\tan^2(e+fx)(g\tan(e+fx))^p - 2a^2\sec^3(e+fx)\tan(e+fx\right.}{a^4}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{2a^2(g\tan(e+fx))^{p+3}}{fg^3(p+3)} - \frac{2a^2\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+5}{2}}(g\tan(e+fx))^{p+2}\text{Hypergeometric2F1}\left(\frac{p+2}{2},\frac{p+5}{2},\frac{p+4}{2},\sin^2(e+fx)\right)}{fg^2(p+2)} + \frac{a^2(g\tan(e+fx)}{fg(p+1)}}{a^4}$$

input
```
Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^2,x]
```

output
```
((a^2*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) - (2*a^2*(Cos[e + f*x]^2)^((
5 + p)/2)*Hypergeometric2F1[(2 + p)/2, (5 + p)/2, (4 + p)/2, Sin[e + f*x]^
2]*Sec[e + f*x]^3*(g*Tan[e + f*x])^(2 + p))/(f*g^2*(2 + p)) + (2*a^2*(g*Ta
n[e + f*x])^(3 + p))/(f*g^3*(3 + p)))/a^4
```

3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3190
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Simp[a^(2*m)   Int[ExpandIntegrand[(g*Tan[e + f*x]
)^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; F
reeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

3.127.4 Maple [F]

$$\int \frac{(g\tan(fx+e))^p}{(a+a\sin(fx+e))^2}dx$$

input
```
int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)
```

output
```
int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x)
```

3.127.5 Fricas [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(g\tan(fx+e))^p}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(-(g*tan(f*x + e))^p/(a^2*cos(f*x + e)^2 - 2*a^2*sin(f*x + e) - 2*
a^2), x)
```

3.127.6 Sympy [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \frac{\int \frac{(g\tan(e+fx))^p}{\sin^2(e+fx)+2\sin(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**2,x)
```

output
```
Integral((g*tan(e + f*x))**p/(sin(e + f*x)**2 + 2*sin(e + f*x) + 1), x)/a*
*2
```

3.127.7 Maxima [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(g\tan(fx+e))^p}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

3.127.8 Giac [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(g\tan(fx+e))^p}{(a\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^2, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^2}\,dx = \int \frac{(g\tan(e+f\,x))^p}{(a+a\,\sin(e+f\,x))^2}\,dx$$

input
```
int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^2,x)
```

output
```
int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^2, x)
```

3.128 $\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx$

3.128.1 Optimal result

Integrand size = 23, antiderivative size = 248

$$\begin{aligned}\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx &= \frac{(g\tan(e+fx))^{1+p}}{a^3fg(1+p)}\\
&- \frac{3\cos^2(e+fx)^{\frac{7+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{2+p}{2},\frac{7+p}{2},\frac{4+p}{2},\sin^2(e+fx)\right)\sec^5(e+fx)(g\tan(e+fx))^{2+p}}{a^3fg^2(2+p)}\\
&+ \frac{5(g\tan(e+fx))^{3+p}}{a^3fg^3(3+p)}\\
&- \frac{\cos^2(e+fx)^{\frac{7+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{4+p}{2},\frac{7+p}{2},\frac{6+p}{2},\sin^2(e+fx)\right)\sec^3(e+fx)(g\tan(e+fx))^{4+p}}{a^3fg^4(4+p)}\\
&+ \frac{4(g\tan(e+fx))^{5+p}}{a^3fg^5(5+p)}\end{aligned}$$

output
```
(g*tan(f*x+e))^(p+1)/a^3/f/g/(p+1)-3*(cos(f*x+e)^2)^(7/2+1/2*p)*hypergeom(
[1+1/2*p, 7/2+1/2*p],[2+1/2*p],sin(f*x+e)^2)*sec(f*x+e)^5*(g*tan(f*x+e))^(
2+p)/a^3/f/g^2/(2+p)+5*(g*tan(f*x+e))^(3+p)/a^3/f/g^3/(3+p)-(cos(f*x+e)^2)
^(7/2+1/2*p)*hypergeom([2+1/2*p, 7/2+1/2*p],[3+1/2*p],sin(f*x+e)^2)*sec(f*
x+e)^3*(g*tan(f*x+e))^(4+p)/a^3/f/g^4/(4+p)+4*(g*tan(f*x+e))^(5+p)/a^3/f/g
^5/(5+p)
```

3.128.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1276 vs. 2(248) = 496.

Time = 17.38 (sec) , antiderivative size = 1276, normalized size of antiderivative = 5.15

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]
```

output
```
(2^(1 + p)*(Cos[(e + f*x)/2] + Sin[(e + f*x)/2])^6*Tan[(e + f*x)/2]*(1 - T
an[(e + f*x)/2]^2)^p*(-(Tan[(e + f*x)/2]/(-1 + Tan[(e + f*x)/2]^2)))^p*(Hy
pergeometric2F1[(1 + p)/2, 2 + p, (3 + p)/2, Tan[(e + f*x)/2]^2]/(1 + p) -
 (4*Hypergeometric2F1[(1 + p)/2, 3 + p, (3 + p)/2, Tan[(e + f*x)/2]^2])/(1
 + p) + (8*Hypergeometric2F1[(1 + p)/2, 4 + p, (3 + p)/2, Tan[(e + f*x)/2]
^2])/(1 + p) - (8*Hypergeometric2F1[(1 + p)/2, 5 + p, (3 + p)/2, Tan[(e +
f*x)/2]^2])/(1 + p) + (4*Hypergeometric2F1[(1 + p)/2, 6 + p, (3 + p)/2, Ta
n[(e + f*x)/2]^2])/(1 + p) - (2*Hypergeometric2F1[(2 + p)/2, 2 + p, (4 + p
)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) + (12*Hypergeometric2F1
[(2 + p)/2, 3 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p
) - (32*Hypergeometric2F1[(2 + p)/2, 4 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]
*Tan[(e + f*x)/2])/(2 + p) + (40*Hypergeometric2F1[(2 + p)/2, 5 + p, (4 +
p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 + p) - (24*Hypergeometric2F
1[(2 + p)/2, 6 + p, (4 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])/(2 +
p) + (Hypergeometric2F1[2 + p, (3 + p)/2, (5 + p)/2, Tan[(e + f*x)/2]^2]*T
an[(e + f*x)/2]^2)/(3 + p) - (12*Hypergeometric2F1[(3 + p)/2, 3 + p, (5 +
p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (48*Hypergeometric
2F1[(3 + p)/2, 4 + p, (5 + p)/2, Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]^2)/(
3 + p) - (80*Hypergeometric2F1[(3 + p)/2, 5 + p, (5 + p)/2, Tan[(e + f*x)/
2]^2]*Tan[(e + f*x)/2]^2)/(3 + p) + (60*Hypergeometric2F1[(3 + p)/2, 6 ...
```

3.128.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3042, 3190, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\tan(e+fx))^p}{(a\sin(e+fx)+a)^3}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{(g\tan(e+fx))^p}{(a\sin(e+fx)+a)^3}dx$$

$\Big\downarrow$ 3190

$$\frac{\int \left(a^3\sec^6(e+fx)(g\tan(e+fx))^p - a^3\sec^3(e+fx)\tan^3(e+fx)(g\tan(e+fx))^p + 3a^3\sec^4(e+fx)\tan^2(e+f\right.}{a^6}$$

$\Big\downarrow$ 2009

$$\frac{\frac{4a^3(g\tan(e+fx))^{p+5}}{fg^5(p+5)} - \frac{a^3\sec^3(e+fx)\cos^2(e+fx)^{\frac{p+7}{2}}(g\tan(e+fx))^{p+4}\text{Hypergeometric2F1}\left(\frac{p+4}{2},\frac{p+7}{2},\frac{p+6}{2},\sin^2(e+fx)\right)}{fg^4(p+4)} + \frac{5a^3(g\tan(e+fx)}{fg^3(p+3)}}{a^6}$$

input
```
Int[(g*Tan[e + f*x])^p/(a + a*Sin[e + f*x])^3,x]
```

output
```
((a^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p)) - (3*a^3*(Cos[e + f*x]^2)^((
7 + p)/2)*Hypergeometric2F1[(2 + p)/2, (7 + p)/2, (4 + p)/2, Sin[e + f*x]^
2]*Sec[e + f*x]^5*(g*Tan[e + f*x])^(2 + p))/(f*g^2*(2 + p)) + (5*a^3*(g*Ta
n[e + f*x])^(3 + p))/(f*g^3*(3 + p)) - (a^3*(Cos[e + f*x]^2)^((7 + p)/2)*H
ypergeometric2F1[(4 + p)/2, (7 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Sec[e +
f*x]^3*(g*Tan[e + f*x])^(4 + p))/(f*g^4*(4 + p)) + (4*a^3*(g*Tan[e + f*x])
^(5 + p))/(f*g^5*(5 + p)))/a^6
```

3.128.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3190
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Simp[a^(2*m)   Int[ExpandIntegrand[(g*Tan[e + f*x]
)^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; F
reeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

3.128.4 Maple [F]

$$\int \frac{(g\tan(fx+e))^p}{(a+a\sin(fx+e))^3}dx$$

input
```
int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)
```

output
```
int((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x)
```

3.128.5 Fricas [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(g\tan(fx+e))^p}{(a\sin(fx+e)+a)^3}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="fricas")
```

output
```
integral(-(g*tan(f*x + e))^p/(3*a^3*cos(f*x + e)^2 - 4*a^3 + (a^3*cos(f*x
+ e)^2 - 4*a^3)*sin(f*x + e)), x)
```

3.128.6 Sympy [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \frac{\int \frac{(g\tan(e+fx))^p}{\sin^3(e+fx)+3\sin^2(e+fx)+3\sin(e+fx)+1}\,dx}{a^3}$$

input
```
integrate((g*tan(f*x+e))**p/(a+a*sin(f*x+e))**3,x)
```

output
```
Integral((g*tan(e + f*x))**p/(sin(e + f*x)**3 + 3*sin(e + f*x)**2 + 3*sin(
e + f*x) + 1), x)/a**3
```

3.128.7 Maxima [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(g\tan{(fx+e)})^p}{(a\sin{(fx+e)}+a)^3}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)
```

3.128.8 Giac [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(g\tan{(fx+e)})^p}{(a\sin{(fx+e)}+a)^3}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+a*sin(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((g*tan(f*x + e))^p/(a*sin(f*x + e) + a)^3, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g\tan(e+fx))^p}{(a+a\sin(e+fx))^3}\,dx = \int \frac{(g\tan(e+f\,x))^p}{(a+a\,\sin{(e+f\,x)})^3}\,dx$$

input
```
int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^3,x)
```

output
```
int((g*tan(e + f*x))^p/(a + a*sin(e + f*x))^3, x)
```

3.129 $\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx$

3.129.1 Optimal result

Integrand size = 23, antiderivative size = 111

$$\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$
$$=\frac{\text{AppellF1}\left(1+p,\frac{1+p}{2},\frac{1}{2}(1-2m+p),2+p,\sin(e+fx),-\sin(e+fx)\right)(1-\sin(e+fx))^{\frac{1+p}{2}}(1+\sin(e+}{fg(1+p)}$$

output
```
AppellF1(p+1,1/2-m+1/2*p,1/2+1/2*p,2+p,-sin(f*x+e),sin(f*x+e))*(1-sin(f*x+
e))^(1/2+1/2*p)*(1+sin(f*x+e))^(1/2-m+1/2*p)*(a+a*sin(f*x+e))^m*(g*tan(f*x
+e))^(p+1)/f/g/(p+1)
```

3.129.2 Mathematica [F]

$$\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

3.129.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.41, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 3199, 152, 152, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m(g\tan(e+fx))^p\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^m(g\tan(e+fx))^pdx$$

$$\Big\downarrow \text{ 3199}$$

$$\frac{(a\sin(e+fx))^{-p-1}(a-a\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx)+a)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\int(a\sin(e+fx))^p(a-a\sin(e}{fg}$$

$$\Big\downarrow \text{ 152}$$

$$\frac{(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx))^{-p-1}(a-a\sin(e+fx))^{\frac{1}{2}(-p-1)+\frac{p+1}{2}}(a\sin(e+fx)+a)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}}{fg}$$

$$\Big\downarrow \text{ 152}$$

$$(1-\sin(e+fx))^{\frac{p+1}{2}}(a\sin(e+fx))^{-p-1}(a-a\sin(e+fx))^{\frac{1}{2}(-p-1)+\frac{p+1}{2}}(g\tan(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{(1-\sin(e+fx))^{\frac{p+1}{2}}(a-a\sin(e+fx))^{\frac{1}{2}(-p-1)+\frac{p+1}{2}}(g\tan(e+fx))^{p+1}(\sin(e+fx)+1)^{\frac{1}{2}(-2m+p+1)}(a\sin(e+fx)}{fg(p+1)}$$

input
```
Int[(a + a*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

output
```
(AppellF1[1 + p, (1 + p)/2, (1 - 2*m + p)/2, 2 + p, Sin[e + f*x], -Sin[e +
 f*x]]*(1 - Sin[e + f*x])^((1 + p)/2)*(1 + Sin[e + f*x])^((1 - 2*m + p)/2)
*(a - a*Sin[e + f*x])^((-1 - p)/2 + (1 + p)/2)*(a + a*Sin[e + f*x])^(-1/2
+ m - p/2 + (1 + p)/2)*(g*Tan[e + f*x])^(1 + p))/(f*g*(1 + p))
```

3.129.3.1 Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 152
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^IntPart[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])
Int[(b*x)^m*(1 + d*(x/c))^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m,
 n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3199
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_), x_Symbol] :> Simp[(g*Tan[e + f*x])^(p + 1)*(a - b*Sin[e + f*x])^
((p + 1)/2)*((a + b*Sin[e + f*x])^((p + 1)/2)/(f*g*(b*Sin[e + f*x])^(p + 1)
))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2)), x], x, b*
Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &
&  !IntegerQ[m] && !IntegerQ[p]
```

3.129.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (g\tan(fx + e))^p \, dx$$

input
```
int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

output
```
int((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

3.129.5 Fricas [F]

$$\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (a\sin\left(fx+e\right)+a)^m(g\tan\left(fx+e\right))^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

3.129.6 Sympy [F]

$$\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (a(\sin\left(e+fx\right)+1))^m\left(g\tan\left(e+fx\right)\right)^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*(g*tan(f*x+e))**p,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*(g*tan(e + f*x))**p, x)
```

3.129.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (a\sin\left(fx+e\right)+a)^m(g\tan\left(fx+e\right))^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

3.129.8 Giac [F]

$$\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (a\sin(fx+e)+a)^m(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (g\tan(e+f\,x))^p\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^m,x)
```

output
```
int((g*tan(e + f*x))^p*(a + a*sin(e + f*x))^m, x)
```

3.130 $\int (a+a\sin(e+fx))^m \tan^3(e+fx)\,dx$

3.130.1 Optimal result

Integrand size = 21, antiderivative size = 163

$$\int (a+a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{a(4+m)\,\text{Hypergeometric2F1}\left(1,-1+m,m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{-1+m}}{4f(1-m)} - \frac{a^2\sin^2(e+fx)(a+a\sin(e+fx))^{-1+m}}{fm(a-a\sin(e+fx))} + \frac{(a+a\sin(e+fx))^{-1+m}\left(a(2-3m-m^2)+2am\sin(e+fx)\right)}{2f(1-m)m(1-\sin(e+fx))}$$

output

```
1/4*a*(4+m)*hypergeom([1, -1+m],[m],1/2+1/2*sin(f*x+e))*(a+a*sin(f*x+e))^(
-1+m)/f/(1-m)-a^2*sin(f*x+e)^2*(a+a*sin(f*x+e))^(-1+m)/f/m/(a-a*sin(f*x+e)
)+1/2*(a+a*sin(f*x+e))^(-1+m)*(a*(-m^2-3*m+2)+2*a*m*sin(f*x+e))/f/(1-m)/m/
(1-sin(f*x+e))
```

3.130.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int (a+a\sin(e+fx))^m \tan^3(e+fx)\,dx = \frac{a(a(1+\sin(e+fx)))^{-1+m}\left(-2(-2+3m+m^2)-m(4+m)\,\text{Hypergeometric2F1}\left(1,-1+m,m,\frac{1}{2}(1+\text{s}\right.\right.}{4f(-1+m)m(-1+\sin(e+f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
(a*(a*(1 + Sin[e + f*x]))^(-1 + m)*(-2*(-2 + 3*m + m^2) - m*(4 + m)*Hyperg
eometric2F1[1, -1 + m, m, (1 + Sin[e + f*x])/2]*(-1 + Sin[e + f*x]) + 4*m*
Sin[e + f*x] + 4*(-1 + m)*Sin[e + f*x]^2))/(4*f*(-1 + m)*m*(-1 + Sin[e + f
*x]))
```

3.130.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3186, 111, 25, 27, 163, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^3(a\sin(e+fx)+a)^m dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a^3\sin^3(e+fx)(\sin(e+fx)a+a)^{m-2}}{(a-a\sin(e+fx))^2}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{111}$$

$$\frac{-\frac{\int -\frac{a^2\sin(e+fx)(\sin(e+fx)a+a)^{m-2}(m\sin(e+fx)a+2a)}{(a-a\sin(e+fx))^2}d(a\sin(e+fx))}{m}-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-1}}{m(a-a\sin(e+fx))}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a^2\sin(e+fx)(\sin(e+fx)a+a)^{m-2}(m\sin(e+fx)a+2a)}{(a-a\sin(e+fx))^2}d(a\sin(e+fx))}{m}-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-1}}{m(a-a\sin(e+fx))}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{a\sin(e+fx)(\sin(e+fx)a+a)^{m-2}(m\sin(e+fx)a+2a)}{(a-a\sin(e+fx))^2}d(a\sin(e+fx))}{m}-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-1}}{m(a-a\sin(e+fx))}}{f}$$

$$\Big\downarrow 163$$

$$\frac{\frac{a\left(\frac{(a\sin(e+fx)+a)^{m-1}\left(2am\sin(e+fx)+a\left(-m^2-3m+2\right)\right)}{2(1-m)(a-a\sin(e+fx))}-\frac{1}{2}am(m+4)\int\frac{(\sin(e+fx)a+a)^{m-2}}{a-a\sin(e+fx)}d(a\sin(e+fx))\right)}{m}-\frac{a^2\sin^2(e+fx)(a\sin(e+fx)+a)^{m-}}{m(a-a\sin(e+fx))}}{f}$$

$$\Big\downarrow 78$$

$$\frac{\frac{a\left(\frac{m(m+4)(a\sin(e+fx)+a)^{m-1}\text{Hypergeometric2F1}\left(1,m-1,m,\frac{\sin(e+fx)a+a}{2a}\right)}{4(1-m)}+\frac{\left(2am\sin(e+fx)+a\left(-m^2-3m+2\right)\right)(a\sin(e+fx)+a)^{m-1}}{2(1-m)(a-a\sin(e+fx))}\right)}{m}-\frac{a^2\sin^2(e+}{m(}}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^3,x]
```

output
```
(-((a^2*Sin[e + f*x]^2*(a + a*Sin[e + f*x])^(-1 + m))/(m*(a - a*Sin[e + f*
x]))) + (a*((m*(4 + m)*Hypergeometric2F1[1, -1 + m, m, (a + a*Sin[e + f*x]
)/(2*a)]*(a + a*Sin[e + f*x])^(-1 + m))/(4*(1 - m)) + ((a + a*Sin[e + f*x]
)^(-1 + m)*(a*(2 - 3*m - m^2) + 2*a*m*Sin[e + f*x]))/(2*(1 - m)*(a - a*Sin
[e + f*x]))))/m)/f
```

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 163
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(m + n
+ 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c - a*d)*
(m + 1)*x)/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)))*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1), x] - Simp[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f
*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*
d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*
d*(b*c - a*d)*(m + 1)*(m + n + 3))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ[m, -
1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.130.4 Maple [F]

$$\int (a + a\sin(fx + e))^m (\tan^3(fx + e))\, dx$$

input
```
int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

output
```
int((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x)
```

3.130.5 Fricas [F]

$$\int (a+a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin{(fx+e)}+a)^m\tan{(fx+e)}^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

3.130.6 Sympy [F]

$$\int (a+a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a(\sin{(e+fx)}+1))^m\tan^3{(e+fx)}\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**3,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x)**3, x)
```

3.130.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin{(fx+e)}+a)^m\tan{(fx+e)}^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

3.130.8 Giac [F]

$$\int (a+a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int (a\sin(fx+e)+a)^m\tan(fx+e)^3\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^3, x)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m\tan^3(e+fx)\,dx=\int \tan(e+f\,x)^3\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^3*(a + a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^3*(a + a*sin(e + f*x))^m, x)
```

3.131 $\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx$

3.131.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx = -\frac{(a+a\sin(e+fx))^m}{2fm} + \frac{\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{1}{2}(1+\sin(e+fx))\right)(a+a\sin(e+fx))^{1+m}}{4af(1+m)}$$

output
```
-1/2*(a+a*sin(f*x+e))^m/f/m+1/4*hypergeom([1, 1+m],[2+m],1/2+1/2*sin(f*x+e
))*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)
```

3.131.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx = \frac{(a(1+\sin(e+fx)))^m\left(-2(1+m)+m\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{1}{2}(1+\sin(e+fx))\right)(1+\text{si}\right.}{4fm(1+m)}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

output
```
((a*(1 + Sin[e + f*x]))^m*(-2*(1 + m) + m*Hypergeometric2F1[1, 1 + m, 2 +
m, (1 + Sin[e + f*x])/2]*(1 + Sin[e + f*x])))/(4*f*m*(1 + m))
```

3.131.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3186, 88, 78}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)(a\sin(e+fx)+a)^m dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{a\sin(e+fx)(\sin(e+fx)a+a)^{m-1}}{a-a\sin(e+fx)}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{88}$$

$$\frac{\frac{1}{2}\int \frac{(\sin(e+fx)a+a)^m}{a-a\sin(e+fx)}d(a\sin(e+fx))-\frac{(a\sin(e+fx)+a)^m}{2m}}{f}$$

$$\Big\downarrow \text{78}$$

$$\frac{\frac{(a\sin(e+fx)+a)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{\sin(e+fx)a+a}{2a}\right)}{4a(m+1)}-\frac{(a\sin(e+fx)+a)^m}{2m}}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x],x]
```

output
```
(-1/2*(a + a*Sin[e + f*x])^m/m + (Hypergeometric2F1[1, 1 + m, 2 + m, (a +
a*Sin[e + f*x])/(2*a)]*(a + a*Sin[e + f*x])^(1 + m))/(4*a*(1 + m)))/f
```

3.131.3.1 Defintions of rubi rules used

rule 78
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 &&  !IntegerQ[m] && IntegerQ[n]
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.131.4 Maple [F]

$$\int (a + a\sin(fx + e))^m \tan(fx + e)\, dx$$

input
```
int((a+a*sin(f*x+e))^m*tan(f*x+e),x)
```

output
```
int((a+a*sin(f*x+e))^m*tan(f*x+e),x)
```

3.131.5 Fricas [F]

$$\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx = \int (a\sin(fx+e)+a)^m \tan(fx+e)\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

3.131.6 Sympy [F]

$$\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx = \int (a(\sin(e+fx)+1))^m \tan(e+fx)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*tan(f*x+e),x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x), x)
```

3.131.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m \tan(e+fx)\,dx = \int (a\sin(fx+e)+a)^m \tan(fx+e)\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

3.131.8 Giac [F]

$$\int (a+a\sin(e+fx))^m\tan(e+fx)\,dx=\int (a\sin(fx+e)+a)^m\tan(fx+e)\ dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e),x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*tan(f*x + e), x)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m\tan(e+fx)\,dx=\int \tan(e+f\,x)\ (a+a\ \sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)*(a + a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)*(a + a*sin(e + f*x))^m, x)
```

3.132 $\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx$

3.132.1 Optimal result

Integrand size = 19, antiderivative size = 43

$$\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}(1,1+m,2+m,1+\sin(e+fx))(a+a\sin(e+fx))^{1+m}}{af(1+m)}$$

output
```
-hypergeom([1, 1+m],[2+m],1+sin(f*x+e))*(a+a*sin(f*x+e))^(1+m)/a/f/(1+m)
```

3.132.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx = -\frac{\text{Hypergeometric2F1}(1,1+m,2+m,1+\sin(e+fx))(a+a\sin(e+fx))^{1+m}}{af(1+m)}$$

input
```
Integrate[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x
])^(1 + m))/(a*f*(1 + m)))
```

3.132.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3186, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m}{\tan(e+fx)}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc(e+fx)(\sin(e+fx)a+a)^m}{a}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{75}$$

$$-\frac{(a\sin(e+fx)+a)^{m+1}\text{Hypergeometric2F1}(1,m+1,m+2,\sin(e+fx)+1)}{af(m+1)}$$

input
```
Int[Cot[e + f*x]*(a + a*Sin[e + f*x])^m,x]
```

output
```
-((Hypergeometric2F1[1, 1 + m, 2 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x
])^(1 + m))/(a*f*(1 + m)))
```

3.132.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.132.4 Maple [F]

$$\int \cot (fx+e) (a+a\sin (fx+e))^m \, dx$$

input
```
int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)*(a+a*sin(f*x+e))^m,x)
```

3.132.5 Fricas [F]

$$\int \cot(e+fx)(a+a\sin(e+fx))^m \, dx = \int (a\sin (fx+e)+a)^m \cot (fx+e) \, dx$$

input
```
integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

3.132.6 Sympy [F]

$$\int \cot(e+fx)(a+a\sin(e+fx))^m \, dx = \int (a(\sin (e+fx)+1))^m \cot (e+fx) \, dx$$

input
```
integrate(cot(f*x+e)*(a+a*sin(f*x+e))**m,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x), x)
```

3.132.7 Maxima [F]

$$\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx = \int (a\sin(fx+e)+a)^m\cot(fx+e)\,dx$$

input
```
integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

3.132.8 Giac [F]

$$\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx = \int (a\sin(fx+e)+a)^m\cot(fx+e)\,dx$$

input
```
integrate(cot(f*x+e)*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e), x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \cot(e+fx)(a+a\sin(e+fx))^m\,dx = \int \cot(e+f\,x)\,\left(a+a\,\sin(e+f\,x)\right)^m\,dx$$

input
```
int(cot(e + f*x)*(a + a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)*(a + a*sin(e + f*x))^m, x)
```

3.133 $\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx$

3.133.1 Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx = -\frac{\csc^2(e+fx)(a+a\sin(e+fx))^{2+m}}{2a^2f} - \frac{(2-m)\,\text{Hypergeometric2F1}(2,2+m,3+m,1+\sin(e+fx))(a+a\sin(e+fx))^{2+m}}{2a^2f(2+m)}$$

output
```
-1/2*csc(f*x+e)^2*(a+a*sin(f*x+e))^(2+m)/a^2/f-1/2*(2-m)*hypergeom([2, 2+m
],[3+m],1+sin(f*x+e))*(a+a*sin(f*x+e))^(2+m)/a^2/f/(2+m)
```

3.133.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx = -\frac{((2+m)\csc^2(e+fx)-(-2+m)\,\text{Hypergeometric2F1}(2,2+m,3+m,1+\sin(e+fx)))\,(1+\sin(e+}{2f(2+m)}$$

input
```
Integrate[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]
```

output
```
-1/2*(((2 + m)*Csc[e + f*x]^2 - (-2 + m)*Hypergeometric2F1[2, 2 + m, 3 + m
, 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])^2*(a*(1 + Sin[e + f*x]))^m)/(f*(2
+ m))
```

3.133.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3186, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m}{\tan(e+fx)^3}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^3(e+fx)(a-a\sin(e+fx))(\sin(e+fx)a+a)^{m+1}}{a^3}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{-\frac{1}{2}(2-m)\int \frac{\csc^2(e+fx)(\sin(e+fx)a+a)^{m+1}}{a^2}d(a\sin(e+fx))-\frac{\csc^2(e+fx)(a\sin(e+fx)+a)^{m+2}}{2a^2}}{f}$$

$$\Big\downarrow \text{75}$$

$$\frac{-\frac{(2-m)(a\sin(e+fx)+a)^{m+2}\,\text{Hypergeometric2F1}(2,m+2,m+3,\sin(e+fx)+1)}{2a^2(m+2)}-\frac{\csc^2(e+fx)(a\sin(e+fx)+a)^{m+2}}{2a^2}}{f}$$

input
```
Int[Cot[e + f*x]^3*(a + a*Sin[e + f*x])^m,x]
```

output
```
(-1/2*(Csc[e + f*x]^2*(a + a*Sin[e + f*x])^(2 + m))/a^2 - ((2 - m)*Hyperge
ometric2F1[2, 2 + m, 3 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(2 + m)
)/(2*a^2*(2 + m)))/f
```

3.133.3.1 Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.133.4 Maple [F]

$$\int \left(\cot^{3}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}dx$$

input
```
int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x)
```

3.133.5 Fricas [F]

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx=\int (a\sin(fx+e)+a)^m\cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

3.133.6 Sympy [F]

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx=\int (a(\sin(e+fx)+1))^m\cot^3(e+fx)\,dx$$

input
```
integrate(cot(f*x+e)**3*(a+a*sin(f*x+e))**m,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**3, x)
```

3.133.7 Maxima [F]

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx=\int (a\sin(fx+e)+a)^m\cot(fx+e)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

3.133.8 Giac [F]

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx=\int (a\sin\left(fx+e\right)+a)^m\cot\left(fx+e\right)^3\,dx$$

input
```
integrate(cot(f*x+e)^3*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^3, x)
```

3.133.9 Mupad [F(-1)]

Timed out.

$$\int \cot^3(e+fx)(a+a\sin(e+fx))^m\,dx=\int \cot(e+f\,x)^3\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

input
```
int(cot(e + f*x)^3*(a + a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^3*(a + a*sin(e + f*x))^m, x)
```

3.134 $\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx$

3.134.1 Optimal result

Integrand size = 21, antiderivative size = 123

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{(9-m)\csc^3(e+fx)(a+a\sin(e+fx))^{3+m}}{12a^3f} - \frac{\csc^4(e+fx)(a+a\sin(e+fx))^{3+m}}{4a^3f} - \frac{(12-9m+m^2)\,\text{Hypergeometric2F1}(3,3+m,4+m,1+\sin(e+fx))(a+a\sin(e+fx))^{3+m}}{12a^3f(3+m)}$$

output
```
1/12*(9-m)*csc(f*x+e)^3*(a+a*sin(f*x+e))^(3+m)/a^3/f-1/4*csc(f*x+e)^4*(a+a
*sin(f*x+e))^(3+m)/a^3/f-1/12*(m^2-9*m+12)*hypergeom([3, 3+m],[4+m],1+sin(
f*x+e))*(a+a*sin(f*x+e))^(3+m)/a^3/f/(3+m)
```

3.134.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.67

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx = -\frac{((3+m)\csc^3(e+fx)(-9+m+3\csc(e+fx)) + (12-9m+m^2)\,\text{Hypergeometric2F1}(3,3+m,4+}{12f(3+m)}$$

input
```
Integrate[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]
```

output
```
-1/12*(((3 + m)*Csc[e + f*x]^3*(-9 + m + 3*Csc[e + f*x]) + (12 - 9*m + m^2
)*Hypergeometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]])*(1 + Sin[e + f*x])
^3*(a*(1 + Sin[e + f*x]))^m)/(f*(3 + m))
```

3.134.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3186, 100, 25, 27, 87, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m}{\tan(e+fx)^5}dx$$

$$\Big\downarrow \text{3186}$$

$$\frac{\int \frac{\csc^5(e+fx)(a-a\sin(e+fx))^2(\sin(e+fx)a+a)^{m+2}}{a^5}d(a\sin(e+fx))}{f}$$

$$\Big\downarrow \text{100}$$

$$\frac{\frac{\int -\frac{\csc^4(e+fx)(a(9-m)-4a\sin(e+fx))(\sin(e+fx)a+a)^{m+2}}{a^3}d(a\sin(e+fx))}{4a}-\frac{\csc^4(e+fx)(a\sin(e+fx)+a)^{m+3}}{4a^3}}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{\csc^4(e+fx)(a(9-m)-4a\sin(e+fx))(\sin(e+fx)a+a)^{m+2}}{a^3}d(a\sin(e+fx))}{4a}-\frac{\csc^4(e+fx)(a\sin(e+fx)+a)^{m+3}}{4a^3}}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{1}{4}\int \frac{\csc^4(e+fx)(a(9-m)-4a\sin(e+fx))(\sin(e+fx)a+a)^{m+2}}{a^4}d(a\sin(e+fx))-\frac{\csc^4(e+fx)(a\sin(e+fx)+a)^{m+3}}{4a^3}}{f}$$

$$\Big\downarrow \text{87}$$

$$\frac{\frac{1}{4}\left(\frac{1}{3}(m^2-9m+12)\int \frac{\csc^3(e+fx)(\sin(e+fx)a+a)^{m+2}}{a^3}d(a\sin(e+fx))+\frac{(9-m)\csc^3(e+fx)(a\sin(e+fx)+a)^{m+3}}{3a^3}\right)-\frac{\csc^4(e+fx)}{}}{f}$$

$$\Big\downarrow \; 75$$

$$\frac{\frac{1}{4}\left(\frac{(9-m)\csc^3(e+fx)(a\sin(e+fx)+a)^{m+3}}{3a^3}-\frac{(m^2-9m+12)(a\sin(e+fx)+a)^{m+3}\text{Hypergeometric2F1}(3,m+3,m+4,\sin(e+fx)+1)}{3a^3(m+3)}\right)-\frac{\csc^4(e}{}}{f}$$

input
```
Int[Cot[e + f*x]^5*(a + a*Sin[e + f*x])^m,x]
```

output
```
(-1/4*(Csc[e + f*x]^4*(a + a*Sin[e + f*x])^(3 + m))/a^3 + (((9 - m)*Csc[e
+ f*x]^3*(a + a*Sin[e + f*x])^(3 + m))/(3*a^3) - ((12 - 9*m + m^2)*Hyperge
ometric2F1[3, 3 + m, 4 + m, 1 + Sin[e + f*x]]*(a + a*Sin[e + f*x])^(3 + m)
)/(3*a^3*(3 + m)))/4)/f
```

3.134.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 100
```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(
p_), x_] :> Simp[(b*c - a*d)^2*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d^2*(d
*e - c*f)*(n + 1))), x] - Simp[1/(d^2*(d*e - c*f)*(n + 1))   Int[(c + d*x)^
(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(
p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x
, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n
 + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p, 1])))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

3.134.4 Maple [F]

$$\int \left(\cot^{5}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}dx$$

input
```
int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x)
```

3.134.5 Fricas [F]

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx=\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{5}\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx = \text{Timed out}$$

input
```
integrate(cot(f*x+e)**5*(a+a*sin(f*x+e))**m,x)
```

output
```
Timed out
```

3.134.7 Maxima [F]

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx = \int (a\sin(fx+e)+a)^m\cot(fx+e)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

3.134.8 Giac [F]

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx = \int (a\sin(fx+e)+a)^m\cot(fx+e)^5\,dx$$

input
```
integrate(cot(f*x+e)^5*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^5, x)
```

3.134.9 Mupad [F(-1)]

Timed out.

$$\int \cot^5(e+fx)(a+a\sin(e+fx))^m\,dx=\int \cot(e+f\,x)^5\,\left(a+a\,\sin\left(e+f\,x\right)\right)^m\,dx$$

input
```
int(cot(e + f*x)^5*(a + a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^5*(a + a*sin(e + f*x))^m, x)
```

3.135 $\int (a + a\sin(e + fx))^m \tan^4(e + fx)\, dx$

3.135.1 Optimal result

Integrand size = 21, antiderivative size = 311

$$\int (a + a\sin(e + fx))^m \tan^4(e + fx)\, dx$$
$$= \frac{2^{-\frac{3}{2}+m}(9 - 12m - 7m^2 + 6m^3 + m^4)\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{5}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \sin(e + fx))\right)\sec(e + fx)(1}{3f(1-m)m}$$
$$- \frac{\sec(e + fx)(a + a\sin(e + fx))^{-1+m}\left(a(6 - m - 7m^2 - m^3) - a(9 - 6m - 8m^2 - m^3)\sin(e + fx)\right)}{3f(1-m)m(1 - \sin(e + fx))}$$
$$+ \frac{a^2\sin(e + fx)(a + a\sin(e + fx))^{-1+m}\tan(e + fx)}{f(1-m)(a - a\sin(e + fx))}$$
$$- \frac{a^2\sin^2(e + fx)(a + a\sin(e + fx))^{-1+m}\tan(e + fx)}{fm(a - a\sin(e + fx))}$$

output
```
1/3*2^(-3/2+m)*(m^4+6*m^3-7*m^2-12*m+9)*hypergeom([1/2, 5/2-m],[3/2],1/2-1
/2*sin(f*x+e))*sec(f*x+e)*(1-sin(f*x+e))*(1+sin(f*x+e))^(1/2-m)*(a+a*sin(f
*x+e))^m/f/(1-m)/m-1/3*sec(f*x+e)*(a+a*sin(f*x+e))^(-1+m)*(a*(-m^3-7*m^2-m
+6)-a*(-m^3-8*m^2-6*m+9)*sin(f*x+e))/f/(1-m)/m/(1-sin(f*x+e))+a^2*sin(f*x+
e)*(a+a*sin(f*x+e))^(-1+m)*tan(f*x+e)/f/(1-m)/(a-a*sin(f*x+e))-a^2*sin(f*x
+e)^2*(a+a*sin(f*x+e))^(-1+m)*tan(f*x+e)/f/m/(a-a*sin(f*x+e))
```

3.135.2 Mathematica [F]

$$\int (a+a\sin(e+fx))^m\tan^4(e+fx)\,dx=\int (a+a\sin(e+fx))^m\tan^4(e+fx)\,dx$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

output
```
Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4, x]
```

3.135.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3042, 3198, 111, 25, 27, 170, 25, 27, 162, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^4(a\sin(e+fx)+a)^m dx$$

$$\Big\downarrow \text{3198}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{a^4\sin^4(e+fx)(\sin(e+fx)a+a)^{m-\frac{5}{2}}}{(a-a\sin(e+fx))^{5/2}}d(a\sin(e+fx))}{af}$$

$$\Big\downarrow \text{111}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(-\frac{\int -\frac{a^3\sin^2(e+fx)(\sin(e+fx)a+a)^{m-\frac{5}{2}}(m\sin(e+fx)a+3a)}{(a-a\sin(e+fx))^{5/2}}d(a\sin(e+fx))}{m}-\right.}{af}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{\int\frac{a^3\sin^2(e+fx)(\sin(e+fx)a+a)^{m-\frac{5}{2}}(m\sin(e+fx)a+3a)}{(a-a\sin(e+fx))^{5/2}}d(a\sin(e+fx))}{m}-\frac{a^3\sin}{}\right.}{af}$$

$\downarrow$ 27

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{a\int\frac{a^2\sin^2(e+fx)(\sin(e+fx)a+a)^{m-\frac{5}{2}}(m\sin(e+fx)a+3a)}{(a-a\sin(e+fx))^{5/2}}d(a\sin(e+fx))}{m}-\frac{a^3\sin}{}\right.}{af}$$

$\downarrow$ 170

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{a\left(\frac{\int-\frac{a^2\sin(e+fx)(\sin(e+fx)a+a)^{m-\frac{5}{2}}\left(2am-a\left(-m^2-3m+3\right)\sin(e+fx)\right)}{(a-a\sin(e+fx))^{5/2}}d(a\sin(}{1-m}\right.}{m}\right.}{af}$$

$\downarrow$ 25

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{a\left(\frac{a^2m\sin^2(e+fx)(a\sin(e+fx)+a)^{m-\frac{3}{2}}}{(1-m)(a-a\sin(e+fx))^{3/2}}-\frac{\int\frac{a^2\sin(e+fx)(\sin(e+fx)a+a)^{m-\frac{5}{2}}(2a}{(a-a\sin(e+f}}{}\right.}{m}\right.}{af}$$

$\downarrow$ 27

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{a\left(\frac{a^2m\sin^2(e+fx)(a\sin(e+fx)+a)^{m-\frac{3}{2}}}{(1-m)(a-a\sin(e+fx))^{3/2}}-\frac{a\int\frac{a\sin(e+fx)(\sin(e+fx)a+a)^{m-\frac{5}{2}}(2a}{(a-a\sin(e+}}{}\right.}{m}\right.}{af}$$

$\downarrow$ 162

3.135. $\int(a+a\sin(e+fx))^m\tan^4(e+fx)\,dx$

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{a\left(\frac{a^2 m\sin^2(e+fx)(a\sin(e+fx)+a)^{m-\frac{3}{2}}}{(1-m)(a-a\sin(e+fx))^{3/2}}-\frac{a\left(\frac{1}{3}\left(m^4+6m^3-7m^2-12m+9\right)\int\frac{(\sin(e+\cdots}{\sqrt{a-\cdots}}\cdots\right.}{\cdots}\right)}{\cdots}\right.$$

$\downarrow$ 80

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{a\left(\frac{a^2 m\sin^2(e+fx)(a\sin(e+fx)+a)^{m-\frac{3}{2}}}{(1-m)(a-a\sin(e+fx))^{3/2}}-\frac{a\left(\frac{2^{m-\frac{5}{2}}\left(m^4+6m^3-7m^2-12m+9\right)(a\,\mathrm{s}\cdots}{\cdots}\right.}{\cdots}\right)}{\cdots}\right.$$

$\downarrow$ 79

$$\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\left(\frac{a\left(\frac{a^2 m\sin^2(e+fx)(a\sin(e+fx)+a)^{m-\frac{3}{2}}}{(1-m)(a-a\sin(e+fx))^{3/2}}-\frac{a\left(\frac{(a\sin(e+fx)+a)^{m-\frac{3}{2}}\left(a\left(-m^3-7m^2\cdots\right.\right.}{3(a-a\,\mathrm{s}\cdots}\right.}{\cdots}\right)}{\cdots}\right.$$

input
```
Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^4,x]
```

3.135. $\int(a+a\sin(e+fx))^m\tan^4(e+fx)\,dx$

output
```
(Sec[e + f*x]*Sqrt[a - a*Sin[e + f*x]]*Sqrt[a + a*Sin[e + f*x]]*(-((a^3*Si
n[e + f*x]^3*(a + a*Sin[e + f*x])^(-3/2 + m))/(m*(a - a*Sin[e + f*x])^(3/2
))) + (a*((a^2*m*Sin[e + f*x]^2*(a + a*Sin[e + f*x])^(-3/2 + m))/((1 - m)*
(a - a*Sin[e + f*x])^(3/2)) - (a*(-1/3*(2^(-3/2 + m)*(9 - 12*m - 7*m^2 + 6
*m^3 + m^4)*Hypergeometric2F1[1/2, 5/2 - m, 3/2, (a - a*Sin[e + f*x])/(2*a
)]*Sqrt[a - a*Sin[e + f*x]]*(a + a*Sin[e + f*x])^(-1/2 + m)*((a + a*Sin[e
+ f*x])/a)^(1/2 - m))/a^2 + ((a + a*Sin[e + f*x])^(-3/2 + m)*(a*(6 - m - 7
*m^2 - m^3) - a*(9 - 6*m - 8*m^2 - m^3)*Sin[e + f*x]))/(3*(a - a*Sin[e + f
*x])^(3/2))))/(1 - m)))/m))/(a*f)
```

3.135.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 162
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(n + 2)
 - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h) + d*e
*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(f*g +
e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)/(b^2*(b
*c - a*d)^2*(m + 1)*(m + 2)))*(a + b*x)^(m + 1)*(c + d*x)^(n + 1), x] + Sim
p[(f*(h/b^2) - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d
*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2))))/(
b^2*(b*c - a*d)^2*(m + 1)*(m + 2)))   Int[(a + b*x)^(m + 2)*(c + d*x)^n, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[m +
 n + 3, 0] &&  !LtQ[n, -2]))
```

rule 170
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.135.4 Maple [F]

$$\int (a+a\sin(fx+e))^m \left(\tan^4(fx+e)\right) dx$$

input
```
int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

output
```
int((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x)
```

3.135.5 Fricas [F]

$$\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int (a\sin(fx+e)+a)^m \tan(fx+e)^4\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

3.135.6 Sympy [F]

$$\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int (a(\sin(e+fx)+1))^m \tan^4(e+fx)\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**4,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x)**4, x)
```

3.135.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx = \int (a\sin(fx+e)+a)^m \tan(fx+e)^4\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

3.135.8 Giac [F]

$$\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx=\int (a\sin\left(fx+e\right)+a)^m \tan\left(fx+e\right)^4\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^4,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^4, x)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m \tan^4(e+fx)\,dx=\int \tan(e+f\,x)^4\,(a+a\,\sin\left(e+f\,x\right))^m\,dx$$

input
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^4*(a + a*sin(e + f*x))^m, x)
```

3.136 $\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx$

3.136.1 Optimal result

Integrand size = 21, antiderivative size = 157

$$\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx = \frac{\sec(e+fx)(a+a\sin(e+fx))^m}{f(1-m)m}$$
$$+\frac{2^{-\frac{1}{2}+m}(1-m-m^2)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)\sec(e+fx)(1+\sin(e+f}{f(1-m)m}$$
$$-\frac{\sec(e+fx)(a+a\sin(e+fx))^{1+m}}{afm}$$

output
```
sec(f*x+e)*(a+a*sin(f*x+e))^m/f/(1-m)/m+2^(-1/2+m)*(-m^2-m+1)*hypergeom([-
1/2, 3/2-m],[1/2],1/2-1/2*sin(f*x+e))*sec(f*x+e)*(1+sin(f*x+e))^(1/2-m)*(a
+a*sin(f*x+e))^m/f/(1-m)/m-sec(f*x+e)*(a+a*sin(f*x+e))^(1+m)/a/f/m
```

3.136.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 1756 vs. $2(157) = 314$.

Time = 22.77 (sec) , antiderivative size = 1756, normalized size of antiderivative = 11.18

$$\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx = \text{Too large to display}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

output
```
(Sqrt[Sec[e + f*x]^2]*(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2*(a + (a*Tan[e
+ f*x])/Sqrt[Sec[e + f*x]^2])^m*(1 + (Sqrt[Sec[e + f*x]^2] + Tan[e + f*x])
^2)^m*((1 + 2*m)*Hypergeometric2F1[-1 + m, -1/2 + m, 1/2 + m, -(Sqrt[Sec[e
 + f*x]^2] + Tan[e + f*x])^2] - 4*(-1 + 2*m)*Hypergeometric2F1[1/2 + m, 1
+ m, 3/2 + m, -(Sqrt[Sec[e + f*x]^2] + Tan[e + f*x])^2]*(1 + 2*Sqrt[Sec[e
+ f*x]^2]*Tan[e + f*x] + 2*Tan[e + f*x]^2)))/(2*f*(-1 + 2*m)*(1 + 2*m)*(Se
c[e + f*x]^2 + Sqrt[Sec[e + f*x]^2]*Tan[e + f*x])*((m*Sqrt[Sec[e + f*x]^2]
*(Sqrt[Sec[e + f*x]^2] + Tan[e + f*x])*(a + (a*Tan[e + f*x])/Sqrt[Sec[e +
f*x]^2])^m*(1 + (Sqrt[Sec[e + f*x]^2] + Tan[e + f*x])^2)^(-1 + m)*((1 + 2*
m)*Hypergeometric2F1[-1 + m, -1/2 + m, 1/2 + m, -(Sqrt[Sec[e + f*x]^2] + T
an[e + f*x])^2] - 4*(-1 + 2*m)*Hypergeometric2F1[1/2 + m, 1 + m, 3/2 + m,
-(Sqrt[Sec[e + f*x]^2] + Tan[e + f*x])^2]*(1 + 2*Sqrt[Sec[e + f*x]^2]*Tan[
e + f*x] + 2*Tan[e + f*x]^2)))/((-1 + 2*m)*(1 + 2*m)) + (Sqrt[Sec[e + f*x]
^2]*Tan[e + f*x]*(a + (a*Tan[e + f*x])/Sqrt[Sec[e + f*x]^2])^m*(1 + (Sqrt[
Sec[e + f*x]^2] + Tan[e + f*x])^2)^m*((1 + 2*m)*Hypergeometric2F1[-1 + m,
-1/2 + m, 1/2 + m, -(Sqrt[Sec[e + f*x]^2] + Tan[e + f*x])^2] - 4*(-1 + 2*m
)*Hypergeometric2F1[1/2 + m, 1 + m, 3/2 + m, -(Sqrt[Sec[e + f*x]^2] + Tan[
e + f*x])^2]*(1 + 2*Sqrt[Sec[e + f*x]^2]*Tan[e + f*x] + 2*Tan[e + f*x]^2))
)/(2*(-1 + 2*m)*(1 + 2*m)*(Sec[e + f*x]^2 + Sqrt[Sec[e + f*x]^2]*Tan[e + f
*x])) + (m*Sqrt[Sec[e + f*x]^2]*(a + (a*Tan[e + f*x])/Sqrt[Sec[e + f*x]...
```

3.136.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 3192, 3042, 3339, 3042, 3168, 80, 79}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(e+fx)^2(a\sin(e+fx)+a)^m dx$$

$$\Big\downarrow \text{3192}$$

$$\frac{\int \sec^2(e+fx)(\sin(e+fx)a+a)^m(a(m+1)+a\sin(e+fx))dx}{am}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{afm}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{(\sin(e+fx)a+a)^m(a(m+1)+a\sin(e+fx))}{\cos(e+fx)^2}dx}{am}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{afm}$$

$$\Big\downarrow \text{3339}$$

$$\frac{\frac{a(-m^2-m+1)\int \sec^2(e+fx)(\sin(e+fx)a+a)^mdx}{1-m}+\frac{a\sec(e+fx)(a\sin(e+fx)+a)^m}{f(1-m)}}{am}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{afm}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{a(-m^2-m+1)\int \frac{(\sin(e+fx)a+a)^m}{\cos(e+fx)^2}dx}{1-m}+\frac{a\sec(e+fx)(a\sin(e+fx)+a)^m}{f(1-m)}}{am}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{afm}$$

$$\Big\downarrow \text{3168}$$

$$\frac{\frac{a^3(-m^2-m+1)\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{(\sin(e+fx)a+a)^{m-\frac{3}{2}}}{(a-a\sin(e+fx))^{3/2}}d\sin(e+fx)}{f(1-m)}+\frac{a\sec(e+fx)(a\sin(e+fx)+a)^m}{f(1-m)}}{am}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{afm}$$

$$\Big\downarrow \text{80}$$

$$\frac{\frac{a^22^{m-\frac{3}{2}}(-m^2-m+1)\sec(e+fx)\sqrt{a-a\sin(e+fx)}(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\int \frac{\left(\frac{1}{2}\sin(e+fx)+\frac{1}{2}\right)^{m-\frac{3}{2}}}{(a-a\sin(e+fx))^{3/2}}d\sin(e+fx)}{f(1-m)}+\frac{a\sec(e+fx)(a\sin(e+fx)+a)^m}{f(1-m)}}{am}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{afm}$$

$$\Big\downarrow \text{79}$$

$$\frac{\frac{a2^{m-\frac{1}{2}}(-m^2-m+1)\sec(e+fx)(\sin(e+fx)+1)^{\frac{1}{2}-m}(a\sin(e+fx)+a)^m\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{2}-m,\frac{1}{2},\frac{1}{2}(1-\sin(e+fx))\right)}{f(1-m)}+\frac{a\sec(e+fx)(a\sin(e+fx)+a)^m}{f(1-m)}}{am}-\frac{\sec(e+fx)(a\sin(e+fx)+a)^{m+1}}{afm}$$

input
```
Int[(a + a*Sin[e + f*x])^m*Tan[e + f*x]^2,x]
```

3.136. $\int (a+a\sin(e+fx))^m\tan^2(e+fx)\,dx$

output
```
-((Sec[e + f*x]*(a + a*Sin[e + f*x])^(1 + m))/(a*f*m)) + ((a*Sec[e + f*x]*
(a + a*Sin[e + f*x])^m)/(f*(1 - m)) + (2^(-1/2 + m)*a*(1 - m - m^2)*Hyperg
eometric2F1[-1/2, 3/2 - m, 1/2, (1 - Sin[e + f*x])/2]*Sec[e + f*x]*(1 + Si
n[e + f*x])^(1/2 - m)*(a + a*Sin[e + f*x])^m)/(f*(1 - m)))/(a*m)
```

3.136.3.1 Defintions of rubi rules used

rule 79
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n))*Hypergeometric2F1[-n, m + 1
, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m, n}, x]
&&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m]
 ||  !(RationalQ[n] && GtQ[-d/(b*c - a*d), 0]))
```

rule 80
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c
 + d*x)^FracPart[n]/((b/(b*c - a*d))^IntPart[n]*(b*((c + d*x)/(b*c - a*d)))
^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c - a*d)) + b*d*(x/(b*c - a*d)
), x]^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integ
erQ[n] && (RationalQ[m] ||  !SimplerQ[n + 1, m + 1])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3168
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.), x_Symbol] :> Simp[a^2*((g*Cos[e + f*x])^(p + 1)/(f*g*(a + b*Sin
[e + f*x])^((p + 1)/2)*(a - b*Sin[e + f*x])^((p + 1)/2)))   Subst[Int[(a +
b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; Fre
eQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

rule 3192
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[-(a + b*Sin[e + f*x])^(m + 1)/(b*f*m*Cos[e + f*x]), x] +
Simp[1/(b*m)   Int[(a + b*Sin[e + f*x])^m*((b*(m + 1) + a*Sin[e + f*x])/Cos
[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] &&
!IntegerQ[m] &&  !LtQ[m, 0]
```

rule 3339
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + S
imp[(a*d*m + b*c*(m + p + 1))/(b*(m + p + 1))   Int[(g*Cos[e + f*x])^p*(a +
 b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[
a^2 - b^2, 0] && NeQ[m + p + 1, 0]
```

3.136.4 Maple [F]

$$\int (a + a\sin(fx + e))^m \left(\tan^2(fx + e)\right) dx$$

input
```
int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

output
```
int((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x)
```

3.136.5 Fricas [F]

$$\int (a + a\sin(e + fx))^m \tan^2(e + fx)\, dx = \int (a\sin(fx + e) + a)^m \tan(fx + e)^2\, dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

3.136.6 Sympy [F]

$$\int (a + a\sin(e + fx))^m \tan^2(e + fx)\, dx = \int (a(\sin(e + fx) + 1))^m \tan^2(e + fx)\, dx$$

input
```
integrate((a+a*sin(f*x+e))**m*tan(f*x+e)**2,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*tan(e + f*x)**2, x)
```

3.136.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx = \int (a\sin(fx+e)+a)^m \tan(fx+e)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

3.136.8 Giac [F]

$$\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx = \int (a\sin(fx+e)+a)^m \tan(fx+e)^2\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m*tan(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*tan(f*x + e)^2, x)
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m \tan^2(e+fx)\,dx = \int \tan(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^m,x)
```

output
```
int(tan(e + f*x)^2*(a + a*sin(e + f*x))^m, x)
```

3.137 $\int (a + a\sin(e + fx))^m \, dx$

3.137.1 Optimal result

Integrand size = 12, antiderivative size = 74

$$\int (a + a\sin(e + fx))^m \, dx = -\frac{2^{\frac{1}{2}+m} \cos(e + fx) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1}{2} - m, \frac{3}{2}, \frac{1}{2}(1 - \sin(e + fx))\right) (1 + \sin(e + fx))^{-\frac{1}{2}-m}(a + a\sin(e+fx))^m}{f}$$

output
```
-2^(1/2+m)*cos(f*x+e)*hypergeom([1/2, 1/2-m],[3/2],1/2-1/2*sin(f*x+e))*(1+
sin(f*x+e))^(-1/2-m)*(a+a*sin(f*x+e))^m/f
```

3.137.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.95

$$\int (a + a\sin(e + fx))^m \, dx = \frac{2^m B_{\frac{1}{2}(1+\sin(e+fx))}\left(\frac{1}{2} + m, \frac{1}{2}\right) \sqrt{\cos^2(e + fx)} \sec(e + fx)(1 + \sin(e + fx))^{-m}(a(1 + \sin(e + fx)))^m}{f}$$

input
```
Integrate[(a + a*Sin[e + f*x])^m,x]
```

output
```
(2^m*Beta[(1 + Sin[e + f*x])/2, 1/2 + m, 1/2]*Sqrt[Cos[e + f*x]^2]*Sec[e +
 f*x]*(a*(1 + Sin[e + f*x]))^m)/(f*(1 + Sin[e + f*x])^m)
```

3.137.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3131, 3042, 3130}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (a\sin(e+fx)+a)^m dx$$

$$\Big\downarrow \text{ 3131}$$

$$(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int (\sin(e+fx)+1)^m dx$$

$$\Big\downarrow \text{ 3042}$$

$$(\sin(e+fx)+1)^{-m}(a\sin(e+fx)+a)^m\int (\sin(e+fx)+1)^m dx$$

$$\Big\downarrow \text{ 3130}$$

$$-\frac{2^{m+\frac{1}{2}}\cos(e+fx)(\sin(e+fx)+1)^{-m-\frac{1}{2}}(a\sin(e+fx)+a)^m\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}-m,\frac{3}{2},\frac{1}{2}(1-\sin(e+f}{f}$$

input
```
Int[(a + a*Sin[e + f*x])^m,x]
```

output
```
-((2^(1/2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Sin[
e + f*x])/2]*(1 + Sin[e + f*x])^(-1/2 - m)*(a + a*Sin[e + f*x])^m)/f)
```

3.137.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3130
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-2^(n +
 1/2))*a^(n - 1/2)*b*(Cos[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]]))*Hypergeome
tric2F1[1/2, 1/2 - n, 3/2, (1/2)*(1 - b*(Sin[c + d*x]/a))], x] /; FreeQ[{a,
 b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

rule 3131
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a^IntPar
t[n]*((a + b*Sin[c + d*x])^FracPart[n]/(1 + (b/a)*Sin[c + d*x])^FracPart[n]
)   Int[(1 + (b/a)*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] &&
EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

3.137.4 Maple [F]

$$\int (a+a\sin(fx+e))^m\,dx$$

input
```
int((a+a*sin(f*x+e))^m,x)
```

output
```
int((a+a*sin(f*x+e))^m,x)
```

3.137.5 Fricas [F]

$$\int (a+a\sin(e+fx))^m\,dx = \int (a\sin(fx+e)+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m, x)
```

3.137.6 Sympy [F]

$$\int (a+a\sin(e+fx))^m\,dx=\int (a\sin{(e+fx)}+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))**m,x)
```

output
```
Integral((a*sin(e + f*x) + a)**m, x)
```

3.137.7 Maxima [F]

$$\int (a+a\sin(e+fx))^m\,dx=\int (a\sin{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m, x)
```

3.137.8 Giac [F]

$$\int (a+a\sin(e+fx))^m\,dx=\int (a\sin{(fx+e)}+a)^m\,dx$$

input
```
integrate((a+a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m, x)
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int (a+a\sin(e+fx))^m\,dx=\int (a+a\,\sin\left(e+f\,x\right))^m\,dx$$

input
```
int((a + a*sin(e + f*x))^m,x)
```

output
```
int((a + a*sin(e + f*x))^m, x)
```

3.138 $\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx$

3.138.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx$$
$$= \frac{2\sqrt{2}\,\text{AppellF1}\left(\frac{3}{2}+m,-\frac{1}{2},2,\frac{5}{2}+m,\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}(}{a^2f(3+2m)}$$

output
```
2*AppellF1(3/2+m,2,-1/2,5/2+m,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*
(a+a*sin(f*x+e))^(2+m)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^2/f/(3+2*m)
```

3.138.2 Mathematica [F]

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx = \int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx$$

input
```
Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

output
```
Integrate[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m, x]
```

3.138.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3198, 154, 27, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{(a\sin(e+fx)+a)^m}{\tan(e+fx)^2}dx$$

$$\Big\downarrow \; 3198$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{\csc^2(e+fx)\sqrt{a-a\sin(e+fx)}(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{a^2}d(a\sin(e+fx))}{af}$$

$$\Big\downarrow \; 154$$

$$\frac{\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \frac{\csc^2(e+fx)\sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{\sqrt{2}a^2}d(a\sin(e+fx))}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \frac{\csc^2(e+fx)\sqrt{1-\sin(e+fx)}(\sin(e+fx)a+a)^{m+\frac{1}{2}}}{a^2}d(a\sin(e+fx))}{af\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$\Big\downarrow \; 153$$

$$\frac{2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))(a\sin(e+fx)+a)^{m+2}\operatorname{AppellF1}\left(m+\frac{3}{2},-\frac{1}{2},2,m+\frac{5}{2},\frac{\sin(e+fx)a+a}{2a},\frac{\sin(e+fx)a}{a}\right.}{a^3f(2m+3)\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

input
```
Int[Cot[e + f*x]^2*(a + a*Sin[e + f*x])^m,x]
```

output
```
(2*Sqrt[2]*AppellF1[3/2 + m, -1/2, 2, 5/2 + m, (a + a*Sin[e + f*x])/(2*a),
 (a + a*Sin[e + f*x])/a]*Sec[e + f*x]*(a - a*Sin[e + f*x])*(a + a*Sin[e +
f*x])^(2 + m))/(a^3*f*(3 + 2*m)*Sqrt[(a - a*Sin[e + f*x])/a])
```

3.138.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.138.4 Maple [F]

$$\int \left(\cot^{2}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}dx$$

input
```
int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x)
```

3.138.5 Fricas [F]

$$\int \cot^{2}(e+fx)(a+a\sin(e+fx))^{m}\,dx = \int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{2}\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

3.138.6 Sympy [F]

$$\int \cot^{2}(e+fx)(a+a\sin(e+fx))^{m}\,dx = \int \left(a(\sin\left(e+fx\right)+1)\right)^{m}\cot^{2}\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**2*(a+a*sin(f*x+e))**m,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**2, x)
```

3.138.7 Maxima [F]

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx=\int (a\sin(fx+e)+a)^m\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

3.138.8 Giac [F]

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx=\int (a\sin(fx+e)+a)^m\cot(fx+e)^2\,dx$$

input
```
integrate(cot(f*x+e)^2*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^2, x)
```

3.138.9 Mupad [F(-1)]

Timed out.

$$\int \cot^2(e+fx)(a+a\sin(e+fx))^m\,dx=\int \cot(e+f\,x)^2\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^2*(a + a*sin(e + f*x))^m, x)
```

3.139 $\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$

3.139.1 Optimal result

Integrand size = 21, antiderivative size = 89

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx = \frac{4\sqrt{2}\,\text{AppellF1}\left(\frac{5}{2}+m,-\frac{3}{2},4,\frac{7}{2}+m,\frac{1}{2}(1+\sin(e+fx)),1+\sin(e+fx)\right)\sec(e+fx)\sqrt{1-\sin(e+fx)}(a}{a^3f(5+2m)}$$

output
```
4*AppellF1(5/2+m,4,-3/2,7/2+m,1+sin(f*x+e),1/2+1/2*sin(f*x+e))*sec(f*x+e)*
(a+a*sin(f*x+e))^(3+m)*2^(1/2)*(1-sin(f*x+e))^(1/2)/a^3/f/(5+2*m)
```

3.139.2 Mathematica [F]

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx = \int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx$$

input
```
Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]
```

output
```
Integrate[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m, x]
```

3.139.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 3198, 154, 27, 153}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(e+fx)(a\sin(e+fx)+a)^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a\sin(e+fx)+a)^m}{\tan(e+fx)^4}dx$$

$$\Big\downarrow \text{ 3198}$$

$$\frac{\sec(e+fx)\sqrt{a-a\sin(e+fx)}\sqrt{a\sin(e+fx)+a}\int \frac{\csc^4(e+fx)(a-a\sin(e+fx))^{3/2}(\sin(e+fx)a+a)^{m+\frac{3}{2}}}{a^4}d(a\sin(e+fx))}{af}$$

$$\Big\downarrow \text{ 154}$$

$$\frac{2\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \frac{\csc^4(e+fx)(1-\sin(e+fx))^{3/2}(\sin(e+fx)a+a)^{m+\frac{3}{2}}}{2\sqrt{2}a^4}d(a\sin(e+fx)}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sec(e+fx)(a-a\sin(e+fx))\sqrt{a\sin(e+fx)+a}\int \frac{\csc^4(e+fx)(1-\sin(e+fx))^{3/2}(\sin(e+fx)a+a)^{m+\frac{3}{2}}}{a^4}d(a\sin(e+fx))}{f\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

$$\Big\downarrow \text{ 153}$$

$$\frac{4\sqrt{2}\sec(e+fx)(a-a\sin(e+fx))(a\sin(e+fx)+a)^{m+3}\operatorname{AppellF1}\left(m+\frac{5}{2},-\frac{3}{2},4,m+\frac{7}{2},\frac{\sin(e+fx)a+a}{2a},\frac{\sin(e+fx)a}{a}\right.}{a^4f(2m+5)\sqrt{\frac{a-a\sin(e+fx)}{a}}}$$

input
```
Int[Cot[e + f*x]^4*(a + a*Sin[e + f*x])^m,x]
```

output
```
(4*Sqrt[2]*AppellF1[5/2 + m, -3/2, 4, 7/2 + m, (a + a*Sin[e + f*x])/(2*a),
 (a + a*Sin[e + f*x])/a]*Sec[e + f*x]*(a - a*Sin[e + f*x])*(a + a*Sin[e +
f*x])^(3 + m))/(a^4*f*(5 + 2*m)*Sqrt[(a - a*Sin[e + f*x])/a])
```

3.139.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 153
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1)*Simp
lify[b/(b*c - a*d)]^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c
 - a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m,
n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[Simplify[b/(
b*c - a*d)], 0] &&  !(GtQ[Simplify[d/(d*a - c*b)], 0] && SimplerQ[c + d*x,
a + b*x])
```

rule 154
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_] :> Simp[(c + d*x)^FracPart[n]/(Simplify[b/(b*c - a*d)]^IntPart[n
]*(b*((c + d*x)/(b*c - a*d)))^FracPart[n])   Int[(a + b*x)^m*Simp[b*(c/(b*c
 - a*d)) + b*d*(x/(b*c - a*d)), x]^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, m, n}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] &&  !G
tQ[Simplify[b/(b*c - a*d)], 0] &&  !SimplerQ[c + d*x, a + b*x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3198
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Simp[Sqrt[a + b*Sin[e + f*x]]*(Sqrt[a - b*Sin[e + f*x]]/(b*
f*Cos[e + f*x]))   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/
2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !IntegerQ[m] && IntegerQ[p/2]
```

3.139.4 Maple [F]

$$\int \left(\cot^{4}\left(fx+e\right)\right)\left(a+a\sin\left(fx+e\right)\right)^{m}dx$$

input
```
int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)
```

output
```
int(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x)
```

3.139.5 Fricas [F]

$$\int \cot^{4}(e+fx)(a+a\sin(e+fx))^{m}\,dx=\int \left(a\sin\left(fx+e\right)+a\right)^{m}\cot\left(fx+e\right)^{4}\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="fricas")
```

output
```
integral((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

3.139.6 Sympy [F]

$$\int \cot^{4}(e+fx)(a+a\sin(e+fx))^{m}\,dx=\int \left(a(\sin\left(e+fx\right)+1)\right)^{m}\cot^{4}\left(e+fx\right)dx$$

input
```
integrate(cot(f*x+e)**4*(a+a*sin(f*x+e))**m,x)
```

output
```
Integral((a*(sin(e + f*x) + 1))**m*cot(e + f*x)**4, x)
```

3.139.7 Maxima [F]

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx = \int (a\sin(fx+e)+a)^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="maxima")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

3.139.8 Giac [F]

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx = \int (a\sin(fx+e)+a)^m \cot(fx+e)^4\,dx$$

input
```
integrate(cot(f*x+e)^4*(a+a*sin(f*x+e))^m,x, algorithm="giac")
```

output
```
integrate((a*sin(f*x + e) + a)^m*cot(f*x + e)^4, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \cot^4(e+fx)(a+a\sin(e+fx))^m\,dx = \int \cot(e+f\,x)^4\,(a+a\,\sin(e+f\,x))^m\,dx$$

input
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^m,x)
```

output
```
int(cot(e + f*x)^4*(a + a*sin(e + f*x))^m, x)
```

3.140 $\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx$

3.140.1 Optimal result

Integrand size = 19, antiderivative size = 88

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx = \frac{(2a+3b)\log(1-\sin(c+dx))}{4d} + \frac{(2a-3b)\log(1+\sin(c+dx))}{4d} + \frac{3b\sin(c+dx)}{2d} + \frac{(a+b\sin(c+dx))\tan^2(c+dx)}{2d}$$

output
```
1/4*(2*a+3*b)*ln(1-sin(d*x+c))/d+1/4*(2*a-3*b)*ln(1+sin(d*x+c))/d+3/2*b*si
n(d*x+c)/d+1/2*(a+b*sin(d*x+c))*tan(d*x+c)^2/d
```

3.140.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx = -\frac{3b\text{arctanh}(\sin(c+dx))}{2d} + \frac{3b\sec(c+dx)\tan(c+dx)}{2d} - \frac{b\sin(c+dx)\tan^2(c+dx)}{d} + \frac{a(2\log(\cos(c+dx))+\tan^2(c+dx))}{2d}$$

input
```
Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

output
```
(-3*b*ArcTanh[Sin[c + d*x]])/(2*d) + (3*b*Sec[c + d*x]*Tan[c + d*x])/(2*d)
 - (b*Sin[c + d*x]*Tan[c + d*x]^2)/d + (a*(2*Log[Cos[c + d*x]] + Tan[c + d
*x]^2))/(2*d)
```

3.140.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 3200, 530, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3(a+b\sin(c+dx))dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b^3\sin^3(c+dx)(a+b\sin(c+dx))}{\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{530}$$

$$\frac{\frac{b^2(a+b\sin(c+dx))}{2(b^2-b^2\sin^2(c+dx))}-\frac{\int \frac{2\sin^2(c+dx)b^4+b^4+2a\sin(c+dx)b^3}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{2b^2}}{d}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\frac{b^2(a+b\sin(c+dx))}{2(b^2-b^2\sin^2(c+dx))}-\frac{\int \left(\frac{3b^4+2a\sin(c+dx)b^3}{b^2-b^2\sin^2(c+dx)}-2b^2\right)d(b\sin(c+dx))}{2b^2}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(a+b\sin(c+dx))}{2(b^2-b^2\sin^2(c+dx))}-\frac{-ab^2\log(b^2-b^2\sin^2(c+dx))+3b^3\text{arctanh}(\sin(c+dx))-2b^3\sin(c+dx)}{2b^2}}{d}$$

input
```
Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^3,x]
```

output
```
(-1/2*(3*b^3*ArcTanh[Sin[c + d*x]] - a*b^2*Log[b^2 - b^2*Sin[c + d*x]^2] -
 2*b^3*Sin[c + d*x])/b^2 + (b^2*(a + b*Sin[c + d*x]))/(2*(b^2 - b^2*Sin[c
+ d*x]^2)))/d
```

3.140.3.1 Defintions of rubi rules used

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.140.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+b\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+b\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
parts	$\frac{a\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{b\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$-iax-\frac{ib\,e^{i(dx+c)}}{2d}+\frac{ib\,e^{-i(dx+c)}}{2d}-\frac{2iac}{d}-\frac{i\left(b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}+2ia\,e^{2i(dx+c)}\right)}{d\left(1+e^{2i(dx+c)}\right)^2}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{d}-3$

input
```
int((a+b*sin(d*x+c))*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+b*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+
1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx$$
$$=\frac{(2\,a-3\,b)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+(2\,a+3\,b)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,(2\,b\,\mathrm{co}}{4\,d\cos\left(dx+c\right)^2}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
1/4*((2*a - 3*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) + (2*a + 3*b)*cos(d*
x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*b*cos(d*x + c)^2 + b)*sin(d*x + c)
+ 2*a)/(d*cos(d*x + c)^2)
```

3.140.6 Sympy [F]

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx=\int (a+b\sin{(c+dx)})\tan^3{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)**3,x)
```

output
```
Integral((a + b*sin(c + d*x))*tan(c + d*x)**3, x)
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.83

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx$$
$$=\frac{(2a-3b)\log{(\sin{(dx+c)}+1)}+(2a+3b)\log{(\sin{(dx+c)}-1)}+4b\sin{(dx+c)}-\frac{2(b\sin(dx+c)+a)}{\sin(dx+c)^2-1}}{4d}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*((2*a - 3*b)*log(sin(d*x + c) + 1) + (2*a + 3*b)*log(sin(d*x + c) - 1)
 + 4*b*sin(d*x + c) - 2*(b*sin(d*x + c) + a)/(sin(d*x + c)^2 - 1))/d
```

3.140.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22388 vs. 2(80) = 160.

Time = 83.24 (sec) , antiderivative size = 22388, normalized size of antiderivative = 254.41

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(3*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c)
+ 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*
d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 3*b*lo
g(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2
*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*ta
n(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2
 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 2*a*log(4*(tan(d*
x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 +
 tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 2*a*tan(
d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 6*b*log(2*(tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2
*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/
2*d*x)^6*tan(1/2*c)^6*tan(c) + 6*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2
*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^6*tan(1/
2*c)^6*tan(c) - 4*a*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(t
an(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^...
```

3.140.9 Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.00

$$\int (a+b\sin(c+dx))\tan^3(c+dx)\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(a+\frac{3b}{2}\right)}{d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(a-\frac{3b}{2}\right)}{d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d} - \frac{3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

input
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

output
```
(log(tan(c/2 + (d*x)/2) - 1)*(a + (3*b)/2))/d + (log(tan(c/2 + (d*x)/2) +
1)*(a - (3*b)/2))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (3*b*tan(c/2 +
 (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(
c/2 + (d*x)/2)^3 + 3*b*tan(c/2 + (d*x)/2)^5)/(d*(tan(c/2 + (d*x)/2)^2 + ta
n(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 - 1))
```

3.141 $\int (a + b\sin(c + dx))\tan(c + dx)\,dx$

3.141.1 Optimal result

Integrand size = 17, antiderivative size = 55

$$\int (a + b\sin(c + dx))\tan(c + dx)\,dx = -\frac{(a + b)\log(1 - \sin(c + dx))}{2d} - \frac{(a - b)\log(1 + \sin(c + dx))}{2d} - \frac{b\sin(c + dx)}{d}$$

output
```
-1/2*(a+b)*ln(1-sin(d*x+c))/d-1/2*(a-b)*ln(1+sin(d*x+c))/d-b*sin(d*x+c)/d
```

3.141.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

$$\int (a + b\sin(c + dx))\tan(c + dx)\,dx = \frac{b\text{arctanh}(\sin(c + dx))}{d} - \frac{a\log(\cos(c + dx))}{d} - \frac{b\sin(c + dx)}{d}$$

input
```
Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

output
```
(b*ArcTanh[Sin[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d - (b*Sin[c + d*x])/d
```

3.141.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3200, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)(a+b\sin(c+dx))dx$$

$$\Big\downarrow \text{ 3200}$$

$$\frac{\int \frac{b\sin(c+dx)(a+b\sin(c+dx))}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 523}$$

$$\frac{\int \left(\frac{b^2+a\sin(c+dx)b}{b^2-b^2\sin^2(c+dx)}-1\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{2}a\log\left(b^2-b^2\sin^2(c+dx)\right)+b\text{arctanh}(\sin(c+dx))-b\sin(c+dx)}{d}$$

input
```
Int[(a + b*Sin[c + d*x])*Tan[c + d*x],x]
```

output
```
(b*ArcTanh[Sin[c + d*x]] - (a*Log[b^2 - b^2*Sin[c + d*x]^2])/2 - b*Sin[c +
 d*x])/d
```

3.141.3.1 Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.141.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.75

method	result
derivativedivides	$\frac{-a\ln(\cos(dx+c))+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{-a\ln(\cos(dx+c))+b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
parts	$\frac{a\ln(1+\tan^2(dx+c))}{2d}+\frac{b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
risch	$iax+\frac{ib\,e^{i(dx+c)}}{2d}-\frac{ib\,e^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{a\ln(e^{i(dx+c)}+i)}{d}+\frac{\ln(e^{i(dx+c)}+i)b}{d}-\frac{a\ln(-i+e^{i(dx+c)})}{d}-\frac{\ln(-$

input
```
int((a+b*sin(d*x+c))*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a*ln(cos(d*x+c))+b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.82

$$\int (a + b\sin(c + dx))\tan(c + dx)\,dx = -\frac{(a - b)\log(\sin(dx + c) + 1) + (a + b)\log(-\sin(dx + c) + 1) + 2\,b\sin(dx + c)}{2\,d}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="fricas")
```

output
```
-1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(-sin(d*x + c) + 1) + 2*b
*sin(d*x + c))/d
```

3.141.6 Sympy [F]

$$\int (a + b\sin(c + dx))\tan(c + dx)\,dx = \int (a + b\sin(c + dx))\tan(c + dx)\,dx$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c),x)
```

output
```
Integral((a + b*sin(c + d*x))*tan(c + d*x), x)
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.78

$$\int (a + b\sin(c + dx))\tan(c + dx)\,dx = -\frac{(a - b)\log(\sin(dx + c) + 1) + (a + b)\log(\sin(dx + c) - 1) + 2\,b\sin(dx + c)}{2\,d}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*((a - b)*log(sin(d*x + c) + 1) + (a + b)*log(sin(d*x + c) - 1) + 2*b*
sin(d*x + c))/d
```

3.141.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. $2(51) = 102$.

Time = 0.45 (sec) , antiderivative size = 1200, normalized size of antiderivative = 21.82

$$\int (a + b\sin(c + dx))\tan(c + dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c),x, algorithm="giac")
```

output
```
-1/2*(b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d
*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + ta
n(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*log(2*(tan(1/2*d*x)^2*tan
(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + ta
n(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*
d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*t
an(1/2*c)^2 + a*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d
*x)^2*tan(c)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 +
 b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) -
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(1/2*d*x)^2 - b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(
1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^
2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2 + a*log(4*(tan(d*x)
^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + t
an(c)^2 + 1))*tan(1/2*d*x)^2 - 4*b*tan(1/2*d*x)^2*tan(1/2*c) + b*log(2*(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d*x)*t
an(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/...
```

3.141.9 Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.35

$$\int (a + b\sin(c + dx))\tan(c + dx)\,dx = \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,\left(a-b\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,\left(a+b\right)}{d} - \frac{b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(tan(c + d*x)*(a + b*sin(c + d*x)),x)
```

output
```
(a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)
)/d - (log(tan(c/2 + (d*x)/2) - 1)*(a + b))/d - (b*sin(c + d*x))/d
```

3.141. $\int (a+b\sin(c+dx))\tan(c+dx)\,dx$

3.142 $\int \cot(c+dx)(a+b\sin(c+dx))\,dx$

3.142.1 Optimal result

Integrand size = 17, antiderivative size = 24

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx = \frac{a\log(\sin(c+dx))}{d} + \frac{b\sin(c+dx)}{d}$$

output
```
a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

3.142.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx = \frac{a\log(\cos(c+dx))}{d} + \frac{a\log(\tan(c+dx))}{d} + \frac{b\cos(dx)\sin(c)}{d} + \frac{b\cos(c)\sin(dx)}{d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

output
```
(a*Log[Cos[c + d*x]])/d + (a*Log[Tan[c + d*x]])/d + (b*Cos[d*x]*Sin[c])/d
+ (b*Cos[c]*Sin[d*x])/d
```

3.142.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3200, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc(c+dx)(a+b\sin(c+dx))}{b}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{a\csc(c+dx)}{b}+1\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log(b\sin(c+dx))+b\sin(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Sin[c + d*x]),x]
```

output
```
(a*Log[b*Sin[c + d*x]] + b*Sin[c + d*x])/d
```

3.142.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.142.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{a\ln(\sin(dx+c))+b\sin(dx+c)}{d}$	23
default	$\frac{a\ln(\sin(dx+c))+b\sin(dx+c)}{d}$	23
risch	$-iax-\frac{2iac}{d}+\frac{a\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{b\sin(dx+c)}{d}$	43

input
```
int(cot(d*x+c)*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sin(d*x+c))+b*sin(d*x+c))
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx=\frac{a\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+b\sin\left(dx+c\right)}{d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
(a*log(1/2*sin(d*x + c)) + b*sin(d*x + c))/d
```

3.142.6 Sympy [F]

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx = \int (a+b\sin(c+dx))\cot(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x)
```

output
```
Integral((a + b*sin(c + d*x))*cot(c + d*x), x)
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx = \frac{a\log(\sin(dx+c))+b\sin(dx+c)}{d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
(a*log(sin(d*x + c)) + b*sin(d*x + c))/d
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.96

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx = \frac{a\log(|\sin(dx+c)|)+b\sin(dx+c)}{d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
(a*log(abs(sin(d*x + c))) + b*sin(d*x + c))/d
```

3.142.9 Mupad [B] (verification not implemented)

Time = 6.00 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.96

$$\int \cot(c+dx)(a+b\sin(c+dx))\,dx = \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} + \frac{b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cot(c + d*x)*(a + b*sin(c + d*x)),x)
```

output
```
(a*log(tan(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (b*s
in(c + d*x))/d
```

3.143 $\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx$

3.143.1 Optimal result

Integrand size = 19, antiderivative size = 54

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx = -\frac{b\csc(c+dx)}{d} - \frac{a\csc^2(c+dx)}{2d} - \frac{a\log(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d}$$

output
```
-b*csc(d*x+c)/d-1/2*a*csc(d*x+c)^2/d-a*ln(sin(d*x+c))/d-b*sin(d*x+c)/d
```

3.143.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx = -\frac{b\csc(c+dx)}{d} - \frac{a(\cot^2(c+dx)+2\log(\cos(c+dx))+2\log(\tan(c+dx)))}{2d} - \frac{b\sin(c+dx)}{d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

output
```
-((b*Csc[c + d*x])/d) - (a*(Cot[c + d*x]^2 + 2*Log[Cos[c + d*x]] + 2*Log[T
an[c + d*x]]))/(2*d) - (b*Sin[c + d*x])/d
```

3.143.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^3(c+dx)(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}{b^3}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{a\csc^3(c+dx)}{b}+\csc^2(c+dx)-\frac{a\csc(c+dx)}{b}-1\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a\log(b\sin(c+dx))-\frac{1}{2}a\csc^2(c+dx)-b\sin(c+dx)-b\csc(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]),x]
```

output
```
(-(b*Csc[c + d*x]) - (a*Csc[c + d*x]^2)/2 - a*Log[b*Sin[c + d*x]] - b*Sin[
c + d*x])/d
```

3.143.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.143.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

method	result	siz
derivativedivides	$\frac{a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)}{d}$	67
default	$\frac{a\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)}{d}$	67
risch	$iax+\frac{ib\,\mathrm{e}^{i(dx+c)}}{2d}-\frac{ib\,\mathrm{e}^{-i(dx+c)}}{2d}+\frac{2iac}{d}-\frac{2i\left(ia\,\mathrm{e}^{2i(dx+c)}+b\,\mathrm{e}^{3i(dx+c)}-b\,\mathrm{e}^{i(dx+c)}\right)}{d\left(\mathrm{e}^{2i(dx+c)}-1\right)^2}-\frac{a\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{d}$	12

input
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+b*(-1/sin(d*x+c)*cos(d*x+c)^4-(2
+cos(d*x+c)^2)*sin(d*x+c)))
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx = -\frac{2\left(a\cos\left(dx+c\right)^2-a\right)\log\left(\frac{1}{2}\sin\left(dx+c\right)\right)+2\left(b\cos\left(dx+c\right)^2-2\,b\right)\sin\left(dx+c\right)-a}{2\left(d\cos\left(dx+c\right)^2-d\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*(a*cos(d*x + c)^2 - a)*log(1/2*sin(d*x + c)) + 2*(b*cos(d*x + c)^2
 - 2*b)*sin(d*x + c) - a)/(d*cos(d*x + c)^2 - d)
```

3.143.6 Sympy [F]

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx = \int (a+b\sin\left(c+dx\right))\cot^3\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)),x)
```

output
```
Integral((a + b*sin(c + d*x))*cot(c + d*x)**3, x)
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx = -\frac{2\,a\log\left(\sin\left(dx+c\right)\right)+2\,b\sin\left(dx+c\right)+\frac{2\,b\sin(dx+c)+a}{\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*a*log(sin(d*x + c)) + 2*b*sin(d*x + c) + (2*b*sin(d*x + c) + a)/si
n(d*x + c)^2)/d
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx$$
$$= -\frac{2\,a\log\left(|\sin\left(dx+c\right)|\right)+2\,b\sin\left(dx+c\right)-\frac{3\,a\sin(dx+c)^2-2\,b\sin(dx+c)-a}{\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*a*log(abs(sin(d*x + c))) + 2*b*sin(d*x + c) - (3*a*sin(d*x + c)^2
- 2*b*sin(d*x + c) - a)/sin(d*x + c)^2)/d
```

3.143.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.70

$$\int \cot^3(c+dx)(a+b\sin(c+dx))\,dx$$
$$= \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d} - \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$
$$- \frac{10\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{2}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$
$$- \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d} - \frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x)),x)
```

output
```
(a*log(tan(c/2 + (d*x)/2)^2 + 1))/d - (b*tan(c/2 + (d*x)/2))/(2*d) - (a/2
+ 2*b*tan(c/2 + (d*x)/2) + (a*tan(c/2 + (d*x)/2)^2)/2 + 10*b*tan(c/2 + (d*
x)/2)^3)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*tan(c/2 + (d*x)/2)^4)) - (a*tan(c/
2 + (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2)))/d
```

3.144 $\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$

3.144.1 Optimal result

Integrand size = 19, antiderivative size = 81

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{2b\csc(c+dx)}{d}+\frac{a\csc^2(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{a\csc^4(c+dx)}{4d}+\frac{a\log(\sin(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

output
```
2*b*csc(d*x+c)/d+a*csc(d*x+c)^2/d-1/3*b*csc(d*x+c)^3/d-1/4*a*csc(d*x+c)^4/
d+a*ln(sin(d*x+c))/d+b*sin(d*x+c)/d
```

3.144.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx = \frac{a\cot^2(c+dx)}{2d}-\frac{a\cot^4(c+dx)}{4d}+\frac{2b\csc(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}+\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\tan(c+dx))}{d}+\frac{b\sin(c+dx)}{d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

output
```
(a*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^4)/(4*d) + (2*b*Csc[c + d*x])/d
 - (b*Csc[c + d*x]^3)/(3*d) + (a*Log[Cos[c + d*x]])/d + (a*Log[Tan[c + d*x
]])/d + (b*Sin[c + d*x])/d
```

3.144.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^5(c+dx)(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)^2}{b^5}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{a\csc^5(c+dx)}{b}+\csc^4(c+dx)-\frac{2a\csc^3(c+dx)}{b}-2\csc^2(c+dx)+\frac{a\csc(c+dx)}{b}+1\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log(b\sin(c+dx))-\frac{1}{4}a\csc^4(c+dx)+a\csc^2(c+dx)+b\sin(c+dx)-\frac{1}{3}b\csc^3(c+dx)+2b\csc(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x]),x]
```

output
```
(2*b*Csc[c + d*x] + a*Csc[c + d*x]^2 - (b*Csc[c + d*x]^3)/3 - (a*Csc[c + d
*x]^4)/4 + a*Log[b*Sin[c + d*x]] + b*Sin[c + d*x])/d
```

3.144.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.144.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\cdots}{d}$
default	$\frac{a\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\cdots}{d}$
risch	$-iax-\frac{ib\,\mathrm{e}^{i(dx+c)}}{2d}+\frac{ib\,\mathrm{e}^{-i(dx+c)}}{2d}-\frac{2iac}{d}+\frac{4i\left(3ia\,\mathrm{e}^{6i(dx+c)}+3b\,\mathrm{e}^{7i(dx+c)}-3ia\,\mathrm{e}^{4i(dx+c)}-7b\,\mathrm{e}^{5i(dx+c)}+3ia\,\mathrm{e}^{2i\cdots}\right)}{3d\left(\mathrm{e}^{2i(dx+c)}-1\right)^4}$

input
```
int(cot(d*x+c)^5*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+b*(-1/3/sin(d*x
+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*cos(d*x
+c)^2)*sin(d*x+c)))
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.36

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx =$$
$$-\frac{12\,a\cos\left(dx+c\right)^2-12\left(a\cos\left(dx+c\right)^4-2\,a\cos\left(dx+c\right)^2+a\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\left(3\,b\cos\left(dx+c\right.\right.}{12\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(dx+c\right)^2+d\right)}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/12*(12*a*cos(d*x + c)^2 - 12*(a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^2 + a
)*log(1/2*sin(d*x + c)) - 4*(3*b*cos(d*x + c)^4 - 12*b*cos(d*x + c)^2 + 8*
b)*sin(d*x + c) - 9*a)/(d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)
```

3.144.6 Sympy [F]

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx = \int \left(a+b\sin\left(c+dx\right)\right)\cot^5\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**5*(a+b*sin(d*x+c)),x)
```

output
```
Integral((a + b*sin(c + d*x))*cot(c + d*x)**5, x)
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$$
$$=\frac{12\,a\log\left(\sin\left(dx+c\right)\right)+12\,b\sin\left(dx+c\right)+\frac{24\,b\sin(dx+c)^3+12\,a\sin(dx+c)^2-4\,b\sin(dx+c)-3\,a}{\sin(dx+c)^4}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(12*a*log(sin(d*x + c)) + 12*b*sin(d*x + c) + (24*b*sin(d*x + c)^3 +
12*a*sin(d*x + c)^2 - 4*b*sin(d*x + c) - 3*a)/sin(d*x + c)^4)/d
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx$$
$$=\frac{12\,a\log\left(|\sin\left(dx+c\right)|\right)+12\,b\sin\left(dx+c\right)-\frac{25\,a\sin(dx+c)^4-24\,b\sin(dx+c)^3-12\,a\sin(dx+c)^2+4\,b\sin(dx+c)+3\,a}{\sin(dx+c)^4}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/12*(12*a*log(abs(sin(d*x + c))) + 12*b*sin(d*x + c) - (25*a*sin(d*x + c)
^4 - 24*b*sin(d*x + c)^3 - 12*a*sin(d*x + c)^2 + 4*b*sin(d*x + c) + 3*a)/s
in(d*x + c)^4)/d
```

3.144.9 Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.56

$$\int \cot^5(c+dx)(a+b\sin(c+dx))\,dx=\frac{7\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d}$$
$$+\frac{46\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{40\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4}-\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{3}-\frac{a}{4}}{d\,\left(16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+16\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}$$
$$-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}{d}+\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,d}$$
$$-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

input
```
int(cot(c + d*x)^5*(a + b*sin(c + d*x)),x)
```

output
```
(7*b*tan(c/2 + (d*x)/2))/(8*d) + ((11*a*tan(c/2 + (d*x)/2)^2)/4 - (2*b*tan
(c/2 + (d*x)/2))/3 - a/4 + 3*a*tan(c/2 + (d*x)/2)^4 + (40*b*tan(c/2 + (d*x
)/2)^3)/3 + 46*b*tan(c/2 + (d*x)/2)^5)/(d*(16*tan(c/2 + (d*x)/2)^4 + 16*ta
n(c/2 + (d*x)/2)^6)) - (a*log(tan(c/2 + (d*x)/2)^2 + 1))/d + (3*a*tan(c/2
+ (d*x)/2)^2)/(16*d) - (a*tan(c/2 + (d*x)/2)^4)/(64*d) - (b*tan(c/2 + (d*x
)/2)^3)/(24*d) + (a*log(tan(c/2 + (d*x)/2)))/d
```

3.145 $\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx$

3.145.1 Optimal result

Integrand size = 19, antiderivative size = 72

$$\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx = ax-\frac{b\cos(c+dx)}{d}-\frac{2b\sec(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

output
```
a*x-b*cos(d*x+c)/d-2*b*sec(d*x+c)/d+1/3*b*sec(d*x+c)^3/d-a*tan(d*x+c)/d+1/
3*a*tan(d*x+c)^3/d
```

3.145.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

$$\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx = \frac{a\arctan(\tan(c+dx))}{d}-\frac{b\cos(c+dx)}{d}-\frac{2b\sec(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

input
```
Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

output
```
(a*ArcTan[Tan[c + d*x]])/d - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (
b*Sec[c + d*x]^3)/(3*d) - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

3.145.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^4(a+b\sin(c+dx))dx$$

$$\Big\downarrow \; 3201$$

$$\int \left(a\tan^4(c+dx)+b\sin(c+dx)\tan^4(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a\tan^3(c+dx)}{3d}-\frac{a\tan(c+dx)}{d}+ax-\frac{b\cos(c+dx)}{d}+\frac{b\sec^3(c+dx)}{3d}-\frac{2b\sec(c+dx)}{d}$$

input
```
Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^4,x]
```

output
```
a*x - (b*Cos[c + d*x])/d - (2*b*Sec[c + d*x])/d + (b*Sec[c + d*x]^3)/(3*d)
 - (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

3.145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.145.4 Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$
default	$\frac{a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$
parts	$\frac{a\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\right.}{d}$
risch	$ax-\frac{b\,e^{i(dx+c)}}{2d}-\frac{b\,e^{-i(dx+c)}}{2d}-\frac{4\left(3ia\,e^{4i(dx+c)}+3b\,e^{5i(dx+c)}+3ia\,e^{2i(dx+c)}+4b\,e^{3i(dx+c)}+2ia+3b\,e^{i(dx+c)}\right)}{3d\left(1+e^{2i(dx+c)}\right)^3}$

input
```
int((a+b*sin(d*x+c))*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+b*(1/3*sin(d*x+c)^6/cos(d*x+c)^
3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx$$
$$=\frac{3\,adx\cos{(dx+c)}^3-3\,b\cos{(dx+c)}^4-6\,b\cos{(dx+c)}^2-\left(4\,a\cos{(dx+c)}^2-a\right)\sin{(dx+c)}+b}{3\,d\cos{(dx+c)}^3}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/3*(3*a*d*x*cos(d*x + c)^3 - 3*b*cos(d*x + c)^4 - 6*b*cos(d*x + c)^2 - (4
*a*cos(d*x + c)^2 - a)*sin(d*x + c) + b)/(d*cos(d*x + c)^3)
```

3.145.6 Sympy [F]

$$\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx = \int (a+b\sin{(c+dx)})\tan^4{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)**4,x)
```

output
```
Integral((a + b*sin(c + d*x))*tan(c + d*x)**4, x)
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx = \frac{(\tan{(dx+c)}^3 + 3\,dx + 3\,c - 3\,\tan{(dx+c)})a - b\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3} + 3\,\cos{(dx+c)}\right)}{3\,d}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/3*((tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a - b*((6*cos(d*x + c
)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/d
```

3.145.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 13744 vs. 2(68) = 136.

Time = 294.68 (sec) , antiderivative size = 13744, normalized size of antiderivative = 190.89

$$\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
1/3*(3*a*d*x*tan(d*x)^3*tan(1/2*d*x)^8*tan(1/2*c)^8*tan(c)^3 - 9*a*d*x*tan
(d*x)^2*tan(1/2*d*x)^8*tan(1/2*c)^8*tan(c)^2 - 6*a*d*x*tan(d*x)^3*tan(1/2*
d*x)^8*tan(1/2*c)^6*tan(c)^3 - 36*a*d*x*tan(d*x)^3*tan(1/2*d*x)^7*tan(1/2*
c)^7*tan(c)^3 - 6*a*d*x*tan(d*x)^3*tan(1/2*d*x)^6*tan(1/2*c)^8*tan(c)^3 -
8*b*tan(d*x)^3*tan(1/2*d*x)^8*tan(1/2*c)^8*tan(c)^3 + 3*a*tan(d*x)^3*tan(1
/2*d*x)^8*tan(1/2*c)^8*tan(c)^2 + 3*a*tan(d*x)^2*tan(1/2*d*x)^8*tan(1/2*c)
^8*tan(c)^3 + 9*a*d*x*tan(d*x)*tan(1/2*d*x)^8*tan(1/2*c)^8*tan(c) + 18*a*d
*x*tan(d*x)^2*tan(1/2*d*x)^8*tan(1/2*c)^6*tan(c)^2 + 108*a*d*x*tan(d*x)^2*
tan(1/2*d*x)^7*tan(1/2*c)^7*tan(c)^2 + 18*a*d*x*tan(d*x)^2*tan(1/2*d*x)^6*
tan(1/2*c)^8*tan(c)^2 + 24*b*tan(d*x)^2*tan(1/2*d*x)^8*tan(1/2*c)^8*tan(c)
^2 + 36*a*d*x*tan(d*x)^3*tan(1/2*d*x)^7*tan(1/2*c)^5*tan(c)^3 + 156*a*d*x*
tan(d*x)^3*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^3 + 16*b*tan(d*x)^3*tan(1/2*
d*x)^8*tan(1/2*c)^6*tan(c)^3 + 36*a*d*x*tan(d*x)^3*tan(1/2*d*x)^5*tan(1/2*
c)^7*tan(c)^3 + 96*b*tan(d*x)^3*tan(1/2*d*x)^7*tan(1/2*c)^7*tan(c)^3 + 16*
b*tan(d*x)^3*tan(1/2*d*x)^6*tan(1/2*c)^8*tan(c)^3 - a*tan(d*x)^3*tan(1/2*d
*x)^8*tan(1/2*c)^8 - 9*a*tan(d*x)^2*tan(1/2*d*x)^8*tan(1/2*c)^8*tan(c) - 6
*a*tan(d*x)^3*tan(1/2*d*x)^8*tan(1/2*c)^6*tan(c)^2 - 36*a*tan(d*x)^3*tan(1
/2*d*x)^7*tan(1/2*c)^7*tan(c)^2 - 6*a*tan(d*x)^3*tan(1/2*d*x)^6*tan(1/2*c)
^8*tan(c)^2 - 9*a*tan(d*x)*tan(1/2*d*x)^8*tan(1/2*c)^8*tan(c)^2 - 6*a*tan(
d*x)^2*tan(1/2*d*x)^8*tan(1/2*c)^6*tan(c)^3 - 36*a*tan(d*x)^2*tan(1/2*d...
```

3.145.9 Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.53

$$\int (a+b\sin(c+dx))\tan^4(c+dx)\,dx = a\,x + \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{3}-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-\frac{32\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{16\,b}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^3\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

input
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

output
```
a*x + ((16*b)/3 + 2*a*tan(c/2 + (d*x)/2) - (14*a*tan(c/2 + (d*x)/2)^3)/3 -
 (14*a*tan(c/2 + (d*x)/2)^5)/3 + 2*a*tan(c/2 + (d*x)/2)^7 - (32*b*tan(c/2
+ (d*x)/2)^2)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^3*(tan(c/2 + (d*x)/2)^2 + 1
))
```

3.146 $\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$

3.146.1 Optimal result

Integrand size = 19, antiderivative size = 38

$$\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx = -ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

output
```
-a*x+b*cos(d*x+c)/d+b*sec(d*x+c)/d+a*tan(d*x+c)/d
```

3.146.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx = -\frac{a\arctan(\tan(c+dx))}{d}+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}+\frac{a\tan(c+dx)}{d}$$

input
```
Integrate[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

output
```
-((a*ArcTan[Tan[c + d*x]])/d) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d +
(a*Tan[c + d*x])/d
```

3.146.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+b\sin(c+dx))dx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a\tan^2(c+dx)+b\sin(c+dx)\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\tan(c+dx)}{d}-ax+\frac{b\cos(c+dx)}{d}+\frac{b\sec(c+dx)}{d}$$

input
```
Int[(a + b*Sin[c + d*x])*Tan[c + d*x]^2,x]
```

output
```
-(a*x) + (b*Cos[c + d*x])/d + (b*Sec[c + d*x])/d + (a*Tan[c + d*x])/d
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.146.4 Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	59
default	$\frac{a(\tan(dx+c)-dx-c)+b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	59
parts	$\frac{a(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)}{d}$	63
risch	$-ax+\frac{b\,e^{i(dx+c)}}{2d}+\frac{b\,e^{-i(dx+c)}}{2d}+\frac{2ia+2b\,e^{i(dx+c)}}{d(1+e^{2i(dx+c)})}$	70

input
```
int((a+b*sin(d*x+c))*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(tan(d*x+c)-d*x-c)+b*(1/cos(d*x+c)*sin(d*x+c)^4+(2+sin(d*x+c)^2)*co
s(d*x+c)))
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx = -\frac{adx\cos(dx+c)-b\cos(dx+c)^2-a\sin(dx+c)-b}{d\cos(dx+c)}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-(a*d*x*cos(d*x + c) - b*cos(d*x + c)^2 - a*sin(d*x + c) - b)/(d*cos(d*x +
 c))
```

3.146.6 Sympy [F]

$$\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx = \int (a+b\sin(c+dx))\tan^2(c+dx)\,dx$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)**2,x)
```

output
```
Integral((a + b*sin(c + d*x))*tan(c + d*x)**2, x)
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.03

$$\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx = -\frac{(dx+c-\tan(dx+c))a-b\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)}{d}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-((d*x + c - tan(d*x + c))*a - b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

3.146.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. 2(38) = 76.

Time = 1.03 (sec) , antiderivative size = 1008, normalized size of antiderivative = 26.53

$$\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(d*x+c))*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-(a*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - a*d*x*tan(1/2*d*x)^4
*tan(1/2*c)^4 - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) - 2*b*
tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + a*tan(d*x)*tan(1/2*d*x)^4*ta
n(1/2*c)^4 + a*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 4*a*d*x*tan(1/2*d*x)^3
*tan(1/2*c)^3 + 2*b*tan(1/2*d*x)^4*tan(1/2*c)^4 - a*d*x*tan(d*x)*tan(1/2*d
*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 4*a*d*x
*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) + 8*b*tan(d*x)*tan(1/2*d*x)^3*t
an(1/2*c)^3*tan(c) - a*d*x*tan(d*x)*tan(1/2*c)^4*tan(c) - 4*a*tan(d*x)*tan
(1/2*d*x)^3*tan(1/2*c)^3 - 4*a*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + a*d*x*
tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)^3*tan(1/2*c) + 4*a*d*x*tan(1/2*d*x)*
tan(1/2*c)^3 - 8*b*tan(1/2*d*x)^3*tan(1/2*c)^3 + a*d*x*tan(1/2*c)^4 - 2*b*
tan(d*x)*tan(1/2*d*x)^4*tan(c) - 4*a*d*x*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)*
tan(c) - 8*b*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c)*tan(c) - 24*b*tan(d*x)*tan
(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 8*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^3*t
an(c) - 2*b*tan(d*x)*tan(1/2*c)^4*tan(c) - a*tan(d*x)*tan(1/2*d*x)^4 - 4*a
*tan(d*x)*tan(1/2*d*x)^3*tan(1/2*c) - 4*a*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)
^3 - a*tan(d*x)*tan(1/2*c)^4 - a*tan(1/2*d*x)^4*tan(c) - 4*a*tan(1/2*d*x)^
3*tan(1/2*c)*tan(c) - 4*a*tan(1/2*d*x)*tan(1/2*c)^3*tan(c) - a*tan(1/2*c)^
4*tan(c) + 2*b*tan(1/2*d*x)^4 + 4*a*d*x*tan(1/2*d*x)*tan(1/2*c) + 8*b*tan(
1/2*d*x)^3*tan(1/2*c) + 24*b*tan(1/2*d*x)^2*tan(1/2*c)^2 + 8*b*tan(1/2*...
```

3.146.9 Mupad [B] (verification not implemented)

Time = 6.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.45

$$\int (a+b\sin(c+dx))\tan^2(c+dx)\,dx = -a\,x-\frac{2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+4\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-1\right)}$$

input
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

output
```
- a*x - (4*b + 2*a*tan(c/2 + (d*x)/2) + 2*a*tan(c/2 + (d*x)/2)^3)/(d*(tan(
c/2 + (d*x)/2)^4 - 1))
```

3.147 $\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$

3.147.1 Optimal result

Integrand size = 19, antiderivative size = 41

$$\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx = -ax - \frac{b\operatorname{arctanh}(\cos(c+dx))}{d} + \frac{b\cos(c+dx)}{d} - \frac{a\cot(c+dx)}{d}$$

output
```
-a*x-b*arctanh(cos(d*x+c))/d+b*cos(d*x+c)/d-a*cot(d*x+c)/d
```

3.147.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.83

$$\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx = \frac{b\cos(c+dx)}{d} - \frac{a\cot(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},-\tan^2(c+dx)\right)}{d} - \frac{b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

output
```
(b*Cos[c + d*x])/d - (a*Cot[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[
c + d*x]^2])/d - (b*Log[Cos[(c + d*x)/2]])/d + (b*Log[Sin[(c + d*x)/2]])/d
```

3.147.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{a+b\sin(c+dx)}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \; 3201$$

$$\int \left(a\cot^2(c+dx)+b\cos(c+dx)\cot(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{a\cot(c+dx)}{d}-ax-\frac{b\,\text{arctanh}(\cos(c+dx))}{d}+\frac{b\cos(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]),x]
```

output
```
-(a*x) - (b*ArcTanh[Cos[c + d*x]])/d + (b*Cos[c + d*x])/d - (a*Cot[c + d*x
])/d
```

3.147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.147.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

method	result	size
derivativedivides	$\frac{a(-\cot(dx+c)-dx-c)+b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$	49
default	$\frac{a(-\cot(dx+c)-dx-c)+b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))}{d}$	49
risch	$-ax+\frac{b\,e^{i(dx+c)}}{2d}+\frac{b\,e^{-i(dx+c)}}{2d}-\frac{2ia}{d(e^{2i(dx+c)}-1)}-\frac{b\ln(e^{i(dx+c)}+1)}{d}+\frac{b\ln(e^{i(dx+c)}-1)}{d}$	91

input
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-cot(d*x+c)-d*x-c)+b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))))
```

3.147.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(41) = 82.

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.05

$$\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx =$$
$$-\frac{b\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-b\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)+2\,a\cos\left(dx+c\right)+2\,(}{2\,d\sin\left(dx+c\right)}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) + 2*a*cos(d*x + c) + 2*(a*d*x - b*cos(d*x + c))*sin(d
*x + c))/(d*sin(d*x + c))
```

3.147.6 Sympy [F]

$$\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx = \int (a+b\sin{(c+dx)})\cot^2{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)),x)
```

output
```
Integral((a + b*sin(c + d*x))*cot(c + d*x)**2, x)
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.32

$$\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx = -\frac{2\left(dx+c+\frac{1}{\tan(dx+c)}\right)a - b(2\cos{(dx+c)} - \log{(\cos{(dx+c)}+1)} + \log{(\cos{(dx+c)}-1)})}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*(2*(d*x + c + 1/tan(d*x + c))*a - b*(2*cos(d*x + c) - log(cos(d*x + c
) + 1) + log(cos(d*x + c) - 1)))/d
```

3.147.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(41) = 82.

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.63

$$\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx = -\frac{6\,(dx+c)a - 6\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right) - 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + \frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-10\,b\tan(\cdots}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{6\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/6*(6*(d*x + c)*a - 6*b*log(abs(tan(1/2*d*x + 1/2*c))) - 3*a*tan(1/2*d*x
 + 1/2*c) + (2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1/2*c)^2 - 10*
b*tan(1/2*d*x + 1/2*c) + 3*a)/(tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*
c)))/d
```

3.147.9 Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.85

$$\int \cot^2(c+dx)(a+b\sin(c+dx))\,dx = \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}+\frac{b\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a}{d\left(2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} + \frac{2a\,\mathrm{atan}\left(\frac{4a^2}{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+4ba}-\frac{4ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+4ba}\right)}{d}$$

input
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x)),x)
```

output
```
(a*tan(c/2 + (d*x)/2))/(2*d) + (b*log(tan(c/2 + (d*x)/2)))/d - (a - 4*b*ta
n(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)/(d*(2*tan(c/2 + (d*x)/2) + 2*ta
n(c/2 + (d*x)/2)^3)) + (2*a*atan((4*a^2)/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2)
) - (4*a*b*tan(c/2 + (d*x)/2))/(4*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.148 $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$

3.148.1 Optimal result

Integrand size = 19, antiderivative size = 82

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx = ax+\frac{3b\text{arctanh}(\cos(c+dx))}{2d}-\frac{3b\cos(c+dx)}{2d}+\frac{a\cot(c+dx)}{d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a\cot^3(c+dx)}{3d}$$

output
```
a*x+3/2*b*arctanh(cos(d*x+c))/d-3/2*b*cos(d*x+c)/d+a*cot(d*x+c)/d-1/2*b*co
s(d*x+c)*cot(d*x+c)^2/d-1/3*a*cot(d*x+c)^3/d
```

3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.52

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx = -\frac{b\cos(c+dx)}{d}-\frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\cot^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},-\tan^2(c+dx)\right)}{3d}+\frac{3b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{3b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

output
```
-((b*Cos[c + d*x])/d) - (b*Csc[(c + d*x)/2]^2)/(8*d) - (a*Cot[c + d*x]^3*H
ypergeometric2F1[-3/2, 1, -1/2, -Tan[c + d*x]^2])/(3*d) + (3*b*Log[Cos[(c
+ d*x)/2]])/(2*d) - (3*b*Log[Sin[(c + d*x)/2]])/(2*d) + (b*Sec[(c + d*x)/2
]^2)/(8*d)
```

3.148.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a\cot^4(c+dx)+b\cos(c+dx)\cot^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cot^3(c+dx)}{3d}+\frac{a\cot(c+dx)}{d}+ax+\frac{3b\text{arctanh}(\cos(c+dx))}{2d}-\frac{3b\cos(c+dx)}{2d}-\frac{b\cos(c+dx)\cot^2(c+dx)}{2d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]),x]
```

output
```
a*x + (3*b*ArcTanh[Cos[c + d*x]])/(2*d) - (3*b*Cos[c + d*x])/(2*d) + (a*Co
t[c + d*x])/d - (b*Cos[c + d*x]*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*x]^3)
/(3*d)
```

3.148.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.148.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^{3}(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos^{5}(dx+c)}{2\sin(dx+c)^{2}}-\frac{\left(\cos^{3}(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
default	$\frac{a\left(-\frac{\left(\cot^{3}(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+b\left(-\frac{\cos^{5}(dx+c)}{2\sin(dx+c)^{2}}-\frac{\left(\cos^{3}(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{d}$
risch	$ax-\frac{b\,e^{i(dx+c)}}{2d}-\frac{b\,e^{-i(dx+c)}}{2d}+\frac{12ia\,e^{4i(dx+c)}+3b\,e^{5i(dx+c)}-12ia\,e^{2i(dx+c)}+8ia-3b\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}-1\right)^{3}}-\frac{3b\ln\left(e^{i(dx+c)}-\right.}{2d}$

input
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+b*(-1/2/sin(d*x+c)^2*cos(d*x+c
)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c))))
```

3.148.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(74) = 148.

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.95

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$$
$$= \frac{16\,a\cos\left(dx+c\right)^3+9\left(b\cos\left(dx+c\right)^2-b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx+c\right)-9\left(b\cos\left(dx+c\right)^2-b\right)\text{lo}}{1}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(16*a*cos(d*x + c)^3 + 9*(b*cos(d*x + c)^2 - b)*log(1/2*cos(d*x + c)
+ 1/2)*sin(d*x + c) - 9*(b*cos(d*x + c)^2 - b)*log(-1/2*cos(d*x + c) + 1/2
)*sin(d*x + c) - 12*a*cos(d*x + c) + 6*(2*a*d*x*cos(d*x + c)^2 - 2*b*cos(d
*x + c)^3 - 2*a*d*x + 3*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 -
 d)*sin(d*x + c))
```

3.148.6 Sympy [F]

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx = \int \left(a+b\sin\left(c+dx\right)\right)\cot^4\left(c+dx\right)\,dx$$

input
```
integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)),x)
```

output
```
Integral((a + b*sin(c + d*x))*cot(c + d*x)**4, x)
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.12

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$$
$$= \frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a+3\,b\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos\left(dx+c\right)+3\,\log\left(\cos\left(dx+c\right)+1\right)-3\,\log\left(\cos\right.\right.}{12\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a + 3*b*(2*c
os(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x + c) + 1
) - 3*log(cos(d*x + c) - 1)))/d
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.72

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+24\,(dx+c)a-36\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a\tan\left(\frac{1}{2}\,dx\right.}{24\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/24*(a*tan(1/2*d*x + 1/2*c)^3 + 3*b*tan(1/2*d*x + 1/2*c)^2 + 24*(d*x + c)
*a - 36*b*log(abs(tan(1/2*d*x + 1/2*c))) - 15*a*tan(1/2*d*x + 1/2*c) - 48*
b/(tan(1/2*d*x + 1/2*c)^2 + 1) + (66*b*tan(1/2*d*x + 1/2*c)^3 + 15*a*tan(1
/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*x + 1/2*c) - a)/tan(1/2*d*x + 1/2*c)^3)/
d
```

3.148.9 Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 225, normalized size of antiderivative = 2.74

$$\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$$
$$= \frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d} - \frac{-5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+17\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{14\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{a}{3}}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$
$$- \frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\,d} + \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d} - \frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{2\,d}$$
$$- \frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\,b\,a}-\frac{6\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+6\,b\,a}\right)}{d}$$

input
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x)),x)
```

output
```
(a*tan(c/2 + (d*x)/2)^3)/(24*d) - (a/3 + b*tan(c/2 + (d*x)/2) - (14*a*tan(
c/2 + (d*x)/2)^2)/3 - 5*a*tan(c/2 + (d*x)/2)^4 + 17*b*tan(c/2 + (d*x)/2)^3
)/(d*(8*tan(c/2 + (d*x)/2)^3 + 8*tan(c/2 + (d*x)/2)^5)) - (5*a*tan(c/2 + (
d*x)/2))/(8*d) + (b*tan(c/2 + (d*x)/2)^2)/(8*d) - (3*b*log(tan(c/2 + (d*x)
/2)))/(2*d) - (2*a*atan((4*a^2)/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2)) - (6*a*
b*tan(c/2 + (d*x)/2))/(6*a*b + 4*a^2*tan(c/2 + (d*x)/2))))/d
```

3.148. $\int \cot^4(c+dx)(a+b\sin(c+dx))\,dx$

3.149 $\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx$

3.149.1 Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx = -ax - \frac{15b\text{arctanh}(\cos(c+dx))}{8d} + \frac{15b\cos(c+dx)}{8d} - \frac{a\cot(c+dx)}{d} + \frac{5b\cos(c+dx)\cot^2(c+dx)}{8d} + \frac{a\cot^3(c+dx)}{3d} - \frac{b\cos(c+dx)\cot^4(c+dx)}{4d} - \frac{a\cot^5(c+dx)}{5d}$$

output
```
-a*x-15/8*b*arctanh(cos(d*x+c))/d+15/8*b*cos(d*x+c)/d-a*cot(d*x+c)/d+5/8*b
*cos(d*x+c)*cot(d*x+c)^2/d+1/3*a*cot(d*x+c)^3/d-1/4*b*cos(d*x+c)*cot(d*x+c
)^4/d-1/5*a*cot(d*x+c)^5/d
```

3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.34

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx = \frac{b\cos(c+dx)}{d}+\frac{9b\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}-\frac{b\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\cot^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},-\tan^2(c+dx)\right)}{5d} - \frac{15b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{8d}+\frac{15b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{9b\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{b\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

output
```
(b*Cos[c + d*x])/d + (9*b*Csc[(c + d*x)/2]^2)/(32*d) - (b*Csc[(c + d*x)/2]
^4)/(64*d) - (a*Cot[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, -Tan[c + d
*x]^2])/(5*d) - (15*b*Log[Cos[(c + d*x)/2]])/(8*d) + (15*b*Log[Sin[(c + d*
x)/2]])/(8*d) - (9*b*Sec[(c + d*x)/2]^2)/(32*d) + (b*Sec[(c + d*x)/2]^4)/(
64*d)
```

3.149.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin(c+dx)}{\tan(c+dx)^6}dx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a\cot^6(c+dx)+b\cos(c+dx)\cot^5(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cot^5(c+dx)}{5d}+\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-ax-\frac{15b\operatorname{arctanh}(\cos(c+dx))}{8d}+\frac{15b\cos(c+dx)}{8d}-\frac{b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{5b\cos(c+dx)\cot^2(c+dx)}{8d}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x]),x]
```

output
```
-(a*x) - (15*b*ArcTanh[Cos[c + d*x]])/(8*d) + (15*b*Cos[c + d*x])/(8*d) -
(a*Cot[c + d*x])/d + (5*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*d) + (a*Cot[c +
d*x]^3)/(3*d) - (b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a*Cot[c + d*x]^5)
/(5*d)
```

3.149.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.149.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.06

method	result
derivativedivides	$\frac{a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c\cdots\right.}{8}\cdots}{d}$
default	$\frac{a\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+b\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(dx+c\cdots\right.}{8}\cdots}{d}$
risch	$-ax+\frac{b\,e^{i(dx+c)}}{2d}+\frac{b\,e^{-i(dx+c)}}{2d}-\frac{360ia\,e^{8i(dx+c)}+135b\,e^{9i(dx+c)}-720ia\,e^{6i(dx+c)}-150b\,e^{7i(dx+c)}+1120ia\,e^{4i(d\cdots}}{60d\left(e^{2i(dx+c)}-1\right)^5\cdots}$

input
```
int(cot(d*x+c)^6*(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+b*(-1/4/sin(d
*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+5/8*co
s(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c))))
```

3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.

Time = 0.29 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.82

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx =$$
$$-\frac{368\,a\cos\left(dx+c\right)^5 - 560\,a\cos\left(dx+c\right)^3 + 225\left(b\cos\left(dx+c\right)^4 - 2\,b\cos\left(dx+c\right)^2 + b\right)\log\left(\frac{1}{2}\,\cos\left(dx+\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/240*(368*a*cos(d*x + c)^5 - 560*a*cos(d*x + c)^3 + 225*(b*cos(d*x + c)^
4 - 2*b*cos(d*x + c)^2 + b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225
*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-1/2*cos(d*x + c) + 1/2)*
sin(d*x + c) + 240*a*cos(d*x + c) + 30*(8*a*d*x*cos(d*x + c)^4 - 8*b*cos(d
*x + c)^5 - 16*a*d*x*cos(d*x + c)^2 + 25*b*cos(d*x + c)^3 + 8*a*d*x - 15*b
*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*
sin(d*x + c))
```

3.149.6 Sympy [F]

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx = \int (a+b\sin(c+dx))\cot^6(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**6*(a+b*sin(d*x+c)),x)
```

output
```
Integral((a + b*sin(c + d*x))*cot(c + d*x)**6, x)
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx =$$
$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a+15\,b\left(\frac{2\left(9\,\cos(dx+c)^3-7\,\cos(dx+c)\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}-16\,\cos(dx+c)+15\right.}{240\,d}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan
(d*x + c)^5)*a + 15*b*(2*(9*cos(d*x + c)^3 - 7*cos(d*x + c))/(cos(d*x + c)
^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c) + 15*log(cos(d*x + c) + 1) -
15*log(cos(d*x + c) - 1)))/d
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.63

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx$$
$$=\frac{6\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-960\,(dx\,+}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/960*(6*a*tan(1/2*d*x + 1/2*c)^5 + 15*b*tan(1/2*d*x + 1/2*c)^4 - 70*a*tan
(1/2*d*x + 1/2*c)^3 - 240*b*tan(1/2*d*x + 1/2*c)^2 - 960*(d*x + c)*a + 180
0*b*log(abs(tan(1/2*d*x + 1/2*c))) + 660*a*tan(1/2*d*x + 1/2*c) + 1920*b/(
tan(1/2*d*x + 1/2*c)^2 + 1) - (4110*b*tan(1/2*d*x + 1/2*c)^5 + 660*a*tan(1
/2*d*x + 1/2*c)^4 - 240*b*tan(1/2*d*x + 1/2*c)^3 - 70*a*tan(1/2*d*x + 1/2*
c)^2 + 15*b*tan(1/2*d*x + 1/2*c) + 6*a)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.149.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.36

$$\int \cot^6(c+dx)(a+b\sin(c+dx))\,dx = \frac{11\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}$$
$$-\frac{22\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-72\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{59\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}-\frac{15\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{32\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{d\left(32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+32\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$
$$-\frac{7\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{96\,d}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{160\,d}-\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{4\,d}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{64\,d}$$
$$+\frac{15\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{8\,d}+\frac{2\,a\,\mathrm{atan}\left(\frac{4\,a^2}{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+\frac{15\,b\,a}{2}}-\frac{15\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2+\frac{15\,b\,a}{2}\right)}\right)}{d}$$

input
```
int(cot(c + d*x)^6*(a + b*sin(c + d*x)),x)
```

output
```
(11*a*tan(c/2 + (d*x)/2))/(16*d) - (a/5 + (b*tan(c/2 + (d*x)/2))/2 - (32*a
*tan(c/2 + (d*x)/2)^2)/15 + (59*a*tan(c/2 + (d*x)/2)^4)/3 + 22*a*tan(c/2 +
 (d*x)/2)^6 - (15*b*tan(c/2 + (d*x)/2)^3)/2 - 72*b*tan(c/2 + (d*x)/2)^5)/(
d*(32*tan(c/2 + (d*x)/2)^5 + 32*tan(c/2 + (d*x)/2)^7)) - (7*a*tan(c/2 + (d
*x)/2)^3)/(96*d) + (a*tan(c/2 + (d*x)/2)^5)/(160*d) - (b*tan(c/2 + (d*x)/2
)^2)/(4*d) + (b*tan(c/2 + (d*x)/2)^4)/(64*d) + (15*b*log(tan(c/2 + (d*x)/2
)))/(8*d) + (2*a*atan((4*a^2)/((15*a*b)/2 + 4*a^2*tan(c/2 + (d*x)/2)) - (1
5*a*b*tan(c/2 + (d*x)/2))/(2*((15*a*b)/2 + 4*a^2*tan(c/2 + (d*x)/2)))))/d
```

3.150 $\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$

3.150.1 Optimal result

Integrand size = 21, antiderivative size = 111

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{(a+b)(a+2b)\log(1-\sin(c+dx))}{2d} + \frac{(a-2b)(a-b)\log(1+\sin(c+dx))}{2d} + \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d} + \frac{\sec^2(c+dx)(a+b\sin(c+dx))^2}{2d}$$

output
```
1/2*(a+b)*(a+2*b)*ln(1-sin(d*x+c))/d+1/2*(a-2*b)*(a-b)*ln(1+sin(d*x+c))/d+
2*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)
)^2/d
```

3.150.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \frac{2(a+b)(a+2b)\log(1-\sin(c+dx)) + 2(a-2b)(a-b)\log(1+\sin(c+dx)) - \frac{(a+b)^2}{-1+\sin(c+dx)} + 8ab\sin(c+}{4d}$$

input
```
Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

output
```
(2*(a + b)*(a + 2*b)*Log[1 - Sin[c + d*x]] + 2*(a - 2*b)*(a - b)*Log[1 + S
in[c + d*x]] - (a + b)^2/(-1 + Sin[c + d*x]) + 8*a*b*Sin[c + d*x] + 2*b^2*
Sin[c + d*x]^2 + (a - b)^2/(1 + Sin[c + d*x]))/(4*d)
```

3.150.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3200, 531, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(c+dx)^3(a+b\sin(c+dx))^2dx$$

$$\Big\downarrow \ 3200$$

$$\frac{\int \frac{b^3\sin^3(c+dx)(a+b\sin(c+dx))^2}{\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \ 531$$

$$\frac{\frac{\int -\frac{2(a+b\sin(c+dx))\left(\sin^2(c+dx)b^4+b^4+a\sin(c+dx)b^3\right)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{2b^2}+\frac{b^2(a+b\sin(c+dx))^2}{2\left(b^2-b^2\sin^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{b^2(a+b\sin(c+dx))^2}{2\left(b^2-b^2\sin^2(c+dx)\right)}-\frac{\int \frac{(a+b\sin(c+dx))\left(\sin^2(c+dx)b^4+b^4+a\sin(c+dx)b^3\right)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{b^2}}{d}$$

$$\Big\downarrow \ 2160$$

$$\frac{\frac{b^2(a+b\sin(c+dx))^2}{2\left(b^2-b^2\sin^2(c+dx)\right)}-\frac{\int \left(-\sin(c+dx)b^3-2ab^2+\frac{3ab^4+\left(a^2+2b^2\right)\sin(c+dx)b^3}{b^2-b^2\sin^2(c+dx)}\right)d(b\sin(c+dx))}{b^2}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(a+b\sin(c+dx))^2}{2(b^2-b^2\sin^2(c+dx))}-\frac{-\frac{1}{2}b^2(a^2+2b^2)\log(b^2-b^2\sin^2(c+dx))+3ab^3\text{arctanh}(\sin(c+dx))-2ab^3\sin(c+dx)-\frac{1}{2}b^4\sin^2(c+dx)}{b^2}}{d}$$

input
```
Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^3,x]
```

output
```
((b^2*(a + b*Sin[c + d*x])^2)/(2*(b^2 - b^2*Sin[c + d*x]^2)) - (3*a*b^3*Ar
cTanh[Sin[c + d*x]] - (b^2*(a^2 + 2*b^2)*Log[b^2 - b^2*Sin[c + d*x]^2])/2
- 2*a*b^3*Sin[c + d*x] - (b^4*Sin[c + d*x]^2)/2)/b^2)/d
```

3.150.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.150.4 Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+2ab\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^2\left(\frac{s}{2}\right.}{d}$
default	$\frac{a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+2ab\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^2\left(\frac{s}{2}\right.}{d}$
parts	$\frac{a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{b^2\left(\frac{\sin^6(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^4(dx+c)\right)}{2}+\sin^2(dx+c)+2\ln(\cos(dx+c))\right)}{d}+\frac{2ab\left(\frac{s}{2}\right.}{}$
risch	$\frac{iab\,e^{-i(dx+c)}}{d}-\frac{2ia^2c}{d}-\frac{b^2e^{2i(dx+c)}}{8d}-2ib^2x-ia^2x-\frac{b^2e^{-2i(dx+c)}}{8d}-\frac{4ib^2c}{d}-\frac{2i\left(ia^2e^{2i(dx+c)}+ib^2e^{2i(dx}\right.}{d(1+}$

input
```
int((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+2*a*b*(1/2*sin(d*x+c)^5/cos(d*x
+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))+b^2*(
1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c
))))
```

3.150.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.26

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx =$$
$$-\frac{2\,b^2\cos(dx+c)^4-b^2\cos(dx+c)^2-2\,(a^2-3\,ab+2\,b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-2\,(a^2+3}{4\,d\cos(d}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*cos(d*x + c)^4 - b^2*cos(d*x + c)^2 - 2*(a^2 - 3*a*b + 2*b^2)*
cos(d*x + c)^2*log(sin(d*x + c) + 1) - 2*(a^2 + 3*a*b + 2*b^2)*cos(d*x + c
)^2*log(-sin(d*x + c) + 1) - 2*a^2 - 2*b^2 - 4*(2*a*b*cos(d*x + c)^2 + a*b
)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.150.6 Sympy [F]

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx=\int (a+b\sin{(c+dx)})^2\tan^3{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**3,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**3, x)
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$$
$$=\frac{b^2\sin{(dx+c)}^2+4\,ab\sin{(dx+c)}+(a^2-3\,ab+2\,b^2)\log{(\sin{(dx+c)}+1)}+(a^2+3\,ab+2\,b^2)\log{(\sin{(}}{2\,d}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 3*a*b + 2*b^2)*log(s
in(d*x + c) + 1) + (a^2 + 3*a*b + 2*b^2)*log(sin(d*x + c) - 1) - (2*a*b*si
n(d*x + c) + a^2 + b^2)/(sin(d*x + c)^2 - 1))/d
```

3.150.8 Giac [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.150.9 Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.09

$$\int (a+b\sin(c+dx))^2\tan^3(c+dx)\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2+4b^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(2a^2+4b^2)+4a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2ab\,\mathrm{ta}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+1\right)}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)(a^2+2b^2)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)(a+b)(a+2b)}{d}$$
$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)(a-b)(a-2b)}{d}$$

input
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b^2) + tan(c/2 + (d*x)/2)^6*(2*a^2 + 4*b^
2) + 4*a^2*tan(c/2 + (d*x)/2)^4 + 2*a*b*tan(c/2 + (d*x)/2)^3 + 2*a*b*tan(c
/2 + (d*x)/2)^5 + 6*a*b*tan(c/2 + (d*x)/2)^7 + 6*a*b*tan(c/2 + (d*x)/2))/(
d*(tan(c/2 + (d*x)/2)^8 - 2*tan(c/2 + (d*x)/2)^4 + 1)) - (log(tan(c/2 + (d
*x)/2)^2 + 1)*(a^2 + 2*b^2))/d + (log(tan(c/2 + (d*x)/2) - 1)*(a + b)*(a +
 2*b))/d + (log(tan(c/2 + (d*x)/2) + 1)*(a - b)*(a - 2*b))/d
```

3.151 $\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx$

3.151.1 Optimal result

Integrand size = 19, antiderivative size = 78

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx = -\frac{(a+b)^2\log(1-\sin(c+dx))}{2d} - \frac{(a-b)^2\log(1+\sin(c+dx))}{2d} - \frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d}$$

output
```
-1/2*(a+b)^2*ln(1-sin(d*x+c))/d-1/2*(a-b)^2*ln(1+sin(d*x+c))/d-2*a*b*sin(d
*x+c)/d-1/2*b^2*sin(d*x+c)^2/d
```

3.151.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.86

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx = -\frac{a^2\log(1-\sin(c+dx))}{2d} - \frac{ab\log(1-\sin(c+dx))}{d} - \frac{b^2\log(1-\sin(c+dx))}{2d} - \frac{a^2\log(1+\sin(c+dx))}{2d} + \frac{ab\log(1+\sin(c+dx))}{d} - \frac{b^2\log(1+\sin(c+dx))}{2d} - \frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d}$$

input
```
Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

output
```
-1/2*(a^2*Log[1 - Sin[c + d*x]])/d - (a*b*Log[1 - Sin[c + d*x]])/d - (b^2*
Log[1 - Sin[c + d*x]])/(2*d) - (a^2*Log[1 + Sin[c + d*x]])/(2*d) + (a*b*Lo
g[1 + Sin[c + d*x]])/d - (b^2*Log[1 + Sin[c + d*x]])/(2*d) - (2*a*b*Sin[c
+ d*x])/d - (b^2*Sin[c + d*x]^2)/(2*d)
```

3.151.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 3200, 525, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b\sin(c+dx)(a+b\sin(c+dx))^2}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{525}$$

$$\frac{-\int -\frac{b\sin(c+dx)\left(a^2+2b\sin(c+dx)a+b^2\right)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))-\frac{1}{2}b^2\sin^2(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b\sin(c+dx)\left(a^2+2b\sin(c+dx)a+b^2\right)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))-\frac{1}{2}b^2\sin^2(c+dx)}{d}$$

$$\Big\downarrow \text{523}$$

$$\frac{\int \left(\frac{2ab^2+\left(a^2+b^2\right)\sin(c+dx)b}{b^2-b^2\sin^2(c+dx)}-2a\right)d(b\sin(c+dx))-\frac{1}{2}b^2\sin^2(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}(a^2+b^2)\log\left(b^2-b^2\sin^2(c+dx)\right)+2ab\text{arctanh}(\sin(c+dx))-2ab\sin(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)}{d}$$

input
```
Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x],x]
```

output
```
(2*a*b*ArcTanh[Sin[c + d*x]] - ((a^2 + b^2)*Log[b^2 - b^2*Sin[c + d*x]^2])
/2 - 2*a*b*Sin[c + d*x] - (b^2*Sin[c + d*x]^2)/2)/d
```

3.151.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.151.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

method	result
derivativedivides	$\frac{-a^2\ln(\cos(dx+c))+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
default	$\frac{-a^2\ln(\cos(dx+c))+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}$
parts	$\frac{a^2\ln(1+\tan^2(dx+c))}{2d}+\frac{b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)}{d}+\frac{2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
risch	$ia^2x+ib^2x+\frac{iab\,\mathrm{e}^{i(dx+c)}}{d}-\frac{iab\,\mathrm{e}^{-i(dx+c)}}{d}+\frac{2ia^2c}{d}+\frac{2ib^2c}{d}-\frac{a^2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{d}+\frac{2\ln\left(\mathrm{e}^{i(dx+c)}+i\right)ab}{d}-$

input
```
int((a+b*sin(d*x+c))^2*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^2*ln(cos(d*x+c))+2*a*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^2
*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c))))
```

3.151.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.95

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx$$
$$=\frac{b^2\cos\left(dx+c\right)^2-4\,ab\sin\left(dx+c\right)-\left(a^2-2\,ab+b^2\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(a^2+2\,ab+b^2\right)\log\left(-\sin\left(d\right.\right.}{2\,d}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="fricas")
```

output
```
1/2*(b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) - (a^2 - 2*a*b + b^2)*log(sin
(d*x + c) + 1) - (a^2 + 2*a*b + b^2)*log(-sin(d*x + c) + 1))/d
```

3.151.6 Sympy [F]

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx=\int (a+b\sin{(c+dx)})^2\tan{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**2*tan(d*x+c),x)
```

output
```
Integral((a + b*sin(c + d*x))**2*tan(c + d*x), x)
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx=$$
$$-\frac{b^2\sin{(dx+c)}^2+4\,ab\sin{(dx+c)}+(a^2-2\,ab+b^2)\log{(\sin{(dx+c)}+1)}+(a^2+2\,ab+b^2)\log{(\sin{(d}}{2\,d}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + (a^2 - 2*a*b + b^2)*log(si
n(d*x + c) + 1) + (a^2 + 2*a*b + b^2)*log(sin(d*x + c) - 1))/d
```

3.151.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6671 vs. $2(72) = 144$.

Time = 1.34 (sec) , antiderivative size = 6671, normalized size of antiderivative = 85.53

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx=\text{Too large to display}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c),x, algorithm="giac")
```

output
```
-1/4*(4*a*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*
c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 4*a
*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*ta
n(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2
*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a^2*log(4*
(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d
*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2
*b^2*log(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c
)^2 + tan(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(c)^2 - b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*a*b*log(
2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + 2*tan(1/2*d
*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(
1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 +
 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 4*a*b*log(2*(tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1
/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))*tan(d*x)^2...
```

3.151.9 Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.92

$$\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\,\left(a^2+b^2\right)}{d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(a+b\right)^2}{d}-\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(a-b\right)^2}{d}$$

input
```
int(tan(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

output
```
(log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + b^2))/d - (log(tan(c/2 + (d*x)/2) -
1)*(a + b)^2)/d - (2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a*b*tan(c/2 + (d*x)/2)^3
 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/
2)^4 + 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)^2)/d
```

3.151. $\int (a+b\sin(c+dx))^2\tan(c+dx)\,dx$

3.152 $\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx$

3.152.1 Optimal result

Integrand size = 19, antiderivative size = 46

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

output
```
a^2*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*b^2*sin(d*x+c)^2/d
```

3.152.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{a^2\log(\sin(c+dx))}{d}+\frac{2ab\sin(c+dx)}{d}+\frac{b^2\sin^2(c+dx)}{2d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

output
```
(a^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d + (b^2*Sin[c + d*x]^2)/
(2*d)
```

3.152.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3200, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\sin(c+dx))^2}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{ 3200}$$

$$\frac{\int \frac{\csc(c+dx)(a+b\sin(c+dx))^2}{b}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(\frac{\csc(c+dx)a^2}{b}+2a+b\sin(c+dx)\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\log(b\sin(c+dx))+2ab\sin(c+dx)+\frac{1}{2}b^2\sin^2(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^2,x]
```

output
```
(a^2*Log[b*Sin[c + d*x]] + 2*a*b*Sin[c + d*x] + (b^2*Sin[c + d*x]^2)/2)/d
```

3.152.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.152.4 Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{a^2 \ln(\sin(dx+c))+2\sin(dx+c)ab-\frac{\left(\cos^2(dx+c)\right)b^2}{2}}{d}$	40
default	$\frac{a^2 \ln(\sin(dx+c))+2\sin(dx+c)ab-\frac{\left(\cos^2(dx+c)\right)b^2}{2}}{d}$	40
risch	$-ia^2x-\frac{b^2 e^{2i(dx+c)}}{8d}-\frac{b^2 e^{-2i(dx+c)}}{8d}-\frac{2ia^2c}{d}+\frac{a^2\ln\left(e^{2i(dx+c)}-1\right)}{d}+\frac{2ab\sin(dx+c)}{d}$	85

input
```
int(cot(d*x+c)*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*ln(sin(d*x+c))+2*sin(d*x+c)*a*b-1/2*cos(d*x+c)^2*b^2)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{b^2\cos\left(dx+c\right)^2-2\,a^2\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)-4\,ab\sin\left(dx+c\right)}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*cos(d*x + c)^2 - 2*a^2*log(1/2*sin(d*x + c)) - 4*a*b*sin(d*x + c
))/d
```

3.152.6 Sympy [F]

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx = \int (a+b\sin{(c+dx)})^2\cot{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*cot(c + d*x), x)
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^2\sin{(dx+c)}^2 + 2\,a^2\log{(\sin{(dx+c)})} + 4\,ab\sin{(dx+c)}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(sin(d*x + c)) + 4*a*b*sin(d*x + c))/d
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{b^2\sin{(dx+c)}^2 + 2\,a^2\log{(|\sin{(dx+c)}|)} + 4\,ab\sin{(dx+c)}}{2\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(b^2*sin(d*x + c)^2 + 2*a^2*log(abs(sin(d*x + c))) + 4*a*b*sin(d*x + c
))/d
```

3.152.9 Mupad [B] (verification not implemented)

Time = 6.16 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.54

$$\int \cot(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$=\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$-\frac{a^2\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}{d}$$

input
```
int(cot(c + d*x)*(a + b*sin(c + d*x))^2,x)
```

output
```
(a^2*log(tan(c/2 + (d*x)/2)))/d + (2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a*b*tan(
c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(2*tan(c/2 + (d*x)/2)^2 +
tan(c/2 + (d*x)/2)^4 + 1)) - (a^2*log(tan(c/2 + (d*x)/2)^2 + 1))/d
```

3.153 $\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$

3.153.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{2ab\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{(a^2-b^2)\log(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d}$$

output
```
-2*a*b*csc(d*x+c)/d-1/2*a^2*csc(d*x+c)^2/d-(a^2-b^2)*ln(sin(d*x+c))/d-2*a*
b*sin(d*x+c)/d-1/2*b^2*sin(d*x+c)^2/d
```

3.153.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{2ab\csc(c+dx)}{d} - \frac{a^2\csc^2(c+dx)}{2d} - \frac{a^2\log(\sin(c+dx))}{d} + \frac{b^2\log(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{b^2\sin^2(c+dx)}{2d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*b*Csc[c + d*x])/d - (a^2*Csc[c + d*x]^2)/(2*d) - (a^2*Log[Sin[c + d*
x]])/d + (b^2*Log[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/d - (b^2*Sin[c +
 d*x]^2)/(2*d)
```

3.153.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(c+dx))^2}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^3(c+dx)(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)}{b^3}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{a^2\csc^3(c+dx)}{b}+2a\csc^2(c+dx)+\frac{(b^2-a^2)\csc(c+dx)}{b}-2a-b\sin(c+dx)\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\left(a^2-b^2\right)\log(b\sin(c+dx))-\frac{1}{2}a^2\csc^2(c+dx)-2ab\sin(c+dx)-2ab\csc(c+dx)-\frac{1}{2}b^2\sin^2(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a*b*Csc[c + d*x] - (a^2*Csc[c + d*x]^2)/2 - (a^2 - b^2)*Log[b*Sin[c +
d*x]] - 2*a*b*Sin[c + d*x] - (b^2*Sin[c + d*x]^2)/2)/d
```

3.153.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.153.4 Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+b^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-(2+\cos^2(dx+c))\sin(dx+c)\right)+b^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(dx+\right.}{d}$
risch	$ia^2x-ib^2x+\frac{b^2e^{2i(dx+c)}}{8d}+\frac{iab\,e^{i(dx+c)}}{d}-\frac{iab\,e^{-i(dx+c)}}{d}+\frac{b^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2c}{d}-\frac{2ib^2c}{d}-\frac{2ia\left(ia\,e^{2i(c\right.}}{}$

input
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+2*a*b*(-1/sin(d*x+c)*cos(d*x+c
)^4-(2+cos(d*x+c)^2)*sin(d*x+c))+b^2*(1/2*cos(d*x+c)^2+ln(sin(d*x+c))))
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.37

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$=\frac{2\,b^2\cos\left(dx+c\right)^4-3\,b^2\cos\left(dx+c\right)^2+2\,a^2+b^2-4\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\log\left(\frac{1}{2}\,\sin\left(dx+c\right.\right.}{4\left(d\cos\left(dx+c\right)^2-d\right)}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/4*(2*b^2*cos(d*x + c)^4 - 3*b^2*cos(d*x + c)^2 + 2*a^2 + b^2 - 4*((a^2 -
 b^2)*cos(d*x + c)^2 - a^2 + b^2)*log(1/2*sin(d*x + c)) - 8*(a*b*cos(d*x +
 c)^2 - 2*a*b)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

3.153.6 Sympy [F]

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx=\int\left(a+b\sin\left(c+dx\right)\right)^2\cot^3\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**3, x)
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.82

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$=-\frac{b^2\sin\left(dx+c\right)^2+4\,ab\sin\left(dx+c\right)+2\left(a^2-b^2\right)\log\left(\sin\left(dx+c\right)\right)+\frac{4\,ab\sin(dx+c)+a^2}{\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(sin(d*x
+ c)) + (4*a*b*sin(d*x + c) + a^2)/sin(d*x + c)^2)/d
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx =$$

$$-\frac{b^2\sin{(dx+c)}^2+4\,ab\sin{(dx+c)}+2\,(a^2-b^2)\log{(|\sin{(dx+c)}|)}-\frac{3\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2-4\,ab\sin(dx+}{\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) + 2*(a^2 - b^2)*log(abs(sin(
d*x + c))) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2 - 4*a*b*sin(d*x
+ c) - a^2)/sin(d*x + c)^2)/d
```

3.153.9 Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.63

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,(a^2-b^2)}{d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{a^2}{2}+8\,b^2\right)+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a^2}{2}+24\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+20\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\,b\,\mathrm{ta}}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}$$

$$-\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(a^2-b^2)}{d}-\frac{a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

input
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x))^2,x)
```

output
```
(log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 - b^2))/d - (tan(c/2 + (d*x)/2)^4*(a^2
/2 + 8*b^2) + a^2*tan(c/2 + (d*x)/2)^2 + a^2/2 + 24*a*b*tan(c/2 + (d*x)/2)
^3 + 20*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))/(d*(4*tan(c/2
 + (d*x)/2)^2 + 8*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6)) - (a^2*t
an(c/2 + (d*x)/2)^2)/(8*d) - (log(tan(c/2 + (d*x)/2))*(a^2 - b^2))/d - (a*
b*tan(c/2 + (d*x)/2))/d
```

3.154 $\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$

3.154.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{4ab\csc(c+dx)}{d} + \frac{(2a^2-b^2)\csc^2(c+dx)}{2d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{a^2\csc^4(c+dx)}{4d} + \frac{(a^2-2b^2)\log(\sin(c+dx))}{d} + \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d}$$

output
```
4*a*b*csc(d*x+c)/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/d-2/3*a*b*csc(d*x+c)^3/d-1
/4*a^2*csc(d*x+c)^4/d+(a^2-2*b^2)*ln(sin(d*x+c))/d+2*a*b*sin(d*x+c)/d+1/2*
b^2*sin(d*x+c)^2/d
```

3.154.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{4ab\csc(c+dx)}{d} + \frac{a^2\csc^2(c+dx)}{d} - \frac{b^2\csc^2(c+dx)}{2d} - \frac{2ab\csc^3(c+dx)}{3d} - \frac{a^2\csc^4(c+dx)}{4d} + \frac{a^2\log(\sin(c+dx))}{d} - \frac{2b^2\log(\sin(c+dx))}{d} + \frac{2ab\sin(c+dx)}{d} + \frac{b^2\sin^2(c+dx)}{2d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

output
```
(4*a*b*Csc[c + d*x])/d + (a^2*Csc[c + d*x]^2)/d - (b^2*Csc[c + d*x]^2)/(2*
d) - (2*a*b*Csc[c + d*x]^3)/(3*d) - (a^2*Csc[c + d*x]^4)/(4*d) + (a^2*Log[
Sin[c + d*x]])/d - (2*b^2*Log[Sin[c + d*x]])/d + (2*a*b*Sin[c + d*x])/d +
(b^2*Sin[c + d*x]^2)/(2*d)
```

3.154.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(c+dx))^2}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^5(c+dx)(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)^2}{b^5}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{a^2\csc^5(c+dx)}{b}+2a\csc^4(c+dx)+\frac{\left(b^4-2a^2b^2\right)\csc^3(c+dx)}{b^3}-4a\csc^2(c+dx)+\frac{\left(a^2-2b^2\right)\csc(c+dx)}{b}+2a+b\sin(c+dx)\right)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}\left(2a^2-b^2\right)\csc^2(c+dx)+\left(a^2-2b^2\right)\log(b\sin(c+dx))-\frac{1}{4}a^2\csc^4(c+dx)+2ab\sin(c+dx)-\frac{2}{3}ab\csc^3(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^2,x]
```

output
```
(4*a*b*Csc[c + d*x] + ((2*a^2 - b^2)*Csc[c + d*x]^2)/2 - (2*a*b*Csc[c + d*
x]^3)/3 - (a^2*Csc[c + d*x]^4)/4 + (a^2 - 2*b^2)*Log[b*Sin[c + d*x]] + 2*a
*b*Sin[c + d*x] + (b^2*Sin[c + d*x]^2)/2)/d
```

3.154.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.154.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right.}{3}\right.\right.}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+2ab\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right.}{3}\right.\right.}{d}$
risch	$-ia^2x+2ib^2x-\frac{b^2e^{2i(dx+c)}}{8d}-\frac{iab\,e^{i(dx+c)}}{d}+\frac{iab\,e^{-i(dx+c)}}{d}-\frac{b^2e^{-2i(dx+c)}}{8d}-\frac{2ia^2c}{d}+\frac{4ib^2c}{d}+\frac{2i(6ia^2}{}$

input
```
int(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+2*a*b*(-1/3/s
in(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3*c
os(d*x+c)^2)*sin(d*x+c))+b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos(d*x+c
)^4-cos(d*x+c)^2-2*ln(sin(d*x+c))))
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.40

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx =$$
$$-\frac{6\,b^2\cos\left(dx+c\right)^6-15\,b^2\cos\left(dx+c\right)^4+6\left(2\,a^2+b^2\right)\cos\left(dx+c\right)^2-9\,a^2+3\,b^2-12\left(\left(a^2-2\,b^2\right)\cos\left(d\right.\right.}{12\left(d\,c\right.}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/12*(6*b^2*cos(d*x + c)^6 - 15*b^2*cos(d*x + c)^4 + 6*(2*a^2 + b^2)*cos(
d*x + c)^2 - 9*a^2 + 3*b^2 - 12*((a^2 - 2*b^2)*cos(d*x + c)^4 - 2*(a^2 - 2
*b^2)*cos(d*x + c)^2 + a^2 - 2*b^2)*log(1/2*sin(d*x + c)) - 8*(3*a*b*cos(d
*x + c)^4 - 12*a*b*cos(d*x + c)^2 + 8*a*b)*sin(d*x + c))/(d*cos(d*x + c)^4
 - 2*d*cos(d*x + c)^2 + d)
```

3.154.6 Sympy [F]

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx = \int (a+b\sin{(c+dx)})^2\cot^5{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**5, x)
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{6\,b^2\sin\left(dx+c\right)^2+24\,ab\sin\left(dx+c\right)+12\left(a^2-2\,b^2\right)\log\left(\sin\left(dx+c\right)\right)+\frac{48\,ab\sin(dx+c)^3-8\,ab\sin(dx+c)+6\,(2\,a^2-b}{\sin(dx+c)^4}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(si
n(d*x + c)) + (48*a*b*sin(d*x + c)^3 - 8*a*b*sin(d*x + c) + 6*(2*a^2 - b^2
)*sin(d*x + c)^2 - 3*a^2)/sin(d*x + c)^4)/d
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{6\,b^2\sin\left(dx+c\right)^2+24\,ab\sin\left(dx+c\right)+12\left(a^2-2\,b^2\right)\log\left(\left|\sin\left(dx+c\right)\right|\right)-\frac{25\,a^2\sin(dx+c)^4-50\,b^2\sin(dx+c)^4-48\,ab}{}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/12*(6*b^2*sin(d*x + c)^2 + 24*a*b*sin(d*x + c) + 12*(a^2 - 2*b^2)*log(ab
s(sin(d*x + c))) - (25*a^2*sin(d*x + c)^4 - 50*b^2*sin(d*x + c)^4 - 48*a*b
*sin(d*x + c)^3 - 12*a^2*sin(d*x + c)^2 + 6*b^2*sin(d*x + c)^2 + 8*a*b*sin
(d*x + c) + 3*a^2)/sin(d*x + c)^4)/d
```

3.154.9 Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.46

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{5a^2}{2}-2b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{23a^2}{4}-4b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(3a^2+30b^2)-\frac{a^2}{4}+\frac{76ab\tan\left(\frac{c}{2}+\right.}{3}}{d\left(16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+16\tan\left(\frac{c}{2}\right.\right.}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)(a^2-2b^2)}{d}-\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a^2-2b^2)}{d}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3a^2}{16}-\frac{b^2}{8}\right)}{d}-\frac{ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12d}+\frac{7ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}$$

input
```
int(cot(c + d*x)^5*(a + b*sin(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^2*((5*a^2)/2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*((23*a^2)
/4 - 4*b^2) + tan(c/2 + (d*x)/2)^6*(3*a^2 + 30*b^2) - a^2/4 + (76*a*b*tan(
c/2 + (d*x)/2)^3)/3 + (356*a*b*tan(c/2 + (d*x)/2)^5)/3 + 92*a*b*tan(c/2 +
(d*x)/2)^7 - (4*a*b*tan(c/2 + (d*x)/2))/3)/(d*(16*tan(c/2 + (d*x)/2)^4 + 3
2*tan(c/2 + (d*x)/2)^6 + 16*tan(c/2 + (d*x)/2)^8)) - (log(tan(c/2 + (d*x)/
2)^2 + 1)*(a^2 - 2*b^2))/d - (a^2*tan(c/2 + (d*x)/2)^4)/(64*d) + (log(tan(
c/2 + (d*x)/2))*(a^2 - 2*b^2))/d + (tan(c/2 + (d*x)/2)^2*((3*a^2)/16 - b^2
/8))/d - (a*b*tan(c/2 + (d*x)/2)^3)/(12*d) + (7*a*b*tan(c/2 + (d*x)/2))/(4
*d)
```

3.155 $\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$

3.155.1 Optimal result

Integrand size = 21, antiderivative size = 149

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = a^2x+\frac{5b^2x}{2}-\frac{2ab\cos(c+dx)}{d}-\frac{4ab\sec(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}-\frac{5b^2\tan(c+dx)}{2d}+\frac{a^2\tan^3(c+dx)}{3d}+\frac{5b^2\tan^3(c+dx)}{6d}-\frac{b^2\sin^2(c+dx)\tan^3(c+dx)}{2d}$$

output
```
a^2*x+5/2*b^2*x-2*a*b*cos(d*x+c)/d-4*a*b*sec(d*x+c)/d+2/3*a*b*sec(d*x+c)^3
/d-a^2*tan(d*x+c)/d-5/2*b^2*tan(d*x+c)/d+1/3*a^2*tan(d*x+c)^3/d+5/6*b^2*ta
n(d*x+c)^3/d-1/2*b^2*sin(d*x+c)^2*tan(d*x+c)^3/d
```

3.155.2 Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.18

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = -\frac{\sec^3(c+dx)\,(200ab-36(2a^2+5b^2)\,(c+dx)\cos(c+dx)+288ab\cos(2(c+dx))-24a^2c\cos(3(c+dx)}{}$$

input
```
Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

output
```
-1/96*(Sec[c + d*x]^3*(200*a*b - 36*(2*a^2 + 5*b^2)*(c + d*x)*Cos[c + d*x]
 + 288*a*b*Cos[2*(c + d*x)] - 24*a^2*c*Cos[3*(c + d*x)] - 60*b^2*c*Cos[3*(
c + d*x)] - 24*a^2*d*x*Cos[3*(c + d*x)] - 60*b^2*d*x*Cos[3*(c + d*x)] + 24
*a*b*Cos[4*(c + d*x)] + 30*b^2*Sin[c + d*x] + 32*a^2*Sin[3*(c + d*x)] + 65
*b^2*Sin[3*(c + d*x)] + 3*b^2*Sin[5*(c + d*x)]))/d
```

3.155.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(c+dx)^4(a+b\sin(c+dx))^2dx$$

$$\Big\downarrow \text{ 3201}$$

$$\int \left(a^2\tan^4(c+dx)+2ab\sin(c+dx)\tan^4(c+dx)+b^2\sin^2(c+dx)\tan^4(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2\tan^3(c+dx)}{3d}-\frac{a^2\tan(c+dx)}{d}+a^2x-\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{4ab\sec(c+dx)}{d}+\frac{5b^2\tan^3(c+dx)}{6d}-\frac{5b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan^3(c+dx)}{2d}+\frac{5b^2x}{2}$$

input
```
Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^4,x]
```

output
```
a^2*x + (5*b^2*x)/2 - (2*a*b*Cos[c + d*x])/d - (4*a*b*Sec[c + d*x])/d + (2
*a*b*Sec[c + d*x]^3)/(3*d) - (a^2*Tan[c + d*x])/d - (5*b^2*Tan[c + d*x])/(
2*d) + (a^2*Tan[c + d*x]^3)/(3*d) + (5*b^2*Tan[c + d*x]^3)/(6*d) - (b^2*Si
n[c + d*x]^2*Tan[c + d*x]^3)/(2*d)
```

3.155.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.155.4 Maple [A] (verified)

Time = 3.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.24

method	result
derivativedivides	$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$
default	$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+2ab\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}$
parts	$\frac{a^2\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{b^2\left(\frac{\sin^7(dx+c)}{3\cos(dx+c)^3}-\frac{4\left(\sin^7(dx+c)\right)}{3\cos(dx+c)}-\frac{4\left(\sin^5(dx+c)+\frac{5\left(\sin^3(dx+c)\right)}{4}\right.}{3}\right.}{d}$
risch	$a^2x+\frac{5b^2x}{2}+\frac{ib^2e^{2i(dx+c)}}{8d}-\frac{ab\,e^{i(dx+c)}}{d}-\frac{ab\,e^{-i(dx+c)}}{d}-\frac{ib^2e^{-2i(dx+c)}}{8d}-\frac{2\left(6ia^2e^{4i(dx+c)}+9ib^2e^{4i(dx+c)}+\right.}{}$

input
```
int((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+2*a*b*(1/3*sin(d*x+c)^6/cos(d
*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*
x+c))+b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)-4/3*(
sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/2*c))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$$
$$= \frac{3\,(2\,a^2+5\,b^2)dx\cos{(dx+c)}^3 - 12\,ab\cos{(dx+c)}^4 - 24\,ab\cos{(dx+c)}^2 + 4\,ab - (3\,b^2\cos{(dx+c)}^4 + 2\,(}{6\,d\cos{(dx+c)}^3}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(3*(2*a^2 + 5*b^2)*d*x*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)^4 - 24*a*b
*cos(d*x + c)^2 + 4*a*b - (3*b^2*cos(d*x + c)^4 + 2*(4*a^2 + 7*b^2)*cos(d*
x + c)^2 - 2*a^2 - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.155.6 Sympy [F]

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \int (a+b\sin{(c+dx)})^2\tan^4{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**4,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**4, x)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.80

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx$$
$$= \frac{2\left(\tan{(dx+c)}^3 + 3\,dx + 3\,c - 3\,\tan{(dx+c)}\right)a^2 + \left(2\,\tan{(dx+c)}^3 + 15\,dx + 15\,c - \frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1} - 12\,\mathrm{ta}\right.}{6\,d}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^2 + (2*tan(d*x +
c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x +
c))*b^2 - 4*a*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d*x + c)))/
d
```

3.155.8 Giac [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.155.9 Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.58

$$\int (a+b\sin(c+dx))^2\tan^4(c+dx)\,dx = \frac{x\,(2\,a^2+5\,b^2)}{2} - \frac{(-2\,a^2-5\,b^2)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,a^2}{3}+\frac{20\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{28\,a^2}{3}+\frac{22\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\frac{64\,a\,b\tan\left(\frac{c}{2}+\cdots\right.}{3}\cdots}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\cdots\right.}$$

input
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

output
```
(x*(2*a^2 + 5*b^2))/2 - (tan(c/2 + (d*x)/2)^3*((8*a^2)/3 + (20*b^2)/3) - t
an(c/2 + (d*x)/2)^9*(2*a^2 + 5*b^2) - (32*a*b)/3 + tan(c/2 + (d*x)/2)^7*((
8*a^2)/3 + (20*b^2)/3) + tan(c/2 + (d*x)/2)^5*((28*a^2)/3 + (22*b^2)/3) -
tan(c/2 + (d*x)/2)*(2*a^2 + 5*b^2) + (32*a*b*tan(c/2 + (d*x)/2)^2)/3 + (64
*a*b*tan(c/2 + (d*x)/2)^4)/3)/(d*(tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)
/2)^4 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)
^10 - 1))
```

3.156 $\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$

3.156.1 Optimal result

Integrand size = 21, antiderivative size = 94

$$\begin{aligned}\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = {} & -a^2x-\frac{3b^2x}{2}+\frac{2ab\cos(c+dx)}{d} \\ & +\frac{2ab\sec(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d} \\ & +\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

output
```
-a^2*x-3/2*b^2*x+2*a*b*cos(d*x+c)/d+2*a*b*sec(d*x+c)/d+a^2*tan(d*x+c)/d+3/
2*b^2*tan(d*x+c)/d-1/2*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

3.156.2 Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$$
$$=\frac{-4(2a^2+3b^2)(c+dx)+b\sec(c+dx)(24a+8a\cos(2(c+dx))+b\sin(3(c+dx)))+(8a^2+9b^2)\tan(c+}{8d}$$

input
```
Integrate[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

output
```
(-4*(2*a^2 + 3*b^2)*(c + d*x) + b*Sec[c + d*x]*(24*a + 8*a*Cos[2*(c + d*x)
] + b*Sin[3*(c + d*x)]) + (8*a^2 + 9*b^2)*Tan[c + d*x])/(8*d)
```

3.156.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \tan(c+dx)^2(a+b\sin(c+dx))^2dx$$

$$\Big\downarrow \; 3201$$

$$\int \left(a^2\tan^2(c+dx)+2ab\sin(c+dx)\tan^2(c+dx)+b^2\sin^2(c+dx)\tan^2(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2\tan(c+dx)}{d}+a^2(-x)+\frac{2ab\cos(c+dx)}{d}+\frac{2ab\sec(c+dx)}{d}+\frac{3b^2\tan(c+dx)}{2d}-\frac{b^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{3b^2x}{2}$$

input
```
Int[(a + b*Sin[c + d*x])^2*Tan[c + d*x]^2,x]
```

output
```
-(a^2*x) - (3*b^2*x)/2 + (2*a*b*Cos[c + d*x])/d + (2*a*b*Sec[c + d*x])/d +
 (a^2*Tan[c + d*x])/d + (3*b^2*Tan[c + d*x])/(2*d) - (b^2*Sin[c + d*x]^2*T
an[c + d*x])/(2*d)
```

3.156.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.156.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos \cdots\right.}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos \cdots\right.}{d}$
parts	$\frac{a^2(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)-\frac{3dx}{2}-\frac{3c}{2}\right)}{d}+\frac{2ab\left(\cdots\right.}{\cdots}$
risch	$-a^2x-\frac{3b^2x}{2}-\frac{ib^2e^{2i(dx+c)}}{8d}+\frac{ab\,e^{i(dx+c)}}{d}+\frac{ab\,e^{-i(dx+c)}}{d}+\frac{ib^2e^{-2i(dx+c)}}{8d}+\frac{2ia^2+2ib^2+4ab\,e^{i(dx+c)}}{d(1+e^{2i(dx+c)})}$

input
```
int((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(1/cos(d*x+c)*sin(d*x+c)^4+(2+sin(d*x+c)
^2)*cos(d*x+c))+b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c))
*cos(d*x+c)-3/2*d*x-3/2*c))
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = -\frac{(2\,a^2+3\,b^2)dx\cos(dx+c)-4\,ab\cos(dx+c)^2-4\,ab-\left(b^2\cos(dx+c)^2+2\,a^2+2\,b^2\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*((2*a^2 + 3*b^2)*d*x*cos(d*x + c) - 4*a*b*cos(d*x + c)^2 - 4*a*b - (b
^2*cos(d*x + c)^2 + 2*a^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

3.156.6 Sympy [F]

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \int (a+b\sin{(c+dx)})^2\tan^2{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**2*tan(d*x+c)**2,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*tan(c + d*x)**2, x)
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = -\frac{2\,(dx+c-\tan{(dx+c)})a^2+\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan{(dx+c)}\right)b^2-4\,ab\left(\frac{1}{\cos(dx+c)}+\cos{(dx}\right.}{2\,d}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(d*x + c - tan(d*x + c))*a^2 + (3*d*x + 3*c - tan(d*x + c)/(tan(d*
x + c)^2 + 1) - 2*tan(d*x + c))*b^2 - 4*a*b*(1/cos(d*x + c) + cos(d*x + c)
))/d
```

3.156.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7670 vs. 2(88) = 176.

Time = 5.68 (sec) , antiderivative size = 7670, normalized size of antiderivative = 81.60

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(d*x+c))^2*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*(2*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 3*b^2*d*
x*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*d*x*tan(d*x)^3*t
an(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^4*ta
n(1/2*c)^4*tan(c) - 2*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c
)^2 - 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 8*a^2*d*
x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 - 12*b^2*d*x*tan(d*x)^3*
tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^3 + 2*a^2*d*x*tan(d*x)*tan(1/2*d*x)^4*t
an(1/2*c)^4*tan(c)^3 + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(
c)^3 - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 + 2*a^2*tan(d
*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 3*b^2*tan(d*x)^3*tan(1/2*d*x)
^4*tan(1/2*c)^4*tan(c)^2 + 2*a^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*ta
n(c)^3 + 3*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^3 - 2*a^2*d*x
*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 - 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x
)^4*tan(1/2*c)^4 - 8*a^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)
 - 12*b^2*d*x*tan(d*x)^3*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c) + 2*a^2*d*x*ta
n(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) + 3*b^2*d*x*tan(d*x)*tan(1/2*d*x
)^4*tan(1/2*c)^4*tan(c) - 8*a*b*tan(d*x)^3*tan(1/2*d*x)^4*tan(1/2*c)^4*tan
(c) + 8*a^2*d*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 + 12*b^2*d
*x*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^3*tan(c)^2 - 2*a^2*d*x*tan(1/2*d*x
)^4*tan(1/2*c)^4*tan(c)^2 - 3*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c...
```

3.156.9 Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.56

$$\int (a+b\sin(c+dx))^2\tan^2(c+dx)\,dx$$
$$=\frac{(2a^2+3b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+(4a^2+2b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+8ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+(2a^2+3b^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$
$$-\frac{x(2a^2+3b^2)}{2}$$

input
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

output
```
(8*a*b + tan(c/2 + (d*x)/2)^3*(4*a^2 + 2*b^2) + tan(c/2 + (d*x)/2)^5*(2*a^
2 + 3*b^2) + tan(c/2 + (d*x)/2)*(2*a^2 + 3*b^2) + 8*a*b*tan(c/2 + (d*x)/2)
^2)/(d*(tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6
 + 1)) - (x*(2*a^2 + 3*b^2))/2
```

3.157 $\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$

3.157.1 Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = -a^2x+\frac{b^2x}{2}-\frac{2ab\operatorname{arctanh}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}-\frac{a^2\cot(c+dx)}{d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
-a^2*x+1/2*b^2*x-2*a*b*arctanh(cos(d*x+c))/d+2*a*b*cos(d*x+c)/d-a^2*cot(d*
x+c)/d+1/2*b^2*cos(d*x+c)*sin(d*x+c)/d
```

3.157.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.49

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = \frac{-4a^2c+2b^2c-4a^2dx+2b^2dx+8ab\cos(c+dx)-2a^2\cot\left(\frac{1}{2}(c+dx)\right)-8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+8ab}{4d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

output
```
(-4*a^2*c + 2*b^2*c - 4*a^2*d*x + 2*b^2*d*x + 8*a*b*Cos[c + d*x] - 2*a^2*C
ot[(c + d*x)/2] - 8*a*b*Log[Cos[(c + d*x)/2]] + 8*a*b*Log[Sin[(c + d*x)/2]
] + b^2*Sin[2*(c + d*x)] + 2*a^2*Tan[(c + d*x)/2])/(4*d)
```

3.157.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(c+dx))^2}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a^2\cot^2(c+dx)+2ab\cos(c+dx)\cot(c+dx)+b^2\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\cot(c+dx)}{d}+a^2(-x)-\frac{2ab\text{arctanh}(\cos(c+dx))}{d}+\frac{2ab\cos(c+dx)}{d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{b^2x}{2}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^2,x]
```

output
```
-(a^2*x) + (b^2*x)/2 - (2*a*b*ArcTanh[Cos[c + d*x]])/d + (2*a*b*Cos[c + d*
x])/d - (a^2*Cot[c + d*x])/d + (b^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.157.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{a^2(-\cot(dx+c)-dx-c)+2ab(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{a^2(-\cot(dx+c)-dx-c)+2ab(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$-a^2x+\frac{b^2x}{2}-\frac{ib^2e^{2i(dx+c)}}{8d}+\frac{ab\,e^{i(dx+c)}}{d}+\frac{ab\,e^{-i(dx+c)}}{d}+\frac{ib^2e^{-2i(dx+c)}}{8d}-\frac{2ia^2}{d\left(e^{2i(dx+c)}-1\right)}-\frac{2ab\ln\left(e^{i(d}\right.}{d}$

input
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-cot(d*x+c)-d*x-c)+2*a*b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c)))+
b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.51

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx =$$
$$-\frac{b^2\cos(dx+c)^3+2ab\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-2ab\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c}{2d\sin(dx+c)}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*cos(d*x + c)^3 + 2*a*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c)
- 2*a*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c) + ((2*a^2 - b^2)*d*x - 4*a*b*cos(d*x + c))*sin(d*x + c))/(d*sin(d*x +
 c))
```

3.157.6 Sympy [F]

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx=\int (a+b\sin{(c+dx)})^2\cot^2{(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**2, x)
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.01

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx = -\frac{4\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^2-(2\,dx+2\,c+\sin{(2\,dx+2\,c)})b^2-4\,ab(2\,\cos{(dx+c)}-\log{(\cos{(dx+c)}+1)}$$

$$4\,d$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/4*(4*(d*x + c + 1/tan(d*x + c))*a^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*
b^2 - 4*a*b*(2*cos(d*x + c) - log(cos(d*x + c) + 1) + log(cos(d*x + c) - 1
)))/d
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-(2\,a^2-b^2)(dx+c)-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + a^2*tan(1/2*d*x + 1/2*c) - (2*
a^2 - b^2)*(d*x + c) - (4*a*b*tan(1/2*d*x + 1/2*c) + a^2)/tan(1/2*d*x + 1/
2*c) - 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*
tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.157.9 Mupad [B] (verification not implemented)

Time = 6.83 (sec) , antiderivative size = 277, normalized size of antiderivative = 3.55

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{b^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right)-2\,a^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+4\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^2}\right.}{d}$$
$$-\frac{a^2\,\cos\left(c+d\,x\right)-\frac{b^2\,\cos(c+d\,x)}{8}+\frac{b^2\,\cos(3\,c+3\,d\,x)}{8}-a\,b\,\sin\left(2\,c+2\,d\,x\right)}{d\,\sin\left(c+d\,x\right)}$$

input
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x))^2,x)
```

output
```
(b^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 4*a*b*sin(c
/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin(c/2 + (d*x)/2) + 4*a*b*
cos(c/2 + (d*x)/2))) - 2*a^2*atan((b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2
+ (d*x)/2) + 4*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - b^2*sin
(c/2 + (d*x)/2) + 4*a*b*cos(c/2 + (d*x)/2))) + 2*a*b*log(sin(c/2 + (d*x)/2
)/cos(c/2 + (d*x)/2)))/d - (a^2*cos(c + d*x) - (b^2*cos(c + d*x))/8 + (b^2
*cos(3*c + 3*d*x))/8 - a*b*sin(2*c + 2*d*x))/(d*sin(c + d*x))
```

3.158 $\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$

3.158.1 Optimal result

Integrand size = 21, antiderivative size = 133

$$\begin{aligned}\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = {} & a^2x-\frac{3b^2x}{2}+\frac{3ab\operatorname{arctanh}(\cos(c+dx))}{d} \\ & -\frac{3ab\cos(c+dx)}{d}+\frac{a^2\cot(c+dx)}{d} \\ & -\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d} \\ & -\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}-\frac{a^2\cot^3(c+dx)}{3d}\end{aligned}$$

output
```
a^2*x-3/2*b^2*x+3*a*b*arctanh(cos(d*x+c))/d-3*a*b*cos(d*x+c)/d+a^2*cot(d*x
+c)/d-3/2*b^2*cot(d*x+c)/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)/d-a*b*cos(d*x+c
)*cot(d*x+c)^2/d-1/3*a^2*cot(d*x+c)^3/d
```

3.158.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(133) = 266.

Time = 6.48 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.20

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{(2a^2-3b^2)(c+dx)}{2d} - \frac{2ab\cos(c+dx)}{d} + \frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6d} - \frac{ab\csc^2\left(\frac{1}{2}(c+dx)\right)}{4d} - \frac{a^2\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d} + \frac{3ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{3ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{ab\sec^2\left(\frac{1}{2}(c+dx)\right)}{4d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d} - \frac{b^2\sin(2(c+dx))}{4d} + \frac{a^2\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

output
```
((2*a^2 - 3*b^2)*(c + d*x))/(2*d) - (2*a*b*Cos[c + d*x])/d + ((4*a^2*Cos[(
c + d*x)/2] - 3*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (a*b*Csc[(
c + d*x)/2]^2)/(4*d) - (a^2*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) +
(3*a*b*Log[Cos[(c + d*x)/2]])/d - (3*a*b*Log[Sin[(c + d*x)/2]])/d + (a*b*S
ec[(c + d*x)/2]^2)/(4*d) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 3*
b^2*Sin[(c + d*x)/2]))/(6*d) - (b^2*Sin[2*(c + d*x)])/(4*d) + (a^2*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

3.158.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(c+dx))^2}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a^2\cot^4(c+dx)+2ab\cos(c+dx)\cot^3(c+dx)+b^2\cos^2(c+dx)\cot^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\cot^3(c+dx)}{3d}+\frac{a^2\cot(c+dx)}{d}+a^2x+\frac{3ab\text{arctanh}(\cos(c+dx))}{d}-\frac{3ab\cos(c+dx)}{d}-\frac{ab\cos(c+dx)\cot^2(c+dx)}{d}-\frac{3b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{3b^2x}{2}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^2,x]
```

output
```
a^2*x - (3*b^2*x)/2 + (3*a*b*ArcTanh[Cos[c + d*x]])/d - (3*a*b*Cos[c + d*x
])/d + (a^2*Cot[c + d*x])/d - (3*b^2*Cot[c + d*x])/(2*d) + (b^2*Cos[c + d*
x]^2*Cot[c + d*x])/(2*d) - (a*b*Cos[c + d*x]*Cot[c + d*x]^2)/d - (a^2*Cot[
c + d*x]^3)/(3*d)
```

3.158.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.158.4 Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+}{d}$
default	$\frac{a^2\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+2ab\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)+}{d}$
risch	$a^2x-\frac{3b^2x}{2}+\frac{ib^2e^{2i(dx+c)}}{8d}-\frac{ab\,e^{i(dx+c)}}{d}-\frac{ab\,e^{-i(dx+c)}}{d}-\frac{ib^2e^{-2i(dx+c)}}{8d}+\frac{4ia^2e^{4i(dx+c)}-2ib^2e^{4i(dx+c)}+2c}{}$

input
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+2*a*b*(-1/2/sin(d*x+c)^2*cos
(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))+b
^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)-3/
2*d*x-3/2*c))
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.64

$$\int\cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$=\frac{3\,b^2\cos\left(dx+c\right)^5+4\left(2\,a^2-3\,b^2\right)\cos\left(dx+c\right)^3+9\left(ab\cos\left(dx+c\right)^2-ab\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\sin\left(dx\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/6*(3*b^2*cos(d*x + c)^5 + 4*(2*a^2 - 3*b^2)*cos(d*x + c)^3 + 9*(a*b*cos(
d*x + c)^2 - a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 9*(a*b*cos(d*
x + c)^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 3*(2*a^2 - 3*b
^2)*cos(d*x + c) + 3*((2*a^2 - 3*b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x +
 c)^3 - (2*a^2 - 3*b^2)*d*x + 6*a*b*cos(d*x + c))*sin(d*x + c))/((d*cos(d*
x + c)^2 - d)*sin(d*x + c))
```

3.158.6 Sympy [F]

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx = \int (a+b\sin(c+dx))^2\cot^4(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**4, x)
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.04

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{2\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^2-3\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)b^2+3\,ab\left(\frac{2\,\cos(dx+c)}{\cos(dx+c)^2-1}-4\,\cos(dx+\right.}{6\,d}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/6*(2*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^2 - 3*(3*d*
x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*b^2 + 3*
a*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*cos(d*x + c) + 3*log(cos(d*x
+ c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.81

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-72\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 + 6*a*b*tan(1/2*d*x + 1/2*c)^2 - 72*a*b*l
og(abs(tan(1/2*d*x + 1/2*c))) - 15*a^2*tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1
/2*d*x + 1/2*c) + 12*(2*a^2 - 3*b^2)*(d*x + c) + 24*(b^2*tan(1/2*d*x + 1/2
*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(
tan(1/2*d*x + 1/2*c)^2 + 1)^2 + (132*a*b*tan(1/2*d*x + 1/2*c)^3 + 15*a^2*t
an(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a*b*tan(1/2*d*x
+ 1/2*c) - a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.158.9 Mupad [B] (verification not implemented)

Time = 8.12 (sec) , antiderivative size = 584, normalized size of antiderivative = 4.39

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^2\,dx =$$

$$-\frac{\frac{5\,b^2\,\cos(c+d\,x)}{16}+\frac{a^2\,\cos(3\,c+3\,d\,x)}{3}-\frac{11\,b^2\,\cos(3\,c+3\,d\,x)}{32}+\frac{b^2\,\cos(5\,c+5\,d\,x)}{32}+\frac{a^2\,\mathrm{atan}\left(\frac{-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b+3\,\cos\left(\frac{c}{2}\right.}{2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2+6\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b-3\,\sin\left(\frac{c}{2}+\right.}\right.}{2}}{}$$

input
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x))^2,x)
```

output
```
-((5*b^2*cos(c + d*x))/16 + (a^2*cos(3*c + 3*d*x))/3 - (11*b^2*cos(3*c + 3
*d*x))/32 + (b^2*cos(5*c + 5*d*x))/32 + (a^2*atan((3*b^2*cos(c/2 + (d*x)/2
) - 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 +
(d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c +
 3*d*x))/2 - (3*b^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)
/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2
+ (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/4 + (3*a*b*sin(c
 + d*x))/2 - (3*a^2*atan((3*b^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)
/2) + 6*a*b*sin(c/2 + (d*x)/2))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2
+ (d*x)/2) + 6*a*b*cos(c/2 + (d*x)/2)))*sin(c + d*x))/2 + (9*b^2*atan((3*b
^2*cos(c/2 + (d*x)/2) - 2*a^2*cos(c/2 + (d*x)/2) + 6*a*b*sin(c/2 + (d*x)/2
))/(2*a^2*sin(c/2 + (d*x)/2) - 3*b^2*sin(c/2 + (d*x)/2) + 6*a*b*cos(c/2 +
(d*x)/2)))*sin(c + d*x))/4 + a*b*sin(2*c + 2*d*x) - (a*b*sin(3*c + 3*d*x))
/2 - (a*b*sin(4*c + 4*d*x))/4 + (9*a*b*sin(c + d*x)*log(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/4 - (3*a*b*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
)*sin(3*c + 3*d*x))/4)/(d*sin(c + d*x)^3)
```

3.159 $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$

3.159.1 Optimal result

Integrand size = 21, antiderivative size = 202

$$\begin{aligned}\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = {}& -a^2x+\frac{5b^2x}{2}-\frac{15ab\operatorname{arctanh}(\cos(c+dx))}{4d} \\ &+\frac{15ab\cos(c+dx)}{4d}-\frac{a^2\cot(c+dx)}{d} \\ &+\frac{5b^2\cot(c+dx)}{2d}+\frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d} \\ &+\frac{a^2\cot^3(c+dx)}{3d}-\frac{5b^2\cot^3(c+dx)}{6d} \\ &+\frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d} \\ &-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}-\frac{a^2\cot^5(c+dx)}{5d}\end{aligned}$$

output
```
-a^2*x+5/2*b^2*x-15/4*a*b*arctanh(cos(d*x+c))/d+15/4*a*b*cos(d*x+c)/d-a^2*
cot(d*x+c)/d+5/2*b^2*cot(d*x+c)/d+5/4*a*b*cos(d*x+c)*cot(d*x+c)^2/d+1/3*a^
2*cot(d*x+c)^3/d-5/6*b^2*cot(d*x+c)^3/d+1/2*b^2*cos(d*x+c)^2*cot(d*x+c)^3/
d-1/2*a*b*cos(d*x+c)*cot(d*x+c)^4/d-1/5*a^2*cot(d*x+c)^5/d
```

3.159.2 Mathematica [A] (verified)

Time = 1.13 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.74

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$= \frac{-480a^2c+1200b^2c-480a^2dx+1200b^2dx+960ab\cos(c+dx)+(-368a^2+560b^2)\cot\left(\frac{1}{2}(c+dx)\right)+27}{}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

output
```
(-480*a^2*c + 1200*b^2*c - 480*a^2*d*x + 1200*b^2*d*x + 960*a*b*Cos[c + d*
x] + (-368*a^2 + 560*b^2)*Cot[(c + d*x)/2] + 270*a*b*Csc[(c + d*x)/2]^2 -
15*a*b*Csc[(c + d*x)/2]^4 - 1800*a*b*Log[Cos[(c + d*x)/2]] + 1800*a*b*Log[
Sin[(c + d*x)/2]] - 270*a*b*Sec[(c + d*x)/2]^2 + 15*a*b*Sec[(c + d*x)/2]^4
 - 328*a^2*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 160*b^2*Csc[c + d*x]^3*Sin[
(c + d*x)/2]^4 + 96*a^2*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6 + (41*a^2*Csc[(c
 + d*x)/2]^4*Sin[c + d*x])/2 - 10*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - (3
*a^2*Csc[(c + d*x)/2]^6*Sin[c + d*x])/2 + 120*b^2*Sin[2*(c + d*x)] + 368*a
^2*Tan[(c + d*x)/2] - 560*b^2*Tan[(c + d*x)/2])/(480*d)
```

3.159.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{(a+b\sin(c+dx))^2}{\tan(c+dx)^6}dx$$
$$\Big\downarrow \; 3201$$
$$\int \left(a^2\cot^6(c+dx)+2ab\cos(c+dx)\cot^5(c+dx)+b^2\cos^2(c+dx)\cot^4(c+dx)\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot(c+dx)}{d}-a^2x-\frac{15ab\text{arctanh}(\cos(c+dx))}{4d}+\frac{15ab\cos(c+dx)}{4d}-\frac{ab\cos(c+dx)\cot^4(c+dx)}{2d}+\frac{5ab\cos(c+dx)\cot^2(c+dx)}{4d}-\frac{5b^2\cot^3(c+dx)}{6d}+\frac{5b^2\cot(c+dx)}{2d}+\frac{b^2\cos^2(c+dx)\cot^3(c+dx)}{2d}+\frac{5b^2x}{2}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^2,x]
```

output
```
-(a^2*x) + (5*b^2*x)/2 - (15*a*b*ArcTanh[Cos[c + d*x]])/(4*d) + (15*a*b*Co
s[c + d*x])/(4*d) - (a^2*Cot[c + d*x])/d + (5*b^2*Cot[c + d*x])/(2*d) + (5
*a*b*Cos[c + d*x]*Cot[c + d*x]^2)/(4*d) + (a^2*Cot[c + d*x]^3)/(3*d) - (5*
b^2*Cot[c + d*x]^3)/(6*d) + (b^2*Cos[c + d*x]^2*Cot[c + d*x]^3)/(2*d) - (a
*b*Cos[c + d*x]*Cot[c + d*x]^4)/(2*d) - (a^2*Cot[c + d*x]^5)/(5*d)
```

3.159.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.159.4 Maple [A] (verified)

Time = 3.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.07

method	result
derivativedivides	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(d\right.}{8}}{}$
default	$\frac{a^2\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+2ab\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3(d\right.}{8}}{}$
risch	$-a^2x+\frac{5b^2x}{2}-\frac{ib^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{ab\,\mathrm{e}^{i(dx+c)}}{d}+\frac{ab\,\mathrm{e}^{-i(dx+c)}}{d}+\frac{ib^2\mathrm{e}^{-2i(dx+c)}}{8d}-\frac{180ia^2\mathrm{e}^{8i(dx+c)}-180ib^2\mathrm{e}^{8i(dx}}{}$

input
```
int(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+2*a*b*(-1/4
/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^5+
5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+b^2*(-1/3
/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos(d*x+c)^5+5
/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c))
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.51

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx =$$

$$-\frac{60\,b^2\cos\left(dx+c\right)^7+92\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^5-140\left(2\,a^2-5\,b^2\right)\cos\left(dx+c\right)^3+225\,\left(ab\cos\left(dx+c\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/120*(60*b^2*cos(d*x + c)^7 + 92*(2*a^2 - 5*b^2)*cos(d*x + c)^5 - 140*(2
*a^2 - 5*b^2)*cos(d*x + c)^3 + 225*(a*b*cos(d*x + c)^4 - 2*a*b*cos(d*x + c
)^2 + a*b)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 225*(a*b*cos(d*x + c
)^4 - 2*a*b*cos(d*x + c)^2 + a*b)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c
) + 60*(2*a^2 - 5*b^2)*cos(d*x + c) + 30*(2*(2*a^2 - 5*b^2)*d*x*cos(d*x +
c)^4 - 8*a*b*cos(d*x + c)^5 - 4*(2*a^2 - 5*b^2)*d*x*cos(d*x + c)^2 + 25*a*
b*cos(d*x + c)^3 + 2*(2*a^2 - 5*b^2)*d*x - 15*a*b*cos(d*x + c))*sin(d*x +
c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

3.159.6 Sympy [F]

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = \int (a+b\sin(c+dx))^2\cot^6(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**2,x)
```

output
```
Integral((a + b*sin(c + d*x))**2*cot(c + d*x)**6, x)
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx =$$

$$-\frac{8\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^2-20\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)b^2+15}{12}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/120*(8*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan(
d*x + c)^5)*a^2 - 20*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x + c)
^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*b^2 + 15*a*b*(2*(9*cos(d*x + c)
^3 - 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x
+ c) + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.67

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$$

$$= \frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+20\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 + 15*a*b*tan(1/2*d*x + 1/2*c)^4 - 35*a
^2*tan(1/2*d*x + 1/2*c)^3 + 20*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*a*b*tan(1/
2*d*x + 1/2*c)^2 + 1800*a*b*log(abs(tan(1/2*d*x + 1/2*c))) + 330*a^2*tan(1
/2*d*x + 1/2*c) - 540*b^2*tan(1/2*d*x + 1/2*c) - 240*(2*a^2 - 5*b^2)*(d*x
+ c) - 480*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c)^2 - b^
2*tan(1/2*d*x + 1/2*c) - 4*a*b)/(tan(1/2*d*x + 1/2*c)^2 + 1)^2 - (4110*a*b
*tan(1/2*d*x + 1/2*c)^5 + 330*a^2*tan(1/2*d*x + 1/2*c)^4 - 540*b^2*tan(1/2
*d*x + 1/2*c)^4 - 240*a*b*tan(1/2*d*x + 1/2*c)^3 - 35*a^2*tan(1/2*d*x + 1/
2*c)^2 + 20*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*a*b*tan(1/2*d*x + 1/2*c) + 3*a
^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.159.9 Mupad [B] (verification not implemented)

Time = 10.57 (sec) , antiderivative size = 888, normalized size of antiderivative = 4.40

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^6*(a + b*sin(c + d*x))^2,x)
```

output
```
((95*b^2*cos(c + d*x))/384 - (5*a^2*cos(c + d*x))/24 + (5*a^2*cos(3*c + 3*
d*x))/48 - (23*a^2*cos(5*c + 5*d*x))/240 - (163*b^2*cos(3*c + 3*d*x))/384
+ (71*b^2*cos(5*c + 5*d*x))/384 - (b^2*cos(7*c + 7*d*x))/128 + (5*a^2*atan
((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 +
(d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*c
os(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/8 - (a^2*atan((10*b^2*cos(c/2 + (d*x
)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/
2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin
(5*c + 5*d*x))/8 - (25*b^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2
 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^
2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(3*c + 3*d*x))/16 +
(5*b^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b
*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2)
 + 15*a*b*cos(c/2 + (d*x)/2)))*sin(5*c + 5*d*x))/16 + (5*a*b*sin(c + d*x))
/4 - (5*a^2*atan((10*b^2*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 1
5*a*b*sin(c/2 + (d*x)/2))/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*
x)/2) + 15*a*b*cos(c/2 + (d*x)/2)))*sin(c + d*x))/4 + (25*b^2*atan((10*b^2
*cos(c/2 + (d*x)/2) - 4*a^2*cos(c/2 + (d*x)/2) + 15*a*b*sin(c/2 + (d*x)/2)
)/(4*a^2*sin(c/2 + (d*x)/2) - 10*b^2*sin(c/2 + (d*x)/2) + 15*a*b*cos(c/2 +
 (d*x)/2)))*sin(c + d*x))/8 + (5*a*b*sin(2*c + 2*d*x))/8 - (5*a*b*sin(3...
```

3.159. $\int \cot^6(c+dx)(a+b\sin(c+dx))^2\,dx$

3.160 $\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$

3.160.1 Optimal result

Integrand size = 21, antiderivative size = 150

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx=\frac{(a+b)^2(2a+5b)\log(1-\sin(c+dx))}{4d}+\frac{(2a-5b)(a-b)^2\log(1+\sin(c+dx))}{4d}+\frac{b(6a^2+5b^2)\sin(c+dx)}{2d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}+\frac{\sec^2(c+dx)(a+b\sin(c+dx))^3}{2d}$$

output
```
1/4*(a+b)^2*(2*a+5*b)*ln(1-sin(d*x+c))/d+1/4*(2*a-5*b)*(a-b)^2*ln(1+sin(d*
x+c))/d+1/2*b*(6*a^2+5*b^2)*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*
sin(d*x+c)^3/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c))^3/d
```

3.160.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.94

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$$
$$=\frac{3(a+b)^2(2a+5b)\log(1-\sin(c+dx))+3(2a-5b)(a-b)^2\log(1+\sin(c+dx))-\frac{3(a+b)^3}{-1+\sin(c+dx)}+12b(3a}{12d}$$

input
```
Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

output
```
(3*(a + b)^2*(2*a + 5*b)*Log[1 - Sin[c + d*x]] + 3*(2*a - 5*b)*(a - b)^2*L
og[1 + Sin[c + d*x]] - (3*(a + b)^3)/(-1 + Sin[c + d*x]) + 12*b*(3*a^2 + 2
*b^2)*Sin[c + d*x] + 18*a*b^2*Sin[c + d*x]^2 + 4*b^3*Sin[c + d*x]^3 + (3*(
a - b)^3)/(1 + Sin[c + d*x]))/(12*d)
```

3.160.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3200, 531, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^3(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^3(a+b\sin(c+dx))^3dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b^3\sin^3(c+dx)(a+b\sin(c+dx))^3}{\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{531}$$

$$\frac{\frac{\int -\frac{(a+b\sin(c+dx))^2\left(2\sin^2(c+dx)b^4+3b^4+2a\sin(c+dx)b^3\right)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{2b^2}+\frac{b^2(a+b\sin(c+dx))^3}{2\left(b^2-b^2\sin^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{b^2(a+b\sin(c+dx))^3}{2\left(b^2-b^2\sin^2(c+dx)\right)}-\frac{\int \frac{(a+b\sin(c+dx))^2\left(2\sin^2(c+dx)b^4+3b^4+2a\sin(c+dx)b^3\right)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{2b^2}}{d}$$

$$\Big\downarrow \text{2160}$$

$$\frac{\frac{b^2(a+b\sin(c+dx))^3}{2(b^2-b^2\sin^2(c+dx))}-\frac{\int\left(-2\sin^2(c+dx)b^4-5b^4-6a\sin(c+dx)b^3-6a^2b^2+\frac{5b^6+9a^2b^4+2a\left(a^2+6b^2\right)\sin(c+dx)b^3}{b^2-b^2\sin^2(c+dx)}\right)d(b\sin(c+dx))}{2b^2}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(a+b\sin(c+dx))^3}{2(b^2-b^2\sin^2(c+dx))}-\frac{b^3(9a^2+5b^2)\text{arctanh}(\sin(c+dx))-ab^2(a^2+6b^2)\log(b^2-b^2\sin^2(c+dx))-b^3(6a^2+5b^2)\sin(c+dx)-3ab^4\sin^2(c+dx)-\frac{2}{3}}{2b^2}}{d}$$

input
```
Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^3,x]
```

output
```
((b^2*(a + b*Sin[c + d*x])^3)/(2*(b^2 - b^2*Sin[c + d*x]^2)) - (b^3*(9*a^2
 + 5*b^2)*ArcTanh[Sin[c + d*x]] - a*b^2*(a^2 + 6*b^2)*Log[b^2 - b^2*Sin[c
+ d*x]^2] - b^3*(6*a^2 + 5*b^2)*Sin[c + d*x] - 3*a*b^4*Sin[c + d*x]^2 - (2
*b^5*Sin[c + d*x]^3)/3)/(2*b^2))/d
```

3.160.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 531
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m, a + b*x^2, x], e = Coeff[Polynomi
alRemainder[x^m, a + b*x^2, x], x, 0], f = Coeff[PolynomialRemainder[x^m, a
 + b*x^2, x], x, 1]}, Simp[(c + d*x)^n*(a*f - b*e*x)*((a + b*x^2)^(p + 1)/(
2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(c + d*x)^(n - 1)*(a + b
*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*(c + d*x)*Qx - a*d*f*n + b*c*e*(2*p
 + 3) + b*d*e*(n + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGt
Q[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && GtQ[n, 1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.160.4 Maple [A] (verified)

Time = 5.46 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.37

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^2b\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2}{}$
default	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^2b\left(\frac{\sin^5(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^3(dx+c)\right)}{2}+\frac{3\sin(dx+c)}{2}-\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3a\,b^2}{}$
parts	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}-\frac{\ln\left(1+\tan^2(dx+c)\right)}{2}\right)}{d}+\frac{b^3\left(\frac{\sin^7(dx+c)}{2\cos(dx+c)^2}+\frac{\left(\sin^5(dx+c)\right)}{2}+\frac{5\left(\sin^3(dx+c)\right)}{6}+\frac{5\sin(dx+c)}{2}-\frac{5\ln(\sec(dx+c)+}{2}\right.}{d}$
risch	$-6ia\,b^2x-\frac{i\,\mathrm{e}^{i(dx+c)}\left(2ia^3\mathrm{e}^{i(dx+c)}+6ib^2a\,\mathrm{e}^{i(dx+c)}+3a^2b\,\mathrm{e}^{2i(dx+c)}+b^3\mathrm{e}^{2i(dx+c)}-3a^2b-b^3\right)}{d\left(1+\mathrm{e}^{2i(dx+c)}\right)^2}+\frac{ib^3\mathrm{e}^{3i(dx+c)}}{24d}-3a$

input
```
int((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+3*a^2*b*(1/2*sin(d*x+c)^5/cos(d
*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(sec(d*x+c)+tan(d*x+c)))+3*a
*b^2*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos
(d*x+c)))+b^3*(1/2*sin(d*x+c)^7/cos(d*x+c)^2+1/2*sin(d*x+c)^5+5/6*sin(d*x+
c)^3+5/2*sin(d*x+c)-5/2*ln(sec(d*x+c)+tan(d*x+c))))
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.29

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = -\frac{18\,ab^2\cos(dx+c)^4-9\,ab^2\cos(dx+c)^2-3\left(2\,a^3-9\,a^2b+12\,ab^2-5\,b^3\right)\cos(dx+c)^2\log(\sin(dx+c)$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/12*(18*a*b^2*cos(d*x + c)^4 - 9*a*b^2*cos(d*x + c)^2 - 3*(2*a^3 - 9*a^2
*b + 12*a*b^2 - 5*b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(2*a^3 + 9
*a^2*b + 12*a*b^2 + 5*b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 6*a^3 -
 18*a*b^2 + 2*(2*b^3*cos(d*x + c)^4 - 9*a^2*b - 3*b^3 - 2*(9*a^2*b + 7*b^3
)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

3.160.6 Sympy [F]

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$$

input
```
integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**3,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**3, x)
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.08

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \frac{4\,b^3\sin(dx+c)^3+18\,ab^2\sin(dx+c)^2+3\left(2\,a^3-9\,a^2b+12\,ab^2-5\,b^3\right)\log(\sin(dx+c)+1)+3\left(2\,a^3+\right.}{12}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 3*(2*a^3 - 9*a^2*b
+ 12*a*b^2 - 5*b^3)*log(sin(d*x + c) + 1) + 3*(2*a^3 + 9*a^2*b + 12*a*b^2
+ 5*b^3)*log(sin(d*x + c) - 1) + 12*(3*a^2*b + 2*b^3)*sin(d*x + c) - 6*(a^
3 + 3*a*b^2 + (3*a^2*b + b^3)*sin(d*x + c))/(sin(d*x + c)^2 - 1))/d
```

3.160.8 Giac [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^3,x, algorithm="giac")
```

output
```
Timed out
```

3.160.9 Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.44

$$\int (a+b\sin(c+dx))^3\tan^3(c+dx)\,dx$$
$$= \frac{(9a^2b+5b^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+(2a^3+12ab^2)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\left(12a^2b+\frac{20b^3}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+(6a^3+1\cdots}{d\left(\tan\left(\frac{c}{2}\cdots\right.\right.}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)(a^3+6ab^2)}{d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)(a-b)^2\left(a-\frac{5b}{2}\right)}{d}$$
$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)(a+b)^2\left(a+\frac{5b}{2}\right)}{d}$$

input
```
int(tan(c + d*x)^3*(a + b*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(9*a^2*b + 5*b^3) + tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 2
*a^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^8*(12
*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^6*(12*a*b^2 + 6*a^3) + tan(c/2 + (d*x
)/2)^9*(9*a^2*b + 5*b^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b - (22*b^3)/3) + t
an(c/2 + (d*x)/2)^3*(12*a^2*b + (20*b^3)/3) + tan(c/2 + (d*x)/2)^7*(12*a^2
*b + (20*b^3)/3))/(d*(tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^4 - 2*ta
n(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) -
(log(tan(c/2 + (d*x)/2)^2 + 1)*(6*a*b^2 + a^3))/d + (log(tan(c/2 + (d*x)/2
) + 1)*(a - b)^2*(a - (5*b)/2))/d + (log(tan(c/2 + (d*x)/2) - 1)*(a + b)^2
*(a + (5*b)/2))/d
```

3.160. $\int (a+b\sin(c+dx))^3 \tan^3(c+dx)\,dx$

3.161 $\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx$

3.161.1 Optimal result

Integrand size = 19, antiderivative size = 105

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = -\frac{(a+b)^3\log(1-\sin(c+dx))}{2d} - \frac{(a-b)^3\log(1+\sin(c+dx))}{2d} - \frac{b(3a^2+b^2)\sin(c+dx)}{d} - \frac{3ab^2\sin^2(c+dx)}{2d} - \frac{b^3\sin^3(c+dx)}{3d}$$

output
```
-1/2*(a+b)^3*ln(1-sin(d*x+c))/d-1/2*(a-b)^3*ln(1+sin(d*x+c))/d-b*(3*a^2+b^
2)*sin(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2/d-1/3*b^3*sin(d*x+c)^3/d
```

3.161.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.86

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = -\frac{3((a+b)^3\log(1-\sin(c+dx))+(a-b)^3\log(1+\sin(c+dx)))+6b(3a^2+b^2)\sin(c+dx)+9ab^2\sin^2(}{6d}$$

input
```
Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

output
```
-1/6*(3*((a + b)^3*Log[1 - Sin[c + d*x]] + (a - b)^3*Log[1 + Sin[c + d*x]]
) + 6*b*(3*a^2 + b^2)*Sin[c + d*x] + 9*a*b^2*Sin[c + d*x]^2 + 2*b^3*Sin[c
+ d*x]^3)/d
```

3.161.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3200, 525, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)(a+b\sin(c+dx))^3dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b\sin(c+dx)(a+b\sin(c+dx))^3}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{525}$$

$$\frac{-\int -\frac{b\sin(c+dx)\left(a^3+3b^2\sin^2(c+dx)a+b\left(3a^2+b^2\right)\sin(c+dx)\right)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))-\frac{1}{3}b^3\sin^3(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b\sin(c+dx)\left(a^3+3b^2\sin^2(c+dx)a+b\left(3a^2+b^2\right)\sin(c+dx)\right)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))-\frac{1}{3}b^3\sin^3(c+dx)}{d}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(-3a^2-3b\sin(c+dx)a-b^2+\frac{b^4+3a^2b^2+a\left(a^2+3b^2\right)\sin(c+dx)b}{b^2-b^2\sin^2(c+dx)}\right)d(b\sin(c+dx))-\frac{1}{3}b^3\sin^3(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\left(3a^2+b^2\right)\text{arctanh}(\sin(c+dx))-b\left(3a^2+b^2\right)\sin(c+dx)-\frac{1}{2}a\left(a^2+3b^2\right)\log\left(b^2-b^2\sin^2(c+dx)\right)-\frac{3}{2}ab^2\sin^2(c}{d}$$

input
```
Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x],x]
```

output
```
(b*(3*a^2 + b^2)*ArcTanh[Sin[c + d*x]] - (a*(a^2 + 3*b^2)*Log[b^2 - b^2*Si
n[c + d*x]^2])/2 - b*(3*a^2 + b^2)*Sin[c + d*x] - (3*a*b^2*Sin[c + d*x]^2)
/2 - (b^3*Sin[c + d*x]^3)/3)/d
```

3.161.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 525
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_))^(n_))/((a_) + (b_.)*(x_)^2), x_Symbol]
:> Simp[d^n*(x^(m + n - 1)/(b*(m + n - 1))), x] + Simp[1/b   Int[x^m*(Expan
dToSum[b*(c + d*x)^n - b*d^n*x^n - a*d^n*x^(n - 2), x]/(a + b*x^2)), x], x]
 /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 1] && IGtQ[m, -2] && NeQ[m + n - 1, 0
]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.161.4 Maple [A] (verified)

Time = 2.59 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05

method	result
derivativedivides	$\frac{-a^3\ln(\cos(dx+c))+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^3\left(-\frac{\left(\mathrm{s}\right.}{}\right.}{d}$
default	$\frac{-a^3\ln(\cos(dx+c))+3a^2b(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+3a\,b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^3\left(-\frac{\left(\mathrm{s}\right.}{}\right.}{d}$
parts	$\frac{a^3\ln(1+\tan^2(dx+c))}{2d}+\frac{b^3\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c))\right)}{d}+\frac{3a\,b^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\mathrm{c}\right.}{d}$
risch	$\frac{6ia\,b^2c}{d}-\frac{3ib\,\mathrm{e}^{-i(dx+c)}a^2}{2d}+3ia\,b^2x+ia^3x+\frac{3ib\,\mathrm{e}^{i(dx+c)}a^2}{2d}+\frac{2ia^3c}{d}-\frac{5ib^3\mathrm{e}^{-i(dx+c)}}{8d}+\frac{5ib^3\mathrm{e}^{i(dx+c)}}{8d}-$

input
```
int((a+b*sin(d*x+c))^3*tan(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3*ln(cos(d*x+c))+3*a^2*b*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+3
*a*b^2*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+b^3*(-1/3*sin(d*x+c)^3-sin(d*x+c
)+ln(sec(d*x+c)+tan(d*x+c))))
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.10

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{9\,ab^2\cos\left(dx+c\right)^2-3\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(-\mathrm{s}\right.}{6\,d}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="fricas")
```

output
```
1/6*(9*a*b^2*cos(d*x + c)^2 - 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(sin(d*
x + c) + 1) - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(-sin(d*x + c) + 1) + 2
*(b^3*cos(d*x + c)^2 - 9*a^2*b - 4*b^3)*sin(d*x + c))/d
```

3.161.6 Sympy [F]

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = \int (a+b\sin{(c+dx)})^3\tan{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**3*tan(d*x+c),x)
```

output
```
Integral((a + b*sin(c + d*x))**3*tan(c + d*x), x)
```

3.161.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx =$$
$$-\frac{2\,b^3\sin{(dx+c)}^3+9\,ab^2\sin{(dx+c)}^2+3\,(a^3-3\,a^2b+3\,ab^2-b^3)\log{(\sin{(dx+c)}+1)}+3\,(a^3+3\,a^2b}{6\,d}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="maxima")
```

output
```
-1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*log(sin(d*x + c) + 1) + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*lo
g(sin(d*x + c) - 1) + 6*(3*a^2*b + b^3)*sin(d*x + c))/d
```

3.161.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47033 vs. 2(97) = 194.

Time = 15.44 (sec) , antiderivative size = 47033, normalized size of antiderivative = 447.93

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = \text{Too large to display}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c),x, algorithm="giac")
```

output
```
-1/12*(18*a^2*b*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(
1/2*c) + 2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*t
an(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 +
 6*b^3*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) +
2*tan(1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*
x) - 2*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan
(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 18*a^2*b
*log(2*(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(
1/2*d*x)*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2
*tan(1/2*c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c
)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 6*b^3*log(2*(t
an(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) - 2*tan(1/2*d*x)*
tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*
c) + 1)/(tan(1/2*d*x)^2*tan(1/2*c)^2 + tan(1/2*d*x)^2 + tan(1/2*c)^2 + 1))
*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 6*a^3*log(4*(tan(d*x)^2
*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + tan(d*x)^2 + tan
(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 18*a*b^2*log
(4*(tan(d*x)^2*tan(c)^2 - 2*tan(d*x)*tan(c) + 1)/(tan(d*x)^2*tan(c)^2 + ta
n(d*x)^2 + tan(c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)...
```

3.161.9 Mupad [B] (verification not implemented)

Time = 6.52 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.15

$$\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\,\left(a^3+3\,a\,b^2\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(6\,a^2\,b+2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(6\,a^2\,b+2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(12\,a^2\,b+\frac{20\,b^3}{3}\right)+6\,a\,b^2\tan(}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(a-b\right)^3}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(a+b\right)^3}{d}$$

input
```
int(tan(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

output
```
(log(tan(c/2 + (d*x)/2)^2 + 1)*(3*a*b^2 + a^3))/d - (tan(c/2 + (d*x)/2)*(6
*a^2*b + 2*b^3) + tan(c/2 + (d*x)/2)^5*(6*a^2*b + 2*b^3) + tan(c/2 + (d*x)
/2)^3*(12*a^2*b + (20*b^3)/3) + 6*a*b^2*tan(c/2 + (d*x)/2)^2 + 6*a*b^2*tan
(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + t
an(c/2 + (d*x)/2)^6 + 1)) - (log(tan(c/2 + (d*x)/2) + 1)*(a - b)^3)/d - (l
og(tan(c/2 + (d*x)/2) - 1)*(a + b)^3)/d
```

3.161. $\int (a+b\sin(c+dx))^3\tan(c+dx)\,dx$

3.162 $\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx$

3.162.1 Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx=\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

output
```
a^3*ln(sin(d*x+c))/d+3*a^2*b*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3
*sin(d*x+c)^3/d
```

3.162.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx=\frac{a^3\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

output
```
(a^3*Log[Sin[c + d*x]])/d + (3*a^2*b*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*
x]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

3.162.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3200, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(c+dx))^3}{\tan(c+dx)}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc(c+dx)(a+b\sin(c+dx))^3}{b}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{\csc(c+dx)a^3}{b}+3a^2+3b\sin(c+dx)a+b^2\sin^2(c+dx)\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\log(b\sin(c+dx))+3a^2b\sin(c+dx)+\frac{3}{2}ab^2\sin^2(c+dx)+\frac{1}{3}b^3\sin^3(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]*(a + b*Sin[c + d*x])^3,x]
```

output
```
(a^3*Log[b*Sin[c + d*x]] + 3*a^2*b*Sin[c + d*x] + (3*a*b^2*Sin[c + d*x]^2)
/2 + (b^3*Sin[c + d*x]^3)/3)/d
```

3.162.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.162.4 Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{a^{3} \ln\left(\sin\left(dx+c\right)\right)+3 \sin\left(dx+c\right) a^{2} b-\frac{3 \left(\cos^{2}\left(dx+c\right)\right) a \,b^{2}}{2}+\frac{\left(\sin^{3}\left(dx+c\right)\right) b^{3}}{3}}{d}$
default	$\frac{a^{3} \ln\left(\sin\left(dx+c\right)\right)+3 \sin\left(dx+c\right) a^{2} b-\frac{3 \left(\cos^{2}\left(dx+c\right)\right) a \,b^{2}}{2}+\frac{\left(\sin^{3}\left(dx+c\right)\right) b^{3}}{3}}{d}$
risch	$-i a^{3} x-\frac{3 a \,b^{2} {\mathrm e}^{2 i \left(dx+c\right)}}{8 d}-\frac{3 a \,b^{2} {\mathrm e}^{-2 i \left(dx+c\right)}}{8 d}-\frac{2 i a^{3} c}{d}+\frac{a^{3} \ln\left({\mathrm e}^{2 i \left(dx+c\right)}-1\right)}{d}+\frac{3 a^{2} b \sin\left(dx+c\right)}{d}+\frac{b^{3} \sin\left(dx+c\right)}{4 d}$

input
```
int(cot(d*x+c)*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*ln(sin(d*x+c))+3*sin(d*x+c)*a^2*b-3/2*cos(d*x+c)^2*a*b^2+1/3*sin(
d*x+c)^3*b^3)
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{9\,ab^2\cos\left(dx+c\right)^2-6\,a^3\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)+2\,\left(b^3\cos\left(dx+c\right)^2-9\,a^2b-b^3\right)\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/6*(9*a*b^2*cos(d*x + c)^2 - 6*a^3*log(1/2*sin(d*x + c)) + 2*(b^3*cos(d*
x + c)^2 - 9*a^2*b - b^3)*sin(d*x + c))/d
```

3.162.6 Sympy [F]

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(a+b\sin\left(c+dx\right)\right)^3\cot\left(c+dx\right)\,dx$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c))**3,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*cot(c + d*x), x)
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.85

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+6\,a^3\log\left(\sin\left(dx+c\right)\right)+18\,a^2b\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(sin(d*x + c
)) + 18*a^2*b*sin(d*x + c))/d
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.87

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+6\,a^3\log\left(\left|\sin\left(dx+c\right)\right|\right)+18\,a^2b\sin\left(dx+c\right)}{6\,d}$$

input
```
integrate(cot(d*x+c)*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*a^3*log(abs(sin(d*x
 + c))) + 18*a^2*b*sin(d*x + c))/d
```

3.162.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.76

$$\int \cot(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b^3\,\sin\left(c+d\,x\right)}{3\,d}-\frac{a^3\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}+\frac{a^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{3\,a\,b^2\,{\cos\left(c+d\,x\right)}^2}{2\,d}-\frac{b^3\,{\cos\left(c+d\,x\right)}^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{3\,a^2\,b\,\sin\left(c+d\,x\right)}{d}$$

input
```
int(cot(c + d*x)*(a + b*sin(c + d*x))^3,x)
```

output
```
(b^3*sin(c + d*x))/(3*d) - (a^3*log(1/cos(c/2 + (d*x)/2)^2))/d + (a^3*log(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (3*a*b^2*cos(c + d*x)^2)/(2*d)
 - (b^3*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (3*a^2*b*sin(c + d*x))/d
```

3.163 $\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$

3.163.1 Optimal result

Integrand size = 21, antiderivative size = 116

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\log(\sin(c+dx))}{d}-\frac{b(3a^2-b^2)\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{b^3\sin^3(c+dx)}{3d}$$

output
```
-3*a^2*b*csc(d*x+c)/d-1/2*a^3*csc(d*x+c)^2/d-a*(a^2-3*b^2)*ln(sin(d*x+c))/
d-b*(3*a^2-b^2)*sin(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2/d-1/3*b^3*sin(d*x+c)^3
/d
```

3.163.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.12

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{3a^2b\csc(c+dx)}{d}-\frac{a^3\csc^2(c+dx)}{2d}-\frac{a^3\log(\sin(c+dx))}{d}+\frac{3ab^2\log(\sin(c+dx))}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{b^3\sin(c+dx)}{d}-\frac{3ab^2\sin^2(c+dx)}{2d}-\frac{b^3\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

output
```
(-3*a^2*b*Csc[c + d*x])/d - (a^3*Csc[c + d*x]^2)/(2*d) - (a^3*Log[Sin[c +
d*x]])/d + (3*a*b^2*Log[Sin[c + d*x]])/d - (3*a^2*b*Sin[c + d*x])/d + (b^3
*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d*x]^2)/(2*d) - (b^3*Sin[c + d*x]^3)/(
3*d)
```

3.163.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(c+dx))^3}{\tan(c+dx)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^3(c+dx)(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)}{b^3}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{a^3\csc^3(c+dx)}{b}+3a^2\csc^2(c+dx)+\frac{\left(3ab^2-a^3\right)\csc(c+dx)}{b}-b^2\sin^2(c+dx)-3a^2\left(1-\frac{b^2}{3a^2}\right)-3ab\sin(c+dx)\right)d(b\sin}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^3\csc^2(c+dx)-b\left(3a^2-b^2\right)\sin(c+dx)-a\left(a^2-3b^2\right)\log(b\sin(c+dx))-3a^2b\csc(c+dx)-\frac{3}{2}ab^2\sin^2(c+d}{d}$$

input
```
Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x])^3,x]
```

output
```
(-3*a^2*b*Csc[c + d*x] - (a^3*Csc[c + d*x]^2)/2 - a*(a^2 - 3*b^2)*Log[b*Si
n[c + d*x]] - b*(3*a^2 - b^2)*Sin[c + d*x] - (3*a*b^2*Sin[c + d*x]^2)/2 -
(b^3*Sin[c + d*x]^3)/3)/d
```

3.163.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.163.4 Maple [A] (verified)

Time = 2.52 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)+3a\,b^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(\right.}{d}$
default	$\frac{a^3\left(-\frac{\left(\cot^2(dx+c)\right)}{2}-\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\cos^4(dx+c)}{\sin(dx+c)}-\left(2+\cos^2(dx+c)\right)\sin(dx+c)\right)+3a\,b^2\left(\frac{\left(\cos^2(dx+c)\right)}{2}+\ln(\sin(\right.}{d}$
risch	$\frac{ib^3e^{-3i(dx+c)}}{24d}+\frac{2ia^3c}{d}-\frac{2ia^2\left(ia\,e^{2i(dx+c)}+3b\,e^{3i(dx+c)}-3b\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}-1\right)^2}+\frac{3a\,b^2e^{2i(dx+c)}}{8d}-\frac{ib^3e^{3i(dx+c)}}{24d}-3ia\,b$

input
```
int(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*cot(d*x+c)^2-ln(sin(d*x+c)))+3*a^2*b*(-1/sin(d*x+c)*cos(d*x
+c)^4-(2+cos(d*x+c)^2)*sin(d*x+c))+3*a*b^2*(1/2*cos(d*x+c)^2+ln(sin(d*x+c)
))+1/3*b^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.32

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$= \frac{18\,ab^2\cos\left(dx+c\right)^4 - 27\,ab^2\cos\left(dx+c\right)^2 + 6\,a^3 + 9\,ab^2 + 12\,\left(a^3 - 3\,ab^2 - \left(a^3 - 3\,ab^2\right)\cos\left(dx+c\right)^2\right)\log}{12\,\left(d\cos\left(dx+c\right)\right.}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(18*a*b^2*cos(d*x + c)^4 - 27*a*b^2*cos(d*x + c)^2 + 6*a^3 + 9*a*b^2
+ 12*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2)*log(1/2*sin(d*x + c)
) + 4*(b^3*cos(d*x + c)^4 + 18*a^2*b - 2*b^3 - (9*a^2*b - b^3)*cos(d*x + c
)^2)*sin(d*x + c))/(d*cos(d*x + c)^2 - d)
```

3.163.6 Sympy [F]

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = \int \left(a+b\sin\left(c+dx\right)\right)^3\cot^3\left(c+dx\right)dx$$

input
```
integrate(cot(d*x+c)**3*(a+b*sin(d*x+c))**3,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**3, x)
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+6\left(a^3-3\,ab^2\right)\log\left(\sin\left(dx+c\right)\right)+6\left(3\,a^2b-b^3\right)\sin\left(dx+c\right)+\frac{3}{6\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 6*(a^3 - 3*a*b^2)*lo
g(sin(d*x + c)) + 6*(3*a^2*b - b^3)*sin(d*x + c) + 3*(6*a^2*b*sin(d*x + c)
 + a^3)/sin(d*x + c)^2)/d
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.13

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{2\,b^3\sin\left(dx+c\right)^3+9\,ab^2\sin\left(dx+c\right)^2+18\,a^2b\sin\left(dx+c\right)-6\,b^3\sin\left(dx+c\right)+6\left(a^3-3\,ab^2\right)\log\left(|\sin(}{6\,d}$$

input
```
integrate(cot(d*x+c)^3*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/6*(2*b^3*sin(d*x + c)^3 + 9*a*b^2*sin(d*x + c)^2 + 18*a^2*b*sin(d*x + c
) - 6*b^3*sin(d*x + c) + 6*(a^3 - 3*a*b^2)*log(abs(sin(d*x + c))) - 3*(3*a
^3*sin(d*x + c)^2 - 9*a*b^2*sin(d*x + c)^2 - 6*a^2*b*sin(d*x + c) - a^3)/s
in(d*x + c)^2)/d
```

3.163.9 Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 312, normalized size of antiderivative = 2.69

$$\int \cot^3(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$= \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-a^3\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,\left(3\,a\,b^2-a^3\right)}{d}$$
$$- \frac{\frac{3\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2} + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{3\,a^3}{2}+24\,a\,b^2\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{a^3}{2}+24\,a\,b^2\right) + \tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(30\,a^2\,b -}{d\,\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 + 12\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 1\right.}$$
$$- \frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d} - \frac{3\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}$$

input
```
int(cot(c + d*x)^3*(a + b*sin(c + d*x))^3,x)
```

output
```
(log(tan(c/2 + (d*x)/2))*(3*a*b^2 - a^3))/d - (log(tan(c/2 + (d*x)/2)^2 +
1)*(3*a*b^2 - a^3))/d - ((3*a^3*tan(c/2 + (d*x)/2)^2)/2 + tan(c/2 + (d*x)/
2)^4*(24*a*b^2 + (3*a^3)/2) + tan(c/2 + (d*x)/2)^6*(24*a*b^2 + a^3/2) + ta
n(c/2 + (d*x)/2)^7*(30*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^3*(42*a^2*b - 8
*b^3) + tan(c/2 + (d*x)/2)^5*(66*a^2*b - (16*b^3)/3) + a^3/2 + 6*a^2*b*tan
(c/2 + (d*x)/2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 12*tan(c/2 + (d*x)/2)^4 + 12
*tan(c/2 + (d*x)/2)^6 + 4*tan(c/2 + (d*x)/2)^8)) - (a^3*tan(c/2 + (d*x)/2)
^2)/(8*d) - (3*a^2*b*tan(c/2 + (d*x)/2))/(2*d)
```

3.164 $\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx$

3.164.1 Optimal result

Integrand size = 21, antiderivative size = 165

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{b(6a^2-b^2)\csc(c+dx)}{d}+\frac{a(2a^2-3b^2)\csc^2(c+dx)}{2d} - \frac{a^2b\csc^3(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d} + \frac{a(a^2-6b^2)\log(\sin(c+dx))}{d} + \frac{b(3a^2-2b^2)\sin(c+dx)}{d} + \frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

output
```
b*(6*a^2-b^2)*csc(d*x+c)/d+1/2*a*(2*a^2-3*b^2)*csc(d*x+c)^2/d-a^2*b*csc(d*
x+c)^3/d-1/4*a^3*csc(d*x+c)^4/d+a*(a^2-6*b^2)*ln(sin(d*x+c))/d+b*(3*a^2-2*
b^2)*sin(d*x+c)/d+3/2*a*b^2*sin(d*x+c)^2/d+1/3*b^3*sin(d*x+c)^3/d
```

3.164.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.18

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{6a^2b\csc(c+dx)}{d}-\frac{b^3\csc(c+dx)}{d}+\frac{a^3\csc^2(c+dx)}{d}-\frac{3ab^2\csc^2(c+dx)}{2d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{a^3\csc^4(c+dx)}{4d}+\frac{a^3\log(\sin(c+dx))}{d}-\frac{6ab^2\log(\sin(c+dx))}{d}+\frac{3a^2b\sin(c+dx)}{d}-\frac{2b^3\sin(c+dx)}{d}+\frac{3ab^2\sin^2(c+dx)}{2d}+\frac{b^3\sin^3(c+dx)}{3d}$$

input
```
Integrate[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

output
```
(6*a^2*b*Csc[c + d*x])/d - (b^3*Csc[c + d*x])/d + (a^3*Csc[c + d*x]^2)/d -
 (3*a*b^2*Csc[c + d*x]^2)/(2*d) - (a^2*b*Csc[c + d*x]^3)/d - (a^3*Csc[c +
d*x]^4)/(4*d) + (a^3*Log[Sin[c + d*x]])/d - (6*a*b^2*Log[Sin[c + d*x]])/d
+ (3*a^2*b*Sin[c + d*x])/d - (2*b^3*Sin[c + d*x])/d + (3*a*b^2*Sin[c + d*x
]^2)/(2*d) + (b^3*Sin[c + d*x]^3)/(3*d)
```

3.164.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(c+dx))^3}{\tan(c+dx)^5}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^5(c+dx)(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)^2}{b^5}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{a^3\csc^5(c+dx)}{b}+3a^2\csc^4(c+dx)+\frac{(3ab^4-2a^3b^2)\csc^3(c+dx)}{b^3}+\frac{(b^4-6a^2b^2)\csc^2(c+dx)}{b^2}+\frac{(a^3-6ab^2)\csc(c+dx)}{b}+b^2\sin^2(c+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{4}a^3\csc^4(c+dx)+b(3a^2-2b^2)\sin(c+dx)+\frac{1}{2}a(2a^2-3b^2)\csc^2(c+dx)+b(6a^2-b^2)\csc(c+dx)+a(a^2-6b}{d}$$

input
```
Int[Cot[c + d*x]^5*(a + b*Sin[c + d*x])^3,x]
```

output
```
(b*(6*a^2 - b^2)*Csc[c + d*x] + (a*(2*a^2 - 3*b^2)*Csc[c + d*x]^2)/2 - a^2
*b*Csc[c + d*x]^3 - (a^3*Csc[c + d*x]^4)/4 + a*(a^2 - 6*b^2)*Log[b*Sin[c +
 d*x]] + b*(3*a^2 - 2*b^2)*Sin[c + d*x] + (3*a*b^2*Sin[c + d*x]^2)/2 + (b^
3*Sin[c + d*x]^3)/3)/d
```

3.164.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.164.4 Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c\right.}{3}\right.\right.}{}$
default	$\frac{a^3\left(-\frac{\left(\cot^4(dx+c)\right)}{4}+\frac{\left(\cot^2(dx+c)\right)}{2}+\ln(\sin(dx+c))\right)+3a^2b\left(-\frac{\cos^6(dx+c)}{3\sin(dx+c)^3}+\frac{\cos^6(dx+c)}{\sin(dx+c)}+\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c\right.}{3}\right.\right.}{}$
risch	$\frac{ib^3e^{3i(dx+c)}}{24d}+6ia\,b^2x-\frac{2i\left(-2ia^3e^{6i(dx+c)}+3ia\,b^2e^{6i(dx+c)}-6a^2b\,e^{7i(dx+c)}+b^3e^{7i(dx+c)}+2ia^3e^{4i(dx+c)}-6ia\,b^2e\right.}{}$

input
```
int(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/4*cot(d*x+c)^4+1/2*cot(d*x+c)^2+ln(sin(d*x+c)))+3*a^2*b*(-1/3
/sin(d*x+c)^3*cos(d*x+c)^6+1/sin(d*x+c)*cos(d*x+c)^6+(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c))+3*a*b^2*(-1/2/sin(d*x+c)^2*cos(d*x+c)^6-1/2*cos
(d*x+c)^4-cos(d*x+c)^2-2*ln(sin(d*x+c)))+b^3*(-1/sin(d*x+c)*cos(d*x+c)^6-(
8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)))
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.36

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx =$$
$$-\frac{18\,ab^2\cos(dx+c)^6-45\,ab^2\cos(dx+c)^4-9\,a^3+9\,ab^2+6\,(2\,a^3+3\,ab^2)\cos(dx+c)^2-12\,((a^3-6\,a}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/12*(18*a*b^2*cos(d*x + c)^6 - 45*a*b^2*cos(d*x + c)^4 - 9*a^3 + 9*a*b^2
 + 6*(2*a^3 + 3*a*b^2)*cos(d*x + c)^2 - 12*((a^3 - 6*a*b^2)*cos(d*x + c)^4
 + a^3 - 6*a*b^2 - 2*(a^3 - 6*a*b^2)*cos(d*x + c)^2)*log(1/2*sin(d*x + c))
 + 4*(b^3*cos(d*x + c)^6 - 3*(3*a^2*b - b^3)*cos(d*x + c)^4 - 24*a^2*b + 8
*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4
- 2*d*cos(d*x + c)^2 + d)
```

3.164.6 Sympy [F]

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx=\int (a+b\sin(c+dx))^3\cot^5(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**5*(a+b*sin(d*x+c))**3,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**5, x)
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.86

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$=\frac{4\,b^3\sin(dx+c)^3+18\,ab^2\sin(dx+c)^2+12\,(a^3-6\,ab^2)\log(\sin(dx+c))+12\,(3\,a^2b-2\,b^3)\sin(dx+c)}{12\,d}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 12*(a^3 - 6*a*b^2)*
log(sin(d*x + c)) + 12*(3*a^2*b - 2*b^3)*sin(d*x + c) - 3*(4*a^2*b*sin(d*x
 + c) - 4*(6*a^2*b - b^3)*sin(d*x + c)^3 + a^3 - 2*(2*a^3 - 3*a*b^2)*sin(d
*x + c)^2)/sin(d*x + c)^4)/d
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.12

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$=\frac{4\,b^3\sin(dx+c)^3+18\,ab^2\sin(dx+c)^2+36\,a^2b\sin(dx+c)-24\,b^3\sin(dx+c)+12\,(a^3-6\,ab^2)\log(|\sin}{}$$

input
```
integrate(cot(d*x+c)^5*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/12*(4*b^3*sin(d*x + c)^3 + 18*a*b^2*sin(d*x + c)^2 + 36*a^2*b*sin(d*x +
c) - 24*b^3*sin(d*x + c) + 12*(a^3 - 6*a*b^2)*log(abs(sin(d*x + c))) - (25
*a^3*sin(d*x + c)^4 - 150*a*b^2*sin(d*x + c)^4 - 72*a^2*b*sin(d*x + c)^3 +
 12*b^3*sin(d*x + c)^3 - 12*a^3*sin(d*x + c)^2 + 18*a*b^2*sin(d*x + c)^2 +
 12*a^2*b*sin(d*x + c) + 3*a^3)/sin(d*x + c)^4)/d
```

3.164.9 Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.57

$$\int \cot^5(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)\,(6\,a\,b^2-a^3)}{d} - \frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3\,a\,b^2}{8}-\frac{3\,a^3}{16}\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(6\,a\,b^2-a^3)}{d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(3\,a^3+90\,a\,b^2) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(18\,a\,b^2-\frac{33\,a^3}{4}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(6\,a\,b^2-\frac{9\,a^3}{4}\right) + \tan\left(\frac{c}{2}+\right.}{} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{21\,a^2\,b}{8}-\frac{b^3}{2}\right)}{d} - \frac{a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8\,d}$$

input
```
int(cot(c + d*x)^5*(a + b*sin(c + d*x))^3,x)
```

output
```
(log(tan(c/2 + (d*x)/2)^2 + 1)*(6*a*b^2 - a^3))/d - (a^3*tan(c/2 + (d*x)/2
)^4)/(64*d) - (tan(c/2 + (d*x)/2)^2*((3*a*b^2)/8 - (3*a^3)/16))/d - (log(t
an(c/2 + (d*x)/2))*(6*a*b^2 - a^3))/d + (tan(c/2 + (d*x)/2)^8*(90*a*b^2 +
3*a^3) - tan(c/2 + (d*x)/2)^4*(18*a*b^2 - (33*a^3)/4) - tan(c/2 + (d*x)/2)
^2*(6*a*b^2 - (9*a^3)/4) + tan(c/2 + (d*x)/2)^6*(78*a*b^2 + (35*a^3)/4) +
tan(c/2 + (d*x)/2)^3*(36*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^9*(138*a^2*b
- 72*b^3) + tan(c/2 + (d*x)/2)^5*(216*a^2*b - 88*b^3) + tan(c/2 + (d*x)/2)
^7*(316*a^2*b - (328*b^3)/3) - a^3/4 - 2*a^2*b*tan(c/2 + (d*x)/2))/(d*(16*
tan(c/2 + (d*x)/2)^4 + 48*tan(c/2 + (d*x)/2)^6 + 48*tan(c/2 + (d*x)/2)^8 +
 16*tan(c/2 + (d*x)/2)^10)) + (tan(c/2 + (d*x)/2)*((21*a^2*b)/8 - b^3/2))/
d - (a^2*b*tan(c/2 + (d*x)/2)^3)/(8*d)
```

3.165 $\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$

3.165.1 Optimal result

Integrand size = 21, antiderivative size = 220

$$\begin{aligned}\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx &= a^3x+\frac{15}{2}ab^2x-\frac{3a^2b\cos(c+dx)}{d}\\&\quad-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}\\&\quad-\frac{6a^2b\sec(c+dx)}{d}-\frac{3b^3\sec(c+dx)}{d}\\&\quad+\frac{a^2b\sec^3(c+dx)}{d}+\frac{b^3\sec^3(c+dx)}{3d}\\&\quad-\frac{a^3\tan(c+dx)}{d}-\frac{15ab^2\tan(c+dx)}{2d}\\&\quad+\frac{a^3\tan^3(c+dx)}{3d}+\frac{5ab^2\tan^3(c+dx)}{2d}\\&\quad-\frac{3ab^2\sin^2(c+dx)\tan^3(c+dx)}{2d}\end{aligned}$$

output
```
a^3*x+15/2*a*b^2*x-3*a^2*b*cos(d*x+c)/d-3*b^3*cos(d*x+c)/d+1/3*b^3*cos(d*x
+c)^3/d-6*a^2*b*sec(d*x+c)/d-3*b^3*sec(d*x+c)/d+a^2*b*sec(d*x+c)^3/d+1/3*b
^3*sec(d*x+c)^3/d-a^3*tan(d*x+c)/d-15/2*a*b^2*tan(d*x+c)/d+1/3*a^3*tan(d*x
+c)^3/d+5/2*a*b^2*tan(d*x+c)^3/d-3/2*a*b^2*sin(d*x+c)^2*tan(d*x+c)^3/d
```

3.165.2 Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.03

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$$
$$= \frac{\sec^3(c+dx)\left(-300a^2b-210b^3+36a(2a^2+15b^2)(c+dx)\cos(c+dx)-3(144a^2b+91b^3)\cos(2(c+dx))\right.}{}$$

input
```
Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

output
```
(Sec[c + d*x]^3*(-300*a^2*b - 210*b^3 + 36*a*(2*a^2 + 15*b^2)*(c + d*x)*Co
s[c + d*x] - 3*(144*a^2*b + 91*b^3)*Cos[2*(c + d*x)] + 24*a^3*c*Cos[3*(c +
 d*x)] + 180*a*b^2*c*Cos[3*(c + d*x)] + 24*a^3*d*x*Cos[3*(c + d*x)] + 180*
a*b^2*d*x*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b^3*Cos[4*(c +
 d*x)] + b^3*Cos[6*(c + d*x)] - 90*a*b^2*Sin[c + d*x] - 32*a^3*Sin[3*(c +
d*x)] - 195*a*b^2*Sin[3*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)]))/(96*d)
```

3.165.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^4(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(c+dx)^4(a+b\sin(c+dx))^3dx$$

$$\Big\downarrow \ 3201$$

$$\int \left(a^3\tan^4(c+dx)+3a^2b\sin(c+dx)\tan^4(c+dx)+3ab^2\sin^2(c+dx)\tan^4(c+dx)+b^3\sin^3(c+dx)\tan^4(c+dx\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3\tan^3(c+dx)}{3d}-\frac{a^3\tan(c+dx)}{d}+a^3x-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{6a^2b\sec(c+dx)}{d}+\frac{5ab^2\tan^3(c+dx)}{2d}-\frac{15ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan^3(c+dx)}{2d}+\frac{15}{2}ab^2x+\frac{b^3\cos^3(c+dx)}{3d}-\frac{3b^3\cos(c+dx)}{d}+\frac{b^3\sec^3(c+dx)}{3d}-\frac{3b^3\sec(c+dx)}{d}$$

input
```
Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^4,x]
```

output
```
a^3*x + (15*a*b^2*x)/2 - (3*a^2*b*Cos[c + d*x])/d - (3*b^3*Cos[c + d*x])/d
 + (b^3*Cos[c + d*x]^3)/(3*d) - (6*a^2*b*Sec[c + d*x])/d - (3*b^3*Sec[c +
d*x])/d + (a^2*b*Sec[c + d*x]^3)/d + (b^3*Sec[c + d*x]^3)/(3*d) - (a^3*Tan
[c + d*x])/d - (15*a*b^2*Tan[c + d*x])/(2*d) + (a^3*Tan[c + d*x]^3)/(3*d)
+ (5*a*b^2*Tan[c + d*x]^3)/(2*d) - (3*a*b^2*Sin[c + d*x]^2*Tan[c + d*x]^3)
/(2*d)
```

3.165.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.165.4 Maple [A] (verified)

Time = 8.79 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^2b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right.}{}$
default	$\frac{a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+dx+c\right)+3a^2b\left(\frac{\sin^6(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^6(dx+c)}{\cos(dx+c)}-\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right.}{}$
parts	$\frac{a^3\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\tan(dx+c)+\arctan(\tan(dx+c))\right)}{d}+\frac{b^3\left(\frac{\sin^8(dx+c)}{3\cos(dx+c)^3}-\frac{5\left(\sin^8(dx+c)\right)}{3\cos(dx+c)}-\frac{5\left(\frac{16}{5}+\sin^6(dx+c)+\frac{6\left(\sin^4(dx\right.}{5}\right.}{}\right.}{d}$
risch	$a^3x+\frac{15ab^2x}{2}+\frac{b^3e^{3i(dx+c)}}{24d}+\frac{3iab^2e^{2i(dx+c)}}{8d}-\frac{3b\,e^{i(dx+c)}a^2}{2d}-\frac{11b^3e^{i(dx+c)}}{8d}-\frac{3b\,e^{-i(dx+c)}a^2}{2d}-\frac{11b^3e^{-}}{8}$

input
```
int((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)+3*a^2*b*(1/3*sin(d*x+c)^6/cos
(d*x+c)^3-sin(d*x+c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(
d*x+c))+3*a*b^2*(1/3*sin(d*x+c)^7/cos(d*x+c)^3-4/3*sin(d*x+c)^7/cos(d*x+c)
-4/3*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/2*d*x+5/
2*c)+b^3*(1/3*sin(d*x+c)^8/cos(d*x+c)^3-5/3*sin(d*x+c)^8/cos(d*x+c)-5/3*(1
6/5+sin(d*x+c)^6+6/5*sin(d*x+c)^4+8/5*sin(d*x+c)^2)*cos(d*x+c)))
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.71

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$$
$$=\frac{2\,b^3\cos\left(dx+c\right)^6+3\left(2\,a^3+15\,ab^2\right)dx\cos\left(dx+c\right)^3-18\left(a^2b+b^3\right)\cos\left(dx+c\right)^4+6\,a^2b+2\,b^3-18\left(2\,a^2\right.}{6\,d\cos\left(dx\right.}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="fricas")
```

output
```
1/6*(2*b^3*cos(d*x + c)^6 + 3*(2*a^3 + 15*a*b^2)*d*x*cos(d*x + c)^3 - 18*(
a^2*b + b^3)*cos(d*x + c)^4 + 6*a^2*b + 2*b^3 - 18*(2*a^2*b + b^3)*cos(d*x
 + c)^2 - (9*a*b^2*cos(d*x + c)^4 - 2*a^3 - 6*a*b^2 + 2*(4*a^3 + 21*a*b^2)
*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

3.165.6 Sympy [F]

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx=\int (a+b\sin{(c+dx)})^3\tan^4{(c+dx)}\,dx$$

input
```
integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**4,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**4, x)
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.76

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx$$
$$=\frac{2\left(\tan{(dx+c)}^3+3\,dx+3\,c-3\,\tan{(dx+c)}\right)a^3+3\left(2\,\tan{(dx+c)}^3+15\,dx+15\,c-\frac{3\,\tan(dx+c)}{\tan(dx+c)^2+1}-12\right.}{}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="maxima")
```

output
```
1/6*(2*(tan(d*x + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a^3 + 3*(2*tan(d*x
+ c)^3 + 15*d*x + 15*c - 3*tan(d*x + c)/(tan(d*x + c)^2 + 1) - 12*tan(d*x
+ c))*a*b^2 + 2*(cos(d*x + c)^3 - (9*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 -
9*cos(d*x + c))*b^3 - 6*a^2*b*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*c
os(d*x + c)))/d
```

3.165.8 Giac [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^4,x, algorithm="giac")
```

output
```
Timed out
```

3.165.9 Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.35

$$\int (a+b\sin(c+dx))^3\tan^4(c+dx)\,dx = \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^2+15b^2)}{2a^3+15ab^2}\right)(2a^2+15b^2)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{2a^3}{3}+5ab^2\right)-16a^2b-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^3+15ab^2)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{2a^3}{3}+5ab^2\right)-}{d\,(}$$

input
```
int(tan(c + d*x)^4*(a + b*sin(c + d*x))^3,x)
```

output
```
(a*atan((a*tan(c/2 + (d*x)/2)*(2*a^2 + 15*b^2))/(15*a*b^2 + 2*a^3))*(2*a^2
 + 15*b^2))/d - (tan(c/2 + (d*x)/2)^3*(5*a*b^2 + (2*a^3)/3) - 16*a^2*b - t
an(c/2 + (d*x)/2)*(15*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^9*(5*a*b^2 + (2*
a^3)/3) - tan(c/2 + (d*x)/2)^11*(15*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^5*
(42*a*b^2 + 12*a^3) + tan(c/2 + (d*x)/2)^7*(42*a*b^2 + 12*a^3) + tan(c/2 +
 (d*x)/2)^4*(48*a^2*b + 32*b^3) - (32*b^3)/3 + 32*a^2*b*tan(c/2 + (d*x)/2)
^6)/(d*(3*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/
2)^12 - 1))
```

3.166 $\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx$

3.166.1 Optimal result

Integrand size = 21, antiderivative size = 146

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = -a^3x-\frac{9}{2}ab^2x+\frac{3a^2b\cos(c+dx)}{d}+\frac{2b^3\cos(c+dx)}{d} - \frac{b^3\cos^3(c+dx)}{3d}+\frac{3a^2b\sec(c+dx)}{d} + \frac{b^3\sec(c+dx)}{d}+\frac{a^3\tan(c+dx)}{d} + \frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}$$

output
```
-a^3*x-9/2*a*b^2*x+3*a^2*b*cos(d*x+c)/d+2*b^3*cos(d*x+c)/d-1/3*b^3*cos(d*x
+c)^3/d+3*a^2*b*sec(d*x+c)/d+b^3*sec(d*x+c)/d+a^3*tan(d*x+c)/d+9/2*a*b^2*t
an(d*x+c)/d-3/2*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d
```

3.166.2 Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.77

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{b\sec(c+dx)\left(108a^2+45b^2+4(9a^2+5b^2)\cos(2(c+dx))-b^2\cos(4(c+dx))+9ab\sin(3(c+dx))\right)+3a(}{24d}$$

input
```
Integrate[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

output
```
(b*Sec[c + d*x]*(108*a^2 + 45*b^2 + 4*(9*a^2 + 5*b^2)*Cos[2*(c + d*x)] - b
^2*Cos[4*(c + d*x)] + 9*a*b*Sin[3*(c + d*x)]) + 3*a*(-4*(2*a^2 + 9*b^2)*(c
 + d*x) + (8*a^2 + 27*b^2)*Tan[c + d*x]))/(24*d)
```

3.166.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tan^2(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(c+dx)^2(a+b\sin(c+dx))^3dx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a^3\tan^2(c+dx)+3a^2b\sin(c+dx)\tan^2(c+dx)+3ab^2\sin^2(c+dx)\tan^2(c+dx)+b^3\sin^3(c+dx)\tan^2(c+dx\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\tan(c+dx)}{d}+a^3(-x)+\frac{3a^2b\cos(c+dx)}{d}+\frac{3a^2b\sec(c+dx)}{d}+\frac{9ab^2\tan(c+dx)}{2d}-\frac{3ab^2\sin^2(c+dx)\tan(c+dx)}{2d}-\frac{9}{2}ab^2x-\frac{b^3\cos^3(c+dx)}{3d}+\frac{2b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

input
```
Int[(a + b*Sin[c + d*x])^3*Tan[c + d*x]^2,x]
```

output
```
-(a^3*x) - (9*a*b^2*x)/2 + (3*a^2*b*Cos[c + d*x])/d + (2*b^3*Cos[c + d*x])
/d - (b^3*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Sec[c + d*x])/d + (b^3*Sec[c +
d*x])/d + (a^3*Tan[c + d*x])/d + (9*a*b^2*Tan[c + d*x])/(2*d) - (3*a*b^2*S
in[c + d*x]^2*Tan[c + d*x])/(2*d)
```

3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.166.4 Maple [A] (verified)

Time = 4.62 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cdots\right.}{d}$
default	$\frac{a^3(\tan(dx+c)-dx-c)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a\,b^2\left(\frac{\sin^5(dx+c)}{\cos(dx+c)}+\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cdots\right.}{d}$
parts	$\frac{a^3(\tan(dx+c)-\arctan(\tan(dx+c)))}{d}+\frac{b^3\left(\frac{\sin^6(dx+c)}{\cos(dx+c)}+\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)\right)}{d}+\frac{3a^2b\left(\frac{\sin^4}{\cos(}\cdots\right.}{}$
risch	$-a^3x-\frac{9a\,b^2x}{2}-\frac{3ia\,b^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{i(dx+c)}a^2}{2d}+\frac{7b^3\mathrm{e}^{i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{-i(dx+c)}a^2}{2d}+\frac{7b^3\mathrm{e}^{-i(dx+c)}}{8d}+\frac{3ia\,b^2}{}\cdots$

input
```
int((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(tan(d*x+c)-d*x-c)+3*a^2*b*(1/cos(d*x+c)*sin(d*x+c)^4+(2+sin(d*x+
c)^2)*cos(d*x+c))+3*a*b^2*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d
*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+b^3*(sin(d*x+c)^6/cos(d*x+c)+(8/3+sin(d*x
+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = -\frac{2\,b^3\cos\left(dx+c\right)^4+3\left(2\,a^3+9\,ab^2\right)dx\cos\left(dx+c\right)-18\,a^2b-6\,b^3-6\left(3\,a^2b+2\,b^3\right)\cos\left(dx+c\right)^2-3\,(3}{6\,d\cos\left(dx+c\right)}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*b^3*cos(d*x + c)^4 + 3*(2*a^3 + 9*a*b^2)*d*x*cos(d*x + c) - 18*a^2
*b - 6*b^3 - 6*(3*a^2*b + 2*b^3)*cos(d*x + c)^2 - 3*(3*a*b^2*cos(d*x + c)^
2 + 2*a^3 + 6*a*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

3.166.6 Sympy [F]

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \int \left(a+b\sin\left(c+dx\right)\right)^3\tan^2\left(c+dx\right)dx$$

input
```
integrate((a+b*sin(d*x+c))**3*tan(d*x+c)**2,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*tan(c + d*x)**2, x)
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.82

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = -\frac{6\left(dx+c-\tan\left(dx+c\right)\right)a^3+9\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)ab^2+2\left(\cos\left(dx+c\right)^3-\frac{}{\cos}\right.}{6\,d}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/6*(6*(d*x + c - tan(d*x + c))*a^3 + 9*(3*d*x + 3*c - tan(d*x + c)/(tan(
d*x + c)^2 + 1) - 2*tan(d*x + c))*a*b^2 + 2*(cos(d*x + c)^3 - 3/cos(d*x +
c) - 6*cos(d*x + c))*b^3 - 18*a^2*b*(1/cos(d*x + c) + cos(d*x + c)))/d
```

3.166.8 Giac [F(-1)]

Timed out.

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \text{Timed out}$$

input
```
integrate((a+b*sin(d*x+c))^3*tan(d*x+c)^2,x, algorithm="giac")
```

output
```
Timed out
```

3.166.9 Mupad [B] (verification not implemented)

Time = 8.47 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.71

$$\int (a+b\sin(c+dx))^3\tan^2(c+dx)\,dx = \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(2\,a^3+9\,a\,b^2\right)+12\,a^2\,b+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\,\left(2\,a^3+9\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(6\,a^3+15\,a\,b^2\right)+\tan}{d\,\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\right.} - \frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+9\,b^2\right)}{2\,a^3+9\,a\,b^2}\right)\,\left(2\,a^2+9\,b^2\right)}{d}$$

input
```
int(tan(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)*(9*a*b^2 + 2*a^3) + 12*a^2*b + tan(c/2 + (d*x)/2)^7*(9
*a*b^2 + 2*a^3) + tan(c/2 + (d*x)/2)^3*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x
)/2)^5*(15*a*b^2 + 6*a^3) + tan(c/2 + (d*x)/2)^2*(24*a^2*b + (32*b^3)/3) +
 (16*b^3)/3 + 12*a^2*b*tan(c/2 + (d*x)/2)^4)/(d*(2*tan(c/2 + (d*x)/2)^2 -
2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1)) - (a*atan((a*tan(c/2 +
 (d*x)/2)*(2*a^2 + 9*b^2))/(9*a*b^2 + 2*a^3))*(2*a^2 + 9*b^2))/d
```

3.167 $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

3.167.1 Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = -a^3x+\frac{3}{2}ab^2x-\frac{3a^2b\operatorname{arctanh}(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}-\frac{b^3\cos^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\cos(c+dx)\sin(c+dx)}{2d}$$

output
```
-a^3*x+3/2*a*b^2*x-3*a^2*b*arctanh(cos(d*x+c))/d+3*a^2*b*cos(d*x+c)/d-1/3*
b^3*cos(d*x+c)^3/d-a^3*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d
```

3.167.2 Mathematica [A] (verified)

Time = 1.24 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{(36a^2b-3b^3)\cos(c+dx)-b^3\cos(3(c+dx))-6a^3\cot\left(\frac{1}{2}(c+dx)\right)+9ab^2\sin(2(c+dx))+6a\left(-2a^2c+\cdots\right.}{12d}$$

input
```
Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

output
```
((36*a^2*b - 3*b^3)*Cos[c + d*x] - b^3*Cos[3*(c + d*x)] - 6*a^3*Cot[(c + d
*x)/2] + 9*a*b^2*Sin[2*(c + d*x)] + 6*a*(-2*a^2*c + 3*b^2*c - 2*a^2*d*x +
3*b^2*d*x - 6*a*b*Log[Cos[(c + d*x)/2]] + 6*a*b*Log[Sin[(c + d*x)/2]] + a^
2*Tan[(c + d*x)/2]))/(12*d)
```

3.167.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(a+b\sin(c+dx))^3}{\tan(c+dx)^2}dx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a^3\cot^2(c+dx)+3a^2b\cos(c+dx)\cot(c+dx)+3ab^2\cos^2(c+dx)+b^3\sin(c+dx)\cos^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\cot(c+dx)}{d}+a^3(-x)-\frac{3a^2b\text{arctanh}(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{3}{2}ab^2x-\frac{b^3\cos^3(c+dx)}{3d}$$

input
```
Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x])^3,x]
```

output
```
-(a^3*x) + (3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cos[c + d*x]])/d + (3*a^2*b*Co
s[c + d*x])/d - (b^3*Cos[c + d*x]^3)/(3*d) - (a^3*Cot[c + d*x])/d + (3*a*b
^2*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.167.4 Maple [A] (verified)

Time = 1.76 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^2b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a\,b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{b^3(\cos^3(d}{3}}{d}$
default	$\frac{a^3(-\cot(dx+c)-dx-c)+3a^2b(\cos(dx+c)+\ln(\csc(dx+c)-\cot(dx+c)))+3a\,b^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{b^3(\cos^3(d}{3}}{d}$
risch	$-a^3x+\frac{3a\,b^2x}{2}-\frac{3ia\,b^2e^{2i(dx+c)}}{8d}+\frac{3b\,e^{i(dx+c)}a^2}{2d}-\frac{b^3e^{i(dx+c)}}{8d}+\frac{3b\,e^{-i(dx+c)}a^2}{2d}-\frac{b^3e^{-i(dx+c)}}{8d}+\frac{3ia\,b^2e^{-}}{8}$

input
```
int(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-cot(d*x+c)-d*x-c)+3*a^2*b*(cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+c))
)+3*a*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/3*b^3*cos(d*x+c)^3)
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{9\,ab^2\cos(dx+c)^3 + 9\,a^2b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c) - 9\,a^2b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx}{6\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/6*(9*a*b^2*cos(d*x + c)^3 + 9*a^2*b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x
 + c) - 9*a^2*b*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(2*a^3 - 3*a
*b^2)*cos(d*x + c) + (2*b^3*cos(d*x + c)^3 - 18*a^2*b*cos(d*x + c) + 3*(2*
a^3 - 3*a*b^2)*d*x)*sin(d*x + c))/(d*sin(d*x + c))
```

3.167.6 Sympy [F]

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = \int (a+b\sin(c+dx))^3\cot^2(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**2*(a+b*sin(d*x+c))**3,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**2, x)
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.93

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = -\frac{4\,b^3\cos(dx+c)^3 + 12\left(dx+c+\frac{1}{\tan(dx+c)}\right)a^3 - 9\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)ab^2 - 18\,a^2b(2\cos(dx+}{12\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/12*(4*b^3*cos(d*x + c)^3 + 12*(d*x + c + 1/tan(d*x + c))*a^3 - 9*(2*d*x
 + 2*c + sin(2*d*x + 2*c))*a*b^2 - 18*a^2*b*(2*cos(d*x + c) - log(cos(d*x
+ c) + 1) + log(cos(d*x + c) - 1)))/d
```

3.167.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(96) = 192.

Time = 0.39 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.95

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$= \frac{18\,a^2b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\left(2\,a^3-3\,ab^2\right)(dx+c)-\frac{3\left(6\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-}{6\,d}$$

input
```
integrate(cot(d*x+c)^2*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/6*(18*a^2*b*log(abs(tan(1/2*d*x + 1/2*c))) + 3*a^3*tan(1/2*d*x + 1/2*c)
- 3*(2*a^3 - 3*a*b^2)*(d*x + c) - 3*(6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/t
an(1/2*d*x + 1/2*c) - 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2
*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/
2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 2*b^3)/(tan(1/2*d*x + 1
/2*c)^2 + 1)^3)/d
```

3.167.9 Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.83

$$\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\left(\frac{a\,b^2\,3\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}$$
$$+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(12\,a^2\,b-\frac{4\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(a^3+6\,a\,b^2\right)-3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(6\,a\,b^2-\right.}{d\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right.}$$
$$+\frac{3\,a^2\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)\left(2\,a^2-3\,b^2\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^2*(a + b*sin(c + d*x))^3,x)
```

output
```
(a^3*tan(c/2 + (d*x)/2))/(2*d) - (log(tan(c/2 + (d*x)/2) - 1i)*((a*b^2*3i)
/2 - a^3*1i))/d + (tan(c/2 + (d*x)/2)*(12*a^2*b - (4*b^3)/3) - tan(c/2 + (
d*x)/2)^6*(6*a*b^2 + a^3) - 3*a^3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2
)^2*(6*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^5*(12*a^2*b - 4*b^3) - a^3 + 24
*a^2*b*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/
2)^3 + 6*tan(c/2 + (d*x)/2)^5 + 2*tan(c/2 + (d*x)/2)^7)) + (3*a^2*b*log(ta
n(c/2 + (d*x)/2)))/d - (a*log(tan(c/2 + (d*x)/2) + 1i)*(2*a^2 - 3*b^2)*1i)
/(2*d)
```

3.167. $\int \cot^2(c+dx)(a+b\sin(c+dx))^3\,dx$

3.168 $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$

3.168.1 Optimal result

Integrand size = 21, antiderivative size = 194

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = a^3x-\frac{9}{2}ab^2x+\frac{9a^2b\text{arctanh}(\cos(c+dx))}{2d} - \frac{b^3\text{arctanh}(\cos(c+dx))}{d}-\frac{9a^2b\cos(c+dx)}{2d} + \frac{b^3\cos(c+dx)}{d}+\frac{b^3\cos^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d} - \frac{9ab^2\cot(c+dx)}{2d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d} - \frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{a^3\cot^3(c+dx)}{3d}$$

output
```
a^3*x-9/2*a*b^2*x+9/2*a^2*b*arctanh(cos(d*x+c))/d-b^3*arctanh(cos(d*x+c))/
d-9/2*a^2*b*cos(d*x+c)/d+b^3*cos(d*x+c)/d+1/3*b^3*cos(d*x+c)^3/d+a^3*cot(d
*x+c)/d-9/2*a*b^2*cot(d*x+c)/d+3/2*a*b^2*cos(d*x+c)^2*cot(d*x+c)/d-3/2*a^2
*b*cos(d*x+c)*cot(d*x+c)^2/d-1/3*a^3*cot(d*x+c)^3/d
```

3.168.2 Mathematica [A] (verified)

Time = 6.59 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.83

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$= \frac{a(2a^2-9b^2)(c+dx)}{2d} + \frac{b(-12a^2+5b^2)\cos(c+dx)}{4d} + \frac{b^3\cos(3(c+dx))}{12d} + \frac{\left(4a^3\cos\left(\frac{1}{2}(c+dx)\right)-9ab^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6d} - \frac{3a^2b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^3\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24d} + \frac{(9a^2b-2b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{(-9a^2b+2b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{3a^2b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^3\sin\left(\frac{1}{2}(c+dx)\right)+9ab^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d} - \frac{3ab^2\sin(2(c+dx))}{4d} + \frac{a^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24d}$$

input
```
Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

output
```
(a*(2*a^2 - 9*b^2)*(c + d*x))/(2*d) + (b*(-12*a^2 + 5*b^2)*Cos[c + d*x])/(
4*d) + (b^3*Cos[3*(c + d*x)])/(12*d) + ((4*a^3*Cos[(c + d*x)/2] - 9*a*b^2*
Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*d) - (3*a^2*b*Csc[(c + d*x)/2]^2)/(
8*d) - (a^3*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*d) + ((9*a^2*b - 2*b^
3)*Log[Cos[(c + d*x)/2]])/(2*d) + ((-9*a^2*b + 2*b^3)*Log[Sin[(c + d*x)/2]
])/(2*d) + (3*a^2*b*Sec[(c + d*x)/2]^2)/(8*d) + (Sec[(c + d*x)/2]*(-4*a^3*
Sin[(c + d*x)/2] + 9*a*b^2*Sin[(c + d*x)/2]))/(6*d) - (3*a*b^2*Sin[2*(c +
d*x)])/(4*d) + (a^3*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*d)
```

3.168.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(a+b\sin(c+dx))^3}{\tan(c+dx)^4}dx$$

$$\Big\downarrow \text{ 3201}$$

$$\int \left(a^3\cot^4(c+dx)+3a^2b\cos(c+dx)\cot^3(c+dx)+3ab^2\cos^2(c+dx)\cot^2(c+dx)+b^3\cos^3(c+dx)\cot(c+dx)\right.$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^3\cot^3(c+dx)}{3d}+\frac{a^3\cot(c+dx)}{d}+a^3x+\frac{9a^2b\text{arctanh}(\cos(c+dx))}{2d}-\frac{9a^2b\cos(c+dx)}{2d}-\frac{3a^2b\cos(c+dx)\cot^2(c+dx)}{2d}-\frac{9ab^2\cot(c+dx)}{2d}+\frac{3ab^2\cos^2(c+dx)\cot(c+dx)}{2d}-\frac{9}{2}ab^2x-\frac{b^3\text{arctanh}(\cos(c+dx))}{d}+\frac{b^3\cos^3(c+dx)}{3d}+\frac{b^3\cos(c+dx)}{d}$$

input
```
Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x])^3,x]
```

output
```
a^3*x - (9*a*b^2*x)/2 + (9*a^2*b*ArcTanh[Cos[c + d*x]])/(2*d) - (b^3*ArcTa
nh[Cos[c + d*x]])/d - (9*a^2*b*Cos[c + d*x])/(2*d) + (b^3*Cos[c + d*x])/d
+ (b^3*Cos[c + d*x]^3)/(3*d) + (a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x
])/(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x])/(2*d) - (3*a^2*b*Cos[c +
d*x]*Cot[c + d*x]^2)/(2*d) - (a^3*Cot[c + d*x]^3)/(3*d)
```

3.168.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.168.4 Maple [A] (verified)

Time = 3.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{}$
default	$\frac{a^3\left(-\frac{\left(\cot^3(dx+c)\right)}{3}+\cot(dx+c)+dx+c\right)+3a^2b\left(-\frac{\cos^5(dx+c)}{2\sin(dx+c)^2}-\frac{\left(\cos^3(dx+c)\right)}{2}-\frac{3\cos(dx+c)}{2}-\frac{3\ln(\csc(dx+c)-\cot(dx+c))}{2}\right)}{}$
risch	$a^3x-\frac{9ab^2x}{2}+\frac{3iab^2e^{2i(dx+c)}}{8d}-\frac{3b\,e^{i(dx+c)}a^2}{2d}+\frac{5b^3e^{i(dx+c)}}{8d}-\frac{3b\,e^{-i(dx+c)}a^2}{2d}+\frac{5b^3e^{-i(dx+c)}}{8d}-\frac{3iab^2e^{-}}{8}$

input
```
int(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/3*cot(d*x+c)^3+cot(d*x+c)+d*x+c)+3*a^2*b*(-1/2/sin(d*x+c)^2*c
os(d*x+c)^5-1/2*cos(d*x+c)^3-3/2*cos(d*x+c)-3/2*ln(csc(d*x+c)-cot(d*x+c)))
+3*a*b^2*(-1/sin(d*x+c)*cos(d*x+c)^5-(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x
+c)-3/2*d*x-3/2*c)+b^3*(1/3*cos(d*x+c)^3+cos(d*x+c)+ln(csc(d*x+c)-cot(d*x+
c))))
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.51

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$=\frac{18\,ab^2\cos\left(dx+c\right)^5+8\left(2\,a^3-9\,ab^2\right)\cos\left(dx+c\right)^3-3\left(9\,a^2b-2\,b^3-\left(9\,a^2b-2\,b^3\right)\cos\left(dx+c\right)^2\right)\log\left(\frac{1}{2}\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(18*a*b^2*cos(d*x + c)^5 + 8*(2*a^3 - 9*a*b^2)*cos(d*x + c)^3 - 3*(9*
a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1
/2)*sin(d*x + c) + 3*(9*a^2*b - 2*b^3 - (9*a^2*b - 2*b^3)*cos(d*x + c)^2)*
log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - 6*(2*a^3 - 9*a*b^2)*cos(d*x +
c) + 2*(2*b^3*cos(d*x + c)^5 + 3*(2*a^3 - 9*a*b^2)*d*x*cos(d*x + c)^2 - 2*
(9*a^2*b - 2*b^3)*cos(d*x + c)^3 - 3*(2*a^3 - 9*a*b^2)*d*x + 3*(9*a^2*b -
2*b^3)*cos(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^2 - d)*sin(d*x + c))
```

3.168.6 Sympy [F]

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = \int (a+b\sin(c+dx))^3\cot^4(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**4*(a+b*sin(d*x+c))**3,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**4, x)
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$= \frac{4\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2-1}{\tan(dx+c)^3}\right)a^3-18\left(3\,dx+3\,c+\frac{3\,\tan(dx+c)^2+2}{\tan(dx+c)^3+\tan(dx+c)}\right)ab^2+2\left(2\,\cos(dx+c)^3+6\,\cos(d\right.}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/12*(4*(3*d*x + 3*c + (3*tan(d*x + c)^2 - 1)/tan(d*x + c)^3)*a^3 - 18*(3*
d*x + 3*c + (3*tan(d*x + c)^2 + 2)/(tan(d*x + c)^3 + tan(d*x + c)))*a*b^2
+ 2*(2*cos(d*x + c)^3 + 6*cos(d*x + c) - 3*log(cos(d*x + c) + 1) + 3*log(c
os(d*x + c) - 1))*b^3 + 9*a^2*b*(2*cos(d*x + c)/(cos(d*x + c)^2 - 1) - 4*c
os(d*x + c) + 3*log(cos(d*x + c) + 1) - 3*log(cos(d*x + c) - 1)))/d
```

3.168.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. 2(178) = 356.

Time = 0.45 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.17

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$= \frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-45\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+108\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,(2}{}$$

input
```
integrate(cot(d*x+c)^4*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/72*(3*a^3*tan(1/2*d*x + 1/2*c)^3 + 27*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 45*
a^3*tan(1/2*d*x + 1/2*c) + 108*a*b^2*tan(1/2*d*x + 1/2*c) + 36*(2*a^3 - 9*
a*b^2)*(d*x + c) - 36*(9*a^2*b - 2*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) + (
198*a^2*b*tan(1/2*d*x + 1/2*c)^9 - 44*b^3*tan(1/2*d*x + 1/2*c)^9 + 45*a^3*
tan(1/2*d*x + 1/2*c)^8 + 108*a*b^2*tan(1/2*d*x + 1/2*c)^8 + 135*a^2*b*tan(
1/2*d*x + 1/2*c)^7 + 156*b^3*tan(1/2*d*x + 1/2*c)^7 + 132*a^3*tan(1/2*d*x
+ 1/2*c)^6 - 324*a*b^2*tan(1/2*d*x + 1/2*c)^6 - 351*a^2*b*tan(1/2*d*x + 1/
2*c)^5 + 156*b^3*tan(1/2*d*x + 1/2*c)^5 + 126*a^3*tan(1/2*d*x + 1/2*c)^4 -
 540*a*b^2*tan(1/2*d*x + 1/2*c)^4 - 315*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 148
*b^3*tan(1/2*d*x + 1/2*c)^3 + 36*a^3*tan(1/2*d*x + 1/2*c)^2 - 108*a*b^2*ta
n(1/2*d*x + 1/2*c)^2 - 27*a^2*b*tan(1/2*d*x + 1/2*c) - 3*a^3)/(tan(1/2*d*x
 + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))^3)/d
```

3.168.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 405, normalized size of antiderivative = 2.09

$$\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,\left(\frac{9\,a^2\,b}{2}-b^3\right)}{d}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\,\left(\frac{a\,b^2\,9\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{3\,a\,b^2}{2}-\frac{5\,a^3}{8}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(12\,a\,b^2-4\,a^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(5\,a^3+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(60\,a\,b^2-14\,a^3\right)+\tan\left(\frac{c}{2}\right.}{d\,\left(8\tan\left(\frac{c}{2}+\frac{d}{2}\right.\right.}$$
$$+\frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d}-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)\,\left(2\,a^2-9\,b^2\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^4*(a + b*sin(c + d*x))^3,x)
```

output
```
(a^3*tan(c/2 + (d*x)/2)^3)/(24*d) - (log(tan(c/2 + (d*x)/2))*((9*a^2*b)/2
- b^3))/d - (log(tan(c/2 + (d*x)/2) + 1i)*((a*b^2*9i)/2 - a^3*1i))/d + (ta
n(c/2 + (d*x)/2)*((3*a*b^2)/2 - (5*a^3)/8))/d - (tan(c/2 + (d*x)/2)^2*(12*
a*b^2 - 4*a^3) - tan(c/2 + (d*x)/2)^8*(12*a*b^2 + 5*a^3) + tan(c/2 + (d*x)
/2)^4*(60*a*b^2 - 14*a^3) + tan(c/2 + (d*x)/2)^6*(36*a*b^2 - (44*a^3)/3) +
 tan(c/2 + (d*x)/2)^7*(51*a^2*b - 32*b^3) + tan(c/2 + (d*x)/2)^3*(57*a^2*b
 - (64*b^3)/3) + tan(c/2 + (d*x)/2)^5*(105*a^2*b - 32*b^3) + a^3/3 + 3*a^2
*b*tan(c/2 + (d*x)/2))/(d*(8*tan(c/2 + (d*x)/2)^3 + 24*tan(c/2 + (d*x)/2)^
5 + 24*tan(c/2 + (d*x)/2)^7 + 8*tan(c/2 + (d*x)/2)^9)) + (3*a^2*b*tan(c/2
+ (d*x)/2)^2)/(8*d) - (a*log(tan(c/2 + (d*x)/2) - 1i)*(2*a^2 - 9*b^2)*1i)/
(2*d)
```

3.168. $\int \cot^4(c+dx)(a+b\sin(c+dx))^3\,dx$

3.169 $\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx$

3.169.1 Optimal result

Integrand size = 21, antiderivative size = 291

$$\begin{aligned}\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx &= -a^3x+\frac{15}{2}ab^2x-\frac{45a^2b\operatorname{arctanh}(\cos(c+dx))}{8d}\\&+\frac{5b^3\operatorname{arctanh}(\cos(c+dx))}{2d}+\frac{45a^2b\cos(c+dx)}{8d}\\&-\frac{5b^3\cos(c+dx)}{2d}-\frac{5b^3\cos^3(c+dx)}{6d}\\&-\frac{a^3\cot(c+dx)}{d}+\frac{15ab^2\cot(c+dx)}{2d}\\&+\frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d}\\&-\frac{b^3\cos^3(c+dx)\cot^2(c+dx)}{2d}\\&+\frac{a^3\cot^3(c+dx)}{3d}-\frac{5ab^2\cot^3(c+dx)}{2d}\\&+\frac{3ab^2\cos^2(c+dx)\cot^3(c+dx)}{2d}\\&-\frac{3a^2b\cos(c+dx)\cot^4(c+dx)}{4d}-\frac{a^3\cot^5(c+dx)}{5d}\end{aligned}$$

output
```
-a^3*x+15/2*a*b^2*x-45/8*a^2*b*arctanh(cos(d*x+c))/d+5/2*b^3*arctanh(cos(d
*x+c))/d+45/8*a^2*b*cos(d*x+c)/d-5/2*b^3*cos(d*x+c)/d-5/6*b^3*cos(d*x+c)^3
/d-a^3*cot(d*x+c)/d+15/2*a*b^2*cot(d*x+c)/d+15/8*a^2*b*cos(d*x+c)*cot(d*x+
c)^2/d-1/2*b^3*cos(d*x+c)^3*cot(d*x+c)^2/d+1/3*a^3*cot(d*x+c)^3/d-5/2*a*b^
2*cot(d*x+c)^3/d+3/2*a*b^2*cos(d*x+c)^2*cot(d*x+c)^3/d-3/4*a^2*b*cos(d*x+c
)*cot(d*x+c)^4/d-1/5*a^3*cot(d*x+c)^5/d
```

3.169.2 Mathematica [A] (verified)

Time = 2.26 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.19

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$= \frac{-600a(2a^2-15b^2)(c+dx)\csc^4(c+dx)+1200b(-9a^2+4b^2)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

output
```
(-600*a*(2*a^2 - 15*b^2)*(c + d*x)*Csc[c + d*x]^4 + 1200*b*(-9*a^2 + 4*b^2
)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 5*Cot[c + d*x]*Csc[c +
 d*x]^4*(-80*a^3 + 285*a*b^2 + 12*b*(60*a^2 - 29*b^2)*Sin[c + d*x]) + Csc[
c + d*x]^5*(5*(40*a^3 - 489*a*b^2)*Cos[3*(c + d*x)] + (-184*a^3 + 1065*a*b
^2)*Cos[5*(c + d*x)] + 5*(-9*a*b^2*Cos[7*(c + d*x)] + 60*a*(2*a^2 - 15*b^2
)*(c + d*x)*Sin[3*(c + d*x)] - 306*a^2*b*Sin[4*(c + d*x)] + 122*b^3*Sin[4*
(c + d*x)] - 24*a^3*c*Sin[5*(c + d*x)] + 180*a*b^2*c*Sin[5*(c + d*x)] - 24
*a^3*d*x*Sin[5*(c + d*x)] + 180*a*b^2*d*x*Sin[5*(c + d*x)] + 36*a^2*b*Sin[
6*(c + d*x)] - 22*b^3*Sin[6*(c + d*x)] - b^3*Sin[8*(c + d*x)])))/(1920*d)
```

3.169.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(a+b\sin(c+dx))^3}{\tan(c+dx)^6}dx$$
$$\Big\downarrow \text{3201}$$
$$\int \left(a^3\cot^6(c+dx)+3a^2b\cos(c+dx)\cot^5(c+dx)+3ab^2\cos^2(c+dx)\cot^4(c+dx)+b^3\cos^3(c+dx)\cot^3(c+dx\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\cot^5(c+dx)}{5d}+\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-a^3x-\frac{45a^2b\text{arctanh}(\cos(c+dx))}{8d}+\frac{45a^2b\cos(c+dx)}{8d}-\frac{3a^2b\cos(c+dx)\cot^4(c+dx)}{4d}+\frac{15a^2b\cos(c+dx)\cot^2(c+dx)}{8d}-\frac{5ab^2\cot^3(c+dx)}{2d}+\frac{15ab^2\cot(c+dx)}{2d}+\frac{3ab^2\cos^2(c+dx)\cot^3(c+dx)}{2d}+\frac{15}{2}ab^2x+\frac{5b^3\text{arctanh}(\cos(c+dx))}{2d}-\frac{5b^3\cos^3(c+dx)}{6d}-\frac{5b^3\cos(c+dx)}{2d}-\frac{b^3\cos^3(c+dx)\cot^2(c+dx)}{2d}$$

input
```
Int[Cot[c + d*x]^6*(a + b*Sin[c + d*x])^3,x]
```

output
```
-(a^3*x) + (15*a*b^2*x)/2 - (45*a^2*b*ArcTanh[Cos[c + d*x]])/(8*d) + (5*b^
3*ArcTanh[Cos[c + d*x]])/(2*d) + (45*a^2*b*Cos[c + d*x])/(8*d) - (5*b^3*Co
s[c + d*x])/(2*d) - (5*b^3*Cos[c + d*x]^3)/(6*d) - (a^3*Cot[c + d*x])/d +
(15*a*b^2*Cot[c + d*x])/(2*d) + (15*a^2*b*Cos[c + d*x]*Cot[c + d*x]^2)/(8*
d) - (b^3*Cos[c + d*x]^3*Cot[c + d*x]^2)/(2*d) + (a^3*Cot[c + d*x]^3)/(3*d
) - (5*a*b^2*Cot[c + d*x]^3)/(2*d) + (3*a*b^2*Cos[c + d*x]^2*Cot[c + d*x]^
3)/(2*d) - (3*a^2*b*Cos[c + d*x]*Cot[c + d*x]^4)/(4*d) - (a^3*Cot[c + d*x]
^5)/(5*d)
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.169.4 Maple [A] (verified)

Time = 6.13 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.99

method	result
derivativedivides	$a^3\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+3a^2b\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3\right.}{8}\right.$
default	$a^3\left(-\frac{\left(\cot^5(dx+c)\right)}{5}+\frac{\left(\cot^3(dx+c)\right)}{3}-\cot(dx+c)-dx-c\right)+3a^2b\left(-\frac{\cos^7(dx+c)}{4\sin(dx+c)^4}+\frac{3\left(\cos^7(dx+c)\right)}{8\sin(dx+c)^2}+\frac{3\left(\cos^5(dx+c)\right)}{8}+\frac{5\left(\cos^3\right.}{8}\right.$
risch	$-a^3x+\frac{15a\,b^2x}{2}-\frac{b^3\mathrm{e}^{3i(dx+c)}}{24d}-\frac{3ia\,b^2\mathrm{e}^{2i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{i(dx+c)}a^2}{2d}-\frac{9b^3\mathrm{e}^{i(dx+c)}}{8d}+\frac{3b\,\mathrm{e}^{-i(dx+c)}a^2}{2d}-\frac{9b^3\mathrm{e}^{-}}{8}$

input
```
int(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/5*cot(d*x+c)^5+1/3*cot(d*x+c)^3-cot(d*x+c)-d*x-c)+3*a^2*b*(-1
/4/sin(d*x+c)^4*cos(d*x+c)^7+3/8/sin(d*x+c)^2*cos(d*x+c)^7+3/8*cos(d*x+c)^
5+5/8*cos(d*x+c)^3+15/8*cos(d*x+c)+15/8*ln(csc(d*x+c)-cot(d*x+c)))+3*a*b^2
*(-1/3/sin(d*x+c)^3*cos(d*x+c)^7+4/3/sin(d*x+c)*cos(d*x+c)^7+4/3*(cos(d*x+
c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/2*d*x+5/2*c)+b^3*(-1/2
/sin(d*x+c)^2*cos(d*x+c)^7-1/2*cos(d*x+c)^5-5/6*cos(d*x+c)^3-5/2*cos(d*x+c
)-5/2*ln(csc(d*x+c)-cot(d*x+c))))
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.42

$$\int\cot^6(c+dx)(a+b\sin(c+dx))^3\,dx =$$
$$-\frac{360\,ab^2\cos\left(dx+c\right)^7+184\left(2\,a^3-15\,ab^2\right)\cos\left(dx+c\right)^5-280\left(2\,a^3-15\,ab^2\right)\cos\left(dx+c\right)^3+75\left(\left(9\,a^2b\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/240*(360*a*b^2*cos(d*x + c)^7 + 184*(2*a^3 - 15*a*b^2)*cos(d*x + c)^5 -
 280*(2*a^3 - 15*a*b^2)*cos(d*x + c)^3 + 75*((9*a^2*b - 4*b^3)*cos(d*x + c
)^4 + 9*a^2*b - 4*b^3 - 2*(9*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*
x + c) + 1/2)*sin(d*x + c) - 75*((9*a^2*b - 4*b^3)*cos(d*x + c)^4 + 9*a^2*
b - 4*b^3 - 2*(9*a^2*b - 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/
2)*sin(d*x + c) + 120*(2*a^3 - 15*a*b^2)*cos(d*x + c) + 10*(8*b^3*cos(d*x
+ c)^7 + 12*(2*a^3 - 15*a*b^2)*d*x*cos(d*x + c)^4 - 8*(9*a^2*b - 4*b^3)*co
s(d*x + c)^5 - 24*(2*a^3 - 15*a*b^2)*d*x*cos(d*x + c)^2 + 25*(9*a^2*b - 4*
b^3)*cos(d*x + c)^3 + 12*(2*a^3 - 15*a*b^2)*d*x - 15*(9*a^2*b - 4*b^3)*cos
(d*x + c))*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(
d*x + c))
```

3.169.6 Sympy [F]

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx = \int (a+b\sin(c+dx))^3\cot^6(c+dx)\,dx$$

input
```
integrate(cot(d*x+c)**6*(a+b*sin(d*x+c))**3,x)
```

output
```
Integral((a + b*sin(c + d*x))**3*cot(c + d*x)**6, x)
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.87

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx =$$

$$-\frac{16\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4-5\,\tan(dx+c)^2+3}{\tan(dx+c)^5}\right)a^3-120\left(15\,dx+15\,c+\frac{15\,\tan(dx+c)^4+10\,\tan(dx+c)^2-2}{\tan(dx+c)^5+\tan(dx+c)^3}\right)ab^2+\cdots}{\cdots}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/240*(16*(15*d*x + 15*c + (15*tan(d*x + c)^4 - 5*tan(d*x + c)^2 + 3)/tan
(d*x + c)^5)*a^3 - 120*(15*d*x + 15*c + (15*tan(d*x + c)^4 + 10*tan(d*x +
c)^2 - 2)/(tan(d*x + c)^5 + tan(d*x + c)^3))*a*b^2 + 20*(4*cos(d*x + c)^3
- 6*cos(d*x + c)/(cos(d*x + c)^2 - 1) + 24*cos(d*x + c) - 15*log(cos(d*x +
 c) + 1) + 15*log(cos(d*x + c) - 1))*b^3 + 45*a^2*b*(2*(9*cos(d*x + c)^3 -
 7*cos(d*x + c))/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1) - 16*cos(d*x + c)
 + 15*log(cos(d*x + c) + 1) - 15*log(cos(d*x + c) - 1)))/d
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.62

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx$$

$$= \frac{6\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+45\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-70\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72}{}$$

input
```
integrate(cot(d*x+c)^6*(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/960*(6*a^3*tan(1/2*d*x + 1/2*c)^5 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 70
*a^3*tan(1/2*d*x + 1/2*c)^3 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 720*a^2*b
*tan(1/2*d*x + 1/2*c)^2 + 120*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^3*tan(1/2
*d*x + 1/2*c) - 3240*a*b^2*tan(1/2*d*x + 1/2*c) - 480*(2*a^3 - 15*a*b^2)*(
d*x + c) + 600*(9*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c))) - 320*(9*a
*b^2*tan(1/2*d*x + 1/2*c)^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 18*b^3*tan
(1/2*d*x + 1/2*c)^4 - 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 24*b^3*tan(1/2*d*x
 + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 14*b^3)/(tan(1/2*d
*x + 1/2*c)^2 + 1)^3 - (12330*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 5480*b^3*tan(
1/2*d*x + 1/2*c)^5 + 660*a^3*tan(1/2*d*x + 1/2*c)^4 - 3240*a*b^2*tan(1/2*d
*x + 1/2*c)^4 - 720*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 120*b^3*tan(1/2*d*x + 1
/2*c)^3 - 70*a^3*tan(1/2*d*x + 1/2*c)^2 + 120*a*b^2*tan(1/2*d*x + 1/2*c)^2
 + 45*a^2*b*tan(1/2*d*x + 1/2*c) + 6*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.169.9 Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.74

$$\int \cot^6(c+dx)(a+b\sin(c+dx))^3\,dx = \frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{160\,d} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}(12\,a\,b^2-22\,a^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(4\,a\,b^2-\frac{26\,a^3}{15}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(96\,a\,b^2-\frac{78\,a^3}{5}\right)+\tan\left(\right.}{}$$

$$+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a\,b^2}{8}-\frac{7\,a^3}{96}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3\,a^2\,b}{4}-\frac{b^3}{8}\right)}{d}$$

$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{45\,a^2\,b}{8}-\frac{5\,b^3}{2}\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)\left(\frac{a\,b^2\,15\mathrm{i}}{2}-a^3\,1\mathrm{i}\right)}{d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{27\,a\,b^2}{8}-\frac{11\,a^3}{16}\right)}{d} + \frac{3\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,d}$$

$$-\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)\left(2\,a^2-15\,b^2\right)\,1\mathrm{i}}{2\,d}$$

input
```
int(cot(c + d*x)^6*(a + b*sin(c + d*x))^3,x)
```

output
```
(a^3*tan(c/2 + (d*x)/2)^5)/(160*d) + (tan(c/2 + (d*x)/2)^10*(12*a*b^2 - 22
*a^3) - tan(c/2 + (d*x)/2)^2*(4*a*b^2 - (26*a^3)/15) + tan(c/2 + (d*x)/2)^
4*(96*a*b^2 - (78*a^3)/5) + tan(c/2 + (d*x)/2)^8*(320*a*b^2 - (191*a^3)/3)
 + tan(c/2 + (d*x)/2)^6*(408*a*b^2 - (296*a^3)/5) + tan(c/2 + (d*x)/2)^3*(
(39*a^2*b)/2 - 4*b^3) + tan(c/2 + (d*x)/2)^9*(216*a^2*b - 196*b^3) + tan(c
/2 + (d*x)/2)^5*((519*a^2*b)/2 - (484*b^3)/3) + tan(c/2 + (d*x)/2)^7*((909
*a^2*b)/2 - 268*b^3) - a^3/5 - (3*a^2*b*tan(c/2 + (d*x)/2))/2)/(d*(32*tan(
c/2 + (d*x)/2)^5 + 96*tan(c/2 + (d*x)/2)^7 + 96*tan(c/2 + (d*x)/2)^9 + 32*
tan(c/2 + (d*x)/2)^11)) + (tan(c/2 + (d*x)/2)^3*((a*b^2)/8 - (7*a^3)/96))/
d - (tan(c/2 + (d*x)/2)^2*((3*a^2*b)/4 - b^3/8))/d + (log(tan(c/2 + (d*x)/
2))*((45*a^2*b)/8 - (5*b^3)/2))/d - (log(tan(c/2 + (d*x)/2) - 1i)*((a*b^2*
15i)/2 - a^3*1i))/d - (tan(c/2 + (d*x)/2)*((27*a*b^2)/8 - (11*a^3)/16))/d
+ (3*a^2*b*tan(c/2 + (d*x)/2)^4)/(64*d) - (a*log(tan(c/2 + (d*x)/2) + 1i)*
(2*a^2 - 15*b^2)*1i)/(2*d)
```

3.170 $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

3.170.1 Optimal result

Integrand size = 21, antiderivative size = 204

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = &-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{16(a+b)^3d} \\ &-\frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx))}{16(a-b)^3d} \\ &+\frac{a^5\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}+\frac{\sec^4(c+dx)(a-b\sin(c+dx))}{4\,(a^2-b^2)\,d} \\ &-\frac{\sec^2(c+dx)\,(4a(2a^2-b^2)-b(9a^2-5b^2)\sin(c+dx))}{8\,(a^2-b^2)^2\,d}\end{aligned}$$

output
```
-1/16*(8*a^2+9*a*b+3*b^2)*ln(1-sin(d*x+c))/(a+b)^3/d-1/16*(8*a^2-9*a*b+3*b
^2)*ln(1+sin(d*x+c))/(a-b)^3/d+a^5*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+1/4*se
c(d*x+c)^4*(a-b*sin(d*x+c))/(a^2-b^2)/d-1/8*sec(d*x+c)^2*(4*a*(2*a^2-b^2)-
b*(9*a^2-5*b^2)*sin(d*x+c))/(a^2-b^2)^2/d
```

3.170.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-\frac{(8a^2+9ab+3b^2)\log(1-\sin(c+dx))}{(a+b)^3} - \frac{(8a^2-9ab+3b^2)\log(1+\sin(c+dx))}{(a-b)^3} + \frac{16a^5\log(a+b\sin(c+dx))}{(a-b)^3(a+b)^3} + \frac{1}{(a+b)(-1+\sin(c+dx))^2} + \frac{\ }{(a+b)}}{16d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

output
```
(-(((8*a^2 + 9*a*b + 3*b^2)*Log[1 - Sin[c + d*x]])/(a + b)^3) - ((8*a^2 -
9*a*b + 3*b^2)*Log[1 + Sin[c + d*x]])/(a - b)^3 + (16*a^5*Log[a + b*Sin[c
+ d*x]])/((a - b)^3*(a + b)^3) + 1/((a + b)*(-1 + Sin[c + d*x])^2) + (7*a
+ 5*b)/((a + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])^2)
+ (-7*a + 5*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

3.170.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.29, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 3200, 601, 25, 2178, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\tan(c+dx)^5}{a+b\sin(c+dx)}dx$$
$$\Big\downarrow \text{3200}$$
$$\frac{\int \frac{b^5\sin^5(c+dx)}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)^3}d(b\sin(c+dx))}{d}$$
$$\Big\downarrow \text{601}$$

$$\frac{\frac{b^4(a-b\sin(c+dx))}{4(a^2-b^2)\left(b^2-b^2\sin^2(c+dx)\right)^2}-\frac{\int -\frac{\frac{ab^6}{a^2-b^2}-4\sin^3(c+dx)b^5-\frac{\left(4a^2-b^2\right)\sin(c+dx)b^5}{a^2-b^2}}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{4b^2}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\frac{ab^6}{a^2-b^2}-4\sin^3(c+dx)b^5-\frac{\left(4a^2-b^2\right)\sin(c+dx)b^5}{a^2-b^2}}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{4b^2}+\frac{b^4(a-b\sin(c+dx))}{4(a^2-b^2)\left(b^2-b^2\sin^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 2178

$$\frac{\frac{\frac{\int -\frac{b^4\left(ab^2\left(7a^2-3b^2\right)-b\left(8a^4-7b^2a^2+3b^4\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2b^2}-\frac{b^4\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\sin^2(c+dx)\right)}}{4b^2}+\frac{b^4(a-b\sin(c+dx))}{4(a^2-b^2)\left(b^2-b^2\sin^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 25

$$\frac{\frac{-\frac{\int \frac{b^4\left(ab^2\left(7a^2-3b^2\right)-b\left(8a^4-7b^2a^2+3b^4\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2b^2}-\frac{b^4\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\sin^2(c+dx)\right)}}{4b^2}+\frac{b^4(a-b\sin(c+dx))}{4(a^2-b^2)\left(b^2-b^2\sin^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 27

$$\frac{\frac{-\frac{b^2\int \frac{ab^2\left(7a^2-3b^2\right)-b\left(8a^4-7b^2a^2+3b^4\right)\sin(c+dx)}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2\left(a^2-b^2\right)^2}-\frac{b^4\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\sin^2(c+dx)\right)}}{4b^2}+\frac{b^4(a-b\sin(c+dx))}{4(a^2-b^2)\left(b^2-b^2\sin^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 657

$$\frac{\frac{-\frac{b^2\int \left(-\frac{8a^5}{(a-b)(a+b)(a+b\sin(c+dx))}-\frac{(a-b)^2\left(8a^2+9ba+3b^2\right)}{2(a+b)(b-b\sin(c+dx))}+\frac{(a+b)^2\left(8a^2-9ba+3b^2\right)}{2(a-b)(\sin(c+dx)b+b)}\right)d(b\sin(c+dx))}{2\left(a^2-b^2\right)^2}-\frac{b^4\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\sin^2(c+dx)\right)}}{4b^2}+ 4(a^2 \cdots}{d}$$

$\downarrow$ 2009

3.170. $\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$

$$\frac{\frac{b^4(a-b\sin(c+dx))}{4(a^2-b^2)(b^2-b^2\sin^2(c+dx))^2}+\frac{-\frac{b^4\left(4a\left(2a^2-b^2\right)-b\left(9a^2-5b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^2\left(b^2-b^2\sin^2(c+dx)\right)}-\frac{b^2\left(\frac{(a-b)^2\left(8a^2+9ab+3b^2\right)\log(b-b\sin(c+dx))}{2(a+b)}+\frac{(a+b)^2\left(8a^2-9ab+3b^2\right)\log}{2(a-b)}\right.}{2\left(a^2-b^2\right)^2}}{4b^2}}{d}$$

input
```
Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

output
```
((b^4*(a - b*Sin[c + d*x]))/(4*(a^2 - b^2)*(b^2 - b^2*Sin[c + d*x]^2)^2) +
 (-1/2*(b^2*(((a - b)^2*(8*a^2 + 9*a*b + 3*b^2)*Log[b - b*Sin[c + d*x]])/(
2*(a + b)) - (8*a^5*Log[a + b*Sin[c + d*x]])/(a^2 - b^2) + ((a + b)^2*(8*a
^2 - 9*a*b + 3*b^2)*Log[b + b*Sin[c + d*x]])/(2*(a - b))))/(a^2 - b^2)^2 -
 (b^4*(4*a*(2*a^2 - b^2) - b*(9*a^2 - 5*b^2)*Sin[c + d*x]))/(2*(a^2 - b^2)
^2*(b^2 - b^2*Sin[c + d*x]^2)))/(4*b^2))/d
```

3.170.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.170.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{\frac{a^5\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a+8b)(\sin(dx+c)-1)^2}-\frac{-7a-5b}{16(a+b)^2(\sin(dx+c)-1)}+\frac{\left(-8a^2-9ab-3b^2\right)\ln(\sin(dx+c)-1)}{16(a+b)^3}+\frac{1}{2(8a-8b)(1+\sin}}{d}$
default	$\frac{\frac{a^5\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}+\frac{1}{2(8a+8b)(\sin(dx+c)-1)^2}-\frac{-7a-5b}{16(a+b)^2(\sin(dx+c)-1)}+\frac{\left(-8a^2-9ab-3b^2\right)\ln(\sin(dx+c)-1)}{16(a+b)^3}+\frac{1}{2(8a-8b)(1+\sin}}{d}$
risch	$-\frac{2ia^5x}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{ia^2c}{d(a^3-3a^2b+3a\,b^2-b^3)}+\frac{3ib^2x}{8(a^3+3a^2b+3a\,b^2+b^3)}+\frac{3ib^2c}{8d(a^3-3a^2b+3a\,b^2-b^3)}-\frac{}{d(a^6-3}$

input
```
int(tan(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^5/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))+1/2/(8*a+8*b)/(sin(d*x+c)-1)^2
-1/16*(-7*a-5*b)/(a+b)^2/(sin(d*x+c)-1)+1/16/(a+b)^3*(-8*a^2-9*a*b-3*b^2)*
ln(sin(d*x+c)-1)+1/2/(8*a-8*b)/(1+sin(d*x+c))^2-1/16*(7*a-5*b)/(a-b)^2/(1+
sin(d*x+c))+1/16/(a-b)^3*(-8*a^2+9*a*b-3*b^2)*ln(1+sin(d*x+c)))
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.28

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{16\,a^5\cos\left(dx+c\right)^4\log\left(b\sin\left(dx+c\right)+a\right)-\left(8\,a^5+15\,a^4b-10\,a^2b^3+3\,b^5\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)\right.}{}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*(16*a^5*cos(d*x + c)^4*log(b*sin(d*x + c) + a) - (8*a^5 + 15*a^4*b -
10*a^2*b^3 + 3*b^5)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (8*a^5 - 15*a^4
*b + 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 4*a^5 - 8
*a^3*b^2 + 4*a*b^4 - 8*(2*a^5 - 3*a^3*b^2 + a*b^4)*cos(d*x + c)^2 - 2*(2*a
^4*b - 4*a^2*b^3 + 2*b^5 - (9*a^4*b - 14*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)*
sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4)
```

3.170.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\tan^5\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.41

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\frac{16\,a^5\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(8\,a^2-9\,ab+3\,b^2\right)\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}-\frac{\left(8\,a^2+9\,ab+3\,b^2\right)\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(\left(9\,a^2b-5\,b^3\right)\sin(dx+c)^3+6\,a^3\right.}{\left(a^4-2\,a^2b^2+b^4\right)\sin(dx+}}{16\,d}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/16*(16*a^5*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
 (8*a^2 - 9*a*b + 3*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 -
b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*
b^2 + b^3) - 2*((9*a^2*b - 5*b^3)*sin(d*x + c)^3 + 6*a^3 - 2*a*b^2 - 4*(2*
a^3 - a*b^2)*sin(d*x + c)^2 - (7*a^2*b - 3*b^3)*sin(d*x + c))/((a^4 - 2*a^
2*b^2 + b^4)*sin(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 +
 b^4)*sin(d*x + c)^2))/d
```

3.170.8 Giac [A] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.68

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\frac{16\,a^5b\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{(8\,a^2-9\,ab+3\,b^2)\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{(8\,a^2+9\,ab+3\,b^2)\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{2\left(6\,a^5\sin(dx+c)^4-9\,a^4b\sin\right.}{}}{}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/16*(16*a^5*b*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5
 - b^7) - (8*a^2 - 9*a*b + 3*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*
b + 3*a*b^2 - b^3) - (8*a^2 + 9*a*b + 3*b^2)*log(abs(sin(d*x + c) - 1))/(a
^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(6*a^5*sin(d*x + c)^4 - 9*a^4*b*sin(d*x
+ c)^3 + 14*a^2*b^3*sin(d*x + c)^3 - 5*b^5*sin(d*x + c)^3 - 4*a^5*sin(d*x
+ c)^2 - 12*a^3*b^2*sin(d*x + c)^2 + 4*a*b^4*sin(d*x + c)^2 + 7*a^4*b*sin(
d*x + c) - 10*a^2*b^3*sin(d*x + c) + 3*b^5*sin(d*x + c) + 8*a^3*b^2 - 2*a*
b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(sin(d*x + c)^2 - 1)^2))/d
```

3.170.9 Mupad [B] (verification not implemented)

Time = 6.80 (sec) , antiderivative size = 498, normalized size of antiderivative = 2.44

$$\int \frac{\tan^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a^5\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{d\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(\frac{1}{a+b}-\frac{7\,b}{8\,(a+b)^2}+\frac{b^2}{4\,(a+b)^3}\right)}{d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(\frac{b^2}{4\,(a-b)^3}+\frac{7\,b}{8\,(a-b)^2}+\frac{1}{a-b}\right)}{d} - \frac{\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{a^4-2\,a^2\,b^2+b^4}+\frac{2\,a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{a^4-2\,a^2\,b^2+b^4}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(a\,b^2-2\,a^3)}{a^4-2\,a^2\,b^2+b^4}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\,(7\,a^2\,b-3\,b^3)}{4\,(a^4-2\,a^2\,b^2+b^4)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,(15\,a^2\,b-11\,b^3)}{4\,(a^4-2\,a^2\,b^2+b^4)}}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(tan(c + d*x)^5/(a + b*sin(c + d*x)),x)
```

output
```
(a^5*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 - b
^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(tan(c/2 + (d*x)/2) - 1)*(1/(a + b) - (
7*b)/(8*(a + b)^2) + b^2/(4*(a + b)^3)))/d - (log(tan(c/2 + (d*x)/2) + 1)*
(b^2/(4*(a - b)^3) + (7*b)/(8*(a - b)^2) + 1/(a - b)))/d - ((2*a^3*tan(c/2
 + (d*x)/2)^2)/(a^4 + b^4 - 2*a^2*b^2) + (2*a^3*tan(c/2 + (d*x)/2)^6)/(a^4
 + b^4 - 2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^4*(a*b^2 - 2*a^3))/(a^4 + b^4
- 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^7*(7*a^2*b - 3*b^3))/(4*(a^4 + b^4 - 2*
a^2*b^2)) + (tan(c/2 + (d*x)/2)^3*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a
^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(15*a^2*b - 11*b^3))/(4*(a^4 + b^4 - 2*a^
2*b^2)) - (b*tan(c/2 + (d*x)/2)*(7*a^2 - 3*b^2))/(4*(a^4 + b^4 - 2*a^2*b^2
)))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x
)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.171 $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

3.171.1 Optimal result

Integrand size = 21, antiderivative size = 126

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{(2a+b)\log(1-\sin(c+dx))}{4(a+b)^2d}+\frac{(2a-b)\log(1+\sin(c+dx))}{4(a-b)^2d} - \frac{a^3\log(a+b\sin(c+dx))}{(a^2-b^2)^2d}+\frac{\sec^2(c+dx)(a-b\sin(c+dx))}{2(a^2-b^2)d}$$

output
```
1/4*(2*a+b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(2*a-b)*ln(1+sin(d*x+c))/(a-b)^
2/d-a^3*ln(a+b*sin(d*x+c))/(a^2-b^2)^2/d+1/2*sec(d*x+c)^2*(a-b*sin(d*x+c))
/(a^2-b^2)/d
```

3.171.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.93

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{(2a+b)\log(1-\sin(c+dx))}{(a+b)^2}+\frac{(2a-b)\log(1+\sin(c+dx))}{(a-b)^2}-\frac{4a^3\log(a+b\sin(c+dx))}{(a-b)^2(a+b)^2}-\frac{1}{(a+b)(-1+\sin(c+dx))}+\frac{1}{(a-b)(1+\sin(c+dx))}}{4d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

output
```
(((2*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^2 + ((2*a - b)*Log[1 + Sin[c +
d*x]])/(a - b)^2 - (4*a^3*Log[a + b*Sin[c + d*x]])/((a - b)^2*(a + b)^2) -
 1/((a + b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/(4*d)
```

3.171.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.26, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3200, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^3}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 3200$$

$$\frac{\int \frac{b^3\sin^3(c+dx)}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \; 601$$

$$\frac{\frac{b^2(a-b\sin(c+dx))}{2(a^2-b^2)(b^2-b^2\sin^2(c+dx))}-\frac{\int -\frac{b^2\left(\frac{ab^2}{a^2-b^2}-\frac{b\left(2a^2-b^2\right)\sin(c+dx)}{a^2-b^2}\right)}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2b^2}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{b^2\left(ab^2-b\left(2a^2-b^2\right)\sin(c+dx)\right)}{\left(a^2-b^2\right)(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2b^2}+\frac{b^2(a-b\sin(c+dx))}{2(a^2-b^2)(b^2-b^2\sin^2(c+dx))}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{ab^2-b\left(2a^2-b^2\right)\sin(c+dx)}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2(a^2-b^2)}+\frac{b^2(a-b\sin(c+dx))}{2(a^2-b^2)(b^2-b^2\sin^2(c+dx))}}{d}$$

$$\Big\downarrow \; 657$$

$$\frac{\frac{\int \left(-\frac{2a^3}{(a-b)(a+b)(a+b\sin(c+dx))}+\frac{-2a^2+ba+b^2}{2(a+b)(b-b\sin(c+dx))}+\frac{(2a-b)(a+b)}{2(a-b)(\sin(c+dx)b+b)}\right)d(b\sin(c+dx))}{2(a^2-b^2)}+\frac{b^2(a-b\sin(c+dx))}{2(a^2-b^2)(b^2-b^2\sin^2(c+dx))}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(a-b\sin(c+dx))}{2(a^2-b^2)(b^2-b^2\sin^2(c+dx))}+\frac{-\frac{2a^3\log(a+b\sin(c+dx))}{a^2-b^2}+\frac{(a-b)(2a+b)\log(b-b\sin(c+dx))}{2(a+b)}+\frac{(2a-b)(a+b)\log(b\sin(c+dx)+b)}{2(a-b)}}{2(a^2-b^2)}}{d}$$

input
```
Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

output
```
((((a - b)*(2*a + b)*Log[b - b*Sin[c + d*x]])/(2*(a + b)) - (2*a^3*Log[a +
 b*Sin[c + d*x]])/(a^2 - b^2) + ((2*a - b)*(a + b)*Log[b + b*Sin[c + d*x]]
)/(2*(a - b)))/(2*(a^2 - b^2)) + (b^2*(a - b*Sin[c + d*x]))/(2*(a^2 - b^2)
*(b^2 - b^2*Sin[c + d*x]^2)))/d
```

3.171.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.171.4 Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{-\frac{a^3\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(2a+b)\ln(\sin(dx+c)-1)}{4(a+b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(2a-b)\ln(1+\sin(dx+c))}{4(a-b)^2}}{d}$
default	$\frac{-\frac{a^3\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{1}{(4a+4b)(\sin(dx+c)-1)}+\frac{(2a+b)\ln(\sin(dx+c)-1)}{4(a+b)^2}+\frac{1}{(4a-4b)(1+\sin(dx+c))}+\frac{(2a-b)\ln(1+\sin(dx+c))}{4(a-b)^2}}{d}$
risch	$-\frac{iax}{a^2+2ab+b^2}+\frac{2ia^3c}{d(a^4-2a^2b^2+b^4)}-\frac{ibc}{2d(a^2+2ab+b^2)}-\frac{ibx}{2(a^2+2ab+b^2)}+\frac{ibx}{2a^2-4ab+2b^2}-\frac{iax}{a^2-2ab+b^2}+$

input
```
int(tan(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a^3/(a+b)^2/(a-b)^2*ln(a+b*sin(d*x+c))-1/(4*a+4*b)/(sin(d*x+c)-1)+1/
4*(2*a+b)/(a+b)^2*ln(sin(d*x+c)-1)+1/(4*a-4*b)/(1+sin(d*x+c))+1/4*(2*a-b)/
(a-b)^2*ln(1+sin(d*x+c)))
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.25

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{4\,a^3\cos\left(dx+c\right)^2\log\left(b\sin\left(dx+c\right)+a\right)-\left(2\,a^3+3\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,a}{4\,(a^4-2\,a^2b^2+b^4)d\,c}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(4*a^3*cos(d*x + c)^2*log(b*sin(d*x + c) + a) - (2*a^3 + 3*a^2*b - b^
3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^3 - 3*a^2*b + b^3)*cos(d*x
+ c)^2*log(-sin(d*x + c) + 1) - 2*a^3 + 2*a*b^2 + 2*(a^2*b - b^3)*sin(d*x
+ c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2)
```

3.171.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx=\int \frac{\tan^3{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.13

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{4\,a^3\log(b\sin(dx+c)+a)}{a^4-2\,a^2b^2+b^4}-\frac{(2\,a-b)\log(\sin(dx+c)+1)}{a^2-2\,ab+b^2}-\frac{(2\,a+b)\log(\sin(dx+c)-1)}{a^2+2\,ab+b^2}-\frac{2\,(b\sin(dx+c)-a)}{(a^2-b^2)\sin(dx+c)^2-a^2+b^2}}{4\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*(4*a^3*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - (2*a - b)*lo
g(sin(d*x + c) + 1)/(a^2 - 2*a*b + b^2) - (2*a + b)*log(sin(d*x + c) - 1)/
(a^2 + 2*a*b + b^2) - 2*(b*sin(d*x + c) - a)/((a^2 - b^2)*sin(d*x + c)^2 -
 a^2 + b^2))/d
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.40

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx =$$

$$-\frac{\frac{4\,a^3b\log(|b\sin(dx+c)+a|)}{a^4b-2\,a^2b^3+b^5} - \frac{(2\,a-b)\log(|\sin(dx+c)+1|)}{a^2-2\,ab+b^2} - \frac{(2\,a+b)\log(|\sin(dx+c)-1|)}{a^2+2\,ab+b^2} + \frac{2\left(a^3\sin(dx+c)^2-a^2b\sin(dx+c)+b^3\sin(dx+c\right.}{(a^4-2\,a^2b^2+b^4)\left(\sin(dx+c)^2-1\right)}}{4\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/4*(4*a^3*b*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5) - (2*
a - b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - (2*a + b)*log(abs(
sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 2*(a^3*sin(d*x + c)^2 - a^2*b*sin
(d*x + c) + b^3*sin(d*x + c) - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x +
c)^2 - 1)))/d
```

3.171.9 Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.72

$$\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)\,(2\,a-b)}{2\,d\,(a-b)^2} - \frac{\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2-b^2} - \frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2-b^2} + \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^2-b^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)} - \frac{a^3\,\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{d\,\left(a^4-2\,a^2\,b^2+b^4\right)} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)\,(2\,a+b)}{2\,d\,(a+b)^2}$$

input
```
int(tan(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

output
```
(log(tan(c/2 + (d*x)/2) + 1)*(2*a - b))/(2*d*(a - b)^2) - ((b*tan(c/2 + (d
*x)/2))/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (b*tan(c/2
+ (d*x)/2)^3)/(a^2 - b^2))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)
^2 + 1)) - (a^3*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/
(d*(a^4 + b^4 - 2*a^2*b^2)) + (log(tan(c/2 + (d*x)/2) - 1)*(2*a + b))/(2*d
*(a + b)^2)
```

3.171. $\int \frac{\tan^3(c+dx)}{a+b\sin(c+dx)}\,dx$

3.172 $\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$

3.172.1 Optimal result

Integrand size = 19, antiderivative size = 74

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\log(1-\sin(c+dx))}{2(a+b)d} - \frac{\log(1+\sin(c+dx))}{2(a-b)d} + \frac{a\log(a+b\sin(c+dx))}{(a^2-b^2)\,d}$$

output
```
-1/2*ln(1-sin(d*x+c))/(a+b)/d-1/2*ln(1+sin(d*x+c))/(a-b)/d+a*ln(a+b*sin(d*
x+c))/(a^2-b^2)/d
```

3.172.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{(-a+b)\log(1-\sin(c+dx)) - (a+b)\log(1+\sin(c+dx)) + 2a\log(a+b\sin(c+dx))}{2(a-b)(a+b)d}$$

input
```
Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
((-a + b)*Log[1 - Sin[c + d*x]] - (a + b)*Log[1 + Sin[c + d*x]] + 2*a*Log[
a + b*Sin[c + d*x]])/(2*(a - b)*(a + b)*d)
```

3.172.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3042, 3200, 587, 16, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b\sin(c+dx)}{(a+b\sin(c+dx))(b^2-b^2\sin^2(c+dx))}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{587}$$

$$\frac{\frac{a\int \frac{1}{a+b\sin(c+dx)}d(b\sin(c+dx))}{a^2-b^2}-\frac{\int \frac{b^2-ab\sin(c+dx)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{a^2-b^2}}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{a\log(a+b\sin(c+dx))}{a^2-b^2}-\frac{\int \frac{b^2-ab\sin(c+dx)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{a^2-b^2}}{d}$$

$$\Big\downarrow \text{452}$$

$$\frac{\frac{a\log(a+b\sin(c+dx))}{a^2-b^2}-\frac{b^2\int \frac{1}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))-a\int \frac{b\sin(c+dx)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{a^2-b^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\log(a+b\sin(c+dx))}{a^2-b^2}-\frac{b\text{arctanh}(\sin(c+dx))-a\int \frac{b\sin(c+dx)}{b^2-b^2\sin^2(c+dx)}d(b\sin(c+dx))}{a^2-b^2}}{d}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{a\log(a+b\sin(c+dx))}{a^2-b^2}-\frac{\frac{1}{2}a\log(b^2-b^2\sin^2(c+dx))+b\text{arctanh}(\sin(c+dx))}{a^2-b^2}}{d}$$

input
```
Int[Tan[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
((a*Log[a + b*Sin[c + d*x]])/(a^2 - b^2) - (b*ArcTanh[Sin[c + d*x]] + (a*L
og[b^2 - b^2*Sin[c + d*x]^2])/2)/(a^2 - b^2))/d
```

3.172.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.172.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{\frac{a\ln(a+b\sin(dx+c))}{(a+b)(a-b)}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}-\frac{\ln(1+\sin(dx+c))}{2a-2b}}{d}$
default	$\frac{\frac{a\ln(a+b\sin(dx+c))}{(a+b)(a-b)}-\frac{\ln(\sin(dx+c)-1)}{2a+2b}-\frac{\ln(1+\sin(dx+c))}{2a-2b}}{d}$
risch	$\frac{ix}{a-b}+\frac{ic}{d(a-b)}+\frac{ix}{a+b}+\frac{ic}{d(a+b)}-\frac{2iax}{a^2-b^2}-\frac{2iac}{d(a^2-b^2)}-\frac{\ln(\mathrm{e}^{i(dx+c)}+i)}{d(a-b)}-\frac{\ln(-i+\mathrm{e}^{i(dx+c)})}{d(a+b)}+\frac{a\ln\left(\mathrm{e}^{2i(dx\right.}}{}$

input
```
int(tan(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a/(a+b)/(a-b)*ln(a+b*sin(d*x+c))-1/(2*a+2*b)*ln(sin(d*x+c)-1)-1/(2*a-
2*b)*ln(1+sin(d*x+c)))
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2\,a\log\left(b\sin\left(dx+c\right)+a\right)-(a+b)\log\left(\sin\left(dx+c\right)+1\right)-(a-b)\log\left(-\sin\left(dx+c\right)+1\right)}{2\,(a^2-b^2)d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*a*log(b*sin(d*x + c) + a) - (a + b)*log(sin(d*x + c) + 1) - (a - b)
*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d)
```

3.172.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.172.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{2a\log(b\sin(dx+c)+a)}{a^2-b^2} - \frac{\log(\sin(dx+c)+1)}{a-b} - \frac{\log(\sin(dx+c)-1)}{a+b}}{2d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*a*log(b*sin(d*x + c) + a)/(a^2 - b^2) - log(sin(d*x + c) + 1)/(a -
b) - log(sin(d*x + c) - 1)/(a + b))/d
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.96

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{2ab\log(|b\sin(dx+c)+a|)}{a^2b-b^3} - \frac{\log(|\sin(dx+c)+1|)}{a-b} - \frac{\log(|\sin(dx+c)-1|)}{a+b}}{2d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(2*a*b*log(abs(b*sin(d*x + c) + a))/(a^2*b - b^3) - log(abs(sin(d*x +
c) + 1))/(a - b) - log(abs(sin(d*x + c) - 1))/(a + b))/d
```

3.172.9 Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.23

$$\int \frac{\tan(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{a\,\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{d\,\left(a^2-b^2\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{d\,\left(a-b\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{d\,\left(a+b\right)}$$

input
```
int(tan(c + d*x)/(a + b*sin(c + d*x)),x)
```

output
```
(a*log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2
)) - log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)) - log(tan(c/2 + (d*x)/2) - 1)
/(d*(a + b))
```

3.173 $\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx$

3.173.1 Optimal result

Integrand size = 19, antiderivative size = 34

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx=\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

output
```
ln(sin(d*x+c))/a/d-ln(a+b*sin(d*x+c))/a/d
```

3.173.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx=\frac{\log(\sin(c+dx))}{ad}-\frac{\log(a+b\sin(c+dx))}{ad}$$

input
```
Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]]/(a*d)
```

3.173.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3042, 3200, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc(c+dx)}{b(a+b\sin(c+dx))}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int \frac{\csc(c+dx)}{b}d(b\sin(c+dx))}{a}-\frac{\int \frac{1}{a+b\sin(c+dx)}d(b\sin(c+dx))}{a}}{d}$$

$$\Big\downarrow \text{14}$$

$$\frac{\frac{\log(b\sin(c+dx))}{a}-\frac{\int \frac{1}{a+b\sin(c+dx)}d(b\sin(c+dx))}{a}}{d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(b\sin(c+dx))}{a}-\frac{\log(a+b\sin(c+dx))}{a}}{d}$$

input
```
Int[Cot[c + d*x]/(a + b*Sin[c + d*x]),x]
```

output
```
(Log[b*Sin[c + d*x]]/a - Log[a + b*Sin[c + d*x]]/a)/d
```

3.173.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.173.4 Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a}+\frac{\ln(\sin(dx+c))}{a}}{d}$	33
default	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a}+\frac{\ln(\sin(dx+c))}{a}}{d}$	33
risch	$\frac{\ln\left(\mathrm{e}^{2i(dx+c)}-1\right)}{da}-\frac{\ln\left(\mathrm{e}^{2i(dx+c)}+\frac{2ia\,\mathrm{e}^{i(dx+c)}}{b}-1\right)}{da}$	57

input
```
int(cot(d*x+c)/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a*ln(a+b*sin(d*x+c))+1/a*ln(sin(d*x+c)))
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\log\left(b\sin\left(dx+c\right)+a\right)-\log\left(-\frac{1}{2}\,\sin\left(dx+c\right)\right)}{ad}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-(log(b*sin(d*x + c) + a) - log(-1/2*sin(d*x + c)))/(a*d)
```

3.173.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cot\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)/(a + b*sin(c + d*x)), x)
```

3.173.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{\log(b\sin(dx+c)+a)}{a}-\frac{\log(\sin(dx+c))}{a}}{d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-(log(b*sin(d*x + c) + a)/a - log(sin(d*x + c))/a)/d
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{\log(|b\sin(dx+c)+a|)}{a}-\frac{\log(|\sin(dx+c)|)}{a}}{d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-(log(abs(b*sin(d*x + c) + a))/a - log(abs(sin(d*x + c)))/a)/d
```

3.173.9 Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.41

$$\int \frac{\cot(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}{a\,d}$$

input
```
int(cot(c + d*x)/(a + b*sin(c + d*x)),x)
```

output
```
(log(tan(c/2 + (d*x)/2)) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d
*x)/2)^2))/(a*d)
```

3.174 $\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$

3.174.1 Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\csc(c+dx)}{a^2d} - \frac{\csc^2(c+dx)}{2ad} - \frac{(a^2-b^2)\log(\sin(c+dx))}{a^3d} + \frac{(a^2-b^2)\log(a+b\sin(c+dx))}{a^3d}$$

output
```
b*csc(d*x+c)/a^2/d-1/2*csc(d*x+c)^2/a/d-(a^2-b^2)*ln(sin(d*x+c))/a^3/d+(a^
2-b^2)*ln(a+b*sin(d*x+c))/a^3/d
```

3.174.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.77

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{-2ab\csc(c+dx)+a^2\csc^2(c+dx)+2(a^2-b^2)\left(\log(\sin(c+dx))-\log(a+b\sin(c+dx))\right)}{2a^3d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

output
```
-1/2*(-2*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - b^2)*(Log[Sin[c
+ d*x]] - Log[a + b*Sin[c + d*x]]))/(a^3*d)
```

3.174.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^3(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^3(c+dx)\left(b^2-b^2\sin^2(c+dx)\right)}{b^3(a+b\sin(c+dx))}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\csc^3(c+dx)}{ab}-\frac{\csc^2(c+dx)}{a^2}+\frac{\left(b^2-a^2\right)\csc(c+dx)}{a^3b}+\frac{a^2-b^2}{a^3(a+b\sin(c+dx))}\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b\csc(c+dx)}{a^2}-\frac{\left(a^2-b^2\right)\log(b\sin(c+dx))}{a^3}+\frac{\left(a^2-b^2\right)\log(a+b\sin(c+dx))}{a^3}-\frac{\csc^2(c+dx)}{2a}}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]),x]
```

output
```
((b*Csc[c + d*x])/a^2 - Csc[c + d*x]^2/(2*a) - ((a^2 - b^2)*Log[b*Sin[c +
d*x]])/a^3 + ((a^2 - b^2)*Log[a + b*Sin[c + d*x]])/a^3)/d
```

3.174.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.174.4 Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{a^3}-\frac{1}{2a\sin(dx+c)^2}+\frac{\left(-a^2+b^2\right)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}}{d}$
default	$\frac{\frac{\left(a^2-b^2\right)\ln(a+b\sin(dx+c))}{a^3}-\frac{1}{2a\sin(dx+c)^2}+\frac{\left(-a^2+b^2\right)\ln(\sin(dx+c))}{a^3}+\frac{b}{a^2\sin(dx+c)}}{d}$
risch	$\frac{2i\left(-ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}\right)}{d\,a^2\left(e^{2i(dx+c)}-1\right)^2}-\frac{\ln\left(e^{2i(dx+c)}-1\right)}{da}+\frac{\ln\left(e^{2i(dx+c)}-1\right)b^2}{a^3d}+\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-\right.}{da}$

input
```
int(cot(d*x+c)^3/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*((a^2-b^2)/a^3*ln(a+b*sin(d*x+c))-1/2/a/sin(d*x+c)^2+1/a^3*(-a^2+b^2)*
ln(sin(d*x+c))+b/a^2/sin(d*x+c))
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2\,ab\sin\left(dx+c\right)-a^2-2\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2-a^2+b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)+2\left(\left(a^2-b^2\right)\cos\left(\right.\right.}{2\left(a^3d\cos\left(dx+c\right)^2-a^3d\right)}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*sin(d*x + c) - a^2 - 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2
)*log(b*sin(d*x + c) + a) + 2*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*log
(-1/2*sin(d*x + c)))/(a^3*d*cos(d*x + c)^2 - a^3*d)
```

3.174.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cot^3\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)), x)
```

3.174.7 Maxima [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{2\left(a^2-b^2\right)\log(b\sin(dx+c)+a)}{a^3}-\frac{2\left(a^2-b^2\right)\log(\sin(dx+c))}{a^3}+\frac{2\,b\sin(dx+c)-a}{a^2\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*(2*(a^2 - b^2)*log(b*sin(d*x + c) + a)/a^3 - 2*(a^2 - b^2)*log(sin(d*x
 + c))/a^3 + (2*b*sin(d*x + c) - a)/(a^2*sin(d*x + c)^2))/d
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.36

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{2\,(a^2-b^2)\log(|\sin(dx+c)|)}{a^3}-\frac{2\,(a^2b-b^3)\log(|b\sin(dx+c)+a|)}{a^3b}-\frac{3\,a^2\sin(dx+c)^2-3\,b^2\sin(dx+c)^2+2\,ab\sin(dx+c)-a^2}{a^3\sin(dx+c)^2}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*(a^2 - b^2)*log(abs(sin(d*x + c)))/a^3 - 2*(a^2*b - b^3)*log(abs(b
*sin(d*x + c) + a))/(a^3*b) - (3*a^2*sin(d*x + c)^2 - 3*b^2*sin(d*x + c)^2
 + 2*a*b*sin(d*x + c) - a^2)/(a^3*sin(d*x + c)^2))/d
```

3.174.9 Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.71

$$\int \frac{\cot^3(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,a\,d} -\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-b^2\right)}{a^3\,d}-\frac{\frac{a}{2}-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a^2\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2} +\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^2-b^2\right)}{a^3\,d}$$

input
```
int(cot(c + d*x)^3/(a + b*sin(c + d*x)),x)
```

output
```
(b*tan(c/2 + (d*x)/2))/(2*a^2*d) - tan(c/2 + (d*x)/2)^2/(8*a*d) - (log(tan
(c/2 + (d*x)/2))*(a^2 - b^2))/(a^3*d) - (a/2 - 2*b*tan(c/2 + (d*x)/2))/(4*
a^2*d*tan(c/2 + (d*x)/2)^2) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2
+ (d*x)/2)^2)*(a^2 - b^2))/(a^3*d)
```

3.175 $\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$

3.175.1 Optimal result

Integrand size = 21, antiderivative size = 148

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{b(2a^2-b^2)\csc(c+dx)}{a^4d}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3d}+\frac{b\csc^3(c+dx)}{3a^2d}-\frac{\csc^4(c+dx)}{4ad}+\frac{(a^2-b^2)^2\log(\sin(c+dx))}{a^5d}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^5d}$$

output
```
-b*(2*a^2-b^2)*csc(d*x+c)/a^4/d+1/2*(2*a^2-b^2)*csc(d*x+c)^2/a^3/d+1/3*b*c
sc(d*x+c)^3/a^2/d-1/4*csc(d*x+c)^4/a/d+(a^2-b^2)^2*ln(sin(d*x+c))/a^5/d-(a
^2-b^2)^2*ln(a+b*sin(d*x+c))/a^5/d
```

3.175.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{12ab(-2a^2+b^2)\csc(c+dx)+6a^2(2a^2-b^2)\csc^2(c+dx)+4a^3b\csc^3(c+dx)-3a^4\csc^4(c+dx)+12(a^2}{12a^5d}$$

input
```
Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

output
```
(12*a*b*(-2*a^2 + b^2)*Csc[c + d*x] + 6*a^2*(2*a^2 - b^2)*Csc[c + d*x]^2 +
 4*a^3*b*Csc[c + d*x]^3 - 3*a^4*Csc[c + d*x]^4 + 12*(a^2 - b^2)^2*(Log[Sin
[c + d*x]] - Log[a + b*Sin[c + d*x]]))/(12*a^5*d)
```

3.175.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^5(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^5(c+dx)\left(b^2-b^2\sin^2(c+dx)\right)^2}{b^5(a+b\sin(c+dx))}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\csc^5(c+dx)}{ab}-\frac{\csc^4(c+dx)}{a^2}+\frac{(b^4-2a^2b^2)\csc^3(c+dx)}{a^3b^3}+\frac{(2a^2b^2-b^4)\csc^2(c+dx)}{a^4b^2}+\frac{(a^2-b^2)^2\csc(c+dx)}{a^5b}-\frac{(a^2-b^2)^2}{a^5(a+b\sin(c+dx))}\right)d(b\sin}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b\csc^3(c+dx)}{3a^2}+\frac{(a^2-b^2)^2\log(b\sin(c+dx))}{a^5}-\frac{(a^2-b^2)^2\log(a+b\sin(c+dx))}{a^5}-\frac{b(2a^2-b^2)\csc(c+dx)}{a^4}+\frac{(2a^2-b^2)\csc^2(c+dx)}{2a^3}-\frac{\csc^4(c+dx)}{4a}}{d}$$

input
```
Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]),x]
```

output
```
(-((b*(2*a^2 - b^2)*Csc[c + d*x])/a^4) + ((2*a^2 - b^2)*Csc[c + d*x]^2)/(2
*a^3) + (b*Csc[c + d*x]^3)/(3*a^2) - Csc[c + d*x]^4/(4*a) + ((a^2 - b^2)^2
*Log[b*Sin[c + d*x]])/a^5 - ((a^2 - b^2)^2*Log[a + b*Sin[c + d*x]])/a^5)/d
```

3.175.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.175.4 Maple [A] (verified)

Time = 1.62 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.93

method	result
derivativedivides	$\frac{-\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx+c))}{a^5}-\frac{1}{4a\sin(dx+c)^4}-\frac{-2a^2+b^2}{2a^3\sin(dx+c)^2}+\frac{(a^4-2a^2b^2+b^4)\ln(\sin(dx+c))}{a^5}-\frac{(2a^2-b^2)b}{a^4\sin(dx+c)}+\frac{b}{3a^2\sin(\cdots}}{d}$
default	$\frac{-\frac{(a^4-2a^2b^2+b^4)\ln(a+b\sin(dx+c))}{a^5}-\frac{1}{4a\sin(dx+c)^4}-\frac{-2a^2+b^2}{2a^3\sin(dx+c)^2}+\frac{(a^4-2a^2b^2+b^4)\ln(\sin(dx+c))}{a^5}-\frac{(2a^2-b^2)b}{a^4\sin(dx+c)}+\frac{b}{3a^2\sin(\cdots}}{d}$
risch	$\frac{2i(6ia^3e^{6i(dx+c)}-3iab^2e^{6i(dx+c)}-6a^2be^{7i(dx+c)}+3b^3e^{7i(dx+c)}-6ia^3e^{4i(dx+c)}+6iab^2e^{4i(dx+c)}+14a^2be^{5i(dx+c)}-9b^3\cdots}{3da^4(e^{2i(dx+c)}-1\cdots}$

input
```
int(cot(d*x+c)^5/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a^4-2*a^2*b^2+b^4)/a^5*ln(a+b*sin(d*x+c))-1/4/a/sin(d*x+c)^4-1/2*(-
2*a^2+b^2)/a^3/sin(d*x+c)^2+(a^4-2*a^2*b^2+b^4)/a^5*ln(sin(d*x+c))-(2*a^2-
b^2)/a^4*b/sin(d*x+c)+1/3*b/a^2/sin(d*x+c)^3)
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.83

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{9\,a^4 - 6\,a^2b^2 - 6\,(2\,a^4 - a^2b^2)\cos\left(dx+c\right)^2 - 12\,\left((a^4 - 2\,a^2b^2 + b^4)\cos\left(dx+c\right)^4 + a^4 - 2\,a^2b^2 + b^4 - 2\,(\cdots\right.}{\cdots}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
1/12*(9*a^4 - 6*a^2*b^2 - 6*(2*a^4 - a^2*b^2)*cos(d*x + c)^2 - 12*((a^4 -
2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b
^2 + b^4)*cos(d*x + c)^2)*log(b*sin(d*x + c) + a) + 12*((a^4 - 2*a^2*b^2 +
 b^4)*cos(d*x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*c
os(d*x + c)^2)*log(-1/2*sin(d*x + c)) - 4*(5*a^3*b - 3*a*b^3 - 3*(2*a^3*b
- a*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^5*d*cos(d*x + c)^4 - 2*a^5*d*cos
(d*x + c)^2 + a^5*d)
```

3.175.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cot^5\left(c+dx\right)}{a+b\sin\left(c+dx\right)}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**5/(a + b*sin(c + d*x)), x)
```

3.175.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.94

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx =$$

$$-\frac{\frac{12\,(a^4-2\,a^2b^2+b^4)\,\log(b\sin(dx+c)+a)}{a^5} - \frac{12\,(a^4-2\,a^2b^2+b^4)\,\log(\sin(dx+c))}{a^5} - \frac{4\,a^2b\sin(dx+c)-12\,(2\,a^2b-b^3)\,\sin(dx+c)^3-3\,a^3+6\,(2\,a^3\cdots}{a^4\sin(dx+c)^4}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
-1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(b*sin(d*x + c) + a)/a^5 - 12*(a^4 -
2*a^2*b^2 + b^4)*log(sin(d*x + c))/a^5 - (4*a^2*b*sin(d*x + c) - 12*(2*a^2
*b - b^3)*sin(d*x + c)^3 - 3*a^3 + 6*(2*a^3 - a*b^2)*sin(d*x + c)^2)/(a^4*
sin(d*x + c)^4))/d
```

3.175.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.36

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{12\,(a^4-2\,a^2b^2+b^4)\log(|\sin(dx+c)|)}{a^5} - \frac{12\,(a^4b-2\,a^2b^3+b^5)\log(|b\sin(dx+c)+a|)}{a^5b} - \frac{25\,a^4\sin(dx+c)^4-50\,a^2b^2\sin(dx+c)^4+25\,b^4\sin(dx+c)^4+}{}}{12\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/12*(12*(a^4 - 2*a^2*b^2 + b^4)*log(abs(sin(d*x + c)))/a^5 - 12*(a^4*b -
2*a^2*b^3 + b^5)*log(abs(b*sin(d*x + c) + a))/(a^5*b) - (25*a^4*sin(d*x +
c)^4 - 50*a^2*b^2*sin(d*x + c)^4 + 25*b^4*sin(d*x + c)^4 + 24*a^3*b*sin(d*
x + c)^3 - 12*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 6*a^2*b^2*sin
(d*x + c)^2 - 4*a^3*b*sin(d*x + c) + 3*a^4)/(a^5*sin(d*x + c)^4))/d
```

3.175.9 Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.90

$$\int \frac{\cot^5(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3}{16a}-\frac{b^2}{8a^3}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64\,a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{b}{8a^2}+\frac{2b\left(\frac{3}{8a}-\frac{b^2}{4a^3}\right)}{a}\right)}{d}$$
$$- \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2ab^2-3a^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(14a^2b-8b^3)+\frac{a^3}{4}-\frac{2a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{3}}{16a^4d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}$$
$$- \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)(a^4-2a^2b^2+b^4)}{a^5\,d}$$
$$+ \frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a^2\,d} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a^4-2a^2b^2+b^4)}{a^5\,d}$$

input
```
int(cot(c + d*x)^5/(a + b*sin(c + d*x)),x)
```

output
```
(tan(c/2 + (d*x)/2)^2*(3/(16*a) - b^2/(8*a^3)))/d - tan(c/2 + (d*x)/2)^4/(
64*a*d) - (tan(c/2 + (d*x)/2)*(b/(8*a^2) + (2*b*(3/(8*a) - b^2/(4*a^3)))/a
))/d - (tan(c/2 + (d*x)/2)^2*(2*a*b^2 - 3*a^3) + tan(c/2 + (d*x)/2)^3*(14*
a^2*b - 8*b^3) + a^3/4 - (2*a^2*b*tan(c/2 + (d*x)/2))/3)/(16*a^4*d*tan(c/2
 + (d*x)/2)^4) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)
*(a^4 + b^4 - 2*a^2*b^2))/(a^5*d) + (b*tan(c/2 + (d*x)/2)^3)/(24*a^2*d) +
(log(tan(c/2 + (d*x)/2))*(a^4 + b^4 - 2*a^2*b^2))/(a^5*d)
```

3.176 $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

3.176.1 Optimal result

Integrand size = 21, antiderivative size = 177

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2a^4\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}\,d} + \frac{a^2b\sec(c+dx)}{(a^2-b^2)^2\,d} + \frac{b\sec(c+dx)}{(a^2-b^2)\,d} - \frac{b\sec^3(c+dx)}{3\,(a^2-b^2)\,d} - \frac{a^3\tan(c+dx)}{(a^2-b^2)^2\,d} + \frac{a\tan^3(c+dx)}{3\,(a^2-b^2)\,d}$$

output
```
2*a^4*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d+a
^2*b*sec(d*x+c)/(a^2-b^2)^2/d+b*sec(d*x+c)/(a^2-b^2)/d-1/3*b*sec(d*x+c)^3/
(a^2-b^2)/d-a^3*tan(d*x+c)/(a^2-b^2)^2/d+1/3*a*tan(d*x+c)^3/(a^2-b^2)/d
```

3.176.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{48a^4\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\sec^3(c+dx)\left(-16a^2b+4b^3+3b(11a^2-5b^2)\cos(c+dx)+12b(-2a^2+b^2)\cos(2(c+dx))+11a^2b\cos(3(c+dx))-5\cdots\right.}{(a-b)^2(a+b)^2}}{24d}$$

input
```
Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

output
```
((48*a^4*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/
2) - (Sec[c + d*x]^3*(-16*a^2*b + 4*b^3 + 3*b*(11*a^2 - 5*b^2)*Cos[c + d*x
] + 12*b*(-2*a^2 + b^2)*Cos[2*(c + d*x)] + 11*a^2*b*Cos[3*(c + d*x)] - 5*b
^3*Cos[3*(c + d*x)] + 6*a*b^2*Sin[c + d*x] + 8*a^3*Sin[3*(c + d*x)] - 2*a*
b^2*Sin[3*(c + d*x)]))/((a - b)^2*(a + b)^2))/(24*d)
```

3.176.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3206, 3042, 3086, 2009, 3087, 15, 3206, 3042, 3086, 24, 3139, 1083, 217, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^4}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 3206$$

$$-\frac{a^2\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \sec^2(c+dx)\tan^2(c+dx)dx}{a^2-b^2}-\frac{b\int \sec(c+dx)\tan^3(c+dx)dx}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a^2\int \frac{\tan(c+dx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \sec(c+dx)^2\tan(c+dx)^2dx}{a^2-b^2}-\frac{b\int \sec(c+dx)\tan(c+dx)^3dx}{a^2-b^2}$$

$$\Big\downarrow \; 3086$$

$$-\frac{b\int \left(\sec^2(c+dx)-1\right)d\sec(c+dx)}{d\left(a^2-b^2\right)}-\frac{a^2\int \frac{\tan(c+dx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \sec(c+dx)^2\tan(c+dx)^2dx}{a^2-b^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{a^2\int \frac{\tan(c+dx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \sec(c+dx)^2\tan(c+dx)^2dx}{a^2-b^2}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3087$$

$$\frac{a\int \tan^2(c+dx)d\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{a^2\int \frac{\tan(c+dx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$\downarrow$ 15

$$-\frac{a^2\int \frac{\tan(c+dx)^2}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$\downarrow$ 3206

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \sec^2(c+dx)dx}{a^2-b^2}-\frac{b\int \sec(c+dx)\tan(c+dx)dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int \sec(c+dx)\tan(c+dx)dx}{a^2-b^2}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$\downarrow$ 3086

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int 1d\sec(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$\downarrow$ 24

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\sec(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$\downarrow$ 3139

$$-\frac{a^2\left(-\frac{2a^2\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{d(a^2-b^2)}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\sec(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$\downarrow$ 1083

3.176. $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

$$-\frac{a^2\left(\frac{4a^2\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2-b^2)}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\sec(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{217}$$

$$-\frac{a^2\left(\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{3/2}}-\frac{b\sec(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{4254}$$

$$-\frac{a^2\left(-\frac{a\int 1d(-\tan(c+dx))}{d(a^2-b^2)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{3/2}}-\frac{b\sec(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \textbf{24}$$

$$-\frac{a^2\left(-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{3/2}}+\frac{a\tan(c+dx)}{d(a^2-b^2)}-\frac{b\sec(c+dx)}{d(a^2-b^2)}\right)}{a^2-b^2}+\frac{a\tan^3(c+dx)}{3d\left(a^2-b^2\right)}-\frac{b\left(\frac{1}{3}\sec^3(c+dx)-\sec(c+dx)\right)}{d\left(a^2-b^2\right)}$$

input
```
Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

output
```
-((b*(-Sec[c + d*x] + Sec[c + d*x]^3/3))/((a^2 - b^2)*d)) + (a*Tan[c + d*x
]^3)/(3*(a^2 - b^2)*d) - (a^2*((-2*a^2*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])
/(2*Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*d) - (b*Sec[c + d*x])/((a^2 - b^
2)*d) + (a*Tan[c + d*x])/((a^2 - b^2)*d)))/(a^2 - b^2)
```

3.176. $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

3.176.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.176. $\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.176.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(32a+32b)}-\frac{16}{(32a+32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a-b}{2(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(32a-32b)}+\cdots}{d}$
default	$\frac{-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(32a+32b)}-\frac{16}{(32a+32b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a-b}{2(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{32}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(32a-32b)}+\cdots}{d}$
risch	$-\frac{2\left(6ia^3e^{4i(dx+c)}-3iab^2e^{4i(dx+c)}-6a^2b\,e^{5i(dx+c)}+3b^3e^{5i(dx+c)}+6ia^3e^{2i(dx+c)}-8a^2b\,e^{3i(dx+c)}+2b^3e^{3i(dx+c)}+4ia^3-\cdots\right.}{3(a^4-2a^2b^2+b^4)\left(1+e^{2i(dx+c)}\right)^3d}$

input
```
int(tan(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-32/3/(tan(1/2*d*x+1/2*c)-1)^3/(32*a+32*b)-16/(32*a+32*b)/(tan(1/2*d*
x+1/2*c)-1)^2-1/2/(a+b)^2*(-2*a-b)/(tan(1/2*d*x+1/2*c)-1)-32/3/(tan(1/2*d*
x+1/2*c)+1)^3/(32*a-32*b)+16/(32*a-32*b)/(tan(1/2*d*x+1/2*c)+1)^2-1/2/(a-b
)^2*(-2*a+b)/(tan(1/2*d*x+1/2*c)+1)+2*a^4/(a-b)^2/(a+b)^2/(a^2-b^2)^(1/2)*
arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

3.176.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.69

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[-\frac{3\sqrt{-a^2+b^2}a^4\cos{(dx+c)}^3\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right.}{6(a}\right.$$
$$-\frac{3\sqrt{a^2-b^2}a^4\arctan\left(-\frac{a\sin(dx+c)+b}{\sqrt{a^2-b^2}\cos(dx+c)}\right)\cos{(dx+c)}^3+a^4b-2a^2b^3+b^5-3(2a^4b-3a^2b^3+b^5)\cos{(}}{3(a^6-3a^4b^2+3a^2b^4-b^6)d\cos}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/6*(3*sqrt(-a^2 + b^2)*a^4*cos(d*x + c)^3*log(((2*a^2 - b^2)*cos(d*x +
c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b
*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
- a^2 - b^2)) + 2*a^4*b - 4*a^2*b^3 + 2*b^5 - 6*(2*a^4*b - 3*a^2*b^3 + b^5
)*cos(d*x + c)^2 - 2*(a^5 - 2*a^3*b^2 + a*b^4 - (4*a^5 - 5*a^3*b^2 + a*b^4
)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos
(d*x + c)^3), -1/3*(3*sqrt(a^2 - b^2)*a^4*arctan(-(a*sin(d*x + c) + b)/(sq
rt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c)^3 + a^4*b - 2*a^2*b^3 + b^5 - 3*
(2*a^4*b - 3*a^2*b^3 + b^5)*cos(d*x + c)^2 - (a^5 - 2*a^3*b^2 + a*b^4 - (4
*a^5 - 5*a^3*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^6 - 3*a^4*b^2
+ 3*a^2*b^4 - b^6)*d*cos(d*x + c)^3)]
```

3.176.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\tan^4{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

3.176.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.176.8 Giac [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.36

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2\left(\frac{3\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^4}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{3a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4ab^2\,\mathrm{ta}\cdots}{(a^4-2a^2b^2\cdots}\cdots\right)}{3d}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
2/3*(3*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x +
1/2*c) + b)/sqrt(a^2 - b^2)))*a^4/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a^2 - b^2)
) + (3*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 10*a^
3*tan(1/2*d*x + 1/2*c)^3 + 4*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a^2*b*tan(1
/2*d*x + 1/2*c)^2 - 6*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2
*c) - 5*a^2*b + 2*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 -
1)^3))/d
```

3.176.9 Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.10

$$\int \frac{\tan^4(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^4-2a^2b^2+b^4}-\frac{2(5a^2b-2b^3)}{3(a^4-2a^2b^2+b^4)}+\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a^4-2a^2b^2+b^4}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2ab^2-5a^3)}{3(a^4-2a^2b^2+b^4)}+\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(2a^2b-b^3)}{a^4-2a^2b^2+b^4}-\frac{2a^2b\tan(}{a^4-2a^}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$
$$+\frac{2a^4\,\mathrm{atan}\left(\frac{\frac{a^4(2a^4b-4a^2b^3+2b^5)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{2a^4}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(tan(c + d*x)^4/(a + b*sin(c + d*x)),x)
```

output
```
((2*a^3*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^2) - (2*(5*a^2*b - 2*b^3)
)/(3*(a^4 + b^4 - 2*a^2*b^2)) + (2*a^3*tan(c/2 + (d*x)/2)^5)/(a^4 + b^4 -
2*a^2*b^2) + (4*tan(c/2 + (d*x)/2)^3*(2*a*b^2 - 5*a^3))/(3*(a^4 + b^4 - 2*
a^2*b^2)) + (4*tan(c/2 + (d*x)/2)^2*(2*a^2*b - b^3))/(a^4 + b^4 - 2*a^2*b^
2) - (2*a^2*b*tan(c/2 + (d*x)/2)^4)/(a^4 + b^4 - 2*a^2*b^2))/(d*(3*tan(c/2
 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) + (2*a
^4*atan(((a^4*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a - b)^(5/2))
 + (2*a^5*tan(c/2 + (d*x)/2)*(a^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a -
b)^(5/2)))/(2*a^4)))/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.177 $\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$

3.177.1 Optimal result

Integrand size = 21, antiderivative size = 96

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2a^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,d} - \frac{b\sec(c+dx)}{(a^2-b^2)\,d} + \frac{a\tan(c+dx)}{(a^2-b^2)\,d}$$

output
```
-2*a^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(3/2)/d-
b*sec(d*x+c)/(a^2-b^2)/d+a*tan(d*x+c)/(a^2-b^2)/d
```

3.177.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.58

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-2a^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\cos(c+dx)+\sqrt{a^2-b^2}(-b+b\cos(c+dx)+a\sin(c+dx))}{(a-b)(a+b)\sqrt{a^2-b^2}d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
(-2*a^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Cos[c + d*x] + Sq
rt[a^2 - b^2]*(-b + b*Cos[c + d*x] + a*Sin[c + d*x]))/((a - b)*(a + b)*Sqr
t[a^2 - b^2]*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + S
in[(c + d*x)/2]))
```

3.177.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3042, 3206, 3042, 3086, 24, 3139, 1083, 217, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^2}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \; 3206$$

$$-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \sec^2(c+dx)dx}{a^2-b^2}-\frac{b\int \sec(c+dx)\tan(c+dx)dx}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int \sec(c+dx)\tan(c+dx)dx}{a^2-b^2}$$

$$\Big\downarrow \; 3086$$

$$-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int 1d\sec(c+dx)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 24$$

$$-\frac{a^2\int \frac{1}{a+b\sin(c+dx)}dx}{a^2-b^2}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3139$$

$$-\frac{2a^2\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{d\left(a^2-b^2\right)}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 1083$$

$$\frac{4a^2\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2-b^2\right)}+\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 217$$

$$\frac{a\int \csc\left(c+dx+\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 4254}$$

$$-\frac{a\int 1d(-\tan(c+dx))}{d\left(a^2-b^2\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{d\left(a^2-b^2\right)^{3/2}}+\frac{a\tan(c+dx)}{d\left(a^2-b^2\right)}-\frac{b\sec(c+dx)}{d\left(a^2-b^2\right)}$$

input
```
Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
(-2*a^2*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/((a^2 -
b^2)^(3/2)*d) - (b*Sec[c + d*x])/((a^2 - b^2)*d) + (a*Tan[c + d*x])/((a^2
- b^2)*d)
```

3.177.3.1 Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

3.177.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{-\frac{8}{(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{8}{(8a+8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$	112
default	$\frac{-\frac{8}{(8a-8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2a^2\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{(a-b)(a+b)\sqrt{a^2-b^2}}-\frac{8}{(8a+8b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$	112
risch	$\frac{-2ia+2b\,e^{i(dx+c)}}{d(-a^2+b^2)\left(1+e^{2i(dx+c)}\right)}+\frac{a^2\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}-\frac{a^2\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}+a^2-b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,(a+b)(a-b)d}$	208

input
```
int(tan(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8/(8*a-8*b)/(tan(1/2*d*x+1/2*c)+1)-2*a^2/(a-b)/(a+b)/(a^2-b^2)^(1/2)
*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-8/(8*a+8*b)/(tan
(1/2*d*x+1/2*c)-1))
```

3.177.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.18

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[\frac{\sqrt{-a^2+b^2}a^2\cos(dx+c)\log\left(\frac{(2a^2-b^2)\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2+2(a\cos(dx+c)\sin(dx+c)+b\cos(dx+c))\sqrt{-a^2+b^2}}{b^2\cos(dx+c)^2-2ab\sin(dx+c)-a^2-b^2}\right)}{2(a^4-2a^2b^2+b^4)d\cos(dx+c)}\right.$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-a^2 + b^2)*a^2*cos(d*x + c)*log(((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(
d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2)) - 2*a^2*b + 2*b^3 + 2*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b
^2 + b^4)*d*cos(d*x + c)), (sqrt(a^2 - b^2)*a^2*arctan(-(a*sin(d*x + c) +
b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*cos(d*x + c) - a^2*b + b^3 + (a^3 - a*b
^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c))]
```

3.177.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.177.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.177.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)a^2}{(a^2-b^2)^{\frac{3}{2}}}+\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b}{(a^2-b^2)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)}\right)}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2
*c) + b)/sqrt(a^2 - b^2)))*a^2/(a^2 - b^2)^(3/2) + (a*tan(1/2*d*x + 1/2*c)
 - b)/((a^2 - b^2)*(tan(1/2*d*x + 1/2*c)^2 - 1)))/d
```

3.177.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.54

$$\int \frac{\tan^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{\frac{2b}{a^2-b^2}-\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a^2-b^2}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)} - \frac{2a^2\,\mathrm{atan}\left(\frac{\frac{a^2\left(2a^2b-2b^3\right)}{(a+b)^{3/2}(a-b)^{3/2}}+\frac{2a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^2-b^2\right)}{(a+b)^{3/2}(a-b)^{3/2}}}{2a^2}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(tan(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

output
```
((2*b)/(a^2 - b^2) - (2*a*tan(c/2 + (d*x)/2))/(a^2 - b^2))/(d*(tan(c/2 + (
d*x)/2)^2 - 1)) - (2*a^2*atan(((a^2*(2*a^2*b - 2*b^3))/((a + b)^(3/2)*(a -
 b)^(3/2)) + (2*a^3*tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)*(a - b)
^(3/2)))/(2*a^2)))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

3.178 $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

3.178.1 Optimal result

Integrand size = 21, antiderivative size = 80

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2\sqrt{a^2-b^2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2d} + \frac{b\,\mathrm{arctanh}(\cos(c+dx))}{a^2d} - \frac{\cot(c+dx)}{ad}$$

output
```
b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-2*arctan((b+a*tan(1/2*d*x+1/2*c
))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^2/d
```

3.178.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.35

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{-4\sqrt{a^2-b^2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) - a\cot\left(\frac{1}{2}(c+dx)\right) + 2b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 2b\log\left(\sin\left(\frac{1}{2}(c+d\right.\right.}{2a^2d}$$

input
```
Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
(-4*Sqrt[a^2 - b^2]*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] - a*C
ot[(c + d*x)/2] + 2*b*Log[Cos[(c + d*x)/2]] - 2*b*Log[Sin[(c + d*x)/2]] +
a*Tan[(c + d*x)/2])/(2*a^2*d)
```

3.178.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.16, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3202, 3042, 3535, 25, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^2(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{ 3202}$$

$$\int \frac{\left(1-\sin^2(c+dx)\right)\csc^2(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1-\sin(c+dx)^2}{\sin(c+dx)^2(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{ 3535}$$

$$\frac{\int -\frac{\csc(c+dx)(b+a\sin(c+dx))}{a+b\sin(c+dx)}dx}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \frac{\csc(c+dx)(b+a\sin(c+dx))}{a+b\sin(c+dx)}dx}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{b+a\sin(c+dx)}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{ 3480}$$

$$-\frac{\frac{(a^2-b^2)\int \frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b\int \csc(c+dx)dx}{a}}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\frac{(a^2-b^2)\int \frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b\int \csc(c+dx)dx}{a}}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{3139}$$

$$-\frac{\frac{2(a^2-b^2)\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}+\frac{b\int \csc(c+dx)dx}{a}}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{1083}$$

$$-\frac{\frac{b\int \csc(c+dx)dx}{a}-\frac{4(a^2-b^2)\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{217}$$

$$-\frac{\frac{b\int \csc(c+dx)dx}{a}+\frac{2\sqrt{a^2-b^2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}}{a}-\frac{\cot(c+dx)}{ad}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{2\sqrt{a^2-b^2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}-\frac{b\,\text{arctanh}(\cos(c+dx))}{ad}}{a}-\frac{\cot(c+dx)}{ad}$$

input
```
Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]),x]
```

output
```
-(((2*Sqrt[a^2 - b^2]*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^
2])])/(a*d) - (b*ArcTanh[Cos[c + d*x]])/(a*d))/a) - Cot[c + d*x]/(a*d)
```

3.178.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.178. $\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$

3.178.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\left(-4a^2+4b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2\sqrt{a^2-b^2}}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{\left(-4a^2+4b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b}{2\sqrt{a^2-b^2}}\right)}{2a^2\sqrt{a^2-b^2}}}{d}$
risch	$-\frac{2i}{da\left(e^{2i(dx+c)}-1\right)}+\frac{b\ln\left(e^{i(dx+c)}+1\right)}{a^2d}-\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}+\frac{ia+\sqrt{-a^2+b^2}}{b}\right)}{d\,a^2}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}-\frac{-ia}{}\right.}{d\,a^2}$

input
```
int(cot(d*x+c)^2/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a*tan(1/2*d*x+1/2*c)-1/2/a/tan(1/2*d*x+1/2*c)-1/a^2*b*ln(tan(1/2*
d*x+1/2*c))+1/2/a^2*(-4*a^2+4*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2
*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
```

3.178.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 314, normalized size of antiderivative = 3.92

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[\frac{b\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)-b\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)\sin(dx+c)+\sqrt{-a^2+b^2}\log\left(\frac{(2a^2-\right.}{2a^2d\sin(d}\right.$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[1/2*(b*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c)
 + 1/2)*sin(d*x + c) + sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2
- 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(
d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2
 - b^2))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c)), 1/2*(b*log
(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) - b*log(-1/2*cos(d*x + c) + 1/2)*sin
(d*x + c) + 2*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2
)*cos(d*x + c)))*sin(d*x + c) - 2*a*cos(d*x + c))/(a^2*d*sin(d*x + c))]
```

3.178.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cot^2{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)), x)
```

3.178.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.178.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.61

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^2} - \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} + \frac{4\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{a^2} - \frac{2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^2 - tan(1/2*d*x + 1/2*c)/a + 4*
(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c)
+ b)/sqrt(a^2 - b^2)))*sqrt(a^2 - b^2)/a^2 - (2*b*tan(1/2*d*x + 1/2*c) - a
)/(a^2*tan(1/2*d*x + 1/2*c)))/d
```

3.178.9 Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.55

$$\int \frac{\cot^2(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{\cot(c+d\,x)}{a\,d} - \frac{b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d} + \frac{\mathrm{atan}\left(\frac{a^3\,\sqrt{b^2-a^2}\,1\mathrm{i}-a\,b^2\,\sqrt{b^2-a^2}\,2\mathrm{i}-b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,4\mathrm{i}+a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}\,3\mathrm{i}}{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4-2\,a^3\,b-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^2\,b^2+2\,a\,b^3+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)b^4}\right)\sqrt{b^2-a^2}\,2\mathrm{i}}{a^2\,d}$$

input
```
int(cot(c + d*x)^2/(a + b*sin(c + d*x)),x)
```

output
```
(atan((a^3*(b^2 - a^2)^(1/2)*1i - a*b^2*(b^2 - a^2)^(1/2)*2i - b^3*tan(c/2
 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i + a^2*b*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1
/2)*3i)/(2*a*b^3 - 2*a^3*b + a^4*tan(c/2 + (d*x)/2) + 4*b^4*tan(c/2 + (d*x
)/2) - 5*a^2*b^2*tan(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*2i)/(a^2*d) - cot(
c + d*x)/(a*d) - (b*log(tan(c/2 + (d*x)/2)))/(a^2*d)
```

3.179 $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

3.179.1 Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2(a^2-b^2)^{3/2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} - \frac{b(3a^2-2b^2)\,\text{arctanh}(\cos(c+dx))}{2a^4d} + \frac{(4a^2-3b^2)\cot(c+dx)}{3a^3d} + \frac{b\cot(c+dx)\csc(c+dx)}{2a^2d} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

output
```
2*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^4/d-1
/2*b*(3*a^2-2*b^2)*arctanh(cos(d*x+c))/a^4/d+1/3*(4*a^2-3*b^2)*cot(d*x+c)/
a^3/d+1/2*b*cot(d*x+c)*csc(d*x+c)/a^2/d-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d
```

3.179.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 350 vs. $2(154) = 308$.

Time = 6.24 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.27

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \frac{2(a^2-b^2)^{3/2}\arctan\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^4d} + \frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-3b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^3d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24ad} + \frac{(-3a^2b+2b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} + \frac{(3a^2b-2b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^4d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+3b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^3d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24ad}$$

input
```
Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

output
```
(2*(a^2 - b^2)^(3/2)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2] + a*Sin[
(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^4*d) + ((4*a^2*Cos[(c + d*x)/2] - 3*b^
2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^3*d) + (b*Csc[(c + d*x)/2]^2)/(
8*a^2*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(24*a*d) + ((-3*a^2*b + 2
*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^4*d) + ((3*a^2*b - 2*b^3)*Log[Sin[(c + d
*x)/2]])/(2*a^4*d) - (b*Sec[(c + d*x)/2]^2)/(8*a^2*d) + (Sec[(c + d*x)/2]*
(-4*a^2*Sin[(c + d*x)/2] + 3*b^2*Sin[(c + d*x)/2]))/(6*a^3*d) + (Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(24*a*d)
```

3.179.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3204, 3042, 3534, 27, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^4(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \text{3204}$$

$$-\frac{\int \frac{\csc^2(c+dx)\left(-3\left(2a^2-b^2\right)\sin^2(c+dx)-ab\sin(c+dx)+2\left(4a^2-3b^2\right)\right)}{a+b\sin(c+dx)}dx}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-3\left(2a^2-b^2\right)\sin(c+dx)^2-ab\sin(c+dx)+2\left(4a^2-3b^2\right)}{\sin(c+dx)^2(a+b\sin(c+dx))}dx}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$$\Big\downarrow \text{3534}$$

$$-\frac{\frac{\int -\frac{3\csc(c+dx)\left(b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}dx}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{3\int \frac{\csc(c+dx)\left(b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}dx}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\frac{3\int \frac{b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\sin(c+dx)}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$\downarrow$ 3480

$$-\frac{-\frac{3\left(\frac{2\left(a^2-b^2\right)^2\int \frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b\left(3a^2-2b^2\right)\int \csc(c+dx)dx}{a}\right)}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$\downarrow$ 3042

$$-\frac{-\frac{3\left(\frac{2\left(a^2-b^2\right)^2\int \frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b\left(3a^2-2b^2\right)\int \csc(c+dx)dx}{a}\right)}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$\downarrow$ 3139

$$-\frac{-\frac{3\left(\frac{4\left(a^2-b^2\right)^2\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}+\frac{b\left(3a^2-2b^2\right)\int \csc(c+dx)dx}{a}\right)}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$\downarrow$ 1083

$$-\frac{-\frac{3\left(\frac{b\left(3a^2-2b^2\right)\int \csc(c+dx)dx}{a}-\frac{8\left(a^2-b^2\right)^2\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

$\downarrow$ 217

$$-\frac{-\frac{3\left(\frac{b\left(3a^2-2b^2\right)\int \csc(c+dx)dx}{a}+\frac{4\left(a^2-b^2\right)^{3/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}\right)}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

3.179. $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

$$\Big\downarrow \text{4257}$$

$$-\frac{-\frac{3\left(\frac{4\left(a^2-b^2\right)^{3/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}-\frac{b\left(3a^2-2b^2\right)\text{arctanh}(\cos(c+dx))}{ad}\right)}{a}-\frac{2\left(4a^2-3b^2\right)\cot(c+dx)}{ad}}{6a^2}+\frac{b\cot(c+dx)\csc(c+dx)}{2a^2d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad}$$

input
```
Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]),x]
```

output
```
-1/6*((-3*((4*(a^2 - b^2)^(3/2)*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqr
t[a^2 - b^2])])/(a*d) - (b*(3*a^2 - 2*b^2)*ArcTanh[Cos[c + d*x]])/(a*d)))/
a - (2*(4*a^2 - 3*b^2)*Cot[c + d*x])/(a*d))/a^2 + (b*Cot[c + d*x]*Csc[c +
d*x])/(2*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d)
```

3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

3.179. $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

rule 3204
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Simp[b*(m - 2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m +
 1)/(6*a^2*f*Sin[e + f*x]^2)), x] - Simp[1/(6*a^2)   Int[((a + b*Sin[e + f*
x])^m/Sin[e + f*x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x]
 - (6*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f
, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.179. $\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$

3.179.4 Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}}{3}-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-5a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^{2}}{8a^{3}}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-5a^{2}+4b^{2}}{8a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b}{8a^{2}\tan\left(\frac{dx}{2}\right.}}{d}$
default	$\frac{\frac{\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^{2}}{3}-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-5a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^{2}}{8a^{3}}-\frac{1}{24a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-5a^{2}+4b^{2}}{8a^{3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b}{8a^{2}\tan\left(\frac{dx}{2}\right.}}{d}$
risch	$-\frac{-12ia^{2}e^{4i(dx+c)}+6ib^{2}e^{4i(dx+c)}+3ab\,e^{5i(dx+c)}+12ia^{2}e^{2i(dx+c)}-12ib^{2}e^{2i(dx+c)}-8ia^{2}+6ib^{2}-3ab\,e^{i(dx+c)}}{3d\,a^{3}\left(e^{2i(dx+c)}-1\right)^{3}}+\frac{\sqrt{-a^{2}}}{}$

input
```
int(cot(d*x+c)^4/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/a^3*(1/3*tan(1/2*d*x+1/2*c)^3*a^2-tan(1/2*d*x+1/2*c)^2*a*b-5*a^2*
tan(1/2*d*x+1/2*c)+4*tan(1/2*d*x+1/2*c)*b^2)-1/24/a/tan(1/2*d*x+1/2*c)^3-1
/8*(-5*a^2+4*b^2)/a^3/tan(1/2*d*x+1/2*c)+1/8/a^2*b/tan(1/2*d*x+1/2*c)^2+1/
2/a^4*b*(3*a^2-2*b^2)*ln(tan(1/2*d*x+1/2*c))+1/8/a^4*(16*a^4-32*a^2*b^2+16
*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1
/2)))
```

3.179.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 633, normalized size of antiderivative = 4.11

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \left[-\frac{6\,a^{2}b\cos\left(dx+c\right)\sin\left(dx+c\right)-4\left(4\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}+6\left(\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-a^{2}+b^{2}\right)\sqrt{\cdot}}{}\right.$$
$$-\frac{6\,a^{2}b\cos\left(dx+c\right)\sin\left(dx+c\right)-4\left(4\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}+12\left(\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}-a^{2}+b^{2}\right)\sqrt{}}{}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4*(4*a^3 - 3*a*b^2)*cos(d*x +
c)^3 + 6*((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2)*sqrt(-a^2 + b^2)*log(((2
*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x
 + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2
 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3
*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) +
 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
 c) + 1/2)*sin(d*x + c) + 12*(a^3 - a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x +
 c)^2 - a^4*d)*sin(d*x + c)), -1/12*(6*a^2*b*cos(d*x + c)*sin(d*x + c) - 4
*(4*a^3 - 3*a*b^2)*cos(d*x + c)^3 + 12*((a^2 - b^2)*cos(d*x + c)^2 - a^2 +
 b^2)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*
x + c)))*sin(d*x + c) - 3*(3*a^2*b - 2*b^3 - (3*a^2*b - 2*b^3)*cos(d*x + c
)^2)*log(1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 3*(3*a^2*b - 2*b^3 - (3*a^
2*b - 2*b^3)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 1
2*(a^3 - a*b^2)*cos(d*x + c))/((a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c)
)]
```

3.179.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cot^4{(c+dx)}}{a+b\sin{(c+dx)}}\,dx$$

input
```
integrate(cot(d*x+c)**4/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**4/(a + b*sin(c + d*x)), x)
```

3.179.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.179.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.77

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}+\frac{12\,(3\,a^2b-2\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^4}+\frac{48\,(a^4-2\,a^2}{}}{}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/24*((a^2*tan(1/2*d*x + 1/2*c)^3 - 3*a*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*
tan(1/2*d*x + 1/2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/a^3 + 12*(3*a^2*b - 2*
b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^4 + 48*(a^4 - 2*a^2*b^2 + b^4)*(pi*f
loor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/
sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^4) - (66*a^2*b*tan(1/2*d*x + 1/2*c)^3
 - 44*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^
2*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^4*tan(1/
2*d*x + 1/2*c)^3))/d
```

3.179.9 Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 654, normalized size of antiderivative = 4.25

$$\int \frac{\cot^4(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a\,d} - \frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{24\,a\,d} + \frac{5\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a\,d} - \frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8\,a\,d} + \frac{3\,b\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2\,a^2\,d} - \frac{b^3\,\ln\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a^4\,d} + \frac{b\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^2\,d} - \frac{b^2\cot\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^3\,d} - \frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^2\,d} + \frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^3\,d} + \frac{\operatorname{atan}\left(\frac{2\,a^5\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}+8\,b^5\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}-7\,a^3\,b^2\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{-a^6+3\,a^4\,b^2-3\,a}}{2i\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^8-5i\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^7\,b-16i\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^6\,b^2+14i\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)a^5\,b^3+}\right.}{}$$

input
```
int(cot(c + d*x)^4/(a + b*sin(c + d*x)),x)
```

output
```
tan(c/2 + (d*x)/2)^3/(24*a*d) - cot(c/2 + (d*x)/2)^3/(24*a*d) + (5*cot(c/2
 + (d*x)/2))/(8*a*d) - (5*tan(c/2 + (d*x)/2))/(8*a*d) + (3*b*log(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/(2*a^2*d) - (b^3*log(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/(a^4*d) + (b*cot(c/2 + (d*x)/2)^2)/(8*a^2*d) - (b^2*cot(
c/2 + (d*x)/2))/(2*a^3*d) - (b*tan(c/2 + (d*x)/2)^2)/(8*a^2*d) + (b^2*tan(
c/2 + (d*x)/2))/(2*a^3*d) + (atan((2*a^5*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3
*a^2*b^4 + 3*a^4*b^2)^(1/2) + 8*b^5*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*
b^4 + 3*a^4*b^2)^(1/2) - 7*a^3*b^2*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(1/2) - 16*a^2*b^3*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b
^4 + 3*a^4*b^2)^(1/2) + 4*a*b^4*cos(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2) + 7*a^4*b*sin(c/2 + (d*x)/2)*(b^6 - a^6 - 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(a^8*sin(c/2 + (d*x)/2)*2i + b^8*sin(c/2 + (d*x)/2)*8i +
a*b^7*cos(c/2 + (d*x)/2)*4i - a^7*b*cos(c/2 + (d*x)/2)*5i - a^3*b^5*cos(c/
2 + (d*x)/2)*13i + a^5*b^3*cos(c/2 + (d*x)/2)*14i - a^2*b^6*sin(c/2 + (d*x
)/2)*28i + a^4*b^4*sin(c/2 + (d*x)/2)*34i - a^6*b^2*sin(c/2 + (d*x)/2)*16i
))*(-(a + b)^3*(a - b)^3)^(1/2)*2i)/(a^4*d)
```

3.180 $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

3.180.1 Optimal result

Integrand size = 21, antiderivative size = 307

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = -\frac{2(a^2-b^2)^{5/2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6d} + \frac{b(15a^4-20a^2b^2+8b^4)\operatorname{arctanh}(\cos(c+dx))}{8a^6d} - \frac{(23a^4-35a^2b^2+15b^4)\cot(c+dx)}{15a^5d} - \frac{\cot(c+dx)\csc(c+dx)}{bd} + \frac{(8a^4-9a^2b^2+4b^4)\cot(c+dx)\csc(c+dx)}{8a^4bd} + \frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d} - \frac{(15a^4-22a^2b^2+10b^4)\cot(c+dx)\csc^2(c+dx)}{30a^3b^2d} + \frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d} - \frac{\cot(c+dx)\csc^4(c+dx)}{5ad}$$

output

```
-2*(a^2-b^2)^(5/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^6/d+
1/8*b*(15*a^4-20*a^2*b^2+8*b^4)*arctanh(cos(d*x+c))/a^6/d-1/15*(23*a^4-35*
a^2*b^2+15*b^4)*cot(d*x+c)/a^5/d-cot(d*x+c)*csc(d*x+c)/b/d+1/8*(8*a^4-9*a^
2*b^2+4*b^4)*cot(d*x+c)*csc(d*x+c)/a^4/b/d+1/2*a*cot(d*x+c)*csc(d*x+c)^2/b
^2/d-1/30*(15*a^4-22*a^2*b^2+10*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d+1/4
*b*cot(d*x+c)*csc(d*x+c)^3/a^2/d-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d
```

3.180.2 Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 504, normalized size of antiderivative = 1.64

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$= \frac{-1920(a^2-b^2)^{5/2}\arctan\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)-32(23a^5-35a^3b^2+15ab^4)\cot\left(\frac{1}{2}(c+dx)\right)-270a^4b\csc^2\left(\frac{1}{2}\right.}{}$$

input
```
Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

output
```
(-1920*(a^2 - b^2)^(5/2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
- 32*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*Cot[(c + d*x)/2] - 270*a^4*b*Csc[(c
+ d*x)/2]^2 + 120*a^2*b^3*Csc[(c + d*x)/2]^2 + 15*a^4*b*Csc[(c + d*x)/2]^4
 + 1800*a^4*b*Log[Cos[(c + d*x)/2]] - 2400*a^2*b^3*Log[Cos[(c + d*x)/2]] +
 960*b^5*Log[Cos[(c + d*x)/2]] - 1800*a^4*b*Log[Sin[(c + d*x)/2]] + 2400*a
^2*b^3*Log[Sin[(c + d*x)/2]] - 960*b^5*Log[Sin[(c + d*x)/2]] + 270*a^4*b*S
ec[(c + d*x)/2]^2 - 120*a^2*b^3*Sec[(c + d*x)/2]^2 - 15*a^4*b*Sec[(c + d*x
)/2]^4 - 656*a^5*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 + 320*a^3*b^2*Csc[c + d
*x]^3*Sin[(c + d*x)/2]^4 + 41*a^5*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 20*a^3
*b^2*Csc[(c + d*x)/2]^4*Sin[c + d*x] - 3*a^5*Csc[(c + d*x)/2]^6*Sin[c + d*
x] + 736*a^5*Tan[(c + d*x)/2] - 1120*a^3*b^2*Tan[(c + d*x)/2] + 480*a*b^4*
Tan[(c + d*x)/2] + 6*a^5*Sec[(c + d*x)/2]^4*Tan[(c + d*x)/2])/(960*a^6*d)
```

3.180.3 Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.12, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.905$, Rules used = {3042, 3205, 27, 3042, 3534, 25, 3042, 3534, 25, 3042, 3534, 27, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{1}{\tan(c+dx)^6(a+b\sin(c+dx))}dx$$

$$\Big\downarrow \; 3205$$

$$\frac{\int \frac{2\csc^4(c+dx)\left(-5\left(4a^4-4b^2a^2+3b^4\right)\sin^2(c+dx)-ab\left(10a^2-b^2\right)\sin(c+dx)+2\left(15a^4-22b^2a^2+10b^4\right)\right)}{a+b\sin(c+dx)}dx}{40a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\csc^4(c+dx)\left(-5\left(4a^4-4b^2a^2+3b^4\right)\sin^2(c+dx)-ab\left(10a^2-b^2\right)\sin(c+dx)+2\left(15a^4-22b^2a^2+10b^4\right)\right)}{a+b\sin(c+dx)}dx}{20a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{\int \frac{-5\left(4a^4-4b^2a^2+3b^4\right)\sin(c+dx)^2-ab\left(10a^2-b^2\right)\sin(c+dx)+2\left(15a^4-22b^2a^2+10b^4\right)}{\sin(c+dx)^4(a+b\sin(c+dx))}dx}{20a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{\int -\frac{\csc^3(c+dx)\left(a\left(28a^2+5b^2\right)\sin(c+dx)b^2-4\left(15a^4-22b^2a^2+10b^4\right)\sin^2(c+dx)b+15\left(8a^4-9b^2a^2+4b^4\right)b\right)}{a+b\sin(c+dx)}dx}{3a}-\frac{2\left(15a^4-22a^2b^2+10b^4\right)\cot(c+dx)\csc^2(c+
}{3ad}}{20a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\int \frac{\csc^3(c+dx)\left(a\left(28a^2+5b^2\right)\sin(c+dx)b^2-4\left(15a^4-22b^2a^2+10b^4\right)\sin^2(c+dx)b+15\left(8a^4-9b^2a^2+4b^4\right)b\right)}{a+b\sin(c+dx)}dx}{3a}-\frac{2\left(15a^4-22a^2b^2+10b^4\right)\cot(c+dx)\csc^2(c+
}{3ad}}{20a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 3042$$

3.180. $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

$$\frac{-\frac{\int \frac{a\left(28a^2+5b^2\right)\sin(c+dx)b^2-4\left(15a^4-22b^2a^2+10b^4\right)\sin(c+dx)^2b+15\left(8a^4-9b^2a^2+4b^4\right)b}{\sin(c+dx)^3(a+b\sin(c+dx))}dx}{3a}-\frac{2\left(15a^4-22a^2b^2+10b^4\right)\cot(c+dx)\csc^2(c+dx)}{3ad}}{20a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 3534$$

$$\frac{-\frac{\frac{\int -\frac{\csc^2(c+dx)\left(-a\left(41a^2-20b^2\right)\sin(c+dx)b^3-15\left(8a^4-9b^2a^2+4b^4\right)\sin^2(c+dx)b^2+8\left(23a^4-35b^2a^2+15b^4\right)b^2\right)}{a+b\sin(c+dx)}dx}{2a}-\frac{15b\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx\cdots}{2ad}}{3a}\cdots}{20a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{-\frac{\int \frac{\csc^2(c+dx)\left(-a\left(41a^2-20b^2\right)\sin(c+dx)b^3-15\left(8a^4-9b^2a^2+4b^4\right)\sin^2(c+dx)b^2+8\left(23a^4-35b^2a^2+15b^4\right)b^2\right)}{a+b\sin(c+dx)}dx}{2a}-\frac{15b\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx\cdots}{2ad}}{3a}\cdots}{20a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{-\frac{\int \frac{-a\left(41a^2-20b^2\right)\sin(c+dx)b^3-15\left(8a^4-9b^2a^2+4b^4\right)\sin(c+dx)^2b^2+8\left(23a^4-35b^2a^2+15b^4\right)b^2}{\sin(c+dx)^2(a+b\sin(c+dx))}dx}{2a}-\frac{15b\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{2ad}}{3a}-\frac{2\left(15a^4\cdots\right.}{\cdots}}{20a^2b^2}+\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 3534$$

3.180. $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

$$-\frac{-\frac{-\frac{\frac{\int -\frac{15\csc(c+dx)\left(\left(15a^4-20b^2a^2+8b^4\right)b^3+a\left(8a^4-9b^2a^2+4b^4\right)\sin(c+dx)b^2\right)}{a+b\sin(c+dx)}dx}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{ad}}{2a}-\frac{15b\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\,\mathrm{c}\cdots}{2ad}}{3a}\cdots}{20a^2b^2}$$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$\big\downarrow$ 27

$$-\frac{-\frac{-\frac{\frac{15\int \frac{\csc(c+dx)\left(\left(15a^4-20b^2a^2+8b^4\right)b^3+a\left(8a^4-9b^2a^2+4b^4\right)\sin(c+dx)b^2\right)}{a+b\sin(c+dx)}dx}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{ad}}{2a}-\frac{15b\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\,\mathrm{c}\cdots}{2ad}}{3a}\cdots}{20a^2b^2}$$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$\big\downarrow$ 3042

$$-\frac{-\frac{-\frac{\frac{15\int \frac{\left(15a^4-20b^2a^2+8b^4\right)b^3+a\left(8a^4-9b^2a^2+4b^4\right)\sin(c+dx)b^2}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{ad}}{2a}-\frac{15b\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx)\csc(c+dx)}{2ad}}{3a}-\cdots}{20a^2b^2}$$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$\big\downarrow$ 3480

$$-\frac{-\frac{-\frac{15\left(\frac{8b^2\left(a^2-b^2\right)^3\int\frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b^3\left(15a^4-20a^2b^2+8b^4\right)\int\csc(c+dx)dx}{a}\right)}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{ad}}{2a}-\frac{15b\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx\cdots}{2ad}}{3a}\cdots}{20a^2b^2}$$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$\big\downarrow$ 3042

3.180. $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

$$-\frac{-\frac{-\frac{15\left(\frac{8b^2\left(a^2-b^2\right)^3\int\frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b^3\left(15a^4-20a^2b^2+8b^4\right)\int\csc(c+dx)dx}{a}\right)}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{ad}}{2a}-\frac{15b\left(8a^4-9a^2b^2+4b^4\right)\cot(c+dx}{2ad}}{3a}$$

$$20a^2b^2$$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 3139$$

$$-\frac{-\frac{-\frac{15\left(\frac{16b^2\left(a^2-b^2\right)^3\int\frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}+\frac{b^3\left(15a^4-20a^2b^2+8b^4\right)\int\csc(c+dx)dx}{a}\right)}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b^4\right)\cot(c}{ad}}{2a}}{3a}$$

$$20a^2b^2$$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 1083$$

$$-\frac{-\frac{-\frac{15\left(\frac{b^3\left(15a^4-20a^2b^2+8b^4\right)\int\csc(c+dx)dx}{a}-\frac{32b^2\left(a^2-b^2\right)^3\int\frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b}{ad}}{2a}}{3a}$$

$$20a^2b^2$$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 217$$

$$-\frac{-\frac{-\frac{15\left(\frac{b^3\left(15a^4-20a^2b^2+8b^4\right)\int\csc(c+dx)dx}{a}+\frac{16b^2\left(a^2-b^2\right)^{5/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}\right)}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{ad}}{2a}-\frac{15b\left(8a^4-9a^2b^2\right.}{}}{3a}$$

$$20a^2b^2$$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

$$\Big\downarrow \; 4257$$

3.180. $\int\frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

$$\frac{b\cot(c+dx)\csc^3(c+dx)}{4a^2d}+\frac{-\frac{-\frac{-\frac{15\left(\frac{16b^2\left(a^2-b^2\right)^{5/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}-\frac{b^3\left(15a^4-20a^2b^2+8b^4\right)\text{arctanh}(\cos(c+dx))}{ad}\right)}{a}-\frac{8b^2\left(23a^4-35a^2b^2+15b^4\right)\cot(c+dx)}{ad}}{2a}-\frac{15b\left(8a \cdots\right.}{\cdots}}{3a}\cdots}{20a^2b^2}$$

$$\frac{a\cot(c+dx)\csc^2(c+dx)}{2b^2d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad}-\frac{\cot(c+dx)\csc(c+dx)}{bd}$$

input
```
Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]),x]
```

output
```
-((Cot[c + d*x]*Csc[c + d*x])/(b*d)) + (a*Cot[c + d*x]*Csc[c + d*x]^2)/(2*
b^2*d) + (b*Cot[c + d*x]*Csc[c + d*x]^3)/(4*a^2*d) - (Cot[c + d*x]*Csc[c +
 d*x]^4)/(5*a*d) + ((-2*(15*a^4 - 22*a^2*b^2 + 10*b^4)*Cot[c + d*x]*Csc[c
+ d*x]^2)/(3*a*d) - (-1/2*((-15*((16*b^2*(a^2 - b^2)^(5/2)*ArcTan[(2*b + 2
*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/(a*d) - (b^3*(15*a^4 - 20*a^2*b
^2 + 8*b^4)*ArcTanh[Cos[c + d*x]])/(a*d)))/a - (8*b^2*(23*a^4 - 35*a^2*b^2
 + 15*b^4)*Cot[c + d*x])/(a*d))/a - (15*b*(8*a^4 - 9*a^2*b^2 + 4*b^4)*Cot[
c + d*x]*Csc[c + d*x])/(2*a*d))/(3*a))/(20*a^2*b^2)
```

3.180.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3205
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*
Sin[e + f*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b
^2*f*m*(m - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*S
in[e + f*x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x] + Simp[1/(20*a^2*b^2*m*
(m - 1))   Int[((a + b*Sin[e + f*x])^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2
*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m -
 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m
 - 1)*(2*m + 1))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f, m}, x] &
& NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.180. $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.180.4 Maple [A] (verified)

Time = 2.15 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.27

method	result
derivativedivides	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{3}+\frac{4a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\,b^3\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{32a^5}}{}$
default	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{3}+\frac{4a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+8a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a\,b^3\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{32a^5}}{}$
risch	$-\frac{1120ia^4e^{4i(dx+c)}-480ib^4e^{2i(dx+c)}-720ia^4e^{6i(dx+c)}-135a^3b\,e^{9i(dx+c)}+60a\,b^3e^{9i(dx+c)}+360ia^4e^{8i(dx+c)}-360ia^2b^2}{}$

input
```
int(cot(d*x+c)^6/(a+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/32/a^5*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-1/2*b*tan(1/2*d*x+1/2*c)^4*a^3
-7/3*tan(1/2*d*x+1/2*c)^3*a^4+4/3*a^2*b^2*tan(1/2*d*x+1/2*c)^3+8*a^3*b*tan
(1/2*d*x+1/2*c)^2-4*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-3
6*a^2*b^2*tan(1/2*d*x+1/2*c)+16*b^4*tan(1/2*d*x+1/2*c))+1/32/a^6*(-64*a^6+
192*a^4*b^2-192*a^2*b^4+64*b^6)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*
x+1/2*c)+2*b)/(a^2-b^2)^(1/2))-1/160/a/tan(1/2*d*x+1/2*c)^5-1/96*(-7*a^2+4
*b^2)/a^3/tan(1/2*d*x+1/2*c)^3-1/32*(22*a^4-36*a^2*b^2+16*b^4)/a^5/tan(1/2
*d*x+1/2*c)+1/64/a^2*b/tan(1/2*d*x+1/2*c)^4-1/8/a^4*b*(2*a^2-b^2)/tan(1/2*
d*x+1/2*c)^2-1/8/a^6*b*(15*a^4-20*a^2*b^2+8*b^4)*ln(tan(1/2*d*x+1/2*c)))
```

3.180.5 Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 1079, normalized size of antiderivative = 3.51

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/240*(16*(23*a^5 - 35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 -
13*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^3 - 120*((a^4 - 2*a^2*b^2 + b^4)*cos(d*
x + c)^4 + a^4 - 2*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^
2)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)
 - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2
+ b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))*sin(d*x + c
) - 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*co
s(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*c
os(d*x + c) + 1/2)*sin(d*x + c) + 15*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*
a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*
b^5)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2)*sin(d*x + c) + 240*(a^5
- 2*a^3*b^2 + a*b^4)*cos(d*x + c) - 30*((9*a^4*b - 4*a^2*b^3)*cos(d*x + c)
^3 - (7*a^4*b - 4*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^6*d*cos(d*x + c
)^4 - 2*a^6*d*cos(d*x + c)^2 + a^6*d)*sin(d*x + c)), -1/240*(16*(23*a^5 -
35*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5 - 80*(7*a^5 - 13*a^3*b^2 + 6*a*b^4)*
cos(d*x + c)^3 - 240*((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2*a^2
*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*arc
tan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))*sin(d*x + c) - 1
5*(15*a^4*b - 20*a^2*b^3 + 8*b^5 + (15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x
 + c)^4 - 2*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*cos(d*x + c)^2)*log(1/2*cos...
```

3.180.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = \int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$$

input
```
integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)),x)
```

output
```
Integral(cot(c + d*x)**6/(a + b*sin(c + d*x)), x)
```

3.180.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.180.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.60

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$$

$$= \frac{\frac{6a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-15a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-70a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+40a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+240a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-120ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^5}}{}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)),x, algorithm="giac")
```

output
```
1/960*((6*a^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 7
0*a^4*tan(1/2*d*x + 1/2*c)^3 + 40*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^3
*b*tan(1/2*d*x + 1/2*c)^2 - 120*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 660*a^4*tan
(1/2*d*x + 1/2*c) - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*b^4*tan(1/2*d*
x + 1/2*c))/a^5 - 120*(15*a^4*b - 20*a^2*b^3 + 8*b^5)*log(abs(tan(1/2*d*x
+ 1/2*c)))/a^6 - 1920*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 -
b^2)))/(sqrt(a^2 - b^2)*a^6) + (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5480*a
^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 2192*b^5*tan(1/2*d*x + 1/2*c)^5 - 660*a^5*
tan(1/2*d*x + 1/2*c)^4 + 1080*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 480*a*b^4*t
an(1/2*d*x + 1/2*c)^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^3*tan
(1/2*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 40*a^3*b^2*tan(1/2*d
*x + 1/2*c)^2 + 15*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^6*tan(1/2*d*x +
1/2*c)^5))/d
```

3.180.9 Mupad [B] (verification not implemented)

Time = 6.77 (sec) , antiderivative size = 1099, normalized size of antiderivative = 3.58

$$\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^6/(a + b*sin(c + d*x)),x)
```

output
```
tan(c/2 + (d*x)/2)^5/(160*a*d) + (tan(c/2 + (d*x)/2)^2*(b/(32*a^2) + (b*(7
/(32*a) - b^2/(8*a^3)))/a))/d - (tan(c/2 + (d*x)/2)*(b^2/(8*a^3) - 11/(16*
a) + (2*b*(b/(16*a^2) + (2*b*(7/(32*a) - b^2/(8*a^3)))/a))/a))/d - (tan(c/
2 + (d*x)/2)^3*(7/(96*a) - b^2/(24*a^3)))/d - (b*tan(c/2 + (d*x)/2)^4)/(64
*a^2*d) - (log(tan(c/2 + (d*x)/2))*((15*a^4*b)/8 + b^5 - (5*a^2*b^3)/2))/(
a^6*d) + (tan(c/2 + (d*x)/2)^2*((7*a^4)/3 - (4*a^2*b^2)/3) - a^4/5 - tan(c
/2 + (d*x)/2)^4*(22*a^4 + 16*b^4 - 36*a^2*b^2) + tan(c/2 + (d*x)/2)^3*(4*a
*b^3 - 8*a^3*b) + (a^3*b*tan(c/2 + (d*x)/2))/2)/(32*a^5*d*tan(c/2 + (d*x)/
2)^5) - (atan((((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^
8*b^4 - 39*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 -
 32*a^10*b^2))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*
x)/2)*(31*a^10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6
 + ((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16*a^6*b^6 + 44*a^8*b^4 - 39*a
^10*b^2)/(4*a^10) - ((2*a^2*b - (tan(c/2 + (d*x)/2)*(24*a^12 - 32*a^10*b^2
))/(4*a^9))*(-(a + b)^5*(a - b)^5)^(1/2))/a^6 + (tan(c/2 + (d*x)/2)*(31*a^
10*b - 32*a^4*b^7 + 96*a^6*b^5 - 98*a^8*b^3))/(4*a^9))*1i)/a^6)/((15*a^10*
b - 8*b^11 + 44*a^2*b^9 - 99*a^4*b^7 + 113*a^6*b^5 - 65*a^8*b^3)/(2*a^10)
+ (tan(c/2 + (d*x)/2)*(16*a^10 - 8*b^10 + 42*a^2*b^8 - 94*a^4*b^6 + 110*a^
6*b^4 - 66*a^8*b^2))/(2*a^9) - ((-(a + b)^5*(a - b)^5)^(1/2)*((8*a^12 - 16
*a^6*b^6 + 44*a^8*b^4 - 39*a^10*b^2)/(4*a^10) + ((2*a^2*b - (tan(c/2 + ...
```

3.180. $\int \frac{\cot^6(c+dx)}{a+b\sin(c+dx)}\,dx$

3.181 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.181.1 Optimal result

Integrand size = 21, antiderivative size = 242

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = & -\frac{a(4a+b)\log(1-\sin(c+dx))}{8(a+b)^4d} \\ & -\frac{a(4a-b)\log(1+\sin(c+dx))}{8(a-b)^4d} \\ & +\frac{a^4(a^2+5b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^4\,d} \\ & -\frac{a^5}{(a^2-b^2)^3\,d(a+b\sin(c+dx))} \\ & +\frac{\sec^4(c+dx)\,(a^2+b^2-2ab\sin(c+dx))}{4\,(a^2-b^2)^2\,d} \\ & -\frac{\sec^2(c+dx)\,(2(2a^4+3a^2b^2-b^4)-ab(9a^2-b^2)\sin(c+dx))}{4\,(a^2-b^2)^3\,d}\end{aligned}$$

output
```
-1/8*a*(4*a+b)*ln(1-sin(d*x+c))/(a+b)^4/d-1/8*a*(4*a-b)*ln(1+sin(d*x+c))/(
a-b)^4/d+a^4*(a^2+5*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d-a^5/(a^2-b^2)^3/
d/(a+b*sin(d*x+c))+1/4*sec(d*x+c)^4*(a^2+b^2-2*a*b*sin(d*x+c))/(a^2-b^2)^2
/d-1/4*sec(d*x+c)^2*(4*a^4+6*a^2*b^2-2*b^4-a*b*(9*a^2-b^2)*sin(d*x+c))/(a^
2-b^2)^3/d
```

3.181.2 Mathematica [A] (verified)

Time = 4.11 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.84

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{-\frac{2a(4a+b)\log(1-\sin(c+dx))}{(a+b)^4} - \frac{2a(4a-b)\log(1+\sin(c+dx))}{(a-b)^4} + \frac{16a^4(a^2+5b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^4} + \frac{1}{(a+b)^2(-1+\sin(c+dx))^2} + \frac{}{(a+b)^3(}}{16d}$$

input
```
Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

output
```
((-2*a*(4*a + b)*Log[1 - Sin[c + d*x]])/(a + b)^4 - (2*a*(4*a - b)*Log[1 +
 Sin[c + d*x]])/(a - b)^4 + (16*a^4*(a^2 + 5*b^2)*Log[a + b*Sin[c + d*x]])
/(a^2 - b^2)^4 + 1/((a + b)^2*(-1 + Sin[c + d*x])^2) + (7*a + 3*b)/((a + b
)^3*(-1 + Sin[c + d*x])) + 1/((a - b)^2*(1 + Sin[c + d*x])^2) + (-7*a + 3*
b)/((a - b)^3*(1 + Sin[c + d*x])) - (16*a^5)/((a^2 - b^2)^3*(a + b*Sin[c +
 d*x])))/(16*d)
```

3.181.3 Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3200, 601, 27, 2178, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\tan(c+dx)^5}{(a+b\sin(c+dx))^2}dx$$
$$\Big\downarrow \text{ 3200}$$
$$\frac{\int \frac{b^5\sin^5(c+dx)}{(a+b\sin(c+dx))^2(b^2-b^2\sin^2(c+dx))^3}d(b\sin(c+dx))}{d}$$
$$\Big\downarrow \text{ 601}$$

$$\frac{\frac{b^4(a^2-2ab\sin(c+dx)+b^2)}{4(a^2-b^2)^2\left(b^2-b^2\sin^2(c+dx)\right)^2}-\frac{\int -\frac{2\left(-\frac{3a\sin^2(c+dx)b^8}{\left(a^2-b^2\right)^2}+\frac{a^3b^6}{\left(a^2-b^2\right)^2}-2\sin^3(c+dx)b^5-\frac{2a^4\sin(c+dx)b^5}{\left(a^2-b^2\right)^2}\right)}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{4b^2}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{-\frac{3a\sin^2(c+dx)b^8}{\left(a^2-b^2\right)^2}+\frac{a^3b^6}{\left(a^2-b^2\right)^2}-2\sin^3(c+dx)b^5-\frac{2a^4\sin(c+dx)b^5}{\left(a^2-b^2\right)^2}}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{2b^2}+\frac{b^4(a^2-2ab\sin(c+dx)+b^2)}{4(a^2-b^2)^2\left(b^2-b^2\sin^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 2178$$

$$\frac{\frac{\frac{\int -\frac{-\frac{a\left(9a^2-b^2\right)\sin^2(c+dx)b^8}{\left(a^2-b^2\right)^3}+\frac{a^3\left(7a^2+b^2\right)b^6}{\left(a^2-b^2\right)^3}-\frac{2a^2\left(2a^2+b^2\right)\sin(c+dx)b^5}{\left(a^2-b^2\right)^2}}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2b^2}-\frac{b^4\left(2\left(2a^4+3a^2b^2-b^4\right)-ab\left(9a^2-b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3\left(b^2-b^2\sin^2(c+dx)\right)}}{2b^2}+\frac{b^4(a^2-2\ldots}{4(a^2-b^2)^2(\ldots}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{-\frac{\int \frac{-\frac{a\left(9a^2-b^2\right)\sin^2(c+dx)b^8}{\left(a^2-b^2\right)^3}+\frac{a^3\left(7a^2+b^2\right)b^6}{\left(a^2-b^2\right)^3}-\frac{2a^2\left(2a^2+b^2\right)\sin(c+dx)b^5}{\left(a^2-b^2\right)^2}}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2b^2}-\frac{b^4\left(2\left(2a^4+3a^2b^2-b^4\right)-ab\left(9a^2-b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3\left(b^2-b^2\sin^2(c+dx)\right)}}{2b^2}+\frac{b^4(a^2-2\ldots}{4(a^2-b^2)^2\ldots}}{d}$$

$$\Big\downarrow \; 2160$$

$$\frac{\frac{-\frac{\int \left(-\frac{4b^4a^5}{(a-b)^3(a+b)^3(a+b\sin(c+dx))^2}-\frac{4b^4\left(a^2+5b^2\right)a^4}{(a-b)^4(a+b)^4(a+b\sin(c+dx))}-\frac{b^4(4a+b)a}{2(a+b)^4(b-b\sin(c+dx))}+\frac{(4a-b)b^4a}{2(a-b)^4(\sin(c+dx)b+b)}\right)d(b\sin(c+dx))}{2b^2}-\frac{b^4\left(2\left(2a^4+3a^2b^2-\ldots\right.\right.}{2\left(a^2-b^2\right)\ldots}}{2b^2}}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{b^4(a^2-2ab\sin(c+dx)+b^2)}{4(a^2-b^2)^2\left(b^2-b^2\sin^2(c+dx)\right)^2}+\frac{-\frac{b^4\left(2\left(2a^4+3a^2b^2-b^4\right)-ab\left(9a^2-b^2\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^3\left(b^2-b^2\sin^2(c+dx)\right)}-\frac{\frac{4a^5b^4}{\left(a^2-b^2\right)^3(a+b\sin(c+dx))}-\frac{4a^4b^4\left(a^2+5b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^4}+a\ldots}{2b^2}}{2b^2}}{d}$$

input
```
Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

3.181. $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

output
```
((b^4*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(4*(a^2 - b^2)^2*(b^2 - b^2*Sin[c
+ d*x]^2)^2) + (-1/2*(b^4*(2*(2*a^4 + 3*a^2*b^2 - b^4) - a*b*(9*a^2 - b^2)
*Sin[c + d*x]))/((a^2 - b^2)^3*(b^2 - b^2*Sin[c + d*x]^2)) - ((a*b^4*(4*a
+ b)*Log[b - b*Sin[c + d*x]])/(2*(a + b)^4) - (4*a^4*b^4*(a^2 + 5*b^2)*Log
[a + b*Sin[c + d*x]])/(a^2 - b^2)^4 + (a*(4*a - b)*b^4*Log[b + b*Sin[c + d
*x]])/(2*(a - b)^4) + (4*a^5*b^4)/((a^2 - b^2)^3*(a + b*Sin[c + d*x])))/(2
*b^2))/(2*b^2))/d
```

3.181.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.181. $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.181.4 Maple [A] (verified)

Time = 2.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{1}{16(a-b)^2(1+\sin(dx+c))^2}-\frac{-3b+7a}{16(a-b)^3(1+\sin(dx+c))}-\frac{a(4a-b)\ln(1+\sin(dx+c))}{8(a-b)^4}+\frac{1}{16(a+b)^2(\sin(dx+c)-1)^2}-\frac{-3b-7a}{16(a+b)^3(\sin(dx+c)-}}{d}$
default	$\frac{\frac{1}{16(a-b)^2(1+\sin(dx+c))^2}-\frac{-3b+7a}{16(a-b)^3(1+\sin(dx+c))}-\frac{a(4a-b)\ln(1+\sin(dx+c))}{8(a-b)^4}+\frac{1}{16(a+b)^2(\sin(dx+c)-1)^2}-\frac{-3b-7a}{16(a+b)^3(\sin(dx+c)-}}{d}$
risch	Expression too large to display

input
```
int(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/16/(a-b)^2/(1+sin(d*x+c))^2-1/16*(-3*b+7*a)/(a-b)^3/(1+sin(d*x+c))-
1/8*a*(4*a-b)/(a-b)^4*ln(1+sin(d*x+c))+1/16/(a+b)^2/(sin(d*x+c)-1)^2-1/16*
(-3*b-7*a)/(a+b)^3/(sin(d*x+c)-1)-1/8*a*(4*a+b)/(a+b)^4*ln(sin(d*x+c)-1)-a
^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))+a^4*(a^2+5*b^2)/(a+b)^4/(a-b)^4*ln(a+b
*sin(d*x+c)))
```

3.181.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(235) = 470$.

Time = 0.48 (sec) , antiderivative size = 555, normalized size of antiderivative = 2.29

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{2\,a^7 - 6\,a^5b^2 + 6\,a^3b^4 - 2\,ab^6 - 2\left(4\,a^7 + 5\,a^5b^2 - 10\,a^3b^4 + ab^6\right)\cos\left(dx+c\right)^4 - 2\left(4\,a^7 - 9\,a^5b^2 + 6\,a^3b^4 - \right.}{}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/8*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 2*(4*a^7 + 5*a^5*b^2 - 10*a
^3*b^4 + a*b^6)*cos(d*x + c)^4 - 2*(4*a^7 - 9*a^5*b^2 + 6*a^3*b^4 - a*b^6)
*cos(d*x + c)^2 + 8*((a^6*b + 5*a^4*b^3)*cos(d*x + c)^4*sin(d*x + c) + (a^
7 + 5*a^5*b^2)*cos(d*x + c)^4)*log(b*sin(d*x + c) + a) - ((4*a^6*b + 15*a^
5*b^2 + 20*a^4*b^3 + 10*a^3*b^4 - a*b^6)*cos(d*x + c)^4*sin(d*x + c) + (4*
a^7 + 15*a^6*b + 20*a^5*b^2 + 10*a^4*b^3 - a^2*b^5)*cos(d*x + c)^4)*log(si
n(d*x + c) + 1) - ((4*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6
)*cos(d*x + c)^4*sin(d*x + c) + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^
3 + a^2*b^5)*cos(d*x + c)^4)*log(-sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3
 + 3*a^2*b^5 - b^7 - (5*a^6*b - 12*a^4*b^3 + 9*a^2*b^5 - 2*b^7)*cos(d*x +
c)^2)*sin(d*x + c))/((a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*c
os(d*x + c)^4*sin(d*x + c) + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*
b^8)*d*cos(d*x + c)^4)
```

3.181.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\tan^5\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

3.181.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(235) = 470$.

Time = 0.25 (sec) , antiderivative size = 505, normalized size of antiderivative = 2.09

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{8\,(a^6+5\,a^4b^2)\log(b\sin(dx+c)+a)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8} - \frac{(4\,a^2-ab)\log(\sin(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} - \frac{(4\,a^2+ab)\log(\sin(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{2\left(7\,a^5\right.}{a^7-3\,a^5b^2+3\,a^3b^4-ab^6+(a^6b-3\,a^4b^3+}}{}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*(8*(a^6 + 5*a^4*b^2)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b^2 + 6*a^4*
b^4 - 4*a^2*b^6 + b^8) - (4*a^2 - a*b)*log(sin(d*x + c) + 1)/(a^4 - 4*a^3*
b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(sin(d*x + c) - 1)/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(7*a^5 + 6*a^3*b^2 - a*b^4 + (4
*a^5 + 9*a^3*b^2 - a*b^4)*sin(d*x + c)^4 + (5*a^4*b - 7*a^2*b^3 + 2*b^5)*s
in(d*x + c)^3 - (12*a^5 + 13*a^3*b^2 - a*b^4)*sin(d*x + c)^2 - (4*a^4*b -
5*a^2*b^3 + b^5)*sin(d*x + c))/(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 + (a^6
*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)^5 + (a^7 - 3*a^5*b^2 + 3*a^
3*b^4 - a*b^6)*sin(d*x + c)^4 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*si
n(d*x + c)^3 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^2 + (a
^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*sin(d*x + c)))/d
```

3.181.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(235) = 470$.

Time = 1.60 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.04

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{8\,(a^6b+5\,a^4b^3)\log(|b\sin(dx+c)+a|)}{a^8b-4\,a^6b^3+6\,a^4b^5-4\,a^2b^7+b^9} - \frac{(4\,a^2-ab)\log(|\sin(dx+c)+1|)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4} - \frac{(4\,a^2+ab)\log(|\sin(dx+c)-1|)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{8\,(a^6b\sin(dx+c)+5\,a^4b^3\sin(dx+c}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)(b}}{}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/8*(8*(a^6*b + 5*a^4*b^3)*log(abs(b*sin(d*x + c) + a))/(a^8*b - 4*a^6*b^3
 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (4*a^2 - a*b)*log(abs(sin(d*x + c) + 1))
/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (4*a^2 + a*b)*log(abs(sin(d
*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^6*b*sin(d
*x + c) + 5*a^4*b^3*sin(d*x + c) + 2*a^7 + 4*a^5*b^2)/((a^8 - 4*a^6*b^2 +
6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c) + a)) + 2*(3*a^6*sin(d*x + c)
^4 + 15*a^4*b^2*sin(d*x + c)^4 - 9*a^5*b*sin(d*x + c)^3 + 10*a^3*b^3*sin(d
*x + c)^3 - a*b^5*sin(d*x + c)^3 - 2*a^6*sin(d*x + c)^2 - 28*a^4*b^2*sin(d
*x + c)^2 - 8*a^2*b^4*sin(d*x + c)^2 + 2*b^6*sin(d*x + c)^2 + 7*a^5*b*sin(
d*x + c) - 6*a^3*b^3*sin(d*x + c) - a*b^5*sin(d*x + c) + 12*a^4*b^2 + 7*a^
2*b^4 - b^6)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c
)^2 - 1)^2))/d
```

3.181.9 Mupad [B] (verification not implemented)

Time = 7.71 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.12

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,(a^6+5\,a^4\,b^2)}{d\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,\left(\frac{1}{(a+b)^2}-\frac{7\,b}{4\,(a+b)^3}+\frac{3\,b^2}{4\,(a+b)^4}\right)}{d}$$
$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,\left(\frac{3\,b^2}{4\,(a-b)^4}+\frac{7\,b}{4\,(a-b)^3}+\frac{1}{(a-b)^2}\right)}{d}$$
$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,(2\,a^3+a\,b^2)}{a^4-2\,a^2\,b^2+b^4}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(a\,b^2-2\,a^3)}{a^4-2\,a^2\,b^2+b^4}+\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,(a\,b^2-2\,a^3)}{a^4-2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\,(2\,a^3+a\,b^2)}{a^4-2\,a^2\,b^2+b^4}-\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,(a^6-3\,a^4\,b^2}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9-3\,a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-8\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+2\,a\tan\left(\frac{c}{2}+\right.\right.}$$

input
```
int(tan(c + d*x)^5/(a + b*sin(c + d*x))^2,x)
```

output
```
(log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^6 + 5*a^4*b^2
))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (log(tan(c/2 + (d
*x)/2) - 1)*(1/(a + b)^2 - (7*b)/(4*(a + b)^3) + (3*b^2)/(4*(a + b)^4)))/d
 - (log(tan(c/2 + (d*x)/2) + 1)*((3*b^2)/(4*(a - b)^4) + (7*b)/(4*(a - b)^
3) + 1/(a - b)^2))/d - ((tan(c/2 + (d*x)/2)^2*(a*b^2 + 2*a^3))/(a^4 + b^4
- 2*a^2*b^2) + (3*tan(c/2 + (d*x)/2)^4*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^2
*b^2) + (3*tan(c/2 + (d*x)/2)^6*(a*b^2 - 2*a^3))/(a^4 + b^4 - 2*a^2*b^2) +
 (tan(c/2 + (d*x)/2)^8*(a*b^2 + 2*a^3))/(a^4 + b^4 - 2*a^2*b^2) - (b*tan(c
/2 + (d*x)/2)^9*(11*a^4 + a^2*b^2))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)
) + (8*b*tan(c/2 + (d*x)/2)^3*(2*a^4 + a^2*b^2))/((a^2 - b^2)*(a^4 + b^4 -
 2*a^2*b^2)) + (8*b*tan(c/2 + (d*x)/2)^7*(2*a^4 + a^2*b^2))/((a^2 - b^2)*(
a^4 + b^4 - 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^5*(13*a^4 - 8*b^4 + 31*a^2
*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)*(11*a
^4 + a^2*b^2))/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(a + 2*b*tan(c/
2 + (d*x)/2) - 3*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^4 + 2*a*t
an(c/2 + (d*x)/2)^6 - 3*a*tan(c/2 + (d*x)/2)^8 + a*tan(c/2 + (d*x)/2)^10 -
 8*b*tan(c/2 + (d*x)/2)^3 + 12*b*tan(c/2 + (d*x)/2)^5 - 8*b*tan(c/2 + (d*x
)/2)^7 + 2*b*tan(c/2 + (d*x)/2)^9))
```

3.181. $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.182 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.182.1 Optimal result

Integrand size = 21, antiderivative size = 161

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{a\log(1-\sin(c+dx))}{2(a+b)^3d} + \frac{a\log(1+\sin(c+dx))}{2(a-b)^3d} - \frac{a^2(a^2+3b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d} + \frac{a^3}{(a^2-b^2)^2\,d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\,(a^2+b^2-2ab\sin(c+dx))}{2\,(a^2-b^2)^2\,d}$$

output
```
1/2*a*ln(1-sin(d*x+c))/(a+b)^3/d+1/2*a*ln(1+sin(d*x+c))/(a-b)^3/d-a^2*(a^2
+3*b^2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d+a^3/(a^2-b^2)^2/d/(a+b*sin(d*x+c)
)+1/2*sec(d*x+c)^2*(a^2+b^2-2*a*b*sin(d*x+c))/(a^2-b^2)^2/d
```

3.182.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{2a\log(1-\sin(c+dx))}{(a+b)^3} + \frac{2a\log(1+\sin(c+dx))}{(a-b)^3} - \frac{4a^2(a^2+3b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^3} - \frac{1}{(a+b)^2(-1+\sin(c+dx))} + \frac{1}{(a-b)^2(1+\sin(c+dx))} + \cdots}{4d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

output
```
((2*a*Log[1 - Sin[c + d*x]])/(a + b)^3 + (2*a*Log[1 + Sin[c + d*x]])/(a -
b)^3 - (4*a^2*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^3 - 1/((a
 + b)^2*(-1 + Sin[c + d*x])) + 1/((a - b)^2*(1 + Sin[c + d*x])) + (4*a^3)/
((a^2 - b^2)^2*(a + b*Sin[c + d*x])))/(4*d)
```

3.182.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3200, 601, 27, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^3}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b^3\sin^3(c+dx)}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{601}$$

$$\frac{\frac{b^2\left(a^2-2ab\sin(c+dx)+b^2\right)}{2(a^2-b^2)^2\left(b^2-b^2\sin^2(c+dx)\right)}-\frac{\int -\frac{2\left(-\frac{a\sin^2(c+dx)b^6}{\left(a^2-b^2\right)^2}+\frac{a^3b^4}{\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)b^3}{a^2-b^2}\right)}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2b^2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-\frac{a\sin^2(c+dx)b^6}{\left(a^2-b^2\right)^2}+\frac{a^3b^4}{\left(a^2-b^2\right)^2}-\frac{a^2\sin(c+dx)b^3}{a^2-b^2}}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{b^2}+\frac{b^2\left(a^2-2ab\sin(c+dx)+b^2\right)}{2(a^2-b^2)^2\left(b^2-b^2\sin^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{2160}$$

$$\frac{\frac{\int \left(-\frac{b^2a^3}{(a-b)^2(a+b)^2(a+b\sin(c+dx))^2}-\frac{b^2\left(a^2+3b^2\right)a^2}{(a-b)^3(a+b)^3(a+b\sin(c+dx))}-\frac{b^2a}{2(a+b)^3(b-b\sin(c+dx))}+\frac{b^2a}{2(a-b)^3(\sin(c+dx)b+b)}\right)d(b\sin(c+dx))}{b^2}+\frac{b^2\left(a^2-2ab\right.}{2(a^2-b^2)^2\left(b^2\right.}}{d}$$

$\downarrow$ 2009

$$\frac{\frac{b^2\left(a^2-2ab\sin(c+dx)+b^2\right)}{2(a^2-b^2)^2\left(b^2-b^2\sin^2(c+dx)\right)}+\frac{-\frac{a^2b^2\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^3}+\frac{a^3b^2}{\left(a^2-b^2\right)^2(a+b\sin(c+dx))}+\frac{ab^2\log(b-b\sin(c+dx))}{2(a+b)^3}+\frac{ab^2\log(b\sin(c+dx)+b)}{2(a-b)^3}}{b^2}}{d}$$

input
```
Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

output
```
((b^2*(a^2 + b^2 - 2*a*b*Sin[c + d*x]))/(2*(a^2 - b^2)^2*(b^2 - b^2*Sin[c
+ d*x]^2)) + ((a*b^2*Log[b - b*Sin[c + d*x]])/(2*(a + b)^3) - (a^2*b^2*(a^
2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^3 + (a*b^2*Log[b + b*Sin[c
 + d*x]])/(2*(a - b)^3) + (a^3*b^2)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])))/
b^2)/d
```

3.182.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.182.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

method	result
derivativedivides	$\frac{\frac{1}{4(a-b)^2(1+\sin(dx+c))}+\frac{a\ln(1+\sin(dx+c))}{2(a-b)^3}-\frac{1}{4(a+b)^2(\sin(dx+c)-1)}+\frac{a\ln(\sin(dx+c)-1)}{2(a+b)^3}+\frac{a^3}{(a+b)^2(a-b)^2(a+b\sin(dx+c))}-\frac{a^2(a^2 \cdots}{\cdots}}{d}$
default	$\frac{\frac{1}{4(a-b)^2(1+\sin(dx+c))}+\frac{a\ln(1+\sin(dx+c))}{2(a-b)^3}-\frac{1}{4(a+b)^2(\sin(dx+c)-1)}+\frac{a\ln(\sin(dx+c)-1)}{2(a+b)^3}+\frac{a^3}{(a+b)^2(a-b)^2(a+b\sin(dx+c))}-\frac{a^2(a^2 \cdots}{\cdots}}{d}$
risch	$-\frac{iax}{a^3-3a^2b+3ab^2-b^3}-\frac{iac}{d(a^3-3a^2b+3ab^2-b^3)}-\frac{iax}{a^3+3a^2b+3ab^2+b^3}-\frac{iac}{d(a^3+3a^2b+3ab^2+b^3)}+\frac{2ia^4x \cdots}{a^6-3a^4b^2+3 \cdots}$

input
```
int(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/(a-b)^2/(1+sin(d*x+c))+1/2*a/(a-b)^3*ln(1+sin(d*x+c))-1/4/(a+b)^2
/(sin(d*x+c)-1)+1/2*a/(a+b)^3*ln(sin(d*x+c)-1)+a^3/(a+b)^2/(a-b)^2/(a+b*si
n(d*x+c))-a^2*(a^2+3*b^2)/(a-b)^3/(a+b)^3*ln(a+b*sin(d*x+c)))
```

3.182.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(159) = 318$.

Time = 0.35 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.41

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{a^5 - 2\,a^3b^2 + ab^4 + 2\,(a^5 - ab^4)\cos{(dx+c)}^2 - 2\,((a^4b + 3\,a^2b^3)\cos{(dx+c)}^2\sin{(dx+c)} + (a^5 + 3\,a^3b^2)}{}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(a^5 - 2*a^3*b^2 + a*b^4 + 2*(a^5 - a*b^4)*cos(d*x + c)^2 - 2*((a^4*b
+ 3*a^2*b^3)*cos(d*x + c)^2*sin(d*x + c) + (a^5 + 3*a^3*b^2)*cos(d*x + c)^
2)*log(b*sin(d*x + c) + a) + ((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(
d*x + c)^2*sin(d*x + c) + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cos(d*x +
c)^2)*log(sin(d*x + c) + 1) + ((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*cos
(d*x + c)^2*sin(d*x + c) + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*cos(d*x +
 c)^2)*log(-sin(d*x + c) + 1) - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c))/((
a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)^2*sin(d*x + c) + (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^2)
```

3.182.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\tan^3{(c+dx)}}{(a+b\sin{(c+dx)})^2}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

3.182.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.70

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\frac{2\,(a^4+3\,a^2b^2)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{a\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{a\log(\sin(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a^3+ab^2-2\,(a^3+ab^2)\sin(\cdots}{a^5-2\,a^3b^2+ab^4-(a^4b-2\,a^2b^3+b^5)\sin(dx+c)^3-(a^5-\cdots}}{2\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
-1/2*(2*(a^4 + 3*a^2*b^2)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6) - a*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - a*
log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3 + a*b^2 - 2
*(a^3 + a*b^2)*sin(d*x + c)^2 - (a^2*b - b^3)*sin(d*x + c))/(a^5 - 2*a^3*b
^2 + a*b^4 - (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)^3 - (a^5 - 2*a^3*b^2 +
 a*b^4)*sin(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*sin(d*x + c)))/d
```

3.182.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.54

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\frac{2\,(a^4b+3\,a^2b^3)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{a\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{a\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\,a^3\sin(dx+c)^2+2\,ab^2\sin(dx+c)^2+a^2b\sin(dx+\cdots}{(a^4-2\,a^2b^2+b^4)\left(b\sin(dx+c)^3+a\sin(dx+\cdots\right.}}{2\,d}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(2*(a^4*b + 3*a^2*b^3)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^
3 + 3*a^2*b^5 - b^7) - a*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - a*log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) -
 (2*a^3*sin(d*x + c)^2 + 2*a*b^2*sin(d*x + c)^2 + a^2*b*sin(d*x + c) - b^3
*sin(d*x + c) - 3*a^3 - a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c)^3
+ a*sin(d*x + c)^2 - b*sin(d*x + c) - a)))/d
```

3.182.9 Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.18

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2-b^2}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^2-b^2}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2+b^2)}{(a^2-b^2)^2}-\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^4-2\,a^2\,b^2+b^4}-\frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{(a^2-b^2)^2}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\right.}$$
$$+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,(a-b)^3}-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,(a^4+3\,a^2\,b^2)}{d\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}$$
$$+\frac{a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,(a+b)^3}$$

input
```
int(tan(c + d*x)^3/(a + b*sin(c + d*x))^2,x)
```

output
```
((2*a*tan(c/2 + (d*x)/2)^2)/(a^2 - b^2) + (2*a*tan(c/2 + (d*x)/2)^4)/(a^2
- b^2) + (4*b*tan(c/2 + (d*x)/2)^3*(a^2 + b^2))/(a^2 - b^2)^2 - (4*a^2*b*t
an(c/2 + (d*x)/2)^5)/(a^4 + b^4 - 2*a^2*b^2) - (4*a^2*b*tan(c/2 + (d*x)/2)
)/(a^2 - b^2)^2)/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2 -
 a*tan(c/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6 - 4*b*tan(c/2 + (d*x)/2)^
3 + 2*b*tan(c/2 + (d*x)/2)^5)) + (a*log(tan(c/2 + (d*x)/2) + 1))/(d*(a - b
)^3) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4 + 3*
a^2*b^2))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (a*log(tan(c/2 + (d*x)
/2) - 1))/(d*(a + b)^3)
```

3.183 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.183.1 Optimal result

Integrand size = 19, antiderivative size = 109

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} - \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{(a^2+b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^2\,d} - \frac{a}{(a^2-b^2)\,d(a+b\sin(c+dx))}$$

output
```
-1/2*ln(1-sin(d*x+c))/(a+b)^2/d-1/2*ln(1+sin(d*x+c))/(a-b)^2/d+(a^2+b^2)*l
n(a+b*sin(d*x+c))/(a^2-b^2)^2/d-a/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

3.183.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.49

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{a((a-b)^2\log(1-\sin(c+dx))+(a+b)^2\log(1+\sin(c+dx))-2(-a^2+b^2+(a^2+b^2)\log(a+b\sin(c}{2(a-b)^2}$$

input
```
Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

output
```
-1/2*(a*((a - b)^2*Log[1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]]
 - 2*(-a^2 + b^2 + (a^2 + b^2)*Log[a + b*Sin[c + d*x]])) + b*((a - b)^2*Lo
g[1 - Sin[c + d*x]] + (a + b)^2*Log[1 + Sin[c + d*x]] - 2*(a^2 + b^2)*Log[
a + b*Sin[c + d*x]])*Sin[c + d*x])/((a - b)^2*(a + b)^2*d*(a + b*Sin[c + d
*x]))
```

3.183.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3200, 594, 25, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b\sin(c+dx)}{(a+b\sin(c+dx))^2(b^2-b^2\sin^2(c+dx))}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{594}$$

$$\frac{\frac{\int -\frac{b^2-ab\sin(c+dx)}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{a^2-b^2}-\frac{a}{(a^2-b^2)(a+b\sin(c+dx))}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{b^2-ab\sin(c+dx)}{(a+b\sin(c+dx))\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{a^2-b^2}-\frac{a}{(a^2-b^2)(a+b\sin(c+dx))}}{d}$$

$$\Big\downarrow \text{657}$$

$$\frac{-\frac{\int \left(\frac{b-a}{2(a+b)(b-b\sin(c+dx))}+\frac{-a^2-b^2}{(a-b)(a+b)(a+b\sin(c+dx))}+\frac{a+b}{2(a-b)(\sin(c+dx)b+b)}\right)d(b\sin(c+dx))}{a^2-b^2}-\frac{a}{(a^2-b^2)(a+b\sin(c+dx))}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a}{(a^2-b^2)(a+b\sin(c+dx))}-\frac{-\frac{(a^2+b^2)\log(a+b\sin(c+dx))}{a^2-b^2}+\frac{(a-b)\log(b-b\sin(c+dx))}{2(a+b)}+\frac{(a+b)\log(b\sin(c+dx)+b)}{2(a-b)}}{a^2-b^2}}{d}$$

input
```
Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

output
```
(-((((a - b)*Log[b - b*Sin[c + d*x]])/(2*(a + b)) - ((a^2 + b^2)*Log[a + b
*Sin[c + d*x]])/(a^2 - b^2) + ((a + b)*Log[b + b*Sin[c + d*x]])/(2*(a - b)
))/(a^2 - b^2)) - a/((a^2 - b^2)*(a + b*Sin[c + d*x])))/d
```

3.183.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.183. $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.183.4 Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{a}{(a+b)(a-b)(a+b\sin(dx+c))}+\frac{\left(a^2+b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^2}-\frac{\ln(1+\sin(dx+c))}{2(a-b)^2}}{d}$
default	$\frac{-\frac{a}{(a+b)(a-b)(a+b\sin(dx+c))}+\frac{\left(a^2+b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^2(a-b)^2}-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^2}-\frac{\ln(1+\sin(dx+c))}{2(a-b)^2}}{d}$
risch	$\frac{ix}{a^2-2ab+b^2}+\frac{ic}{d(a^2-2ab+b^2)}+\frac{ix}{a^2+2ab+b^2}+\frac{ic}{d(a^2+2ab+b^2)}-\frac{2ia^2x}{a^4-2a^2b^2+b^4}-\frac{2ia^2c}{d(a^4-2a^2b^2+b^4)}-\frac{2ib}{a^4-2a}$

input
```
int(tan(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/(a+b)/(a-b)/(a+b*sin(d*x+c))+(a^2+b^2)/(a+b)^2/(a-b)^2*ln(a+b*sin(
d*x+c))-1/2/(a+b)^2*ln(sin(d*x+c)-1)-1/2/(a-b)^2*ln(1+sin(d*x+c)))
```

3.183.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.79

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{2\,a^3-2\,ab^2-2\left(a^3+ab^2+(a^2b+b^3)\sin\left(dx+c\right)\right)\log\left(b\sin\left(dx+c\right)+a\right)+\left(a^3+2\,a^2b+ab^2+(a^2b+\right.}{2\left((a^4b-2\,a^2b^3+b\right.}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^3 - 2*a*b^2 - 2*(a^3 + a*b^2 + (a^2*b + b^3)*sin(d*x + c))*log(b
*sin(d*x + c) + a) + (a^3 + 2*a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*sin(
d*x + c))*log(sin(d*x + c) + 1) + (a^3 - 2*a^2*b + a*b^2 + (a^2*b - 2*a*b^
2 + b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/((a^4*b - 2*a^2*b^3 + b^5)*
d*sin(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)
```

3.183.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\tan{(c+dx)}}{(a+b\sin{(c+dx)})^2}\,dx$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.183.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{2(a^2+b^2)\log(b\sin(dx+c)+a)}{a^4-2a^2b^2+b^4} - \frac{2a}{a^3-ab^2+(a^2b-b^3)\sin(dx+c)} - \frac{\log(\sin(dx+c)+1)}{a^2-2ab+b^2} - \frac{\log(\sin(dx+c)-1)}{a^2+2ab+b^2}}{2d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*(2*(a^2 + b^2)*log(b*sin(d*x + c) + a)/(a^4 - 2*a^2*b^2 + b^4) - 2*a/(
a^3 - a*b^2 + (a^2*b - b^3)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^2 - 2
*a*b + b^2) - log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

3.183.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.43

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{2(a^2b+b^3)\log(|b\sin(dx+c)+a|)}{a^4b-2a^2b^3+b^5} - \frac{\log(|\sin(dx+c)+1|)}{a^2-2ab+b^2} - \frac{\log(|\sin(dx+c)-1|)}{a^2+2ab+b^2} - \frac{2(a^2b\sin(dx+c)+b^3\sin(dx+c)+2a^3)}{(a^4-2a^2b^2+b^4)(b\sin(dx+c)+a)}}{2d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*(a^2*b + b^3)*log(abs(b*sin(d*x + c) + a))/(a^4*b - 2*a^2*b^3 + b^5
) - log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2) - log(abs(sin(d*x + c)
- 1))/(a^2 + 2*a*b + b^2) - 2*(a^2*b*sin(d*x + c) + b^3*sin(d*x + c) + 2*a
^3)/((a^4 - 2*a^2*b^2 + b^4)*(b*sin(d*x + c) + a)))/d
```

3.183.9 Mupad [B] (verification not implemented)

Time = 6.63 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.45

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,(a^2+b^2)}{d\,(a^4-2\,a^2\,b^2+b^4)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)}{d\,(a-b)^2} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)}{d\,(a+b)^2} + \frac{2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,(a^2-b^2)\,\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}$$

input
```
int(tan(c + d*x)/(a + b*sin(c + d*x))^2,x)
```

output
```
(log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 + b^2))/(d*
(a^4 + b^4 - 2*a^2*b^2)) - log(tan(c/2 + (d*x)/2) + 1)/(d*(a - b)^2) - log
(tan(c/2 + (d*x)/2) - 1)/(d*(a + b)^2) + (2*b*tan(c/2 + (d*x)/2))/(d*(a^2
- b^2)*(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2))
```

3.184 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.184.1 Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx=\frac{\log(\sin(c+dx))}{a^2d}-\frac{\log(a+b\sin(c+dx))}{a^2d}+\frac{1}{ad(a+b\sin(c+dx))}$$

output
```
ln(sin(d*x+c))/a^2/d-ln(a+b*sin(d*x+c))/a^2/d+1/a/d/(a+b*sin(d*x+c))
```

3.184.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx=\frac{\log(\sin(c+dx))-\log(a+b\sin(c+dx))+\frac{a}{a+b\sin(c+dx)}}{a^2d}$$

input
```
Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

output
```
(Log[Sin[c + d*x]] - Log[a + b*Sin[c + d*x]] + a/(a + b*Sin[c + d*x]))/(a^
2*d)
```

3.184.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 3200, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\tan(c+dx)(a+b\sin(c+dx))^2}dx$$
$$\Big\downarrow \text{3200}$$
$$\frac{\int \frac{\csc(c+dx)}{b(a+b\sin(c+dx))^2}d(b\sin(c+dx))}{d}$$
$$\Big\downarrow \text{54}$$
$$\frac{\int \left(\frac{\csc(c+dx)}{a^2b}-\frac{1}{a^2(a+b\sin(c+dx))}-\frac{1}{a(a+b\sin(c+dx))^2}\right)d(b\sin(c+dx))}{d}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{\log(b\sin(c+dx))}{a^2}-\frac{\log(a+b\sin(c+dx))}{a^2}+\frac{1}{a(a+b\sin(c+dx))}}{d}$$

input
```
Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^2,x]
```

output
```
(Log[b*Sin[c + d*x]]/a^2 - Log[a + b*Sin[c + d*x]]/a^2 + 1/(a*(a + b*Sin[c
 + d*x])))/d
```

3.184.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.184.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a^2}+\frac{1}{a(a+b\sin(dx+c))}+\frac{\ln(\sin(dx+c))}{a^2}}{d}$	49
default	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a^2}+\frac{1}{a(a+b\sin(dx+c))}+\frac{\ln(\sin(dx+c))}{a^2}}{d}$	49
risch	$\frac{2i e^{i(dx+c)}}{da\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{a^2d}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{a^2d}$	105

input
```
int(cot(d*x+c)/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^2*ln(a+b*sin(d*x+c))+1/a/(a+b*sin(d*x+c))+1/a^2*ln(sin(d*x+c)))
```

3.184.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.30

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{(b\sin(dx+c)+a)\log(b\sin(dx+c)+a)-(b\sin(dx+c)+a)\log\left(-\frac{1}{2}\sin(dx+c)\right)-a}{a^2bd\sin(dx+c)+a^3d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-((b*sin(d*x + c) + a)*log(b*sin(d*x + c) + a) - (b*sin(d*x + c) + a)*log(
-1/2*sin(d*x + c)) - a)/(a^2*b*d*sin(d*x + c) + a^3*d)
```

3.184.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)/(a + b*sin(c + d*x))**2, x)
```

3.184.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.89

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{1}{ab\sin(dx+c)+a^2}-\frac{\log(b\sin(dx+c)+a)}{a^2}+\frac{\log(\sin(dx+c))}{a^2}}{d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
(1/(a*b*sin(d*x + c) + a^2) - log(b*sin(d*x + c) + a)/a^2 + log(sin(d*x +
c))/a^2)/d
```

3.184.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{b\left(\frac{\log\left(\left|-\frac{a}{b\sin(dx+c)+a}+1\right|\right)}{a^2b} + \frac{1}{(b\sin(dx+c)+a)ab}\right)}{d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
b*(log(abs(-a/(b*sin(d*x + c) + a) + 1))/(a^2*b) + 1/((b*sin(d*x + c) + a)
*a*b))/d
```

3.184.9 Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.98

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^2\,d} - \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{a^2\,d} - \frac{2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^3+2\,b\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

input
```
int(cot(c + d*x)/(a + b*sin(c + d*x))^2,x)
```

output
```
log(tan(c/2 + (d*x)/2))/(a^2*d) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c
/2 + (d*x)/2)^2)/(a^2*d) - (2*b*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x
)/2)^2 + a^3 + 2*a^2*b*tan(c/2 + (d*x)/2)))
```

3.185 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.185.1 Optimal result

Integrand size = 21, antiderivative size = 114

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{2b\csc(c+dx)}{a^3d} - \frac{\csc^2(c+dx)}{2a^2d} - \frac{(a^2-3b^2)\log(\sin(c+dx))}{a^4d} + \frac{(a^2-3b^2)\log(a+b\sin(c+dx))}{a^4d} - \frac{a^2-b^2}{a^3d(a+b\sin(c+dx))}$$

output
```
2*b*csc(d*x+c)/a^3/d-1/2*csc(d*x+c)^2/a^2/d-(a^2-3*b^2)*ln(sin(d*x+c))/a^4
/d+(a^2-3*b^2)*ln(a+b*sin(d*x+c))/a^4/d+(-a^2+b^2)/a^3/d/(a+b*sin(d*x+c))
```

3.185.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.84

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{-4ab\csc(c+dx)+a^2\csc^2(c+dx)+2(a^2-3b^2)\log(\sin(c+dx))-2(a^2-3b^2)\log(a+b\sin(c+dx))}{2a^4d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

output
```
-1/2*(-4*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 3*b^2)*Log[Sin[c
 + d*x]] - 2*(a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]] + (2*a*(a - b)*(a + b))
/(a + b*Sin[c + d*x]))/(a^4*d)
```

3.185.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^3(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^3(c+dx)(b^2-b^2\sin^2(c+dx))}{b^3(a+b\sin(c+dx))^2}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\csc^3(c+dx)}{a^2b}-\frac{2\csc^2(c+dx)}{a^3}+\frac{(3b^2-a^2)\csc(c+dx)}{a^4b}+\frac{a^2-3b^2}{a^4(a+b\sin(c+dx))}+\frac{a^2-b^2}{a^3(a+b\sin(c+dx))^2}\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2b\csc(c+dx)}{a^3}-\frac{\csc^2(c+dx)}{2a^2}-\frac{(a^2-3b^2)\log(b\sin(c+dx))}{a^4}+\frac{(a^2-3b^2)\log(a+b\sin(c+dx))}{a^4}-\frac{a^2-b^2}{a^3(a+b\sin(c+dx))}}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^2,x]
```

output
```
((2*b*Csc[c + d*x])/a^3 - Csc[c + d*x]^2/(2*a^2) - ((a^2 - 3*b^2)*Log[b*Si
n[c + d*x]])/a^4 + ((a^2 - 3*b^2)*Log[a + b*Sin[c + d*x]])/a^4 - (a^2 - b^
2)/(a^3*(a + b*Sin[c + d*x])))/d
```

3.185.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.185.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.92

method	result
derivativedivides	$\frac{-\frac{1}{2a^2\sin(dx+c)^2}+\frac{(-a^2+3b^2)\ln(\sin(dx+c))}{a^4}+\frac{2b}{a^3\sin(dx+c)}+\frac{(a^2-3b^2)\ln(a+b\sin(dx+c))}{a^4}-\frac{a^2-b^2}{a^3(a+b\sin(dx+c))}}{d}$
default	$\frac{-\frac{1}{2a^2\sin(dx+c)^2}+\frac{(-a^2+3b^2)\ln(\sin(dx+c))}{a^4}+\frac{2b}{a^3\sin(dx+c)}+\frac{(a^2-3b^2)\ln(a+b\sin(dx+c))}{a^4}-\frac{a^2-b^2}{a^3(a+b\sin(dx+c))}}{d}$
risch	$-\frac{2i(-3iab\,e^{4i(dx+c)}-3b^2e^{5i(dx+c)}+3iab\,e^{2i(dx+c)}-4a^2e^{3i(dx+c)}+6b^2e^{3i(dx+c)}-3\,e^{i(dx+c)}b^2+a^2e^{5i(dx+c)}+a^2e^{i(dx+c}}{(e^{2i(dx+c)}-1)^2(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)})d\,a^3}$

input
```
int(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^2/sin(d*x+c)^2+(-a^2+3*b^2)/a^4*ln(sin(d*x+c))+2*b/a^3/sin(d*x
+c)+(a^2-3*b^2)/a^4*ln(a+b*sin(d*x+c))-(a^2-b^2)/a^3/(a+b*sin(d*x+c)))
```

3.185.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(112) = 224$.

Time = 0.30 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.27

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{3\,a^2b\sin\left(dx+c\right)-3\,a^3+6\,ab^2+2\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2+2\left(a^3-3\,ab^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right.\right.}{}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/2*(3*a^2*b*sin(d*x + c) - 3*a^3 + 6*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x +
 c)^2 + 2*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - 3*b^3
 - (a^2*b - 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) -
 2*(a^3 - 3*a*b^2 - (a^3 - 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - 3*b^3 - (a^2
*b - 3*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*sin(d*x + c)))/(a^5*d*c
os(d*x + c)^2 - a^5*d + (a^4*b*d*cos(d*x + c)^2 - a^4*b*d)*sin(d*x + c))
```

3.185.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\cot^3\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**2, x)
```

3.185.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.02

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{3\,ab\sin(dx+c)-2\,\left(a^2-3\,b^2\right)\sin(dx+c)^2-a^2}{a^3b\sin(dx+c)^3+a^4\sin(dx+c)^2}+\frac{2\,\left(a^2-3\,b^2\right)\log(b\sin(dx+c)+a)}{a^4}-\frac{2\,\left(a^2-3\,b^2\right)\log(\sin(dx+c))}{a^4}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*((3*a*b*sin(d*x + c) - 2*(a^2 - 3*b^2)*sin(d*x + c)^2 - a^2)/(a^3*b*si
n(d*x + c)^3 + a^4*sin(d*x + c)^2) + 2*(a^2 - 3*b^2)*log(b*sin(d*x + c) +
a)/a^4 - 2*(a^2 - 3*b^2)*log(sin(d*x + c))/a^4)/d
```

3.185.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.45

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\frac{2\,(a^2-3\,b^2)\log(|\sin(dx+c)|)}{a^4} - \frac{2\,(a^2b-3\,b^3)\log(|b\sin(dx+c)+a|)}{a^4b} + \frac{2\,(a^2b\sin(dx+c)-3\,b^3\sin(dx+c)+2\,a^3-4\,ab^2)}{(b\sin(dx+c)+a)a^4} - \frac{3\,a^2\sin(dx+c)^2-9}{}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/2*(2*(a^2 - 3*b^2)*log(abs(sin(d*x + c)))/a^4 - 2*(a^2*b - 3*b^3)*log(a
bs(b*sin(d*x + c) + a))/(a^4*b) + 2*(a^2*b*sin(d*x + c) - 3*b^3*sin(d*x +
c) + 2*a^3 - 4*a*b^2)/((b*sin(d*x + c) + a)*a^4) - (3*a^2*sin(d*x + c)^2 -
 9*b^2*sin(d*x + c)^2 + 4*a*b*sin(d*x + c) - a^2)/(a^4*sin(d*x + c)^2))/d
```

3.185.9 Mupad [B] (verification not implemented)

Time = 6.25 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.06

$$\begin{aligned}
&\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx \\
&= \frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d} \\
&\quad - \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{a^2}{2}-8\,b^2\right)+\frac{a^2}{2}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(3\,a^2\,b-2\,b^3\right)}{a}-3\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(4\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,a^4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+8\,b\,a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)} \\
&\quad - \frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,a^2\,d} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-3\,b^2\right)}{a^4\,d} \\
&\quad + \frac{\ln\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,\left(a^2-3\,b^2\right)}{a^4\,d}
\end{aligned}$$

input
```
int(cot(c + d*x)^3/(a + b*sin(c + d*x))^2,x)
```

output
```
(b*tan(c/2 + (d*x)/2))/(a^3*d) - (tan(c/2 + (d*x)/2)^2*(a^2/2 - 8*b^2) + a
^2/2 - (4*tan(c/2 + (d*x)/2)^3*(3*a^2*b - 2*b^3))/a - 3*a*b*tan(c/2 + (d*x
)/2))/(d*(4*a^4*tan(c/2 + (d*x)/2)^2 + 4*a^4*tan(c/2 + (d*x)/2)^4 + 8*a^3*
b*tan(c/2 + (d*x)/2)^3)) - tan(c/2 + (d*x)/2)^2/(8*a^2*d) - (log(tan(c/2 +
 (d*x)/2))*(a^2 - 3*b^2))/(a^4*d) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*ta
n(c/2 + (d*x)/2)^2)*(a^2 - 3*b^2))/(a^4*d)
```

3.185. $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.186 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.186.1 Optimal result

Integrand size = 21, antiderivative size = 188

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = &-\frac{4b(a^2-b^2)\csc(c+dx)}{a^5d}+\frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d}\\&+\frac{2b\csc^3(c+dx)}{3a^3d}-\frac{\csc^4(c+dx)}{4a^2d}\\&+\frac{(a^4-6a^2b^2+5b^4)\log(\sin(c+dx))}{a^6d}\\&-\frac{(a^4-6a^2b^2+5b^4)\log(a+b\sin(c+dx))}{a^6d}\\&+\frac{(a^2-b^2)^2}{a^5d(a+b\sin(c+dx))}\end{aligned}$$

output
```
-4*b*(a^2-b^2)*csc(d*x+c)/a^5/d+1/2*(2*a^2-3*b^2)*csc(d*x+c)^2/a^4/d+2/3*b
*csc(d*x+c)^3/a^3/d-1/4*csc(d*x+c)^4/a^2/d+(a^4-6*a^2*b^2+5*b^4)*ln(sin(d*
x+c))/a^6/d-(a^4-6*a^2*b^2+5*b^4)*ln(a+b*sin(d*x+c))/a^6/d+(a^2-b^2)^2/a^5
/d/(a+b*sin(d*x+c))
```

3.186.2 Mathematica [A] (verified)

Time = 6.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.99

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{4(a-b)b(a+b)\csc(c+dx)}{a^5d} + \frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4d} + \frac{2b\csc^3(c+dx)}{3a^3d} - \frac{\csc^4(c+dx)}{4a^2d} + \frac{(a^4-6a^2b^2+5b^4)\log(\sin(c+dx))}{a^6d} - \frac{(a^4-6a^2b^2+5b^4)\log(a+b\sin(c+dx))}{a^6d} + \frac{(a^2-b^2)^2}{a^5d(a+b\sin(c+dx))}$$

input
```
Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

output
```
(-4*(a - b)*b*(a + b)*Csc[c + d*x])/(a^5*d) + ((2*a^2 - 3*b^2)*Csc[c + d*x
]^2)/(2*a^4*d) + (2*b*Csc[c + d*x]^3)/(3*a^3*d) - Csc[c + d*x]^4/(4*a^2*d)
 + ((a^4 - 6*a^2*b^2 + 5*b^4)*Log[Sin[c + d*x]])/(a^6*d) - ((a^4 - 6*a^2*b
^2 + 5*b^4)*Log[a + b*Sin[c + d*x]])/(a^6*d) + (a^2 - b^2)^2/(a^5*d*(a + b
*Sin[c + d*x]))
```

3.186.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^5(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^5(c+dx)\left(b^2-b^2\sin^2(c+dx)\right)^2}{b^5(a+b\sin(c+dx))^2}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\csc^5(c+dx)}{a^2b}-\frac{2\csc^4(c+dx)}{a^3}+\frac{(3b^4-2a^2b^2)\csc^3(c+dx)}{a^4b^3}+\frac{4(a^2-b^2)\csc^2(c+dx)}{a^5}+\frac{(a^4-6b^2a^2+5b^4)\csc(c+dx)}{a^6b}+\frac{-a^4+6b^2a^2-5b^4}{a^6(a+b\sin(c+dx))}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2b\csc^3(c+dx)}{3a^3}-\frac{\csc^4(c+dx)}{4a^2}+\frac{(a^2-b^2)^2}{a^5(a+b\sin(c+dx))}-\frac{4b(a^2-b^2)\csc(c+dx)}{a^5}+\frac{(2a^2-3b^2)\csc^2(c+dx)}{2a^4}+\frac{(a^4-6a^2b^2+5b^4)\log(b\sin(c+dx))}{a^6}}{d}$$

input
```
Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^2,x]
```

output
```
((-4*b*(a^2 - b^2)*Csc[c + d*x])/a^5 + ((2*a^2 - 3*b^2)*Csc[c + d*x]^2)/(2
*a^4) + (2*b*Csc[c + d*x]^3)/(3*a^3) - Csc[c + d*x]^4/(4*a^2) + ((a^4 - 6*
a^2*b^2 + 5*b^4)*Log[b*Sin[c + d*x]])/a^6 - ((a^4 - 6*a^2*b^2 + 5*b^4)*Log
[a + b*Sin[c + d*x]])/a^6 + (a^2 - b^2)^2/(a^5*(a + b*Sin[c + d*x])))/d
```

3.186.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.186. $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.186.4 Maple [A] (verified)

Time = 4.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{-\frac{\left(a^4-6a^2b^2+5b^4\right)\ln(a+b\sin(dx+c))}{a^6}+\frac{a^4-2a^2b^2+b^4}{a^5(a+b\sin(dx+c))}-\frac{1}{4a^2\sin(dx+c)^4}-\frac{-2a^2+3b^2}{2a^4\sin(dx+c)^2}+\frac{\left(a^4-6a^2b^2+5b^4\right)\ln(\sin(dx+c))}{a^6}+}{d}$
default	$\frac{-\frac{\left(a^4-6a^2b^2+5b^4\right)\ln(a+b\sin(dx+c))}{a^6}+\frac{a^4-2a^2b^2+b^4}{a^5(a+b\sin(dx+c))}-\frac{1}{4a^2\sin(dx+c)^4}-\frac{-2a^2+3b^2}{2a^4\sin(dx+c)^2}+\frac{\left(a^4-6a^2b^2+5b^4\right)\ln(\sin(dx+c))}{a^6}+}{d}$
risch	$\frac{2i\left(-18\,\mathrm{e}^{i(dx+c)}b^2a^2-15ia\,b^3\mathrm{e}^{2i(dx+c)}-18ia^3b\,\mathrm{e}^{8i(dx+c)}-45ia\,b^3\mathrm{e}^{6i(dx+c)}+45ia\,b^3\mathrm{e}^{4i(dx+c)}+15ia\,b^3\mathrm{e}^{8i(dx+c)}-44ia^3b\right.}{}$

input
```
int(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a^4-6*a^2*b^2+5*b^4)/a^6*ln(a+b*sin(d*x+c))+(a^4-2*a^2*b^2+b^4)/a^5
/(a+b*sin(d*x+c))-1/4/a^2/sin(d*x+c)^4-1/2*(-2*a^2+3*b^2)/a^4/sin(d*x+c)^2
+(a^4-6*a^2*b^2+5*b^4)/a^6*ln(sin(d*x+c))+2/3*b/a^3/sin(d*x+c)^3-4*b*(a^2-
b^2)/a^5/sin(d*x+c))
```

3.186.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(182) = 364$.

Time = 0.33 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.88

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{21\,a^5-82\,a^3b^2+60\,ab^4+12\left(a^5-6\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^4-2\left(18\,a^5-77\,a^3b^2+60\,ab^4\right)\cos\left(dx+c\right)}{}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/12*(21*a^5 - 82*a^3*b^2 + 60*a*b^4 + 12*(a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(
d*x + c)^4 - 2*(18*a^5 - 77*a^3*b^2 + 60*a*b^4)*cos(d*x + c)^2 - 12*(a^5 -
 6*a^3*b^2 + 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5
 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4
*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos
(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 12*(a^5 - 6*a^3*b^2 +
 5*a*b^4 + (a^5 - 6*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 - 2*(a^5 - 6*a^3*b^2
 + 5*a*b^4)*cos(d*x + c)^2 + (a^4*b - 6*a^2*b^3 + 5*b^5 + (a^4*b - 6*a^2*b
^3 + 5*b^5)*cos(d*x + c)^4 - 2*(a^4*b - 6*a^2*b^3 + 5*b^5)*cos(d*x + c)^2)
*sin(d*x + c))*log(-1/2*sin(d*x + c)) - (31*a^4*b - 30*a^2*b^3 - 6*(6*a^4*
b - 5*a^2*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(a^7*d*cos(d*x + c)^4 - 2*a^7
*d*cos(d*x + c)^2 + a^7*d + (a^6*b*d*cos(d*x + c)^4 - 2*a^6*b*d*cos(d*x +
c)^2 + a^6*b*d)*sin(d*x + c))
```

3.186.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\cot^5\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**2, x)
```

3.186.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{5\,a^3b\sin(dx+c)+12\,(a^4-6\,a^2b^2+5\,b^4)\sin(dx+c)^4-3\,a^4-6\,(6\,a^3b-5\,ab^3)\sin(dx+c)^3+2\,(6\,a^4-5\,a^2b^2)\sin(dx+c)^2}{a^5b\sin(dx+c)^5+a^6\sin(dx+c)^4} - \frac{12\,(a^4-6\,a^2b^2+5\,b^4)\log(b \cdots}{a^6} \cdots}{12\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/12*((5*a^3*b*sin(d*x + c) + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*sin(d*x + c)^4
- 3*a^4 - 6*(6*a^3*b - 5*a*b^3)*sin(d*x + c)^3 + 2*(6*a^4 - 5*a^2*b^2)*sin
(d*x + c)^2)/(a^5*b*sin(d*x + c)^5 + a^6*sin(d*x + c)^4) - 12*(a^4 - 6*a^2
*b^2 + 5*b^4)*log(b*sin(d*x + c) + a)/a^6 + 12*(a^4 - 6*a^2*b^2 + 5*b^4)*l
og(sin(d*x + c))/a^6)/d
```

3.186.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.48

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{12\,(a^4-6\,a^2b^2+5\,b^4)\log(|\sin(dx+c)|)}{a^6} - \frac{12\,(a^4b-6\,a^2b^3+5\,b^5)\log(|b\sin(dx+c)+a|)}{a^6b} + \frac{12\,(a^4b\sin(dx+c)-6\,a^2b^3\sin(dx+c)+5\,b^5\sin(dx+c)+2}{(b\sin(dx+c)+a)a^6}}{}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/12*(12*(a^4 - 6*a^2*b^2 + 5*b^4)*log(abs(sin(d*x + c)))/a^6 - 12*(a^4*b
- 6*a^2*b^3 + 5*b^5)*log(abs(b*sin(d*x + c) + a))/(a^6*b) + 12*(a^4*b*sin(
d*x + c) - 6*a^2*b^3*sin(d*x + c) + 5*b^5*sin(d*x + c) + 2*a^5 - 8*a^3*b^2
 + 6*a*b^4)/((b*sin(d*x + c) + a)*a^6) - (25*a^4*sin(d*x + c)^4 - 150*a^2*
b^2*sin(d*x + c)^4 + 125*b^4*sin(d*x + c)^4 + 48*a^3*b*sin(d*x + c)^3 - 48
*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 18*a^2*b^2*sin(d*x + c)^2
- 8*a^3*b*sin(d*x + c) + 3*a^4)/(a^6*sin(d*x + c)^4))/d
```

3.186.9 Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.34

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(3a^4-62a^2b^2+64b^4)-\frac{a^4}{4}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{11a^4}{4}-\frac{10a^2b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(20ab^3-62\right.}{d\left(16a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+16a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+32ba^5\tan\left(\frac{c}{2}\right.\right.}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64a^2d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{\frac{a^2}{16}+\frac{b^2}{8}}{a^4}+\frac{1}{8a^2}-\frac{b^2}{2a^4}\right)}{d}$$

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{b(32a^2+64b^2)}{64a^5}-\frac{b}{4a^3}+\frac{4b\left(\frac{\frac{a^2}{8}+\frac{b^2}{4}}{a^4}+\frac{1}{4a^2}-\frac{b^2}{a^4}\right)}{a}\right)}{d}$$

$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a^4-6a^2b^2+5b^4)}{a^6d}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{12a^3d}$$

$$-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)(a^4-6a^2b^2+5b^4)}{a^6d}$$

input
```
int(cot(c + d*x)^5/(a + b*sin(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^4*(3*a^4 + 64*b^4 - 62*a^2*b^2) - a^4/4 + tan(c/2 + (d
*x)/2)^2*((11*a^4)/4 - (10*a^2*b^2)/3) + tan(c/2 + (d*x)/2)^3*(20*a*b^3 -
(62*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(60*a^4*b + 32*b^5 - 96*a^2*b^3))/a
+ (5*a^3*b*tan(c/2 + (d*x)/2))/6)/(d*(16*a^6*tan(c/2 + (d*x)/2)^4 + 16*a^6
*tan(c/2 + (d*x)/2)^6 + 32*a^5*b*tan(c/2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/
2)^4/(64*a^2*d) + (tan(c/2 + (d*x)/2)^2*((a^2/16 + b^2/8)/a^4 + 1/(8*a^2)
- b^2/(2*a^4)))/d - (tan(c/2 + (d*x)/2)*((b*(32*a^2 + 64*b^2))/(64*a^5) -
b/(4*a^3) + (4*b*((a^2/8 + b^2/4)/a^4 + 1/(4*a^2) - b^2/a^4))/a))/d + (log
(tan(c/2 + (d*x)/2))*(a^4 + 5*b^4 - 6*a^2*b^2))/(a^6*d) + (b*tan(c/2 + (d*
x)/2)^3)/(12*a^3*d) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/
2)^2)*(a^4 + 5*b^4 - 6*a^2*b^2))/(a^6*d)
```

3.187 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.187.1 Optimal result

Integrand size = 21, antiderivative size = 333

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx &= \frac{2a^5\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} + \frac{8a^3b^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} \\ &\quad + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))^2} + \frac{\cos(c+dx)}{12(a+b)^2d(1-\sin(c+dx))} \\ &\quad - \frac{(3a+b)\cos(c+dx)}{4(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{12(a-b)^2d(1+\sin(c+dx))^2} \\ &\quad - \frac{\cos(c+dx)}{12(a-b)^2d(1+\sin(c+dx))} + \frac{(3a-b)\cos(c+dx)}{4(a-b)^3d(1+\sin(c+dx))} \\ &\quad + \frac{a^4b\cos(c+dx)}{(a^2-b^2)^3\,d(a+b\sin(c+dx))}\end{aligned}$$

output
```
2*a^5*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/d+8
*a^3*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2)/
d+1/12*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))^2+1/12*cos(d*x+c)/(a+b)^2/d/(1-
sin(d*x+c))-1/4*(3*a+b)*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/12*cos(d*x+c
)/(a-b)^2/d/(1+sin(d*x+c))^2-1/12*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))+1/4*
(3*a-b)*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+a^4*b*cos(d*x+c)/(a^2-b^2)^3/d
/(a+b*sin(d*x+c))
```

3.187.2 Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.02

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$= \frac{\frac{24a^3(a^2+4b^2)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2\sin\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{}{(a+b)^3}}{}$$

input
```
Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

output
```
((24*a^3*(a^2 + 4*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(7/2) + 1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) +
 (2*Sin[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
- (4*(4*a + b)*Sin[(c + d*x)/2])/((a + b)^3*(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])) + (2*Sin[(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^3) - 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (4*(-4
*a + b)*Sin[(c + d*x)/2])/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])
) + (12*a^4*b*Cos[c + d*x])/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])))/(1
2*d)
```

3.187.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(c+dx)^4}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \; 3210$$

$$\int \left(\frac{a^4}{(a^2-b^2)^2 (a+b\sin(c+dx))^2} + \frac{4a^3b^2}{(a^2-b^2)^3 (a+b\sin(c+dx))} + \frac{3a+b}{4(a+b)^3(\sin(c+dx)-1)} + \frac{b-}{4(a-b)^3(\sin(}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{2a^5 \arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{7/2}} + \frac{a^4 b\cos(c+dx)}{d\,(a^2-b^2)^3 (a+b\sin(c+dx))} + \frac{8a^3b^2 \arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d\,(a^2-b^2)^{7/2}} - \frac{(3a+b)\cos(c+dx)}{4d(a+b)^3(1-\sin(c+dx))} + \frac{\cos(c+dx)}{12d(a+b)^2(1-\sin(c+dx))} + \frac{(3a-b)\cos(c+dx)}{4d(a-b)^3(\sin(c+dx)+1)} - \frac{\cos(c+dx)}{12d(a-b)^2(\sin(c+dx)+1)} + \frac{\cos(c+dx)}{12d(a+b)^2(1-\sin(c+dx))^2} - \frac{\cos(c+dx)}{12d(a-b)^2(\sin(c+dx)+1)^2}$$

input
```
Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

output
```
(2*a^5*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2
)*d) + (8*a^3*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2
- b^2)^(7/2)*d) + Cos[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])^2) + Cos
[c + d*x]/(12*(a + b)^2*d*(1 - Sin[c + d*x])) - ((3*a + b)*Cos[c + d*x])/(
4*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[
c + d*x])^2) - Cos[c + d*x]/(12*(a - b)^2*d*(1 + Sin[c + d*x])) + ((3*a -
b)*Cos[c + d*x])/(4*(a - b)^3*d*(1 + Sin[c + d*x])) + (a^4*b*Cos[c + d*x])
/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3210
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

3.187.4 Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{1}{3(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{a}{(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2a^3\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}\right.}{(a-}$
default	$-\frac{1}{3(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{a}{(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2a^3\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}\right.}{(a-}$
risch	$14ia^4b\,e^{6i(dx+c)}-2ib^5e^{4i(dx+c)}+\frac{2ib^5e^{2i(dx+c)}}{3}+\frac{22a^5e^{i(dx+c)}}{3}-\frac{2ib^5}{3}-\frac{4a\,b^4e^{i(dx+c)}}{3}+\frac{14ia^2b^3e^{4i(dx+c)}}{3}+\frac{70ia^4b\,e^{2i(dx+c)}}{3}$

input
```
int(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tan(1/2*d*x+1/2*c)
-1)^2+a/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)+2*a^3/(a-b)^3/(a+b)^3*((tan(1/2*d*x
+1/2*c)*b^2+a*b)/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^2+4*
b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2)))-1/3/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/(a-b)^2/(tan(1/2*d*x+1/2*c)+
1)^2+a/(a-b)^3/(tan(1/2*d*x+1/2*c)+1))
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.45

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$=\left[-\frac{2\,a^6b-6\,a^4b^3+6\,a^2b^5-2\,b^7-2\left(7\,a^6b+2\,a^4b^3-10\,a^2b^5+b^7\right)\cos\left(dx+c\right)^4-2\left(7\,a^6b-16\,a^4b^3+11\right.}{}\right.$$
$$-\frac{a^6b-3\,a^4b^3+3\,a^2b^5-b^7-\left(7\,a^6b+2\,a^4b^3-10\,a^2b^5+b^7\right)\cos\left(dx+c\right)^4-\left(7\,a^6b-16\,a^4b^3+11\,a^2b^5-\right.}{3\,(}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/6*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 - 2*(7*a^6*b + 2*a^4*b^3 -
10*a^2*b^5 + b^7)*cos(d*x + c)^4 - 2*(7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 -
2*b^7)*cos(d*x + c)^2 - 3*((a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c)
 + (a^6 + 4*a^4*b^2)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*
x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2)) - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7
- 7*a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4
*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a
^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3), -1/3*(a
^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 - (7*a^6*b + 2*a^4*b^3 - 10*a^2*b^5 + b
^7)*cos(d*x + c)^4 - (7*a^6*b - 16*a^4*b^3 + 11*a^2*b^5 - 2*b^7)*cos(d*x +
 c)^2 + 3*((a^5*b + 4*a^3*b^3)*cos(d*x + c)^3*sin(d*x + c) + (a^6 + 4*a^4*
b^2)*cos(d*x + c)^3)*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^
2 - b^2)*cos(d*x + c))) - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6 - (4*a^7 -
7*a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b - 4*a
^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^3*sin(d*x + c) + (a^9
 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3)]
```

3.187.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\tan^4{(c+dx)}}{(a+b\sin{(c+dx)})^2}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

3.187.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.187.8 Giac [A] (verification not implemented)

Time = 1.10 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.22

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{2\left(\frac{3\,(a^5+4\,a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{a^2-b^2}}+\frac{3\,(a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^4b)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right.}\right.}{}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
2/3*(3*(a^5 + 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan
((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*sqrt(a^2 - b^2)) + 3*(a^3*b^2*tan(1/2*d*x + 1/2*c) + a^4*b)/((a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*
d*x + 1/2*c) + a)) + (3*a^4*tan(1/2*d*x + 1/2*c)^5 + 9*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^5 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 6*a*b^3*tan(1/2*d*x + 1/2*c
)^4 - 10*a^4*tan(1/2*d*x + 1/2*c)^3 - 18*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 +
4*b^4*tan(1/2*d*x + 1/2*c)^3 + 24*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^4*tan
(1/2*d*x + 1/2*c) + 9*a^2*b^2*tan(1/2*d*x + 1/2*c) - 10*a^3*b - 2*a*b^3)/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.187.9 Mupad [B] (verification not implemented)

Time = 9.79 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.17

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{\frac{2\,(13\,a^4\,b+2\,a^2\,b^3)}{3\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(-3\,a^5+14\,a^3\,b^2+4\,a\,b^4)}{3\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,(a^4\,b+4\,a^2\,b^3)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(29\,a^4\,b+16\,a^2\,b^3)}{3\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\right.}$$
$$+\frac{2\,a^3\,\mathrm{atan}\left(\frac{\frac{a^3\,(a^2+4\,b^2)\,(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7)}{(a+b)^{7/2}\,(a-b)^{7/2}}+\frac{2\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2+4\,b^2)\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}{(a+b)^{7/2}\,(a-b)^{7/2}}}{2\,a^5+8\,a^3\,b^2}\right)\,(a^2+4\,b^2)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

input
```
int(tan(c + d*x)^4/(a + b*sin(c + d*x))^2,x)
```

output
```
((2*(13*a^4*b + 2*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*t
an(c/2 + (d*x)/2)*(4*a*b^4 - 3*a^5 + 14*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(a^4*b + 4*a^2*b^3))/(a^6 - b^6
+ 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2 + (d*x)/2)^2*(29*a^4*b + 16*a^2*b^3)
)/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^5*(8*a*b
^4 + 7*a^5 + 30*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan
(c/2 + (d*x)/2)^3*(4*a*b^4 - 7*a^5 + 48*a^3*b^2))/(3*(a^6 - b^6 + 3*a^2*b^
4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^4*(11*a^4*b - 8*b^5 + 42*a^2*b^3))
/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*a^3*tan(c/2 + (d*x)/2)^7*(a^
2 + 4*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/(d*(a + 2*b*tan(c/2 + (d*
x)/2) - 2*a*tan(c/2 + (d*x)/2)^2 + 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 +
(d*x)/2)^8 - 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 - 2*b*tan
(c/2 + (d*x)/2)^7)) + (2*a^3*atan(((a^3*(a^2 + 4*b^2)*(2*a^6*b - 2*b^7 + 6
*a^2*b^5 - 6*a^4*b^3))/((a + b)^(7/2)*(a - b)^(7/2)) + (2*a^4*tan(c/2 + (d
*x)/2)*(a^2 + 4*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(
a - b)^(7/2)))/(2*a^5 + 8*a^3*b^2))*(a^2 + 4*b^2))/(d*(a + b)^(7/2)*(a - b
)^(7/2))
```

3.188 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.188.1 Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{2a^3\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}d} - \frac{4ab^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}d} + \frac{\cos(c+dx)}{2(a+b)^2d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{2(a-b)^2d(1+\sin(c+dx))} - \frac{a^2b\cos(c+dx)}{(a^2-b^2)^2d(a+b\sin(c+dx))}$$

output
```
-2*a^3*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d-
4*a*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(5/2)/d
+1/2*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/2*cos(d*x+c)/(a-b)^2/d/(1+sin(d
*x+c))-a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

3.188.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.84

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{-\frac{2a(a^2+2b^2)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{1}{(a+b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{1}{(a-b)^2\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}\right.\right.}\right.}{d}$$

input
```
Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

output
```
((-2*a*(a^2 + 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^
2 - b^2)^(5/2) + Sin[(c + d*x)/2]*(1/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])) + 1/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))) - (a^2
*b*Cos[c + d*x])/((a - b)^2*(a + b)^2*(a + b*Sin[c + d*x])))/d
```

3.188.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^2}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3210}$$

$$\int \left(-\frac{a^2}{(a^2-b^2)(a+b\sin(c+dx))^2}-\frac{2ab^2}{(a^2-b^2)^2(a+b\sin(c+dx))}-\frac{1}{2(a+b)^2(\sin(c+dx)-1)}+\frac{}{2(a-b)^2(\sin$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4ab^2\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}}-\frac{a^2b\cos(c+dx)}{d(a^2-b^2)^2(a+b\sin(c+dx))}-\frac{2a^3\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{5/2}}+\frac{\cos(c+dx)}{2d(a+b)^2(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^2(\sin(c+dx)+1)}$$

input
```
Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

output
```
(-2*a^3*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(5/
2)*d) - (4*a*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 -
 b^2)^(5/2)*d) + Cos[c + d*x]/(2*(a + b)^2*d*(1 - Sin[c + d*x])) - Cos[c +
 d*x]/(2*(a - b)^2*d*(1 + Sin[c + d*x])) - (a^2*b*Cos[c + d*x])/((a^2 - b^
2)^2*d*(a + b*Sin[c + d*x]))
```

3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3210
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

3.188.4 Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^2+2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}\right.}{2\sqrt{a^2}}\right.}{\sqrt{a^2-b^2}}\right.}{(a-b)^2(a+b)^2}}{d}$
default	$\frac{-\frac{1}{(a-b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{(a+b)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2a\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^2+2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}\right.}{2\sqrt{a^2}}\right.}{\sqrt{a^2-b^2}}\right.}{(a-b)^2(a+b)^2}}{d}$
risch	$\frac{2i\left(3a^3e^{i(dx+c)}+4ia^2b\,e^{2i(dx+c)}-ib^3e^{2i(dx+c)}+2\,e^{3i(dx+c)}a\,b^2+2ia^2b+ib^3+a^3e^{3i(dx+c)}\right)}{\left(1+e^{2i(dx+c)}\right)\left(-ib\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}+ib\right)\left(a^2-b^2\right)^2d}+\frac{a^3\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+}}{b\sqrt{-}}\right.}{\sqrt{-a^2+b^2}\,(a+b)^2(a}$

input
```
int(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-1/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-2*
a/(a-b)^2/(a+b)^2*((tan(1/2*d*x+1/2*c)*b^2+a*b)/(a*tan(1/2*d*x+1/2*c)^2+2*
b*tan(1/2*d*x+1/2*c)+a)+(a^2+2*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/
2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.84

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \left[-\frac{2\,a^4b-4\,a^2b^3+2\,b^5+2\,(2\,a^4b-a^2b^3-b^5)\cos{(dx+c)}^2+((a^3b+2\,ab^3)\cos{(dx+c)}\sin{(dx+c)}+(a}{2\,((a^6b-3\,a^4b^3+3\,a^2b^5-}\right.$$
$$-\frac{a^4b-2\,a^2b^3+b^5+(2\,a^4b-a^2b^3-b^5)\cos{(dx+c)}^2-((a^3b+2\,ab^3)\cos{(dx+c)}\sin{(dx+c)}+(a^4+2}{(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)d\cos{(dx+c)}\sin{(dx+c)}+}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/2*(2*a^4*b - 4*a^2*b^3 + 2*b^5 + 2*(2*a^4*b - a^2*b^3 - b^5)*cos(d*x +
 c)^2 + ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(-a^2 + b^2)*log(-((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2 - 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))
*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2))
- 2*(a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b - 3*a^4*b^3 + 3*a^2*b^
5 - b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^
6)*d*cos(d*x + c)), -(a^4*b - 2*a^2*b^3 + b^5 + (2*a^4*b - a^2*b^3 - b^5)*
cos(d*x + c)^2 - ((a^3*b + 2*a*b^3)*cos(d*x + c)*sin(d*x + c) + (a^4 + 2*a
^2*b^2)*cos(d*x + c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a
^2 - b^2)*cos(d*x + c))) - (a^5 - 2*a^3*b^2 + a*b^4)*sin(d*x + c))/((a^6*b
 - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^7 - 3*a^5
*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c))]
```

3.188.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(tan(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.188.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.188.8 Giac [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.26

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{2\left(\frac{(a^3+2ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{a^2-b^2}}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}\right.}{d}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-2*((a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*t
an(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(a
^2 - b^2)) + (a^3*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3
+ a^2*b*tan(1/2*d*x + 1/2*c)^2 + 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/
2*d*x + 1/2*c) - 4*a*b^2*tan(1/2*d*x + 1/2*c) - 3*a^2*b)/((a*tan(1/2*d*x +
 1/2*c)^4 + 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)*(a^
4 - 2*a^2*b^2 + b^4)))/d
```

3.188.9 Mupad [B] (verification not implemented)

Time = 8.03 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.56

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\frac{6a^2b}{(a^2-b^2)^2}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4ab^2-a^3)}{(a^2-b^2)^2}-\frac{2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2+2b^2)}{a^4-2a^2b^2+b^4}-\frac{2a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a^2+2b^2)}{(a^2-b^2)^2}}{d\left(-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)} - \frac{2a\,\mathrm{atan}\left(\frac{\frac{a(a^2+2b^2)(2a^4b-4a^2b^3+2b^5)}{(a+b)^{5/2}(a-b)^{5/2}}+\frac{2a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^2+2b^2)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{5/2}}}{2a^3+4ab^2}\right)(a^2+2b^2)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

input
```
int(tan(c + d*x)^2/(a + b*sin(c + d*x))^2,x)
```

output
```
- ((6*a^2*b)/(a^2 - b^2)^2 + (2*tan(c/2 + (d*x)/2)*(4*a*b^2 - a^3))/(a^2 -
 b^2)^2 - (2*b*tan(c/2 + (d*x)/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 - 2*a^2*b^2)
 - (2*a*tan(c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a^2 - b^2)^2)/(d*(a + 2*b*tan
(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (d*x)/2)^3)) - (2
*a*atan(((a*(a^2 + 2*b^2)*(2*a^4*b + 2*b^5 - 4*a^2*b^3))/((a + b)^(5/2)*(a
 - b)^(5/2)) + (2*a^2*tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(a^4 + b^4 - 2*a^2*
b^2))/((a + b)^(5/2)*(a - b)^(5/2)))/(4*a*b^2 + 2*a^3))*(a^2 + 2*b^2))/(d*
(a + b)^(5/2)*(a - b)^(5/2))
```

3.189 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2} \, dx$

3.189.1 Optimal result

Integrand size = 21, antiderivative size = 115

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2} \, dx = -\frac{2(a^2-2b^2)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\sqrt{a^2-b^2}d} + \frac{2b\text{arctanh}(\cos(c+dx))}{a^3d} - \frac{2\cot(c+dx)}{a^2d} + \frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

output
```
2*b*arctanh(cos(d*x+c))/a^3/d-2*cot(d*x+c)/a^2/d+cot(d*x+c)/a/d/(a+b*sin(d
*x+c))-2*(a^2-2*b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a^3/
d/(a^2-b^2)^(1/2)
```

3.189.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.21

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2} \, dx = -\frac{\frac{4(a^2-2b^2)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + a\cot\left(\frac{1}{2}(c+dx)\right) - 4b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 4b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^3d}$$

input
```
Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

output
```
-1/2*((4*(a^2 - 2*b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + a*Cot[(c + d*x)/2] - 4*b*Log[Cos[(c + d*x)/2]] + 4*b*Log[
Sin[(c + d*x)/2]] + (2*a*b*Cos[c + d*x])/(a + b*Sin[c + d*x]) - a*Tan[(c +
 d*x)/2])/(a^3*d)
```

3.189.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.40, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3202, 3042, 3535, 3042, 3535, 25, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^2(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3202}$$

$$\int \frac{\left(1-\sin^2(c+dx)\right)\csc^2(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1-\sin(c+dx)^2}{\sin(c+dx)^2(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3535}$$

$$\frac{\int \frac{\csc^2(c+dx)\left(2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{2\left(a^2-b^2\right)-\left(a^2-b^2\right)\sin(c+dx)^2}{\sin(c+dx)^2(a+b\sin(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 3535}$$

$$\frac{\frac{\int -\frac{\csc(c+dx)\left(2b\left(a^2-b^2\right)+a\sin(c+dx)\left(a^2-b^2\right)\right)}{a+b\sin(c+dx)}dx}{a}-\frac{2\left(a^2-b^2\right)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{\csc(c+dx)\left(2b\left(a^2-b^2\right)+a\sin(c+dx)\left(a^2-b^2\right)\right)}{a+b\sin(c+dx)}dx}{a}-\frac{2(a^2-b^2)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\int \frac{2b\left(a^2-b^2\right)+a\sin(c+dx)\left(a^2-b^2\right)}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}-\frac{2(a^2-b^2)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3480}$$

$$\frac{-\frac{\frac{\left(a^2-2b^2\right)\left(a^2-b^2\right)\int \frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{2b\left(a^2-b^2\right)\int \csc(c+dx)dx}{a}}{a}-\frac{2(a^2-b^2)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\frac{\left(a^2-2b^2\right)\left(a^2-b^2\right)\int \frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{2b\left(a^2-b^2\right)\int \csc(c+dx)dx}{a}}{a}-\frac{2(a^2-b^2)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3139}$$

$$\frac{-\frac{\frac{2\left(a^2-2b^2\right)\left(a^2-b^2\right)\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}+\frac{2b\left(a^2-b^2\right)\int \csc(c+dx)dx}{a}}{a}-\frac{2(a^2-b^2)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{\frac{2b\left(a^2-b^2\right)\int \csc(c+dx)dx}{a}-\frac{4\left(a^2-2b^2\right)\left(a^2-b^2\right)\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}}{a}-\frac{2(a^2-b^2)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{217}$$

$$\frac{-\frac{\frac{2b\left(a^2-b^2\right)\int \csc(c+dx)dx}{a}+\frac{2\left(a^2-2b^2\right)\sqrt{a^2-b^2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}}{a}-\frac{2(a^2-b^2)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{4257}$$

3.189. $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

$$-\frac{\frac{-\frac{2\left(a^2-2b^2\right)\sqrt{a^2-b^2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}-\frac{2b\left(a^2-b^2\right)\text{arctanh}(\cos(c+dx))}{ad}}{a}-\frac{2\left(a^2-b^2\right)\cot(c+dx)}{ad}}{a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{ad(a+b\sin(c+dx))}$$

input
```
Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^2,x]
```

output
```
(-(((2*(a^2 - 2*b^2)*Sqrt[a^2 - b^2]*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(
2*Sqrt[a^2 - b^2])])/(a*d) - (2*b*(a^2 - b^2)*ArcTanh[Cos[c + d*x]])/(a*d)
)/a) - (2*(a^2 - b^2)*Cot[c + d*x])/(a*d))/(a*(a^2 - b^2)) + Cot[c + d*x]/
(a*d*(a + b*Sin[c + d*x]))
```

3.189.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.189.4 Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.36

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\right.}{2\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}}\right.}{a^3}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}-\frac{1}{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2+ab}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{\left(a^2-2b^2\right)\arctan\left(\frac{2a\tan\left(\frac{dx}{2}+\right.}{2\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}}\right.}{a^3}}{d}$
risch	$-\frac{2\left(-3a\,e^{i(dx+c)}+2ib\,e^{2i(dx+c)}-2ib+a\,e^{3i(dx+c)}\right)}{\left(e^{2i(dx+c)}-1\right)\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)a^2d}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2+b^2}-a^2+b^2}{b\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}\,da}-\frac{2\ln\left(e^{i(dx+c)}+\frac{ia\sqrt{-a^2-}}{b\sqrt{-}}\right.}{\sqrt{-a^2+b^2}\,d}$

input
```
int(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a^2*tan(1/2*d*x+1/2*c)-1/2/a^2/tan(1/2*d*x+1/2*c)-2*b/a^3*ln(tan(
1/2*d*x+1/2*c))-2/a^3*((tan(1/2*d*x+1/2*c)*b^2+a*b)/(a*tan(1/2*d*x+1/2*c)^
2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^2-2*b^2)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*ta
n(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

3.189.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(110) = 220$.

Time = 0.35 (sec) , antiderivative size = 768, normalized size of antiderivative = 6.68

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \left[\frac{4\left(a^3b-ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(a^2b-2\,b^3-\left(a^2b-2\,b^3\right)\cos\left(dx+c\right)^2+\left(a^3-2\,ab^2\right)\sin\left(dx+\right.\right.}{}\right.$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/2*(4*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (a^2*b - 2*b^3 - (a^2*
b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x + c))*sqrt(-a^2 + b^2)
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a
*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 2*(a^4 - a^2*b^2)*cos(d*x +
c) - 2*(a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*s
in(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 2*(a^2*b^2 - b^4 - (a^2*b^2 - b
^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(-1/2*cos(d*x + c) +
 1/2))/((a^5*b - a^3*b^3)*d*cos(d*x + c)^2 - (a^6 - a^4*b^2)*d*sin(d*x + c
) - (a^5*b - a^3*b^3)*d), (2*(a^3*b - a*b^3)*cos(d*x + c)*sin(d*x + c) - (
a^2*b - 2*b^3 - (a^2*b - 2*b^3)*cos(d*x + c)^2 + (a^3 - 2*a*b^2)*sin(d*x +
 c))*sqrt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x
 + c))) + (a^4 - a^2*b^2)*cos(d*x + c) - (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*
cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + (a^2*b^2 - b^4 - (a^2*b^2 - b^4)*cos(d*x + c)^2 + (a^3*b - a*b^3)*sin(d
*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^5*b - a^3*b^3)*d*cos(d*x + c)^2
 - (a^6 - a^4*b^2)*d*sin(d*x + c) - (a^5*b - a^3*b^3)*d)]
```

3.189.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**2, x)
```

3.189.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.189.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.90

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx =$$

$$-\frac{\frac{12\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^3}-\frac{3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}+\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)(a^2-2\,b^2)}{\sqrt{a^2-b^2}a^3}-\frac{4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\right.}}{6\,d}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/6*(12*b*log(abs(tan(1/2*d*x + 1/2*c)))/a^3 - 3*tan(1/2*d*x + 1/2*c)/a^2
 + 12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1
/2*c) + b)/sqrt(a^2 - b^2)))*(a^2 - 2*b^2)/(sqrt(a^2 - b^2)*a^3) - (4*a*b*
tan(1/2*d*x + 1/2*c)^3 - 3*a^2*tan(1/2*d*x + 1/2*c)^2 - 4*b^2*tan(1/2*d*x
+ 1/2*c)^2 - 14*a*b*tan(1/2*d*x + 1/2*c) - 3*a^2)/((a*tan(1/2*d*x + 1/2*c)
^3 + 2*b*tan(1/2*d*x + 1/2*c)^2 + a*tan(1/2*d*x + 1/2*c))*a^3))/d
```

3.189.9 Mupad [B] (verification not implemented)

Time = 8.18 (sec) , antiderivative size = 1616, normalized size of antiderivative = 14.05

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2/(a + b*sin(c + d*x))^2,x)
```

output
```
-(a^4*cos(c + d*x) - b^4/2 - b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)
) + (a^2*b^2)/2 + (b^4*cos(2*c + 2*d*x))/2 - a^2*b^2*cos(c + d*x) - a*b^3*
sin(2*c + 2*d*x) + a^3*b*sin(2*c + 2*d*x) + a^2*b^2*log(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)) + b^4*log(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(
2*c + 2*d*x) + b^3*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3
*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 -
 a^2)^(1/2)*4i + a^2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c
/2 + (d*x)/2) + 8*b^4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*
a^3*b*cos(c/2 + (d*x)/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2
)*2i - (a^2*b^2*cos(2*c + 2*d*x))/2 - a*b^3*sin(c + d*x) + a^3*b*sin(c + d
*x) - a^2*b*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*sin(c/
2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(
1/2)*4i + a^2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2 + (d
*x)/2) + 8*b^4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^3*b*c
os(c/2 + (d*x)/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/2)*1i -
a^3*sin(c + d*x)*atan((a^3*cos(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*1i - b^3*s
in(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*8i - a*b^2*cos(c/2 + (d*x)/2)*(b^2 - a
^2)^(1/2)*4i + a^2*b*sin(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2)*4i)/(a^4*sin(c/2
 + (d*x)/2) + 8*b^4*sin(c/2 + (d*x)/2) + 4*a*b^3*cos(c/2 + (d*x)/2) - 3*a^
3*b*cos(c/2 + (d*x)/2) - 8*a^2*b^2*sin(c/2 + (d*x)/2)))*(b^2 - a^2)^(1/...
```

3.189. $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.190 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.190.1 Optimal result

Integrand size = 21, antiderivative size = 238

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{2(a^4-5a^2b^2+4b^4)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}d} - \frac{b(3a^2-4b^2)\operatorname{arctanh}(\cos(c+dx))}{a^5d} + \frac{(7a^2-12b^2)\cot(c+dx)}{3a^4d} - \frac{(a^2-2b^2)\cot(c+dx)\csc(c+dx)}{a^3bd} + \frac{(3a^2-4b^2)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))} - \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

output
```
-b*(3*a^2-4*b^2)*arctanh(cos(d*x+c))/a^5/d+1/3*(7*a^2-12*b^2)*cot(d*x+c)/a
^4/d-(a^2-2*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d+1/3*(3*a^2-4*b^2)*cot(d*x+c
)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))-1/3*cot(d*x+c)*csc(d*x+c)^2/a/d/(a+b
*sin(d*x+c))+2*(a^4-5*a^2*b^2+4*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-
b^2)^(1/2))/a^5/d/(a^2-b^2)^(1/2)
```

3.190.2 Mathematica [A] (verified)

Time = 6.34 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.69

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$
$$= \frac{2(a^4-5a^2b^2+4b^4)\arctan\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^5\sqrt{a^2-b^2}d} + \frac{\left(4a^2\cos\left(\frac{1}{2}(c+dx)\right)-9b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{6a^4d} + \frac{b\csc^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^2d} + \frac{(-3a^2b+4b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} + \frac{(3a^2b-4b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a^5d} - \frac{b\sec^2\left(\frac{1}{2}(c+dx)\right)}{4a^3d} + \frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-4a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6a^4d} + \frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{a^4d(a+b\sin(c+dx))} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^2d}$$

input
```
Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

output
```
(2*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2]
+ a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]*d) + ((4*a^2
*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(6*a^4*d) +
(b*Csc[(c + d*x)/2]^2)/(4*a^3*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(
24*a^2*d) + ((-3*a^2*b + 4*b^3)*Log[Cos[(c + d*x)/2]])/(a^5*d) + ((3*a^2*b
 - 4*b^3)*Log[Sin[(c + d*x)/2]])/(a^5*d) - (b*Sec[(c + d*x)/2]^2)/(4*a^3*d
) + (Sec[(c + d*x)/2]*(-4*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c + d*x)/2]))/
(6*a^4*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(a^4*d*(a + b*Sin[c +
d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^2*d)
```

3.190.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3203, 3042, 3534, 27, 3042, 3534, 27, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^4(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{ 3203}$$

$$\frac{\int \frac{\csc^3(c+dx)\left(-\left(\left(3a^2-8b^2\right)\sin^2(c+dx)\right)-ab\sin(c+dx)+6\left(a^2-2b^2\right)\right)}{a+b\sin(c+dx)}dx}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-\left(\left(3a^2-8b^2\right)\sin(c+dx)^2\right)-ab\sin(c+dx)+6\left(a^2-2b^2\right)}{\sin(c+dx)^3(a+b\sin(c+dx))}dx}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{\frac{\int -\frac{2\csc^2(c+dx)\left(-2a\sin(c+dx)b^2-3\left(a^2-2b^2\right)\sin^2(c+dx)b+\left(7a^2-12b^2\right)b\right)}{a+b\sin(c+dx)}dx}{2a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{\csc^2(c+dx)\left(-2a\sin(c+dx)b^2-3\left(a^2-2b^2\right)\sin^2(c+dx)b+\left(7a^2-12b^2\right)b\right)}{a+b\sin(c+dx)}dx}{a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{\int \frac{-2a\sin(c+dx)b^2-3\left(a^2-2b^2\right)\sin(c+dx)^2b+\left(7a^2-12b^2\right)b}{\sin(c+dx)^2(a+b\sin(c+dx))}dx}{a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{ 3534}$$

3.190. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

$$\frac{-\frac{\frac{\int -\frac{3\csc(c+dx)\left(\left(3a^2-4b^2\right)b^2+a\left(a^2-2b^2\right)\sin(c+dx)b\right)}{a+b\sin(c+dx)}dx}{a}-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}}{a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{-\frac{3\int \frac{\csc(c+dx)\left(\left(3a^2-4b^2\right)b^2+a\left(a^2-2b^2\right)\sin(c+dx)b\right)}{a+b\sin(c+dx)}dx}{a}-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}}{a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{-\frac{3\int \frac{\left(3a^2-4b^2\right)b^2+a\left(a^2-2b^2\right)\sin(c+dx)b}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}}{a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3480$$

$$\frac{-\frac{-\frac{3\left(\frac{b^2\left(3a^2-4b^2\right)\int \csc(c+dx)dx}{a}+\frac{b\left(a^4-5a^2b^2+4b^4\right)\int \frac{1}{a+b\sin(c+dx)}dx}{a}\right)}{a}-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}}{a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{-\frac{3\left(\frac{b^2\left(3a^2-4b^2\right)\int \csc(c+dx)dx}{a}+\frac{b\left(a^4-5a^2b^2+4b^4\right)\int \frac{1}{a+b\sin(c+dx)}dx}{a}\right)}{a}-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}}{a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}+\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3139$$

3.190. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

$$-\frac{-\frac{-\frac{3\left(\frac{b^2\left(3a^2-4b^2\right)\int \csc(c+dx)dx}{a}+\frac{2b\left(a^4-5a^2b^2+4b^4\right)\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}}{a}-\frac{3(a^2-2b}{}}{3a^2b}$$

$$\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$\Big\downarrow$ 1083

$$-\frac{-\frac{-\frac{3\left(\frac{b^2\left(3a^2-4b^2\right)\int \csc(c+dx)dx}{a}-\frac{4b\left(a^4-5a^2b^2+4b^4\right)\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}}{a}-\frac{3}{}}{3a^2b}$$

$$\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$\Big\downarrow$ 217

$$-\frac{-\frac{-\frac{3\left(\frac{b^2\left(3a^2-4b^2\right)\int \csc(c+dx)dx}{a}+\frac{2b\left(a^4-5a^2b^2+4b^4\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}\right)}{a}-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}}{a}-\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}}{3a^2b}$$

$$\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

$\Big\downarrow$ 4257

$$\frac{\left(3a^2-4b^2\right)\cot(c+dx)\csc(c+dx)}{3a^2bd(a+b\sin(c+dx))}+$$

$$-\frac{\frac{3\left(a^2-2b^2\right)\cot(c+dx)\csc(c+dx)}{ad}-\frac{-\frac{b\left(7a^2-12b^2\right)\cot(c+dx)}{ad}-\frac{3\left(\frac{2b\left(a^4-5a^2b^2+4b^4\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{b^2\left(3a^2-4b^2\right)\text{arctanh}(\cos(c}{ad}\right)}{a}}{a}}{3a^2b}$$

$$\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))}$$

input

```
Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^2,x]
```

3.190. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

output
```
(-(((-3*((2*b*(a^4 - 5*a^2*b^2 + 4*b^4)*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2]
)/(2*Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (b^2*(3*a^2 - 4*b^2)*ArcTa
nh[Cos[c + d*x]])/(a*d)))/a - (b*(7*a^2 - 12*b^2)*Cot[c + d*x])/(a*d))/a)
- (3*(a^2 - 2*b^2)*Cot[c + d*x]*Csc[c + d*x])/(a*d))/(3*a^2*b) + ((3*a^2 -
 4*b^2)*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*b*d*(a + b*Sin[c + d*x])) - (Cot
[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x]))
```

3.190.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3203
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Simp[(3*a^2 + b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e
+ f*x])^(m + 1)/(3*a^2*b*f*(m + 1)*Sin[e + f*x]^2)), x] - Simp[1/(3*a^2*b*(
m + 1))   Int[((a + b*Sin[e + f*x])^(m + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^
2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[
e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[m, -1] && IntegerQ[2*m]
```

3.190. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
 - a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.190.4 Maple [A] (verified)

Time = 2.96 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2}{8a^4}+\frac{\frac{2\left(b^2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab\left(a^2-b^2\right)\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{2\left(a^4 - \cdots\right.}{\cdots}}{a^5}}{\cdots}$
default	$\frac{\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+12\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2}{8a^4}+\frac{\frac{2\left(b^2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+ab\left(a^2-b^2\right)\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a}+\frac{2\left(a^4 - \cdots\right.}{\cdots}}{a^5}}{\cdots}$
risch	$\frac{-14ia^2b\,\mathrm{e}^{4i(dx+c)}+24ib^3\mathrm{e}^{4i(dx+c)}-4\,\mathrm{e}^{7i(dx+c)}a\,b^2+20\,\mathrm{e}^{5i(dx+c)}a\,b^2-28\,\mathrm{e}^{3i(dx+c)}a\,b^2+\frac{50ia^2b\,\mathrm{e}^{2i(dx+c)}}{3}+2ia^2b\,\mathrm{e}^{6i(dx+\cdots}}{\left(\mathrm{e}^{2i(dx+c)}-1\right)^3\left(b\,\mathrm{e}^{2i\cdots}\right.}$

input
```
int(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/a^4*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)^2*a*b-5*a^
2*tan(1/2*d*x+1/2*c)+12*tan(1/2*d*x+1/2*c)*b^2)+2/a^5*((b^2*(a^2-b^2)*tan(
1/2*d*x+1/2*c)+a*b*(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*
c)+a)+(a^4-5*a^2*b^2+4*b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/
2*c)+2*b)/(a^2-b^2)^(1/2)))-1/24/a^2/tan(1/2*d*x+1/2*c)^3-1/8*(-5*a^2+12*b
^2)/a^4/tan(1/2*d*x+1/2*c)+1/4*b/a^3/tan(1/2*d*x+1/2*c)^2+1/a^5*b*(3*a^2-4
*b^2)*ln(tan(1/2*d*x+1/2*c)))
```

3.190.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(227) = 454$.

Time = 0.38 (sec) , antiderivative size = 1149, normalized size of antiderivative = 4.83

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
[-1/6*(4*(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 3*((a^2*b - 4*b^3)*cos(d*x +
 c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a*b^2
- (a^3 - 4*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a
^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x +
 c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 -
 2*a*b*sin(d*x + c) - a^2 - b^2)) - 6*(a^4 - 2*a^2*b^2)*cos(d*x + c) + 3*(
(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*
b^4)*cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c
)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 3*((3*a^2*b^2 - 4*b^4)*co
s(d*x + c)^4 + 3*a^2*b^2 - 4*b^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 +
(3*a^3*b - 4*a*b^3 - (3*a^3*b - 4*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log
(-1/2*cos(d*x + c) + 1/2) + 2*((7*a^3*b - 12*a*b^3)*cos(d*x + c)^3 - 3*(3*
a^3*b - 4*a*b^3)*cos(d*x + c))*sin(d*x + c))/(a^5*b*d*cos(d*x + c)^4 - 2*a
^5*b*d*cos(d*x + c)^2 + a^5*b*d - (a^6*d*cos(d*x + c)^2 - a^6*d)*sin(d*x +
 c)), -1/6*(4*(2*a^4 - 3*a^2*b^2)*cos(d*x + c)^3 + 6*((a^2*b - 4*b^3)*cos(
d*x + c)^4 + a^2*b - 4*b^3 - 2*(a^2*b - 4*b^3)*cos(d*x + c)^2 + (a^3 - 4*a
*b^2 - (a^3 - 4*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*arcta
n(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c))) - 6*(a^4 - 2*a^2*b
^2)*cos(d*x + c) + 3*((3*a^2*b^2 - 4*b^4)*cos(d*x + c)^4 + 3*a^2*b^2 - 4*b
^4 - 2*(3*a^2*b^2 - 4*b^4)*cos(d*x + c)^2 + (3*a^3*b - 4*a*b^3 - (3*a^3...
```

3.190.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx$$

input
```
integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**2, x)
```

3.190.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.190.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.50

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \frac{\frac{24\,(3\,a^2b-4\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^5} + \frac{48\,(a^4-5\,a^2b^2+4\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^5} + \frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,\cdots}{\cdots}}{\cdots}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
1/24*(24*(3*a^2*b - 4*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^5 + 48*(a^4 -
5*a^2*b^2 + 4*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan((a*ta
n(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^5) + (a^4*tan
(1/2*d*x + 1/2*c)^3 - 6*a^3*b*tan(1/2*d*x + 1/2*c)^2 - 15*a^4*tan(1/2*d*x
 + 1/2*c) + 36*a^2*b^2*tan(1/2*d*x + 1/2*c))/a^6 + 48*(a^2*b^2*tan(1/2*d*x
 + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c) + a^3*b - a*b^3)/((a*tan(1/2*d*x + 1/2
*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^5) - (132*a^2*b*tan(1/2*d*x + 1/2*
c)^3 - 176*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 36
*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 6*a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^5*t
an(1/2*d*x + 1/2*c)^3))/d
```

3.190.9 Mupad [B] (verification not implemented)

Time = 6.81 (sec) , antiderivative size = 973, normalized size of antiderivative = 4.09

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4/(a + b*sin(c + d*x))^2,x)
```

output
```
(tan(c/2 + (d*x)/2)^3*(28*a^2*b - 40*b^3) - tan(c/2 + (d*x)/2)^2*(8*a*b^2
- (14*a^3)/3) - a^3/3 + (4*a^2*b*tan(c/2 + (d*x)/2))/3 + (tan(c/2 + (d*x)/
2)^4*(5*a^4 - 16*b^4 + 4*a^2*b^2))/a)/(d*(8*a^5*tan(c/2 + (d*x)/2)^3 + 8*a
^5*tan(c/2 + (d*x)/2)^5 + 16*a^4*b*tan(c/2 + (d*x)/2)^4)) + tan(c/2 + (d*x
)/2)^3/(24*a^2*d) - (tan(c/2 + (d*x)/2)*((16*a^2 + 32*b^2)/(64*a^4) + 3/(8
*a^2) - (2*b^2)/a^4))/d - (b*tan(c/2 + (d*x)/2)^2)/(4*a^3*d) + (b*log(tan(
c/2 + (d*x)/2))*(3*a^2 - 4*b^2))/(a^5*d) + (atan((((b^2 - a^2)^(1/2)*(a^2
- 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5
*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((2*a^2*b - (2*tan(c/2 + (d*x)/2)
*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a^2 - 4*b^2))/a^5)*1i)/a^5
+ ((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))/a^8
+ (2*tan(c/2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 - ((2*a^2
*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/2)*(a
^2 - 4*b^2))/a^5)*1i)/a^5)/((4*(3*a^6*b - 16*b^7 + 32*a^2*b^5 - 19*a^4*b^3
))/a^8 + (4*tan(c/2 + (d*x)/2)*(2*a^6 - 16*b^6 + 28*a^2*b^4 - 14*a^4*b^2))
/a^7 - ((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 + 8*a^5*b^4 - 8*a^7*b^2))
/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b + 16*a^3*b^5 - 20*a^5*b^3))/a^7 + ((
2*a^2*b - (2*tan(c/2 + (d*x)/2)*(3*a^10 - 4*a^8*b^2))/a^7)*(b^2 - a^2)^(1/
2)*(a^2 - 4*b^2))/a^5))/a^5 + ((b^2 - a^2)^(1/2)*(a^2 - 4*b^2)*((2*(a^9 +
8*a^5*b^4 - 8*a^7*b^2))/a^8 + (2*tan(c/2 + (d*x)/2)*(5*a^7*b + 16*a^3*b...
```

3.191 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.191.1 Optimal result

Integrand size = 21, antiderivative size = 424

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = &-\frac{2(a^2-6b^2)\left(a^2-b^2\right)^{3/2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^7d}\\
&+\frac{b(15a^4-40a^2b^2+24b^4)\,\mathrm{arctanh}(\cos(c+dx))}{4a^7d}\\
&-\frac{(38a^4-135a^2b^2+90b^4)\cot(c+dx)}{15a^6d}\\
&+\frac{(4a^4-17a^2b^2+12b^4)\cot(c+dx)\csc(c+dx)}{4a^5bd}\\
&-\frac{(15a^4-82a^2b^2+60b^4)\cot(c+dx)\csc^2(c+dx)}{30a^4b^2d}\\
&-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}\\
&+\frac{(2a^4-12a^2b^2+9b^4)\cot(c+dx)\csc^2(c+dx)}{6a^3b^2d(a+b\sin(c+dx))}\\
&+\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}\end{aligned}$$

output
```
-2*(a^2-6*b^2)*(a^2-b^2)^(3/2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(
1/2))/a^7/d+1/4*b*(15*a^4-40*a^2*b^2+24*b^4)*arctanh(cos(d*x+c))/a^7/d-1/1
5*(38*a^4-135*a^2*b^2+90*b^4)*cot(d*x+c)/a^6/d+1/4*(4*a^4-17*a^2*b^2+12*b^
4)*cot(d*x+c)*csc(d*x+c)/a^5/b/d-1/30*(15*a^4-82*a^2*b^2+60*b^4)*cot(d*x+c
)*csc(d*x+c)^2/a^4/b^2/d-1/2*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))+1/
6*a*cot(d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))+1/6*(2*a^4-12*a^2*b^2+9
*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))+3/10*b*cot(d*x+c)
*csc(d*x+c)^3/a^2/d/(a+b*sin(d*x+c))-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(a+b*
sin(d*x+c))
```

3.191.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 0.85

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{1920(a^2-6b^2)\left(a^2-b^2\right)^{3/2}\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-240b(15a^4-40a^2b^2+24b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{}$$

input
```
Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

output
```
-1/960*(1920*(a^2 - 6*b^2)*(a^2 - b^2)^(3/2)*ArcTan[(b + a*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]] - 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Cos[(c + d*
x)/2]] + 240*b*(15*a^4 - 40*a^2*b^2 + 24*b^4)*Log[Sin[(c + d*x)/2]] + (2*a
*Cot[c + d*x]*Csc[c + d*x]^5*(196*a^5 - 735*a^3*b^2 + 540*a*b^4 - 12*(16*a
^5 - 85*a^3*b^2 + 60*a*b^4)*Cos[2*(c + d*x)] + (92*a^5 - 285*a^3*b^2 + 180
*a*b^4)*Cos[4*(c + d*x)] + 1162*a^4*b*Sin[c + d*x] - 3060*a^2*b^3*Sin[c +
d*x] + 1800*b^5*Sin[c + d*x] - 562*a^4*b*Sin[3*(c + d*x)] + 1470*a^2*b^3*S
in[3*(c + d*x)] - 900*b^5*Sin[3*(c + d*x)] + 76*a^4*b*Sin[5*(c + d*x)] - 2
70*a^2*b^3*Sin[5*(c + d*x)] + 180*b^5*Sin[5*(c + d*x)]))/(b + a*Csc[c + d*
x]))/(a^7*d)
```

3.191.3 Rubi [A] (verified)

Time = 3.25 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.19, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.048, Rules used = {3042, 3205, 27, 3042, 3534, 27, 3042, 3534, 25, 3042, 3534, 25, 3042, 3534, 27, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^6(a+b\sin(c+dx))^2}dx$$

$$\Big\downarrow \text{3205}$$

$$\frac{\int \frac{4\csc^4(c+dx)\left(-\left(\left(10a^4-45b^2a^2+36b^4\right)\sin^2(c+dx)\right)-ab\left(10a^2-3b^2\right)\sin(c+dx)+3\left(5a^4-22b^2a^2+15b^4\right)\right)}{(a+b\sin(c+dx))^2}dx}{120a^2b^2}+\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\csc^4(c+dx)\left(-\left(\left(10a^4-45b^2a^2+36b^4\right)\sin^2(c+dx)\right)-ab\left(10a^2-3b^2\right)\sin(c+dx)+3\left(5a^4-22b^2a^2+15b^4\right)\right)}{(a+b\sin(c+dx))^2}dx}{30a^2b^2}+\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-\left(\left(10a^4-45b^2a^2+36b^4\right)\sin(c+dx)^2\right)-ab\left(10a^2-3b^2\right)\sin(c+dx)+3\left(5a^4-22b^2a^2+15b^4\right)}{\sin(c+dx)^4(a+b\sin(c+dx))^2}dx}{30a^2b^2}+\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{3\csc^4(c+dx)\left(15a^6-97b^2a^4+142b^4a^2-b\left(5a^4-8b^2a^2+3b^4\right)\sin(c+dx)a-60b^6-5\left(2a^6-14b^2a^4+21b^4a^2-9b^6\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{a(a^2-b^2)}+\frac{5(2a^4-12a^2b^2+9b^4)\cot(c}{ad(a+b\sin(c+}}{30a^2b^2}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$\downarrow$ 27

$$\frac{\frac{3\int \frac{\csc^4(c+dx)\left(15a^6-97b^2a^4+142b^4a^2-b\left(5a^4-8b^2a^2+3b^4\right)\sin(c+dx)a-60b^6-5\left(2a^6-14b^2a^4+21b^4a^2-9b^6\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{a(a^2-b^2)}+\frac{5(2a^4-12a^2b^2+9b^4)\cot(c}{ad(a+b\sin(c+}}{30a^2b^2}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$\downarrow$ 3042

$$\frac{\frac{3\int \frac{15a^6-97b^2a^4+142b^4a^2-b\left(5a^4-8b^2a^2+3b^4\right)\sin(c+dx)a-60b^6-5\left(2a^6-14b^2a^4+21b^4a^2-9b^6\right)\sin(c+dx)^2}{\sin(c+dx)^4(a+b\sin(c+dx))}dx}{a(a^2-b^2)}+\frac{5(2a^4-12a^2b^2+9b^4)\cot(c+dx)\csc^2(c}{ad(a+b\sin(c+dx))}}{30a^2b^2}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$\downarrow$ 3534

$$\frac{\frac{3\left(\frac{\int -\frac{\csc^3(c+dx)\left(-a\left(16a^4-31b^2a^2+15b^4\right)\sin(c+dx)b^2-2\left(15a^6-97b^2a^4+142b^4a^2-60b^6\right)\sin^2(c+dx)b+15\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)b\right)}{a+b\sin(c+dx)}dx}{3a}-\frac{\left(15a^6-97a^4b\right.}{}\right.}{a(a^2-b^2)}}{30a^2b^2}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$\downarrow$ 25

3.191. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

$$\frac{\frac{3\left(-\frac{\int \frac{\csc^3(c+dx)\left(-a\left(16a^4-31b^2a^2+15b^4\right)\sin(c+dx)b^2-2\left(15a^6-97b^2a^4+142b^4a^2-60b^6\right)\sin^2(c+dx)b+15\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)b\right)}{a+b\sin(c+dx)}dx}{3a}-\frac{\left(15a^6-97a^4b\cdots\right.}{\cdots}\right.}{a(a^2-b^2)}}{30a^2b^2}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{3\left(-\frac{\int \frac{-a\left(16a^4-31b^2a^2+15b^4\right)\sin(c+dx)b^2-2\left(15a^6-97b^2a^4+142b^4a^2-60b^6\right)\sin(c+dx)^2b+15\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)b}{\sin(c+dx)^3(a+b\sin(c+dx))}dx}{3a}-\frac{\left(15a^6-97a^4b^2+142a^2b^4-\cdots\right.}{3a\cdots}\right.}{a(a^2-b^2)}}{30a^2b^2}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{3\left(-\frac{\frac{\int -\frac{\csc^2(c+dx)\left(-a\left(73a^4-133b^2a^2+60b^4\right)\sin(c+dx)b^3-15\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)\sin^2(c+dx)b^2+4\left(38a^6-173b^2a^4+225b^4a^2-90b^6\right)b^2\right)}{a+b\sin(c+dx)}dx}{2a}-\frac{15b\left(4a\cdots\right.}{\cdots}}{3a}\right.}{a(a^2-b^2)}}{30a\cdots}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{3\left(-\frac{-\frac{\int \frac{\csc^2(c+dx)\left(-a\left(73a^4-133b^2a^2+60b^4\right)\sin(c+dx)b^3-15\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)\sin^2(c+dx)b^2+4\left(38a^6-173b^2a^4+225b^4a^2-90b^6\right)b^2\right)}{a+b\sin(c+dx)}dx}{2a}-\frac{15b\left(4a\cdots\right.}{\cdots}}{3a}\right.}{a(a^2-b^2)}}{30a\cdots}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.191. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

$$\frac{3\left(-\frac{-\frac{\int \frac{-a\left(73a^4-133b^2a^2+60b^4\right)\sin(c+dx)b^3-15\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)\sin(c+dx)^2b^2+4\left(38a^6-173b^2a^4+225b^4a^2-90b^6\right)b^2}{\sin(c+dx)^2(a+b\sin(c+dx))}dx}{2a}-\frac{15b\left(4a^6-21a^4b^2+29\right.}{}}{3a}\right.}{a(a^2-b^2)}$$

$$30a^2b^2$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3534$$

$$\frac{3\left(-\frac{-\frac{\int -\frac{15\csc(c+dx)\left(\left(15a^6-55b^2a^4+64b^4a^2-24b^6\right)b^3+a\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)\sin(c+dx)b^2\right)}{a+b\sin(c+dx)}dx}{a}-\frac{4b^2\left(38a^6-173a^4b^2+225a^2b^4-90b^6\right)\cot(c+dx)}{ad}-}{2a}}{3a}\right.}{a(a^2-b^2)}$$

$$3$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(-\frac{-\frac{-\frac{15\int \frac{\csc(c+dx)\left(\left(15a^6-55b^2a^4+64b^4a^2-24b^6\right)b^3+a\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)\sin(c+dx)b^2\right)}{a+b\sin(c+dx)}dx}{a}-\frac{4b^2\left(38a^6-173a^4b^2+225a^2b^4-90b^6\right)\cot(c+dx)}{ad}-}{2a}}{3a}\right.}{a(a^2-b^2)}$$

$$3$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 3042$$

3.191. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

$$3\left(-\frac{-\frac{-\frac{15\int\frac{\left(15a^6-55b^2a^4+64b^4a^2-24b^6\right)b^3+a\left(4a^6-21b^2a^4+29b^4a^2-12b^6\right)\sin(c+dx)b^2}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}}{2a}-\frac{4b^2\left(38a^6-173a^4b^2+225a^2b^4-90b^6\right)\cot(c+dx)}{ad}-\frac{15b\left(4a^6-21\right.}{}}{3a}}{a(a^2-b^2)}\right.$$

$$30a^2b$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3480}$$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{4b^2\left(a^2-6b^2\right)\left(a^2-b^2\right)^3\int\frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b^3\left(15a^6-55a^4b^2+64a^2b^4-24b^6\right)\int\csc(c+dx)dx}{a}\right)}{a}}{2a}-\frac{4b^2\left(38a^6-173a^4b^2+225a^2b^4-90b^6\right)\cot(c+dx)}{ad}}{3a}}{a(a^2-b^2)}\right.$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{4b^2\left(a^2-6b^2\right)\left(a^2-b^2\right)^3\int\frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b^3\left(15a^6-55a^4b^2+64a^2b^4-24b^6\right)\int\csc(c+dx)dx}{a}\right)}{a}}{2a}-\frac{4b^2\left(38a^6-173a^4b^2+225a^2b^4-90b^6\right)\cot(c+dx)}{ad}}{3a}}{a(a^2-b^2)}\right.$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \text{3139}$$

3.191. $\int\frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{8b^2\left(a^2-6b^2\right)\left(a^2-b^2\right)^3\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}+\frac{b^3\left(15a^6-55a^4b^2+64a^2b^4-24b^6\right)\int \csc(c+dx)dx}{a}\right)}{a}-\frac{4b^2\left(38a^6\right.}{}}{2a}}{3a}\right.$$

$$\overline{\qquad a(a^2-b}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 1083$$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{b^3\left(15a^6-55a^4b^2+64a^2b^4-24b^6\right)\int \csc(c+dx)dx}{a}-\frac{16b^2\left(a^2-6b^2\right)\left(a^2-b^2\right)^3\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}-}{2a}}{3a}\right.$$

$$\overline{\qquad a(a}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

$$\Big\downarrow \; 217$$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{b^3\left(15a^6-55a^4b^2+64a^2b^4-24b^6\right)\int \csc(c+dx)dx}{a}+\frac{8b^2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{5/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}\right)}{a}-\frac{4b^2\left(38a^6-173a^4b^2+225a^2b^4-90\right.}{ad}}{2a}}{3a}\right.$$

$$\overline{\qquad a(a^2-b^2)}$$

$$\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

3.191. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

$\big\downarrow$ 4257

$$\frac{\frac{3b\cot(c+dx)\csc^3(c+dx)}{10a^2d(a+b\sin(c+dx))}+\frac{\frac{5\left(2a^4-12a^2b^2+9b^4\right)\cot(c+dx)\csc^2(c+dx)}{ad(a+b\sin(c+dx))}+3\left(-\frac{-\frac{-\frac{15\left(\frac{8b^2\left(a^2-6b^2\right)\left(a^2-b^2\right)^{5/2}\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}-\frac{b^3\left(15a^6-55a^4b^2+64a^2b^4-24b^6\right)\cdots}{ad}\right)}{a}\cdots}{2a}\cdots}{\cdots}\right)}{\cdots}}{\cdots}$$

$$\frac{a\cot(c+dx)\csc^2(c+dx)}{6b^2d(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))}-\frac{\cot(c+dx)\csc(c+dx)}{2bd(a+b\sin(c+dx))}$$

input
```
Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^2,x]
```

output
```
-1/2*(Cot[c + d*x]*Csc[c + d*x])/(b*d*(a + b*Sin[c + d*x])) + (a*Cot[c + d
*x]*Csc[c + d*x]^2)/(6*b^2*d*(a + b*Sin[c + d*x])) + (3*b*Cot[c + d*x]*Csc
[c + d*x]^3)/(10*a^2*d*(a + b*Sin[c + d*x])) - (Cot[c + d*x]*Csc[c + d*x]^
4)/(5*a*d*(a + b*Sin[c + d*x])) + ((3*(-1/3*((15*a^6 - 97*a^4*b^2 + 142*a^
2*b^4 - 60*b^6)*Cot[c + d*x]*Csc[c + d*x]^2)/(a*d) - (-1/2*((-15*((8*b^2*(
a^2 - 6*b^2)*(a^2 - b^2)^(5/2)*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt
[a^2 - b^2])])/(a*d) - (b^3*(15*a^6 - 55*a^4*b^2 + 64*a^2*b^4 - 24*b^6)*Ar
cTanh[Cos[c + d*x]])/(a*d)))/a - (4*b^2*(38*a^6 - 173*a^4*b^2 + 225*a^2*b^
4 - 90*b^6)*Cot[c + d*x])/(a*d))/a - (15*b*(4*a^6 - 21*a^4*b^2 + 29*a^2*b^
4 - 12*b^6)*Cot[c + d*x]*Csc[c + d*x])/(2*a*d))/(3*a)))/(a*(a^2 - b^2)) +
(5*(2*a^4 - 12*a^2*b^2 + 9*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(a*d*(a + b*S
in[c + d*x])))/(30*a^2*b^2)
```

3.191.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.191. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3205
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*
Sin[e + f*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b
^2*f*m*(m - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*S
in[e + f*x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x] + Simp[1/(20*a^2*b^2*m*
(m - 1))   Int[((a + b*Sin[e + f*x])^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2
*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m -
 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m
 - 1)*(2*m + 1))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f, m}, x] &
& NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.191. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.191.4 Maple [A] (verified)

Time = 5.90 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{3}+4a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+16a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16a\,b^3\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{32a^6}$
default	$\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{3}+4a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+16a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-16a\,b^3\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{32a^6}$
risch	Expression too large to display

input
```
int(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/32/a^6*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-b*tan(1/2*d*x+1/2*c)^4*a^3-7/3
*tan(1/2*d*x+1/2*c)^3*a^4+4*a^2*b^2*tan(1/2*d*x+1/2*c)^3+16*a^3*b*tan(1/2*
d*x+1/2*c)^2-16*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-108*a
^2*b^2*tan(1/2*d*x+1/2*c)+80*b^4*tan(1/2*d*x+1/2*c))-2/a^7*((b^2*(a^4-2*a^
2*b^2+b^4)*tan(1/2*d*x+1/2*c)+b*a*(a^4-2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*
c)^2+2*b*tan(1/2*d*x+1/2*c)+a)+(a^6-8*a^4*b^2+13*a^2*b^4-6*b^6)/(a^2-b^2)^
(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/160/a^2/
tan(1/2*d*x+1/2*c)^5-1/96*(-7*a^2+12*b^2)/a^4/tan(1/2*d*x+1/2*c)^3-1/32*(2
2*a^4-108*a^2*b^2+80*b^4)/a^6/tan(1/2*d*x+1/2*c)+1/32*b/a^3/tan(1/2*d*x+1/
2*c)^4-1/2/a^5*b*(a^2-b^2)/tan(1/2*d*x+1/2*c)^2-1/4/a^7*b*(15*a^4-40*a^2*b
^2+24*b^4)*ln(tan(1/2*d*x+1/2*c)))
```

3.191.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 964 vs. $2(401) = 802$.

Time = 0.62 (sec) , antiderivative size = 2011, normalized size of antiderivative = 4.74

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2} \, dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="fricas")
```

output
```
[1/120*(2*(92*a^6 - 285*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^5 - 40*(7*a^6
- 27*a^4*b^2 + 18*a^2*b^4)*cos(d*x + c)^3 + 60*((a^4*b - 7*a^2*b^3 + 6*b^5
)*cos(d*x + c)^6 - a^4*b + 7*a^2*b^3 - 6*b^5 - 3*(a^4*b - 7*a^2*b^3 + 6*b^
5)*cos(d*x + c)^4 + 3*(a^4*b - 7*a^2*b^3 + 6*b^5)*cos(d*x + c)^2 - (a^5 -
7*a^3*b^2 + 6*a*b^4 + (a^5 - 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^4 - 2*(a^5
- 7*a^3*b^2 + 6*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(
((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(
d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 30*(4*a^6 - 17*a^4*b^2 + 12*a^2*b
^4)*cos(d*x + c) + 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^6 -
 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*c
os(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x + c)^2 - (15*
a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x
 + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^2)*sin(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) - 15*((15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*co
s(d*x + c)^6 - 15*a^4*b^2 + 40*a^2*b^4 - 24*b^6 - 3*(15*a^4*b^2 - 40*a^2*b
^4 + 24*b^6)*cos(d*x + c)^4 + 3*(15*a^4*b^2 - 40*a^2*b^4 + 24*b^6)*cos(d*x
 + c)^2 - (15*a^5*b - 40*a^3*b^3 + 24*a*b^5 + (15*a^5*b - 40*a^3*b^3 + 24*
a*b^5)*cos(d*x + c)^4 - 2*(15*a^5*b - 40*a^3*b^3 + 24*a*b^5)*cos(d*x + c)^
2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 2*(4*(38*a^5*b - 135*a^...
```

3.191.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx=\int \frac{\cot^6{(c+dx)}}{(a+b\sin{(c+dx)})^2}\,dx$$

input
```
integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**2,x)
```

output
```
Integral(cot(c + d*x)**6/(a + b*sin(c + d*x))**2, x)
```

3.191.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.191.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.41

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = -\frac{\frac{120\,(15\,a^4b-40\,a^2b^3+24\,b^5)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^7}+\frac{960\,(a^6-8\,a^4b^2+13\,a^2b^4-6\,b^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}a^7}}{}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/480*(120*(15*a^4*b - 40*a^2*b^3 + 24*b^5)*log(abs(tan(1/2*d*x + 1/2*c))
)/a^7 + 960*(a^6 - 8*a^4*b^2 + 13*a^2*b^4 - 6*b^6)*(pi*floor(1/2*(d*x + c)
/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/
(sqrt(a^2 - b^2)*a^7) + 960*(a^4*b^2*tan(1/2*d*x + 1/2*c) - 2*a^2*b^4*tan(
1/2*d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c) + a^5*b - 2*a^3*b^3 + a*b^5)/(
(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)*a^7) - (3*a^8*ta
n(1/2*d*x + 1/2*c)^5 - 15*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 35*a^8*tan(1/2*d*
x + 1/2*c)^3 + 60*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^7*b*tan(1/2*d*x +
 1/2*c)^2 - 240*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 330*a^8*tan(1/2*d*x + 1/2
*c) - 1620*a^6*b^2*tan(1/2*d*x + 1/2*c) + 1200*a^4*b^4*tan(1/2*d*x + 1/2*c
))/a^10 - (4110*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 10960*a^2*b^3*tan(1/2*d*x +
 1/2*c)^5 + 6576*b^5*tan(1/2*d*x + 1/2*c)^5 - 330*a^5*tan(1/2*d*x + 1/2*c)
^4 + 1620*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)
^4 - 240*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b^3*tan(1/2*d*x + 1/2*c)^3
 + 35*a^5*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*b^2*tan(1/2*d*x + 1/2*c)^2 + 15*
a^4*b*tan(1/2*d*x + 1/2*c) - 3*a^5)/(a^7*tan(1/2*d*x + 1/2*c)^5))/d
```

3.191.9 Mupad [B] (verification not implemented)

Time = 6.66 (sec) , antiderivative size = 1424, normalized size of antiderivative = 3.36

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^6/(a + b*sin(c + d*x))^2,x)
```

output
```
tan(c/2 + (d*x)/2)^5/(160*a^2*d) + (tan(c/2 + (d*x)/2)*(1/(4*a^2) + b^2/(2
*a^4) - (4*b*((b*(64*a^2 + 128*b^2))/(256*a^5) - b/(8*a^3) + (4*b*((64*a^2
 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/a + ((64*a^2 + 128
*b^2)*((64*a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/(32*a^2)
))/d - (tan(c/2 + (d*x)/2)^3*((64*a^2 + 128*b^2)/(3072*a^4) + 5/(96*a^2) -
 b^2/(6*a^4)))/d - (tan(c/2 + (d*x)/2)^3*((31*a^4*b)/3 - 8*a^2*b^3) + tan(
c/2 + (d*x)/2)^4*(48*a*b^4 + (59*a^5)/3 - 72*a^3*b^2) + tan(c/2 + (d*x)/2)
^5*(124*a^4*b + 224*b^5 - 360*a^2*b^3) + a^5/5 - tan(c/2 + (d*x)/2)^2*((32
*a^5)/15 - 2*a^3*b^2) - (3*a^4*b*tan(c/2 + (d*x)/2))/5 + (2*tan(c/2 + (d*x
)/2)^6*(11*a^6 + 32*b^6 - 24*a^2*b^4 - 22*a^4*b^2))/a)/(d*(32*a^7*tan(c/2
+ (d*x)/2)^5 + 32*a^7*tan(c/2 + (d*x)/2)^7 + 64*a^6*b*tan(c/2 + (d*x)/2)^6
)) + (tan(c/2 + (d*x)/2)^2*((b*(64*a^2 + 128*b^2))/(512*a^5) - b/(16*a^3)
+ (2*b*((64*a^2 + 128*b^2)/(1024*a^4) + 5/(32*a^2) - b^2/(2*a^4)))/a))/d -
 (log(tan(c/2 + (d*x)/2))*(15*a^4*b + 24*b^5 - 40*a^2*b^3))/(4*a^7*d) - (b
*tan(c/2 + (d*x)/2)^4)/(32*a^3*d) - (atan((((a^2 - 6*b^2)*(-(a + b)^3*(a -
 b)^3)^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 47*a^11*b^2)/(2*a^12) +
(tan(c/2 + (d*x)/2)*(23*a^11*b - 96*a^5*b^7 + 208*a^7*b^5 - 134*a^9*b^3))/
(2*a^11) + ((2*a^2*b - (tan(c/2 + (d*x)/2)*(12*a^14 - 16*a^12*b^2))/(2*a^1
1))*(a^2 - 6*b^2)*(-(a + b)^3*(a - b)^3)^(1/2))/a^7)*1i)/a^7 + ((a^2 - 6*b
^2)*(-(a + b)^3*(a - b)^3)^(1/2)*((4*a^13 - 48*a^7*b^6 + 92*a^9*b^4 - 4...
```

3.191. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^2}\,dx$

3.192 $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.192.1 Optimal result

Integrand size = 21, antiderivative size = 321

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= -\frac{(8a^2-5ab-b^2)\log(1-\sin(c+dx))}{16(a+b)^5d} - \frac{(8a^2+5ab-b^2)\log(1+\sin(c+dx))}{16(a-b)^5d} \\ &\quad + \frac{a^3(a^4+13a^2b^2+10b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^5\,d} - \frac{a^5}{2\,(a^2-b^2)^3\,d(a+b\sin(c+dx))^2} \\ &\quad - \frac{a^4(a^2+5b^2)}{(a^2-b^2)^4\,d(a+b\sin(c+dx))} + \frac{\sec^4(c+dx)\,(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx))}{4\,(a^2-b^2)^3\,d} \\ &\quad - \frac{\sec^2(c+dx)\,(8a^3(a^2+5b^2)-b(27a^4+22a^2b^2-b^4)\sin(c+dx))}{8\,(a^2-b^2)^4\,d}\end{aligned}$$

output
```
-1/16*(8*a^2-5*a*b-b^2)*ln(1-sin(d*x+c))/(a+b)^5/d-1/16*(8*a^2+5*a*b-b^2)*
ln(1+sin(d*x+c))/(a-b)^5/d+a^3*(a^4+13*a^2*b^2+10*b^4)*ln(a+b*sin(d*x+c))/
(a^2-b^2)^5/d-1/2*a^5/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2-a^4*(a^2+5*b^2)/(a^
2-b^2)^4/d/(a+b*sin(d*x+c))+1/4*sec(d*x+c)^4*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*
sin(d*x+c))/(a^2-b^2)^3/d-1/8*sec(d*x+c)^2*(8*a^3*(a^2+5*b^2)-b*(27*a^4+22
*a^2*b^2-b^4)*sin(d*x+c))/(a^2-b^2)^4/d
```

3.192.2 Mathematica [A] (verified)

Time = 6.24 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.95

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{(8a^2-5ab-b^2)\log(1-\sin(c+dx))}{16(a+b)^5d} - \frac{(8a^2+5ab-b^2)\log(1+\sin(c+dx))}{16(a-b)^5d} + \frac{a^3(a^4+13a^2b^2+10b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^5\,d} + \frac{1}{16(a+b)^3d(1-\sin(c+dx))^2} - \frac{7a+b}{16(a+b)^4d(1-\sin(c+dx))} + \frac{1}{16(a-b)^3d(1+\sin(c+dx))^2} - \frac{7a-b}{16(a-b)^4d(1+\sin(c+dx))} - \frac{a^5}{2\,(a^2-b^2)^3\,d(a+b\sin(c+dx))^2} - \frac{a^4(a^2+5b^2)}{(a^2-b^2)^4\,d(a+b\sin(c+dx))}$$

input
```
Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

output
```
-1/16*((8*a^2 - 5*a*b - b^2)*Log[1 - Sin[c + d*x]])/((a + b)^5*d) - ((8*a^
2 + 5*a*b - b^2)*Log[1 + Sin[c + d*x]])/(16*(a - b)^5*d) + (a^3*(a^4 + 13*
a^2*b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/((a^2 - b^2)^5*d) + 1/(16*(a +
b)^3*d*(1 - Sin[c + d*x])^2) - (7*a + b)/(16*(a + b)^4*d*(1 - Sin[c + d*x]
)) + 1/(16*(a - b)^3*d*(1 + Sin[c + d*x])^2) - (7*a - b)/(16*(a - b)^4*d*(
1 + Sin[c + d*x])) - a^5/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) - (a^4
*(a^2 + 5*b^2))/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

3.192.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 3200, 601, 25, 2178, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(c+dx)^5}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{b^5\sin^5(c+dx)}{(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)^3}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{601}$$

$$\frac{\frac{b^4\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4(a^2-b^2)^3\left(b^2-b^2\sin^2(c+dx)\right)^2}-\frac{\int -\frac{-\frac{a\left(23a^2-3b^2\right)\sin^2(c+dx)b^8}{\left(a^2-b^2\right)^3}+\frac{a^3\left(3a^2+b^2\right)b^6}{\left(a^2-b^2\right)^3}-\frac{\left(4a^6-12b^2a^4+21b^4a^2-b^6\right)\sin^3(c+dx)b^5}{\left(a^2-b^2\right)^3}-\frac{a^2\left(4a^4+3b^2a^2-\right.}{\left(a^2\right.}}{(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)^2}}{4b^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-\frac{a\left(23a^2-3b^2\right)\sin^2(c+dx)b^8}{\left(a^2-b^2\right)^3}+\frac{a^3\left(3a^2+b^2\right)b^6}{\left(a^2-b^2\right)^3}-\frac{\left(4a^6-12b^2a^4+21b^4a^2-b^6\right)\sin^3(c+dx)b^5}{\left(a^2-b^2\right)^3}-\frac{a^2\left(4a^4+3b^2a^2-3b^4\right)\sin(c+dx)b^5}{\left(a^2-b^2\right)^3}}{(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{4b^2}+\frac{b^4\left(a\left(a^2-\right.\right.}{4(a^2-}}{d}$$

$$\Big\downarrow \text{2178}$$

$$\frac{\frac{\frac{\int -\frac{-\frac{\left(27a^4+22b^2a^2-b^4\right)\sin^3(c+dx)b^9}{\left(a^2-b^2\right)^4}-\frac{a\left(65a^4-14b^2a^2-3b^4\right)\sin^2(c+dx)b^8}{\left(a^2-b^2\right)^4}+\frac{a^3\left(21a^4+26b^2a^2+b^4\right)b^6}{\left(a^2-b^2\right)^4}-\frac{a^2\left(8a^6+b^2a^4-54b^4a^2-3b^6\right)\sin(c+dx)b^5}{\left(a^2-b^2\right)^4}}{(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+}{2b^2}}{4b^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{-\frac{\int \frac{-\frac{\left(27a^4+22b^2a^2-b^4\right)\sin^3(c+dx)b^9}{\left(a^2-b^2\right)^4}-\frac{a\left(65a^4-14b^2a^2-3b^4\right)\sin^2(c+dx)b^8}{\left(a^2-b^2\right)^4}+\frac{a^3\left(21a^4+26b^2a^2+b^4\right)b^6}{\left(a^2-b^2\right)^4}-\frac{a^2\left(8a^6+b^2a^4-54b^4a^2-3b^6\right)\sin(c+dx)b^5}{\left(a^2-b^2\right)^4}}{(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+}{2b^2}}{4b^2}}{d}$$

$$\Big\downarrow \text{2160}$$

3.192. $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$-\frac{\frac{\frac{\int \left(-\frac{8b^4a^5}{\left(a^2-b^2\right)^3(a+b\sin(c+dx))^3}-\frac{8b^4\left(a^2+5b^2\right)a^4}{\left(a^2-b^2\right)^4(a+b\sin(c+dx))^2}-\frac{8b^4\left(a^4+13b^2a^2+10b^4\right)a^3}{\left(a^2-b^2\right)^5(a+b\sin(c+dx))}+\frac{b^4\left(-8a^2+5ba+b^2\right)}{2(a+b)^5(b-b\sin(c+dx))}+\frac{b^4\left(-8a^2-5ba+b^2\right)}{2(b-a)^5(\sin(c+dx)b+b)}\right)d(b\sin(c+dx \cdots}{2b^2}}{4b^2}}{d}$$

$\big\downarrow$ 2009

$$\frac{\frac{b^4\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\sin(c+dx)\right)}{4\left(a^2-b^2\right)^3\left(b^2-b^2\sin^2(c+dx)\right)^2}+\frac{-\frac{b^4\left(8a^3\left(a^2+5b^2\right)-b\left(27a^4+22a^2b^2-b^4\right)\sin(c+dx)\right)}{2\left(a^2-b^2\right)^4\left(b^2-b^2\sin^2(c+dx)\right)}-\frac{\frac{b^4\left(8a^2-5ab-b^2\right)\log(b-b\sin(c+dx))}{2(a+b)^5}+\frac{b^4\left(8a^2+5ab-b^2\right)}{2(a \cdots}{\cdots}}{\cdots}}{d}$$

input
```
Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

output
```
((b^4*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c + d*x]))/(4*(a^2 - b^2)^3*(
b^2 - b^2*Sin[c + d*x]^2)^2) + (-1/2*(b^4*(8*a^3*(a^2 + 5*b^2) - b*(27*a^4
 + 22*a^2*b^2 - b^4)*Sin[c + d*x]))/((a^2 - b^2)^4*(b^2 - b^2*Sin[c + d*x]
^2)) - ((b^4*(8*a^2 - 5*a*b - b^2)*Log[b - b*Sin[c + d*x]])/(2*(a + b)^5)
- (8*a^3*b^4*(a^4 + 13*a^2*b^2 + 10*b^4)*Log[a + b*Sin[c + d*x]])/(a^2 - b
^2)^5 + (b^4*(8*a^2 + 5*a*b - b^2)*Log[b + b*Sin[c + d*x]])/(2*(a - b)^5)
+ (4*a^5*b^4)/((a^2 - b^2)^3*(a + b*Sin[c + d*x])^2) + (8*a^4*b^4*(a^2 + 5
*b^2))/((a^2 - b^2)^4*(a + b*Sin[c + d*x])))/(2*b^2))/(4*b^2))/d
```

3.192.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2178
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Qx = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x^2, x], R = Coeff[Po
lynomialRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 0], S = Coeff[Polynomia
lRemainder[(d + e*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*S - b*R*x)*((a +
b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*b*(p + 1))   Int[(d + e*x
)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*b*(p + 1)*Qx)/(d + e*x)^m + (b*R*(
2*p + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, d, e}, x] && PolyQ[Pq, x
] && NeQ[b*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.192.4 Maple [A] (verified)

Time = 4.14 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{1}{16(a-b)^3(1+\sin(dx+c))^2}-\frac{-b+7a}{16(a-b)^4(1+\sin(dx+c))}+\frac{(-8a^2-5ab+b^2)\ln(1+\sin(dx+c))}{16(a-b)^5}+\frac{1}{16(a+b)^3(\sin(dx+c)-1)^2}-\frac{-b-\cdots}{16(a+b)^4(\sin\cdots}$
default	$\frac{1}{16(a-b)^3(1+\sin(dx+c))^2}-\frac{-b+7a}{16(a-b)^4(1+\sin(dx+c))}+\frac{(-8a^2-5ab+b^2)\ln(1+\sin(dx+c))}{16(a-b)^5}+\frac{1}{16(a+b)^3(\sin(dx+c)-1)^2}-\frac{-b-\cdots}{16(a+b)^4(\sin\cdots}$
risch	Expression too large to display

input
```
int(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/16/(a-b)^3/(1+sin(d*x+c))^2-1/16*(-b+7*a)/(a-b)^4/(1+sin(d*x+c))+1/
16/(a-b)^5*(-8*a^2-5*a*b+b^2)*ln(1+sin(d*x+c))+1/16/(a+b)^3/(sin(d*x+c)-1)
^2-1/16*(-b-7*a)/(a+b)^4/(sin(d*x+c)-1)+1/16/(a+b)^5*(-8*a^2+5*a*b+b^2)*ln
(sin(d*x+c)-1)-1/2*a^5/(a+b)^3/(a-b)^3/(a+b*sin(d*x+c))^2-a^4*(a^2+5*b^2)/
(a+b)^4/(a-b)^4/(a+b*sin(d*x+c))+a^3*(a^4+13*a^2*b^2+10*b^4)/(a+b)^5/(a-b)
^5*ln(a+b*sin(d*x+c)))
```

3.192.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 981 vs. $2(311) = 622$.

Time = 0.76 (sec) , antiderivative size = 981, normalized size of antiderivative = 3.06

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{4\,a^9 - 16\,a^7b^2 + 24\,a^5b^4 - 16\,a^3b^6 + 4\,ab^8 - 4\,(6\,a^9 + 35\,a^7b^2 - 39\,a^5b^4 - 3\,a^3b^6 + ab^8)\cos{(dx+c)}^4 - 1}{}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/16*(4*a^9 - 16*a^7*b^2 + 24*a^5*b^4 - 16*a^3*b^6 + 4*a*b^8 - 4*(6*a^9 +
 35*a^7*b^2 - 39*a^5*b^4 - 3*a^3*b^6 + a*b^8)*cos(d*x + c)^4 - 16*(a^9 - 3
*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*cos(d*x + c)^2 - 16*((a^7*b^2 + 13*a^5*b^4
 + 10*a^3*b^6)*cos(d*x + c)^6 - 2*(a^8*b + 13*a^6*b^3 + 10*a^4*b^5)*cos(d*
x + c)^4*sin(d*x + c) - (a^9 + 14*a^7*b^2 + 23*a^5*b^4 + 10*a^3*b^6)*cos(d
*x + c)^4)*log(b*sin(d*x + c) + a) + ((8*a^7*b^2 + 45*a^6*b^3 + 104*a^5*b^
4 + 125*a^4*b^5 + 80*a^3*b^6 + 23*a^2*b^7 - b^9)*cos(d*x + c)^6 - 2*(8*a^8
*b + 45*a^7*b^2 + 104*a^6*b^3 + 125*a^5*b^4 + 80*a^4*b^5 + 23*a^3*b^6 - a*
b^8)*cos(d*x + c)^4*sin(d*x + c) - (8*a^9 + 45*a^8*b + 112*a^7*b^2 + 170*a
^6*b^3 + 184*a^5*b^4 + 148*a^4*b^5 + 80*a^3*b^6 + 22*a^2*b^7 - b^9)*cos(d*
x + c)^4)*log(sin(d*x + c) + 1) + ((8*a^7*b^2 - 45*a^6*b^3 + 104*a^5*b^4 -
 125*a^4*b^5 + 80*a^3*b^6 - 23*a^2*b^7 + b^9)*cos(d*x + c)^6 - 2*(8*a^8*b
- 45*a^7*b^2 + 104*a^6*b^3 - 125*a^5*b^4 + 80*a^4*b^5 - 23*a^3*b^6 + a*b^8
)*cos(d*x + c)^4*sin(d*x + c) - (8*a^9 - 45*a^8*b + 112*a^7*b^2 - 170*a^6*
b^3 + 184*a^5*b^4 - 148*a^4*b^5 + 80*a^3*b^6 - 22*a^2*b^7 + b^9)*cos(d*x +
 c)^4)*log(-sin(d*x + c) + 1) - 2*(2*a^8*b - 8*a^6*b^3 + 12*a^4*b^5 - 8*a^
2*b^7 + 2*b^9 + (8*a^8*b + 59*a^6*b^3 - 45*a^4*b^5 - 23*a^2*b^7 + b^9)*cos
(d*x + c)^4 - (11*a^8*b - 36*a^6*b^3 + 42*a^4*b^5 - 20*a^2*b^7 + 3*b^9)*co
s(d*x + c)^2)*sin(d*x + c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b
^8 + 5*a^2*b^10 - b^12)*d*cos(d*x + c)^6 - 2*(a^11*b - 5*a^9*b^3 + 10*a...
```

3.192.6 Sympy [F]

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

input
```
integrate(tan(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

3.192.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 730 vs. $2(311) = 622$.

Time = 0.23 (sec) , antiderivative size = 730, normalized size of antiderivative = 2.27

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{16\,(a^7+13\,a^5b^2+10\,a^3b^4)\log(b\sin(dx+c)+a)}{a^{10}-5\,a^8b^2+10\,a^6b^4-10\,a^4b^6+5\,a^2b^8-b^{10}} - \frac{(8\,a^2+5\,ab-b^2)\log(\sin(dx+c)+1)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5} - \frac{(8\,a^2-5\,ab-b^2)\log(\sin(dx+c)-1)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} - \frac{\cdots}{a^{10}-4\,a\cdots}}{\cdots}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/16*(16*(a^7 + 13*a^5*b^2 + 10*a^3*b^4)*log(b*sin(d*x + c) + a)/(a^10 - 5
*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^2 + 5*a*b -
b^2)*log(sin(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*
b^4 - b^5) - (8*a^2 - 5*a*b - b^2)*log(sin(d*x + c) - 1)/(a^5 + 5*a^4*b +
10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(18*a^7 + 72*a^5*b^2 + 6*a^3*
b^4 + (8*a^6*b + 67*a^4*b^3 + 22*a^2*b^5 - b^7)*sin(d*x + c)^5 + 2*(6*a^7
+ 41*a^5*b^2 + 2*a^3*b^4 - a*b^6)*sin(d*x + c)^4 - (5*a^6*b + 159*a^4*b^3
+ 27*a^2*b^5 + b^7)*sin(d*x + c)^3 - 4*(8*a^7 + 37*a^5*b^2 + 4*a^3*b^4 - a
*b^6)*sin(d*x + c)^2 - (a^6*b - 86*a^4*b^3 - 11*a^2*b^5)*sin(d*x + c))/(a^
10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8 + (a^8*b^2 - 4*a^6*b^4 +
6*a^4*b^6 - 4*a^2*b^8 + b^10)*sin(d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^
5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)^5 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4
 - 16*a^4*b^6 + 9*a^2*b^8 - 2*b^10)*sin(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3
+ 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)^3 - (2*a^10 - 9*a^8*b^2 + 16
*a^6*b^4 - 14*a^4*b^6 + 6*a^2*b^8 - b^10)*sin(d*x + c)^2 + 2*(a^9*b - 4*a^
7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*sin(d*x + c)))/d
```

3.192.8 Giac [A] (verification not implemented)

Time = 1.77 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.82

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\frac{16\,(a^7b+13\,a^5b^3+10\,a^3b^5)\log(|b\sin(dx+c)+a|)}{a^{10}b-5\,a^8b^3+10\,a^6b^5-10\,a^4b^7+5\,a^2b^9-b^{11}} - \frac{(8\,a^2+5\,ab-b^2)\log(|\sin(dx+c)+1|)}{a^5-5\,a^4b+10\,a^3b^2-10\,a^2b^3+5\,ab^4-b^5} - \frac{(8\,a^2-5\,ab-b^2)\log(|\sin(dx+c)-1|)}{a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5} - \frac{2\left(8\,a \cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(tan(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/16*(16*(a^7*b + 13*a^5*b^3 + 10*a^3*b^5)*log(abs(b*sin(d*x + c) + a))/(a
^10*b - 5*a^8*b^3 + 10*a^6*b^5 - 10*a^4*b^7 + 5*a^2*b^9 - b^11) - (8*a^2 +
 5*a*b - b^2)*log(abs(sin(d*x + c) + 1))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*
a^2*b^3 + 5*a*b^4 - b^5) - (8*a^2 - 5*a*b - b^2)*log(abs(sin(d*x + c) - 1)
)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) - 2*(8*a^6*b*s
in(d*x + c)^5 + 67*a^4*b^3*sin(d*x + c)^5 + 22*a^2*b^5*sin(d*x + c)^5 - b^
7*sin(d*x + c)^5 + 12*a^7*sin(d*x + c)^4 + 82*a^5*b^2*sin(d*x + c)^4 + 4*a
^3*b^4*sin(d*x + c)^4 - 2*a*b^6*sin(d*x + c)^4 - 5*a^6*b*sin(d*x + c)^3 -
159*a^4*b^3*sin(d*x + c)^3 - 27*a^2*b^5*sin(d*x + c)^3 - b^7*sin(d*x + c)^
3 - 32*a^7*sin(d*x + c)^2 - 148*a^5*b^2*sin(d*x + c)^2 - 16*a^3*b^4*sin(d*
x + c)^2 + 4*a*b^6*sin(d*x + c)^2 - a^6*b*sin(d*x + c) + 86*a^4*b^3*sin(d*
x + c) + 11*a^2*b^5*sin(d*x + c) + 18*a^7 + 72*a^5*b^2 + 6*a^3*b^4)/((a^8
- 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c)^3 + a*sin(d*x +
 c)^2 - b*sin(d*x + c) - a)^2))/d
```

3.192.9 Mupad [B] (verification not implemented)

Time = 10.69 (sec) , antiderivative size = 1229, normalized size of antiderivative = 3.83

$$\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(tan(c + d*x)^5/(a + b*sin(c + d*x))^3,x)
```

output
```
((tan(c/2 + (d*x)/2)^2*(a*b^6 - 2*a^7 + 38*a^3*b^4 + 11*a^5*b^2))/(a^8 + b
^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) - (4*tan(c/2 + (d*x)/2)^4*(4*a*b^6
 - a^7 + 33*a^3*b^4 + 12*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*
a^6*b^2) - (4*tan(c/2 + (d*x)/2)^8*(4*a*b^6 - a^7 + 33*a^3*b^4 + 12*a^5*b^
2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) + (tan(c/2 + (d*x)/2)^
10*(a*b^6 - 2*a^7 + 38*a^3*b^4 + 11*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a
^4*b^4 - 4*a^6*b^2) + (2*tan(c/2 + (d*x)/2)^6*(7*a*b^6 + 6*a^7 + 118*a^3*b
^4 + 13*a^5*b^2))/(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) + (b*tan
(c/2 + (d*x)/2)^11*(37*a^6 + a^2*b^4 + 58*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*
b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (b*tan(c/2 + (d*x)/2)^5*(7*a^6 + 14*b^6 -
57*a^2*b^4 + 132*a^4*b^2))/(2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b
^2)) + (b*tan(c/2 + (d*x)/2)^7*(7*a^6 + 14*b^6 - 57*a^2*b^4 + 132*a^4*b^2)
)/(2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (b*tan(c/2 + (d*x)
/2)^3*(83*a^6 - 4*b^6 - 17*a^2*b^4 + 226*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b
^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (b*tan(c/2 + (d*x)/2)^9*(83*a^6 - 4*b^6 - 1
7*a^2*b^4 + 226*a^4*b^2))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^
2)) + (b*tan(c/2 + (d*x)/2)*(37*a^6 + a^2*b^4 + 58*a^4*b^2))/(4*(a^8 + b^8
 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))/(d*(tan(c/2 + (d*x)/2)^6*(4*a^2 +
24*b^2) - tan(c/2 + (d*x)/2)^10*(2*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(2*
a^2 - 4*b^2) + a^2*tan(c/2 + (d*x)/2)^12 + a^2 - tan(c/2 + (d*x)/2)^4*(...
```

3.192. $\int \frac{\tan^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.193 $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.193.1 Optimal result

Integrand size = 21, antiderivative size = 232

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{(2a-b)\log(1-\sin(c+dx))}{4(a+b)^4d} + \frac{(2a+b)\log(1+\sin(c+dx))}{4(a-b)^4d} - \frac{a(a^4+8a^2b^2+3b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^4\,d} + \frac{a^3}{2\,(a^2-b^2)^2\,d(a+b\sin(c+dx))^2} + \frac{a^2(a^2+3b^2)}{(a^2-b^2)^3\,d(a+b\sin(c+dx))} + \frac{\sec^2(c+dx)\,(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx))}{2\,(a^2-b^2)^3\,d}$$

output
```
1/4*(2*a-b)*ln(1-sin(d*x+c))/(a+b)^4/d+1/4*(2*a+b)*ln(1+sin(d*x+c))/(a-b)^
4/d-a*(a^4+8*a^2*b^2+3*b^4)*ln(a+b*sin(d*x+c))/(a^2-b^2)^4/d+1/2*a^3/(a^2-
b^2)^2/d/(a+b*sin(d*x+c))^2+a^2*(a^2+3*b^2)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
+1/2*sec(d*x+c)^2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*sin(d*x+c))/(a^2-b^2)^3/d
```

3.193.2 Mathematica [A] (verified)

Time = 1.51 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.84

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{(2a-b)\log(1-\sin(c+dx))}{(a+b)^4} + \frac{(2a+b)\log(1+\sin(c+dx))}{(a-b)^4} - \frac{4a\left(a^4+8a^2b^2+3b^4\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^4} - \frac{1}{(a+b)^3(-1+\sin(c+dx))} + \frac{1}{(a-b)^3(1+\text{s}}}{4d}$$

input
```
Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

output
```
(((2*a - b)*Log[1 - Sin[c + d*x]])/(a + b)^4 + ((2*a + b)*Log[1 + Sin[c +
d*x]])/(a - b)^4 - (4*a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])
/(a^2 - b^2)^4 - 1/((a + b)^3*(-1 + Sin[c + d*x])) + 1/((a - b)^3*(1 + Sin
[c + d*x])) + (2*a^3)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) + (4*a^2*(a^2
 + 3*b^2))/((a^2 - b^2)^3*(a + b*Sin[c + d*x])))/(4*d)
```

3.193.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3200, 601, 25, 2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(c+dx)^3}{(a+b\sin(c+dx))^3}dx$$
$$\Big\downarrow \; 3200$$
$$\frac{\int \frac{b^3\sin^3(c+dx)}{(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)^2}d(b\sin(c+dx))}{d}$$
$$\Big\downarrow \; 601$$

$$\frac{\frac{b^2\left(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx)\right)}{2(a^2-b^2)^3\left(b^2-b^2\sin^2(c+dx)\right)}-\frac{\int -\frac{-\frac{(3a^2+b^2)\sin^3(c+dx)b^7}{(a^2-b^2)^3}-\frac{a(7a^2-3b^2)\sin^2(c+dx)b^6}{(a^2-b^2)^3}+\frac{a^3(3a^2+b^2)b^4}{(a^2-b^2)^3}-\frac{a^2(2a^4-3b^2a^2-3b^4)\sin(c+dx)b^3}{(a^2-b^2)^3}}{(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)}}{2b^2}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{-\frac{(3a^2+b^2)\sin^3(c+dx)b^7}{(a^2-b^2)^3}-\frac{a(7a^2-3b^2)\sin^2(c+dx)b^6}{(a^2-b^2)^3}+\frac{a^3(3a^2+b^2)b^4}{(a^2-b^2)^3}-\frac{a^2(2a^4-3b^2a^2-3b^4)\sin(c+dx)b^3}{(a^2-b^2)^3}}{(a+b\sin(c+dx))^3\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2b^2}+\frac{b^2\left(a(a^2+3b^2)-b(3a^2+b^2\right.}{2(a^2-b^2)^3\left(b^2-b^2\sin^2\right.}}{d}$$

$$\Big\downarrow \; 2160$$

$$\frac{\frac{\int \left(-\frac{2b^2a^3}{(a^2-b^2)^2(a+b\sin(c+dx))^3}-\frac{2b^2(a^2+3b^2)a^2}{(a^2-b^2)^3(a+b\sin(c+dx))^2}-\frac{2b^2(a^4+8b^2a^2+3b^4)a}{(a^2-b^2)^4(a+b\sin(c+dx))}+\frac{b^2(b-2a)}{2(a+b)^4(b-b\sin(c+dx))}+\frac{b^2(2a+b)}{2(a-b)^4(\sin(c+dx)b+b)}\right)d(b\sin(c+d}{2b^2}}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{b^2\left(a(a^2+3b^2)-b(3a^2+b^2)\sin(c+dx)\right)}{2(a^2-b^2)^3\left(b^2-b^2\sin^2(c+dx)\right)}+\frac{\frac{2a^2b^2(a^2+3b^2)}{(a^2-b^2)^3(a+b\sin(c+dx))}-\frac{2ab^2(a^4+8a^2b^2+3b^4)\log(a+b\sin(c+dx))}{(a^2-b^2)^4}+\frac{a^3b^2}{(a^2-b^2)^2(a+b\sin(c+dx))^2}+\frac{b^2(2a-b)}{}}{2b^2}}{d}$$

input
```
Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

output
```
((b^2*(a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Sin[c + d*x]))/(2*(a^2 - b^2)^3*(
b^2 - b^2*Sin[c + d*x]^2)) + (((2*a - b)*b^2*Log[b - b*Sin[c + d*x]])/(2*(
a + b)^4) - (2*a*b^2*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[a + b*Sin[c + d*x]])/(a
^2 - b^2)^4 + (b^2*(2*a + b)*Log[b + b*Sin[c + d*x]])/(2*(a - b)^4) + (a^3
*b^2)/((a^2 - b^2)^2*(a + b*Sin[c + d*x])^2) + (2*a^2*b^2*(a^2 + 3*b^2))/(
(a^2 - b^2)^3*(a + b*Sin[c + d*x]))))/(2*b^2))/d
```

3.193. $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.193.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.193. $\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.193.4 Maple [A] (verified)

Time = 2.19 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{a^3}{2(a+b)^2(a-b)^2(a+b\sin(dx+c))^2}-\frac{a\left(a^4+8a^2b^2+3b^4\right)\ln(a+b\sin(dx+c))}{(a+b)^4(a-b)^4}+\frac{a^2\left(a^2+3b^2\right)}{(a+b)^3(a-b)^3(a+b\sin(dx+c))}-\frac{1}{4(a+b)^3(\sin(dx+c)-1)}}{d}$
default	$\frac{\frac{a^3}{2(a+b)^2(a-b)^2(a+b\sin(dx+c))^2}-\frac{a\left(a^4+8a^2b^2+3b^4\right)\ln(a+b\sin(dx+c))}{(a+b)^4(a-b)^4}+\frac{a^2\left(a^2+3b^2\right)}{(a+b)^3(a-b)^3(a+b\sin(dx+c))}-\frac{1}{4(a+b)^3(\sin(dx+c)-1)}}{d}$
risch	Expression too large to display

input
```
int(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*a^3/(a+b)^2/(a-b)^2/(a+b*sin(d*x+c))^2-a*(a^4+8*a^2*b^2+3*b^4)/(a
+b)^4/(a-b)^4*ln(a+b*sin(d*x+c))+a^2*(a^2+3*b^2)/(a+b)^3/(a-b)^3/(a+b*sin(
d*x+c))-1/4/(a+b)^3/(sin(d*x+c)-1)+1/4*(2*a-b)/(a+b)^4*ln(sin(d*x+c)-1)+1/
4/(a-b)^3/(1+sin(d*x+c))+1/4*(2*a+b)/(a-b)^4*ln(1+sin(d*x+c)))
```

3.193.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 788 vs. $2(224) = 448$.

Time = 0.50 (sec) , antiderivative size = 788, normalized size of antiderivative = 3.40

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx =$$
$$-\frac{2\,a^7-6\,a^5b^2+6\,a^3b^4-2\,ab^6+2\,(3\,a^7+7\,a^5b^2-11\,a^3b^4+ab^6)\cos{(dx+c)}^2+4\,((a^5b^2+8\,a^3b^4+3\,ab}{}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 + 2*(3*a^7 + 7*a^5*b^2 - 11*
a^3*b^4 + a*b^6)*cos(d*x + c)^2 + 4*((a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*cos(d
*x + c)^4 - 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^2*sin(d*x + c)
- (a^7 + 9*a^5*b^2 + 11*a^3*b^4 + 3*a*b^6)*cos(d*x + c)^2)*log(b*sin(d*x +
 c) + a) - ((2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5 + 6*a*b^6 + b
^7)*cos(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*
a^2*b^5 + a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (2*a^7 + 9*a^6*b + 18*a^5*b
^2 + 23*a^4*b^3 + 22*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*cos(d*x + c)^2)
*log(sin(d*x + c) + 1) - ((2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14*a^2*b^5
 + 6*a*b^6 - b^7)*cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 1
4*a^3*b^4 + 6*a^2*b^5 - a*b^6)*cos(d*x + c)^2*sin(d*x + c) - (2*a^7 - 9*a^
6*b + 18*a^5*b^2 - 23*a^4*b^3 + 22*a^3*b^4 - 15*a^2*b^5 + 6*a*b^6 - b^7)*c
os(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7 - (2*a^6*b + 7*a^4*b^3 - 8*a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x + c
))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^4
- 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)^2*s
in(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10
)*d*cos(d*x + c)^2)
```

3.193.6 Sympy [F]

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

input
```
integrate(tan(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.90

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx =$$

$$-\frac{\frac{4\left(a^5+8a^3b^2+3ab^4\right)\log(b\sin(dx+c)+a)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} - \frac{(2a+b)\log(\sin(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} - \frac{(2a-b)\log(\sin(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} - \frac{\cdots}{a^8-3a^6b^2+3a^4b^4-a^2b^6-(a^6\cdots}}{\cdots}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
-1/4*(4*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(b*sin(d*x + c) + a)/(a^8 - 4*a^6*b
^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (2*a + b)*log(sin(d*x + c) + 1)/(a^4 -
 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*log(sin(d*x + c) - 1)/(a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 2*(4*a^5 + 8*a^3*b^2 - (2*a^4*
b + 9*a^2*b^3 + b^5)*sin(d*x + c)^3 - (3*a^5 + 10*a^3*b^2 - a*b^4)*sin(d*x
 + c)^2 + (a^4*b + 11*a^2*b^3)*sin(d*x + c))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4
- a^2*b^6 - (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*sin(d*x + c)^4 - 2*(a^
7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*sin(d*x + c)^3 - (a^8 - 4*a^6*b^2 + 6
*a^4*b^4 - 4*a^2*b^6 + b^8)*sin(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*
b^5 - a*b^7)*sin(d*x + c)))/d
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(224) = 448$.

Time = 0.81 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.00

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\frac{4\left(a^5b+8a^3b^3+3ab^5\right)\log(|b\sin(dx+c)+a|)}{a^8b-4a^6b^3+6a^4b^5-4a^2b^7+b^9} - \frac{(2a+b)\log(|\sin(dx+c)+1|)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4} - \frac{(2a-b)\log(|\sin(dx+c)-1|)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} + \frac{2\left(a^5\sin(dx+c)^2+8a^3b^2\sin\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(tan(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/4*(4*(a^5*b + 8*a^3*b^3 + 3*a*b^5)*log(abs(b*sin(d*x + c) + a))/(a^8*b
- 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9) - (2*a + b)*log(abs(sin(d*x + c
) + 1))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (2*a - b)*log(abs(si
n(d*x + c) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(a^5*sin(
d*x + c)^2 + 8*a^3*b^2*sin(d*x + c)^2 + 3*a*b^4*sin(d*x + c)^2 - 3*a^4*b*s
in(d*x + c) + 2*a^2*b^3*sin(d*x + c) + b^5*sin(d*x + c) - 6*a^3*b^2 - 6*a*
b^4)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(sin(d*x + c)^2 - 1)
) - 2*(3*a^5*b^2*sin(d*x + c)^2 + 24*a^3*b^4*sin(d*x + c)^2 + 9*a*b^6*sin(
d*x + c)^2 + 8*a^6*b*sin(d*x + c) + 52*a^4*b^3*sin(d*x + c) + 12*a^2*b^5*s
in(d*x + c) + 6*a^7 + 26*a^5*b^2 + 4*a^3*b^4)/((a^8 - 4*a^6*b^2 + 6*a^4*b^
4 - 4*a^2*b^6 + b^8)*(b*sin(d*x + c) + a)^2))/d
```

3.193.9 Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 690, normalized size of antiderivative = 2.97

$$\int \frac{\tan^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-1\right)\,(2a-b)}{2d\,(a+b)^4} - \frac{\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,(-a^5+6a^3b^2+7ab^4)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(a^5+2a^3b^2+9ab^4)}{a^6-3a^4b^2+3a^2b^4-b^6} - \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,(3a^4b+13a^2b^3-4b^5)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{(a^2-b^2)\,(}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8 - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(2a^2+8b^2) + 4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 + 4b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6 + \right.} + \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\right)\,(2a+b)}{2d\,(a-b)^4} - \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)\,(a^5+8a^3b^2+3ab^4)}{d\,(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)}$$

input
```
int(tan(c + d*x)^3/(a + b*sin(c + d*x))^3,x)
```

output
```
(log(tan(c/2 + (d*x)/2) - 1)*(2*a - b))/(2*d*(a + b)^4) - ((2*tan(c/2 + (d
*x)/2)^6*(7*a*b^4 - a^5 + 6*a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)
- (4*tan(c/2 + (d*x)/2)^4*(9*a*b^4 + a^5 + 2*a^3*b^2))/(a^6 - b^6 + 3*a^2*
b^4 - 3*a^4*b^2) - (tan(c/2 + (d*x)/2)^5*(3*a^4*b - 4*b^5 + 13*a^2*b^3))/(
a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (2*tan(c/2 + (d*x)/2)^2*(7*a*b^4 - a^
5 + 6*a^3*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b*tan(c/2 + (d*x)
/2)^7*(7*a^4 + 5*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (b*tan(c/
2 + (d*x)/2)^3*(3*a^4 - 4*b^4 + 13*a^2*b^2))/((a^2 - b^2)*(a^4 + b^4 - 2*a
^2*b^2)) + (a*tan(c/2 + (d*x)/2)*(5*a*b^3 + 7*a^3*b))/((a^2 - b^2)*(a^4 +
b^4 - 2*a^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^4*(2*
a^2 + 8*b^2) + 4*b^2*tan(c/2 + (d*x)/2)^2 + 4*b^2*tan(c/2 + (d*x)/2)^6 + a
^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c
/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))) + (log(tan(c/2 + (d*x)/2) + 1
)*(2*a + b))/(2*d*(a - b)^4) - (log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2
 + (d*x)/2)^2)*(3*a*b^4 + a^5 + 8*a^3*b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*
a^4*b^4 - 4*a^6*b^2))
```

3.194 $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.194.1 Optimal result

Integrand size = 19, antiderivative size = 149

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\log(1-\sin(c+dx))}{2(a+b)^3d}-\frac{\log(1+\sin(c+dx))}{2(a-b)^3d}+\frac{a(a^2+3b^2)\log(a+b\sin(c+dx))}{(a^2-b^2)^3\,d}-\frac{a}{2\,(a^2-b^2)\,d(a+b\sin(c+dx))^2}-\frac{a^2+b^2}{(a^2-b^2)^2\,d(a+b\sin(c+dx))}$$

output
```
-1/2*ln(1-sin(d*x+c))/(a+b)^3/d-1/2*ln(1+sin(d*x+c))/(a-b)^3/d+a*(a^2+3*b^
2)*ln(a+b*sin(d*x+c))/(a^2-b^2)^3/d-1/2*a/(a^2-b^2)/d/(a+b*sin(d*x+c))^2+(
-a^2-b^2)/(a^2-b^2)^2/d/(a+b*sin(d*x+c))
```

3.194.2 Mathematica [A] (verified)

Time = 1.46 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.43

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{-\frac{\log(1-\sin(c+dx))}{(a+b)^2}+\frac{\log(1+\sin(c+dx))}{(a-b)^2}-\frac{4ab\log(a+b\sin(c+dx))}{(a^2-b^2)^2}+\frac{2b}{(a^2-b^2)(a+b\sin(c+dx))}+a\left(\frac{\log(1-\sin(c+dx))}{(a+b)^3}-\frac{\log(1+\sin(c}{(a-b)^3}\right.}{2bd}$$

input
```
Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

output
```
(-(Log[1 - Sin[c + d*x]]/(a + b)^2) + Log[1 + Sin[c + d*x]]/(a - b)^2 - (4
*a*b*Log[a + b*Sin[c + d*x]])/(a^2 - b^2)^2 + (2*b)/((a^2 - b^2)*(a + b*Si
n[c + d*x])) + a*(Log[1 - Sin[c + d*x]]/(a + b)^3 - Log[1 + Sin[c + d*x]]/
(a - b)^3 + (b*(2*(3*a^2 + b^2)*Log[a + b*Sin[c + d*x]] + ((a^2 - b^2)*(-5
*a^2 + b^2 - 4*a*b*Sin[c + d*x]))/(a + b*Sin[c + d*x])^2))/(a^2 - b^2)^3))
/(2*b*d)
```

3.194.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3042, 3200, 594, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3200}$$

$$\frac{\int \frac{b\sin(c+dx)}{(a+b\sin(c+dx))^3(b^2-b^2\sin^2(c+dx))}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 594}$$

$$\frac{\frac{\int -\frac{2\left(b^2-ab\sin(c+dx)\right)}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{2(a^2-b^2)}-\frac{a}{2(a^2-b^2)(a+b\sin(c+dx))^2}}{d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{-\frac{\int \frac{b^2-ab\sin(c+dx)}{(a+b\sin(c+dx))^2\left(b^2-b^2\sin^2(c+dx)\right)}d(b\sin(c+dx))}{a^2-b^2}-\frac{a}{2(a^2-b^2)(a+b\sin(c+dx))^2}}{d}$$

$$\Big\downarrow \text{ 657}$$

$$-\frac{\frac{\int \left(\frac{b-a}{2(a+b)^2(b-b\sin(c+dx))}-\frac{a\left(a^2+3b^2\right)}{(a-b)^2(a+b)^2(a+b\sin(c+dx))}+\frac{a+b}{2(a-b)^2(\sin(c+dx)b+b)}+\frac{-a^2-b^2}{(a-b)(a+b)(a+b\sin(c+dx))^2}\right)d(b\sin(c+dx))}{a^2-b^2}-\frac{a}{2(a^2-b^2)(a+b\,\mathrm{s}}}{d}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\frac{a}{2(a^2-b^2)(a+b\sin(c+dx))^2}-\frac{\frac{a^2+b^2}{\left(a^2-b^2\right)(a+b\sin(c+dx))}-\frac{a\left(a^2+3b^2\right)\log(a+b\sin(c+dx))}{\left(a^2-b^2\right)^2}+\frac{(a-b)\log(b-b\sin(c+dx))}{2(a+b)^2}+\frac{(a+b)\log(b\sin(c+dx)+b)}{2(a-b)^2}}{a^2-b^2}}{d}$$

input
```
Int[Tan[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

output
```
(-1/2*a/((a^2 - b^2)*(a + b*Sin[c + d*x])^2) - (((a - b)*Log[b - b*Sin[c +
 d*x]])/(2*(a + b)^2) - (a*(a^2 + 3*b^2)*Log[a + b*Sin[c + d*x]])/(a^2 - b
^2)^2 + ((a + b)*Log[b + b*Sin[c + d*x]])/(2*(a - b)^2) + (a^2 + b^2)/((a^
2 - b^2)*(a + b*Sin[c + d*x])))/(a^2 - b^2))/d
```

3.194.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 594
```
Int[(x_)*((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/((n + 1)*(b*c^2 + a*d^2)))
, x] + Simp[1/((n + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^(n + 1)*(a + b*x^2)
^p*(a*d*(n + 1) + b*c*(n + 2*p + 3)*x), x], x] /; FreeQ[{a, b, c, d, p}, x]
 && LtQ[n, -1] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.194.4 Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{a}{2(a+b)(a-b)(a+b\sin(dx+c))^2}-\frac{a^2+b^2}{(a+b)^2(a-b)^2(a+b\sin(dx+c))}+\frac{a\left(a^2+3b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^3}-\frac{\ln(1+\sin(dx-}{2(a-b)^3}}{d}$
default	$\frac{-\frac{a}{2(a+b)(a-b)(a+b\sin(dx+c))^2}-\frac{a^2+b^2}{(a+b)^2(a-b)^2(a+b\sin(dx+c))}+\frac{a\left(a^2+3b^2\right)\ln(a+b\sin(dx+c))}{(a+b)^3(a-b)^3}-\frac{\ln(\sin(dx+c)-1)}{2(a+b)^3}-\frac{\ln(1+\sin(dx-}{2(a-b)^3}}{d}$
risch	$\frac{ix}{a^3+3a^2b+3a\,b^2+b^3}+\frac{ic}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{ix}{a^3-3a^2b+3a\,b^2-b^3}+\frac{ic}{d(a^3-3a^2b+3a\,b^2-b^3)}-\frac{2ia^3x}{a^6-3a^4b^2+3a^2}$

input
```
int(tan(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a/(a+b)/(a-b)/(a+b*sin(d*x+c))^2-(a^2+b^2)/(a+b)^2/(a-b)^2/(a+b*
sin(d*x+c))+a*(a^2+3*b^2)/(a+b)^3/(a-b)^3*ln(a+b*sin(d*x+c))-1/2/(a+b)^3*l
n(sin(d*x+c)-1)-1/2/(a-b)^3*ln(1+sin(d*x+c)))
```

3.194.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(143) = 286$.

Time = 0.36 (sec) , antiderivative size = 462, normalized size of antiderivative = 3.10

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{3\,a^5-2\,a^3b^2-ab^4-2\left(a^5+4\,a^3b^2+3\,ab^4-(a^3b^2+3\,ab^4)\cos\left(dx+c\right)^2+2\left(a^4b+3\,a^2b^3\right)\sin\left(dx+c\right)\right)\mathrm{l}}{}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2*(3*a^5 - 2*a^3*b^2 - a*b^4 - 2*(a^5 + 4*a^3*b^2 + 3*a*b^4 - (a^3*b^2 +
 3*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^2*b^3)*sin(d*x + c))*log(b*sin(d
*x + c) + a) + (a^5 + 3*a^4*b + 4*a^3*b^2 + 4*a^2*b^3 + 3*a*b^4 + b^5 - (a
^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cos(d*x + c)^2 + 2*(a^4*b + 3*a^3*b^2
+ 3*a^2*b^3 + a*b^4)*sin(d*x + c))*log(sin(d*x + c) + 1) + (a^5 - 3*a^4*b
+ 4*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4 - b^5 - (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 -
 b^5)*cos(d*x + c)^2 + 2*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*sin(d*x +
 c))*log(-sin(d*x + c) + 1) + 2*(a^4*b - b^5)*sin(d*x + c))/((a^6*b^2 - 3*
a^4*b^4 + 3*a^2*b^6 - b^8)*d*cos(d*x + c)^2 - 2*(a^7*b - 3*a^5*b^3 + 3*a^3
*b^5 - a*b^7)*d*sin(d*x + c) - (a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d)
```

3.194.6 Sympy [F]

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\tan\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.53

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{2\,(a^3+3\,ab^2)\log(b\sin(dx+c)+a)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{3\,a^3+ab^2+2\,(a^2b+b^3)\sin(dx+c)}{a^6-2\,a^4b^2+a^2b^4+(a^4b^2-2\,a^2b^4+b^6)\sin(dx+c)^2+2\,(a^5b-2\,a^3b^3+ab^5)\sin(dx+c)} - \frac{\log(\sin(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}}{2\,d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*(2*(a^3 + 3*a*b^2)*log(b*sin(d*x + c) + a)/(a^6 - 3*a^4*b^2 + 3*a^2*b^
4 - b^6) - (3*a^3 + a*b^2 + 2*(a^2*b + b^3)*sin(d*x + c))/(a^6 - 2*a^4*b^2
 + a^2*b^4 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sin(d*x + c)^2 + 2*(a^5*b - 2*a^3
*b^3 + a*b^5)*sin(d*x + c)) - log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b
^2 - b^3) - log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

3.194.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.72

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{2\,(a^3b+3\,ab^3)\log(|b\sin(dx+c)+a|)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{\log(|\sin(dx+c)+1|)}{a^3-3\,a^2b+3\,ab^2-b^3} - \frac{\log(|\sin(dx+c)-1|)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a^3b^2\sin(dx+c)^2+9\,ab^4\sin(dx+c)^2+8\,a^4b\sin(dx+c)+}{(a^6-3\,a^4b^2+3\,a^2b^4-b}}{2\,d}$$

input
```
integrate(tan(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2*(2*(a^3*b + 3*a*b^3)*log(abs(b*sin(d*x + c) + a))/(a^6*b - 3*a^4*b^3 +
 3*a^2*b^5 - b^7) - log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2*b + 3*a*b^2 -
b^3) - log(abs(sin(d*x + c) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a^3
*b^2*sin(d*x + c)^2 + 9*a*b^4*sin(d*x + c)^2 + 8*a^4*b*sin(d*x + c) + 18*a
^2*b^3*sin(d*x + c) - 2*b^5*sin(d*x + c) + 6*a^5 + 7*a^3*b^2 - a*b^4)/((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*sin(d*x + c) + a)^2))/d
```

3.194.9 Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.04

$$\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(3\,a^2\,b^2+b^4)}{a\,(a^4-2\,a^2\,b^2+b^4)} + \frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4-2\,a^2\,b^2+b^4} + \frac{4\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{a^4-2\,a^2\,b^2+b^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2+4\,b^2)+a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+a^2+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-1\right)}{d\,(a+b)^3} - \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\right)}{d\,(a-b)^3} + \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)\,(a^3+3\,a\,b^2)}{d\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}$$

input
```
int(tan(c + d*x)/(a + b*sin(c + d*x))^3,x)
```

output
```
((2*tan(c/2 + (d*x)/2)^2*(b^4 + 3*a^2*b^2))/(a*(a^4 + b^4 - 2*a^2*b^2)) +
(4*a^2*b*tan(c/2 + (d*x)/2))/(a^4 + b^4 - 2*a^2*b^2) + (4*a^2*b*tan(c/2 +
(d*x)/2)^3)/(a^4 + b^4 - 2*a^2*b^2))/(d*(tan(c/2 + (d*x)/2)^2*(2*a^2 + 4*b
^2) + a^2*tan(c/2 + (d*x)/2)^4 + a^2 + 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*
tan(c/2 + (d*x)/2))) - log(tan(c/2 + (d*x)/2) - 1)/(d*(a + b)^3) - log(tan
(c/2 + (d*x)/2) + 1)/(d*(a - b)^3) + (log(a + 2*b*tan(c/2 + (d*x)/2) + a*t
an(c/2 + (d*x)/2)^2)*(3*a*b^2 + a^3))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^
2))
```

3.194. $\int \frac{\tan(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.195 $\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.195.1 Optimal result

Integrand size = 19, antiderivative size = 75

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\log(\sin(c+dx))}{a^3d} - \frac{\log(a+b\sin(c+dx))}{a^3d} + \frac{1}{2ad(a+b\sin(c+dx))^2} + \frac{1}{a^2d(a+b\sin(c+dx))}$$

output
```
ln(sin(d*x+c))/a^3/d-ln(a+b*sin(d*x+c))/a^3/d+1/2/a/d/(a+b*sin(d*x+c))^2+1
/a^2/d/(a+b*sin(d*x+c))
```

3.195.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{2\log(\sin(c+dx)) - 2\log(a+b\sin(c+dx)) + \frac{a(3a+2b\sin(c+dx))}{(a+b\sin(c+dx))^2}}{2a^3d}$$

input
```
Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

output
```
(2*Log[Sin[c + d*x]] - 2*Log[a + b*Sin[c + d*x]] + (a*(3*a + 2*b*Sin[c + d
*x]))/(a + b*Sin[c + d*x])^2)/(2*a^3*d)
```

3.195.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 3200, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3200}$$

$$\frac{\int \frac{\csc(c+dx)}{b(a+b\sin(c+dx))^3}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{\int \left(\frac{\csc(c+dx)}{a^3b}-\frac{1}{a^3(a+b\sin(c+dx))}-\frac{1}{a^2(a+b\sin(c+dx))^2}-\frac{1}{a(a+b\sin(c+dx))^3}\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{\log(b\sin(c+dx))}{a^3}-\frac{\log(a+b\sin(c+dx))}{a^3}+\frac{1}{a^2(a+b\sin(c+dx))}+\frac{1}{2a(a+b\sin(c+dx))^2}}{d}$$

input
```
Int[Cot[c + d*x]/(a + b*Sin[c + d*x])^3,x]
```

output
```
(Log[b*Sin[c + d*x]]/a^3 - Log[a + b*Sin[c + d*x]]/a^3 + 1/(2*a*(a + b*Sin
[c + d*x])^2) + 1/(a^2*(a + b*Sin[c + d*x])))/d
```

3.195.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.195.4 Maple [A] (verified)

Time = 1.47 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a^3}+\frac{1}{a^2(a+b\sin(dx+c))}+\frac{1}{2a(a+b\sin(dx+c))^2}+\frac{\ln(\sin(dx+c))}{a^3}}{d}$	66
default	$\frac{-\frac{\ln(a+b\sin(dx+c))}{a^3}+\frac{1}{a^2(a+b\sin(dx+c))}+\frac{1}{2a(a+b\sin(dx+c))^2}+\frac{\ln(\sin(dx+c))}{a^3}}{d}$	66
risch	$\frac{2i\left(3ia\,e^{2i(dx+c)}+b\,e^{3i(dx+c)}-b\,e^{i(dx+c)}\right)}{\left(b\,e^{2i(dx+c)}-b+2ia\,e^{i(dx+c)}\right)^2 a^2 d}+\frac{\ln\left(e^{2i(dx+c)}-1\right)}{a^3 d}-\frac{\ln\left(e^{2i(dx+c)}+\frac{2ia\,e^{i(dx+c)}}{b}-1\right)}{a^3 d}$	133

input
```
int(cot(d*x+c)/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a^3*ln(a+b*sin(d*x+c))+1/a^2/(a+b*sin(d*x+c))+1/2/a/(a+b*sin(d*x+c
))^2+1/a^3*ln(sin(d*x+c)))
```

3.195.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(73) = 146.

Time = 0.31 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.05

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{2\,ab\sin\left(dx+c\right)+3\,a^2+2\left(b^2\cos\left(dx+c\right)^2-2\,ab\sin\left(dx+c\right)-a^2-b^2\right)\log\left(b\sin\left(dx+c\right)+a\right)-2\,(b}{2\left(a^3b^2d\cos\left(dx+c\right)^2-2\,a^4bd\sin\left(dx+c\right)-(a\right.}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(2*a*b*sin(d*x + c) + 3*a^2 + 2*(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x +
 c) - a^2 - b^2)*log(b*sin(d*x + c) + a) - 2*(b^2*cos(d*x + c)^2 - 2*a*b*s
in(d*x + c) - a^2 - b^2)*log(-1/2*sin(d*x + c)))/(a^3*b^2*d*cos(d*x + c)^2
 - 2*a^4*b*d*sin(d*x + c) - (a^5 + a^3*b^2)*d)
```

3.195.6 Sympy [F]

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\cot\left(c+dx\right)}{\left(a+b\sin\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)/(a + b*sin(c + d*x))**3, x)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{2\,b\sin(dx+c)+3\,a}{a^2b^2\sin(dx+c)^2+2\,a^3b\sin(dx+c)+a^4}-\frac{2\,\log(b\sin(dx+c)+a)}{a^3}+\frac{2\,\log(\sin(dx+c))}{a^3}}{2\,d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((2*b*sin(d*x + c) + 3*a)/(a^2*b^2*sin(d*x + c)^2 + 2*a^3*b*sin(d*x +
c) + a^4) - 2*log(b*sin(d*x + c) + a)/a^3 + 2*log(sin(d*x + c))/a^3)/d
```

3.195.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\frac{2\log(|b\sin(dx+c)+a|)}{a^3} - \frac{2\log(|\sin(dx+c)|)}{a^3} - \frac{2ab\sin(dx+c)+3a^2}{(b\sin(dx+c)+a)^2a^3}}{2d}$$

input
```
integrate(cot(d*x+c)/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(2*log(abs(b*sin(d*x + c) + a))/a^3 - 2*log(abs(sin(d*x + c)))/a^3 -
(2*a*b*sin(d*x + c) + 3*a^2)/((b*sin(d*x + c) + a)^2*a^3))/d
```

3.195.9 Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.92

$$\int \frac{\cot(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{a^3\,d} - \frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}{a^3\,d}$$
$$-\frac{6\,b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{d\,\left(a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^5+4\,a^4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a^4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4\,a^3\,b^2\tan\right.}$$
$$-\frac{4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4+4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4\,a^2\,b^2\tan\right.}$$
$$-\frac{4\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4+4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+4\,a^3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+4\,a^2\,b^2\tan\right.}$$

input
```
int(cot(c + d*x)/(a + b*sin(c + d*x))^3,x)
```

output
```
log(tan(c/2 + (d*x)/2))/(a^3*d) - log(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c
/2 + (d*x)/2)^2)/(a^3*d) - (6*b^2*tan(c/2 + (d*x)/2)^2)/(d*(2*a^5*tan(c/2
+ (d*x)/2)^2 + a^5*tan(c/2 + (d*x)/2)^4 + a^5 + 4*a^3*b^2*tan(c/2 + (d*x)/
2)^2 + 4*a^4*b*tan(c/2 + (d*x)/2) + 4*a^4*b*tan(c/2 + (d*x)/2)^3)) - (4*b*
tan(c/2 + (d*x)/2))/(d*(2*a^4*tan(c/2 + (d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2
)^4 + a^4 + 4*a^2*b^2*tan(c/2 + (d*x)/2)^2 + 4*a^3*b*tan(c/2 + (d*x)/2) +
4*a^3*b*tan(c/2 + (d*x)/2)^3)) - (4*b*tan(c/2 + (d*x)/2)^3)/(d*(2*a^4*tan(
c/2 + (d*x)/2)^2 + a^4*tan(c/2 + (d*x)/2)^4 + a^4 + 4*a^2*b^2*tan(c/2 + (d
*x)/2)^2 + 4*a^3*b*tan(c/2 + (d*x)/2) + 4*a^3*b*tan(c/2 + (d*x)/2)^3))
```

3.196 $\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3} dx$

3.196.1 Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3} dx = \frac{3b\csc(c+dx)}{a^4d} - \frac{\csc^2(c+dx)}{2a^3d} - \frac{(a^2-6b^2)\log(\sin(c+dx))}{a^5d} + \frac{(a^2-6b^2)\log(a+b\sin(c+dx))}{a^5d} - \frac{a^2-b^2}{2a^3d(a+b\sin(c+dx))^2} - \frac{a^2-3b^2}{a^4d(a+b\sin(c+dx))}$$

output
```
3*b*csc(d*x+c)/a^4/d-1/2*csc(d*x+c)^2/a^3/d-(a^2-6*b^2)*ln(sin(d*x+c))/a^5
/d+(a^2-6*b^2)*ln(a+b*sin(d*x+c))/a^5/d+1/2*(-a^2+b^2)/a^3/d/(a+b*sin(d*x+
c))^2+(-a^2+3*b^2)/a^4/d/(a+b*sin(d*x+c))
```

3.196.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.83

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3} dx = -\frac{-6ab\csc(c+dx) + a^2\csc^2(c+dx) + 2(a^2-6b^2)\log(\sin(c+dx)) - 2(a^2-6b^2)\log(a+b\sin(c+dx))}{2a^5d}$$

input
```
Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

output
```
-1/2*(-6*a*b*Csc[c + d*x] + a^2*Csc[c + d*x]^2 + 2*(a^2 - 6*b^2)*Log[Sin[c
 + d*x]] - 2*(a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]] + (a^2*(a - b)*(a + b))
/(a + b*Sin[c + d*x])^2 + (2*a*(a^2 - 3*b^2))/(a + b*Sin[c + d*x]))/(a^5*d
)
```

3.196.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^3(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3200}$$

$$\frac{\int \frac{\csc^3(c+dx)\left(b^2-b^2\sin^2(c+dx)\right)}{b^3(a+b\sin(c+dx))^3}d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{522}$$

$$\frac{\int \left(\frac{\csc^3(c+dx)}{a^3b}-\frac{3\csc^2(c+dx)}{a^4}+\frac{\left(6b^2-a^2\right)\csc(c+dx)}{a^5b}+\frac{a^2-6b^2}{a^5(a+b\sin(c+dx))}+\frac{a^2-3b^2}{a^4(a+b\sin(c+dx))^2}+\frac{a^2-b^2}{a^3(a+b\sin(c+dx))^3}\right)d(b\sin(c+dx))}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{3b\csc(c+dx)}{a^4}-\frac{\csc^2(c+dx)}{2a^3}-\frac{\left(a^2-6b^2\right)\log(b\sin(c+dx))}{a^5}+\frac{\left(a^2-6b^2\right)\log(a+b\sin(c+dx))}{a^5}-\frac{a^2-3b^2}{a^4(a+b\sin(c+dx))}-\frac{a^2-b^2}{2a^3(a+b\sin(c+dx))^2}}{d}$$

input
```
Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x])^3,x]
```

output
```
((3*b*Csc[c + d*x])/a^4 - Csc[c + d*x]^2/(2*a^3) - ((a^2 - 6*b^2)*Log[b*Si
n[c + d*x]])/a^5 + ((a^2 - 6*b^2)*Log[a + b*Sin[c + d*x]])/a^5 - (a^2 - b^
2)/(2*a^3*(a + b*Sin[c + d*x])^2) - (a^2 - 3*b^2)/(a^4*(a + b*Sin[c + d*x]
)))/d
```

3.196.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.196.4 Maple [A] (verified)

Time = 5.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{\left(a^2-6b^2\right)\ln(a+b\sin(dx+c))}{a^5}-\frac{a^2-3b^2}{a^4(a+b\sin(dx+c))}-\frac{a^2-b^2}{2a^3(a+b\sin(dx+c))^2}-\frac{1}{2a^3\sin(dx+c)^2}+\frac{\left(-a^2+6b^2\right)\ln(\sin(dx+c))}{a^5}+\frac{3b}{a^4\sin(dx}}{d}$
default	$\frac{\frac{\left(a^2-6b^2\right)\ln(a+b\sin(dx+c))}{a^5}-\frac{a^2-3b^2}{a^4(a+b\sin(dx+c))}-\frac{a^2-b^2}{2a^3(a+b\sin(dx+c))^2}-\frac{1}{2a^3\sin(dx+c)^2}+\frac{\left(-a^2+6b^2\right)\ln(\sin(dx+c))}{a^5}+\frac{3b}{a^4\sin(dx}}{d}$
risch	$-\frac{2i\left(3ia^3e^{6i(dx+c)}-18ia\,b^2e^{6i(dx+c)}+a^2b\,e^{7i(dx+c)}-6b^3e^{7i(dx+c)}-10ia^3e^{4i(dx+c)}+36ia\,b^2e^{4i(dx+c)}+5a^2b\,e^{5i(dx+c)}+\right.}{\left(e^{2i(dx+c)}-1\right)^2\left(b\,e^{2i(dx+c)}-\right.}$

input
```
int(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*((a^2-6*b^2)/a^5*ln(a+b*sin(d*x+c))-(a^2-3*b^2)/a^4/(a+b*sin(d*x+c))-1
/2*(a^2-b^2)/a^3/(a+b*sin(d*x+c))^2-1/2/a^3/sin(d*x+c)^2+(-a^2+6*b^2)/a^5*
ln(sin(d*x+c))+3/a^4*b/sin(d*x+c))
```

3.196.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(141) = 282$.

Time = 0.33 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.79

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{4\,a^4-18\,a^2b^2-3\,(a^4-6\,a^2b^2)\cos{(dx+c)}^2-2\,((a^2b^2-6\,b^4)\cos{(dx+c)}^4+a^4-5\,a^2b^2-6\,b^4-(a^4}{}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/2*(4*a^4 - 18*a^2*b^2 - 3*(a^4 - 6*a^2*b^2)*cos(d*x + c)^2 - 2*((a^2*b^
2 - 6*b^4)*cos(d*x + c)^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 1
2*b^4)*cos(d*x + c)^2 + 2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c
)^2)*sin(d*x + c))*log(b*sin(d*x + c) + a) + 2*((a^2*b^2 - 6*b^4)*cos(d*x
+ c)^4 + a^4 - 5*a^2*b^2 - 6*b^4 - (a^4 - 4*a^2*b^2 - 12*b^4)*cos(d*x + c)
^2 + 2*(a^3*b - 6*a*b^3 - (a^3*b - 6*a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*
log(-1/2*sin(d*x + c)) - 2*(a^3*b + 6*a*b^3 + (a^3*b - 6*a*b^3)*cos(d*x +
c)^2)*sin(d*x + c))/(a^5*b^2*d*cos(d*x + c)^4 - (a^7 + 2*a^5*b^2)*d*cos(d*
x + c)^2 + (a^7 + a^5*b^2)*d - 2*(a^6*b*d*cos(d*x + c)^2 - a^6*b*d)*sin(d*
x + c))
```

3.196.6 Sympy [F]

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\cot^3{(c+dx)}}{(a+b\sin{(c+dx)})^3}\,dx$$

input
```
integrate(cot(d*x+c)**3/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**3/(a + b*sin(c + d*x))**3, x)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{4\,a^2 b\sin(dx+c)-2\,(a^2b-6\,b^3)\sin(dx+c)^3-a^3-3\,(a^3-6\,ab^2)\sin(dx+c)^2}{a^4b^2\sin(dx+c)^4+2\,a^5b\sin(dx+c)^3+a^6\sin(dx+c)^2}+\frac{2\,(a^2-6\,b^2)\log(b\sin(dx+c)+a)}{a^5}-\frac{2\,(a^2-6\,b^2)\log(\sin(dx+c))}{a^5}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/2*((4*a^2*b*sin(d*x + c) - 2*(a^2*b - 6*b^3)*sin(d*x + c)^3 - a^3 - 3*(a
^3 - 6*a*b^2)*sin(d*x + c)^2)/(a^4*b^2*sin(d*x + c)^4 + 2*a^5*b*sin(d*x +
c)^3 + a^6*sin(d*x + c)^2) + 2*(a^2 - 6*b^2)*log(b*sin(d*x + c) + a)/a^5 -
 2*(a^2 - 6*b^2)*log(sin(d*x + c))/a^5)/d
```

3.196.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.06

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\frac{2\,(a^2-6\,b^2)\log(|\sin(dx+c)|)}{a^5}-\frac{2\,(a^2b-6\,b^3)\log(|b\sin(dx+c)+a|)}{a^5b}+\frac{2\,a^2b\sin(dx+c)^3-12\,b^3\sin(dx+c)^3+3\,a^3\sin(dx+c)^2-18\,ab^2\sin(dx+c}{\left(b\sin(dx+c)^2+a\sin(dx+c)\right)^2a^4}}{2\,d}$$

input
```
integrate(cot(d*x+c)^3/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(2*(a^2 - 6*b^2)*log(abs(sin(d*x + c)))/a^5 - 2*(a^2*b - 6*b^3)*log(a
bs(b*sin(d*x + c) + a))/(a^5*b) + (2*a^2*b*sin(d*x + c)^3 - 12*b^3*sin(d*x
 + c)^3 + 3*a^3*sin(d*x + c)^2 - 18*a*b^2*sin(d*x + c)^2 - 4*a^2*b*sin(d*x
 + c) + a^3)/((b*sin(d*x + c)^2 + a*sin(d*x + c))^2*a^4))/d
```

3.196.9 Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.30

$$\int \frac{\cot^3(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(22\,a\,b^2-a^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(26\,a^2\,b-8\,b^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(22\,a^2\,b-32\,b^3)-\frac{a^3}{2}+4\,a^2\,b}{d\left(4\,a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+4\,a^6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(8\,a^6+16\,a^4\,b^2)+16\,a^5\,b\tan\left(\frac{c}{2}+\right.\right.}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,a^3\,d}+\frac{3\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\,a^4\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a^2-6\,b^2)}{a^5\,d}$$
$$+\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)(a^2-6\,b^2)}{a^5\,d}$$

input
```
int(cot(c + d*x)^3/(a + b*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^2*(22*a*b^2 - a^3) + tan(c/2 + (d*x)/2)^3*(26*a^2*b -
8*b^3) + tan(c/2 + (d*x)/2)^5*(22*a^2*b - 32*b^3) - a^3/2 + 4*a^2*b*tan(c/
2 + (d*x)/2) - (tan(c/2 + (d*x)/2)^4*(a^4 + 112*b^4 - 96*a^2*b^2))/(2*a))/
(d*(4*a^6*tan(c/2 + (d*x)/2)^2 + 4*a^6*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d
*x)/2)^4*(8*a^6 + 16*a^4*b^2) + 16*a^5*b*tan(c/2 + (d*x)/2)^3 + 16*a^5*b*t
an(c/2 + (d*x)/2)^5)) - tan(c/2 + (d*x)/2)^2/(8*a^3*d) + (3*b*tan(c/2 + (d
*x)/2))/(2*a^4*d) - (log(tan(c/2 + (d*x)/2))*(a^2 - 6*b^2))/(a^5*d) + (log
(a + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^2 - 6*b^2))/(a^5*
d)
```

3.197 $\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.197.1 Optimal result

Integrand size = 21, antiderivative size = 221

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx =& -\frac{2b(3a^2-5b^2)\csc(c+dx)}{a^6d}+\frac{(a^2-3b^2)\csc^2(c+dx)}{a^5d}\\&+\frac{b\csc^3(c+dx)}{a^4d}-\frac{\csc^4(c+dx)}{4a^3d}\\&+\frac{(a^4-12a^2b^2+15b^4)\log(\sin(c+dx))}{a^7d}\\&-\frac{(a^4-12a^2b^2+15b^4)\log(a+b\sin(c+dx))}{a^7d}\\&+\frac{(a^2-b^2)^2}{2a^5d(a+b\sin(c+dx))^2}+\frac{a^4-6a^2b^2+5b^4}{a^6d(a+b\sin(c+dx))}\end{aligned}$$

output
```
-2*b*(3*a^2-5*b^2)*csc(d*x+c)/a^6/d+(a^2-3*b^2)*csc(d*x+c)^2/a^5/d+b*csc(d
*x+c)^3/a^4/d-1/4*csc(d*x+c)^4/a^3/d+(a^4-12*a^2*b^2+15*b^4)*ln(sin(d*x+c)
)/a^7/d-(a^4-12*a^2*b^2+15*b^4)*ln(a+b*sin(d*x+c))/a^7/d+1/2*(a^2-b^2)^2/a
^5/d/(a+b*sin(d*x+c))^2+(a^4-6*a^2*b^2+5*b^4)/a^6/d/(a+b*sin(d*x+c))
```

3.197.2 Mathematica [A] (verified)

Time = 4.83 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.88

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{-8ab(3a^2-5b^2)\csc(c+dx)+4a^2(a^2-3b^2)\csc^2(c+dx)+4a^3b\csc^3(c+dx)-a^4\csc^4(c+dx)+4(a^4-}{}$$

input
```
Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

output
```
(-8*a*b*(3*a^2 - 5*b^2)*Csc[c + d*x] + 4*a^2*(a^2 - 3*b^2)*Csc[c + d*x]^2
+ 4*a^3*b*Csc[c + d*x]^3 - a^4*Csc[c + d*x]^4 + 4*(a^4 - 12*a^2*b^2 + 15*b
^4)*Log[Sin[c + d*x]] - 4*(a^4 - 12*a^2*b^2 + 15*b^4)*Log[a + b*Sin[c + d*
x]] + (2*(a^3 - a*b^2)^2)/(a + b*Sin[c + d*x])^2 + (4*a*(a^4 - 6*a^2*b^2 +
 5*b^4))/(a + b*Sin[c + d*x]))/(4*a^7*d)
```

3.197.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\tan(c+dx)^5(a+b\sin(c+dx))^3}dx$$
$$\Big\downarrow \ 3200$$
$$\frac{\int \frac{\csc^5(c+dx)\left(b^2-b^2\sin^2(c+dx)\right)^2}{b^5(a+b\sin(c+dx))^3}d(b\sin(c+dx))}{d}$$
$$\Big\downarrow \ 522$$

$$\frac{\int \left(\frac{\csc^5(c+dx)}{a^3b} - \frac{3\csc^4(c+dx)}{a^4} + \frac{2(3b^2-a^2)\csc^3(c+dx)}{a^5b} + \frac{2(3a^2b^2-5b^4)\csc^2(c+dx)}{a^6b^2} + \frac{(a^4-12b^2a^2+15b^4)\csc(c+dx)}{a^7b} + \frac{-a^4+12b^2a^2-1}{a^7(a+b\sin(c+d}\right.}{d}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{b\csc^3(c+dx)}{a^4} - \frac{\csc^4(c+dx)}{4a^3} - \frac{2b(3a^2-5b^2)\csc(c+dx)}{a^6} + \frac{(a^2-b^2)^2}{2a^5(a+b\sin(c+dx))^2} + \frac{(a^2-3b^2)\csc^2(c+dx)}{a^5} + \frac{(a^4-12a^2b^2+15b^4)\log(b\sin(c+d}{a^7}}{d}$$

input
```
Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x])^3,x]
```

output
```
((-2*b*(3*a^2 - 5*b^2)*Csc[c + d*x])/a^6 + ((a^2 - 3*b^2)*Csc[c + d*x]^2)/
a^5 + (b*Csc[c + d*x]^3)/a^4 - Csc[c + d*x]^4/(4*a^3) + ((a^4 - 12*a^2*b^2
 + 15*b^4)*Log[b*Sin[c + d*x]])/a^7 - ((a^4 - 12*a^2*b^2 + 15*b^4)*Log[a +
 b*Sin[c + d*x]])/a^7 + (a^2 - b^2)^2/(2*a^5*(a + b*Sin[c + d*x])^2) + (a^
4 - 6*a^2*b^2 + 5*b^4)/(a^6*(a + b*Sin[c + d*x])))/d
```

3.197.3.1 Defintions of rubi rules used

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

3.197.4 Maple [A] (verified)

Time = 10.75 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{-\frac{\left(a^4-12a^2b^2+15b^4\right)\ln(a+b\sin(dx+c))}{a^7}+\frac{a^4-6a^2b^2+5b^4}{a^6(a+b\sin(dx+c))}+\frac{a^4-2a^2b^2+b^4}{2a^5(a+b\sin(dx+c))^2}-\frac{1}{4a^3\sin(dx+c)^4}-\frac{-2a^2+6b^2}{2a^5\sin(dx+c)^2}+\frac{(a^4-12}{}}{d}$
default	$\frac{-\frac{\left(a^4-12a^2b^2+15b^4\right)\ln(a+b\sin(dx+c))}{a^7}+\frac{a^4-6a^2b^2+5b^4}{a^6(a+b\sin(dx+c))}+\frac{a^4-2a^2b^2+b^4}{2a^5(a+b\sin(dx+c))^2}-\frac{1}{4a^3\sin(dx+c)^4}-\frac{-2a^2+6b^2}{2a^5\sin(dx+c)^2}+\frac{(a^4-12}{}}{d}$
risch	$2i\left(-12b^3a^2e^{11i(dx+c)}+30b\,a^4e^{5i(dx+c)}+11b\,a^4e^{9i(dx+c)}-60b^3a^2e^{7i(dx+c)}+40b^3a^2e^{9i(dx+c)}+60b^3a^2e^{5i(dx+c)}-b\,a^4e^{i(}\right.$

input
```
int(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-(a^4-12*a^2*b^2+15*b^4)/a^7*ln(a+b*sin(d*x+c))+(a^4-6*a^2*b^2+5*b^4)
/a^6/(a+b*sin(d*x+c))+1/2*(a^4-2*a^2*b^2+b^4)/a^5/(a+b*sin(d*x+c))^2-1/4/a
^3/sin(d*x+c)^4-1/2*(-2*a^2+6*b^2)/a^5/sin(d*x+c)^2+(a^4-12*a^2*b^2+15*b^4
)/a^7*ln(sin(d*x+c))+1/a^4*b/sin(d*x+c)^3-2*b*(3*a^2-5*b^2)/a^6/sin(d*x+c)
)
```

3.197.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 754 vs. 2(217) = 434.

Time = 0.35 (sec) , antiderivative size = 754, normalized size of antiderivative = 3.41

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx =$$
$$-\frac{9\,a^6-77\,a^4b^2+90\,a^2b^4+6\left(a^6-12\,a^4b^2+15\,a^2b^4\right)\cos\left(dx+c\right)^4-\left(16\,a^6-149\,a^4b^2+180\,a^2b^4\right)\cos\left(d\right.}{}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
-1/4*(9*a^6 - 77*a^4*b^2 + 90*a^2*b^4 + 6*(a^6 - 12*a^4*b^2 + 15*a^2*b^4)*
cos(d*x + c)^4 - (16*a^6 - 149*a^4*b^2 + 180*a^2*b^4)*cos(d*x + c)^2 + 4*(
(a^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*b^2 - 3*a^2*
b^4 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x + c)^4 + (2
*a^6 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^5*b - 12*a^3
*b^3 + 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(a^5*
b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(b*sin(d*x + c
) + a) - 4*((a^4*b^2 - 12*a^2*b^4 + 15*b^6)*cos(d*x + c)^6 - a^6 + 11*a^4*
b^2 - 3*a^2*b^4 - 15*b^6 - (a^6 - 9*a^4*b^2 - 21*a^2*b^4 + 45*b^6)*cos(d*x
 + c)^4 + (2*a^6 - 21*a^4*b^2 - 6*a^2*b^4 + 45*b^6)*cos(d*x + c)^2 - 2*(a^
5*b - 12*a^3*b^3 + 15*a*b^5 + (a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)
^4 - 2*(a^5*b - 12*a^3*b^3 + 15*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*log(-
1/2*sin(d*x + c)) - 2*(5*a^5*b + 14*a^3*b^3 - 30*a*b^5 - 2*(a^5*b - 12*a^3
*b^3 + 15*a*b^5)*cos(d*x + c)^4 - 2*(2*a^5*b + 19*a^3*b^3 - 30*a*b^5)*cos(
d*x + c)^2)*sin(d*x + c))/(a^7*b^2*d*cos(d*x + c)^6 - (a^9 + 3*a^7*b^2)*d*
cos(d*x + c)^4 + (2*a^9 + 3*a^7*b^2)*d*cos(d*x + c)^2 - (a^9 + a^7*b^2)*d
- 2*(a^8*b*d*cos(d*x + c)^4 - 2*a^8*b*d*cos(d*x + c)^2 + a^8*b*d)*sin(d*x
+ c))
```

3.197.6 Sympy [F]

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\cot^5{(c+dx)}}{(a+b\sin{(c+dx)})^3}\,dx$$

input
```
integrate(cot(d*x+c)**5/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**5/(a + b*sin(c + d*x))**3, x)
```

3.197.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.07

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{2\,a^4b\sin(dx+c)+4\left(a^4b-12\,a^2b^3+15\,b^5\right)\sin(dx+c)^5-a^5+6\left(a^5-12\,a^3b^2+15\,ab^4\right)\sin(dx+c)^4-4\left(4\,a^4b-5\,a^2b^3\right)\sin(dx+c)^3+\left(4\,a^5-5\,a^3b^2\right)\sin(d}{a^6b^2\sin(dx+c)^6+2\,a^7b\sin(dx+c)^5+a^8\sin(dx+c)^4}}{4\,d}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
1/4*((2*a^4*b*sin(d*x + c) + 4*(a^4*b - 12*a^2*b^3 + 15*b^5)*sin(d*x + c)^
5 - a^5 + 6*(a^5 - 12*a^3*b^2 + 15*a*b^4)*sin(d*x + c)^4 - 4*(4*a^4*b - 5*
a^2*b^3)*sin(d*x + c)^3 + (4*a^5 - 5*a^3*b^2)*sin(d*x + c)^2)/(a^6*b^2*sin
(d*x + c)^6 + 2*a^7*b*sin(d*x + c)^5 + a^8*sin(d*x + c)^4) - 4*(a^4 - 12*a
^2*b^2 + 15*b^4)*log(b*sin(d*x + c) + a)/a^7 + 4*(a^4 - 12*a^2*b^2 + 15*b^
4)*log(sin(d*x + c))/a^7)/d
```

3.197.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.48

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{12\,(a^4-12\,a^2b^2+15\,b^4)\log(|\sin(dx+c)|)}{a^7} - \frac{12\,(a^4b-12\,a^2b^3+15\,b^5)\log(|b\sin(dx+c)+a|)}{a^7b} + \frac{6\left(3\,a^4b^2\sin(dx+c)^2-36\,a^2b^4\sin(dx+c)^2+45\,b^6\,\mathrm{s}\ldots\right.}{\ldots}}{\ldots}$$

input
```
integrate(cot(d*x+c)^5/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/12*(12*(a^4 - 12*a^2*b^2 + 15*b^4)*log(abs(sin(d*x + c)))/a^7 - 12*(a^4*
b - 12*a^2*b^3 + 15*b^5)*log(abs(b*sin(d*x + c) + a))/(a^7*b) + 6*(3*a^4*b
^2*sin(d*x + c)^2 - 36*a^2*b^4*sin(d*x + c)^2 + 45*b^6*sin(d*x + c)^2 + 8*
a^5*b*sin(d*x + c) - 84*a^3*b^3*sin(d*x + c) + 100*a*b^5*sin(d*x + c) + 6*
a^6 - 50*a^4*b^2 + 56*a^2*b^4)/((b*sin(d*x + c) + a)^2*a^7) - (25*a^4*sin(
d*x + c)^4 - 300*a^2*b^2*sin(d*x + c)^4 + 375*b^4*sin(d*x + c)^4 + 72*a^3*
b*sin(d*x + c)^3 - 120*a*b^3*sin(d*x + c)^3 - 12*a^4*sin(d*x + c)^2 + 36*a
^2*b^2*sin(d*x + c)^2 - 12*a^3*b*sin(d*x + c) + 3*a^4)/(a^7*sin(d*x + c)^4
))/d
```

3.197.9 Mupad [B] (verification not implemented)

Time = 6.69 (sec) , antiderivative size = 563, normalized size of antiderivative = 2.55

$$\int \frac{\cot^5(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{23a^5}{4}-172a^3b^2+272ab^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(27a^4b-40a^2b^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(-134a^4b}{d\left(16a^8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+16a^8\tan\right.}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{64a^3d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{3(a^2+4b^2)}{32a^5}+\frac{3}{32a^3}-\frac{9b^2}{8a^5}\right)}{d}$$
$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{6b\left(\frac{3(a^2+4b^2)}{16a^5}+\frac{3}{16a^3}-\frac{9b^2}{4a^5}\right)}{a}-\frac{192a^2b+128b^3}{256a^6}+\frac{9b(a^2+4b^2)}{8a^6}\right)}{d}$$
$$+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(a^4-12a^2b^2+15b^4)}{a^7d}+\frac{b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{8a^4d}$$
$$-\frac{\ln\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)(a^4-12a^2b^2+15b^4)}{a^7d}$$

input
```
int(cot(c + d*x)^5/(a + b*sin(c + d*x))^3,x)
```

output
```
(tan(c/2 + (d*x)/2)^4*(272*a*b^4 + (23*a^5)/4 - 172*a^3*b^2) - tan(c/2 + (
d*x)/2)^3*(27*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^5*(128*b^5 - 134*a^
4*b + 200*a^2*b^3) - tan(c/2 + (d*x)/2)^7*(106*a^4*b + 192*b^5 - 336*a^2*b
^3) - a^5/4 + tan(c/2 + (d*x)/2)^2*((5*a^5)/2 - 5*a^3*b^2) + a^4*b*tan(c/2
 + (d*x)/2) + (tan(c/2 + (d*x)/2)^6*(3*a^6 - 352*b^6 + 768*a^2*b^4 - 276*a
^4*b^2))/a)/(d*(16*a^8*tan(c/2 + (d*x)/2)^4 + 16*a^8*tan(c/2 + (d*x)/2)^8
+ tan(c/2 + (d*x)/2)^6*(32*a^8 + 64*a^6*b^2) + 64*a^7*b*tan(c/2 + (d*x)/2)
^5 + 64*a^7*b*tan(c/2 + (d*x)/2)^7)) - tan(c/2 + (d*x)/2)^4/(64*a^3*d) + (
tan(c/2 + (d*x)/2)^2*((3*(a^2 + 4*b^2))/(32*a^5) + 3/(32*a^3) - (9*b^2)/(8
*a^5)))/d - (tan(c/2 + (d*x)/2)*((6*b*((3*(a^2 + 4*b^2))/(16*a^5) + 3/(16*
a^3) - (9*b^2)/(4*a^5)))/a - (192*a^2*b + 128*b^3)/(256*a^6) + (9*b*(a^2 +
 4*b^2))/(8*a^6)))/d + (log(tan(c/2 + (d*x)/2))*(a^4 + 15*b^4 - 12*a^2*b^2
))/(a^7*d) + (b*tan(c/2 + (d*x)/2)^3)/(8*a^4*d) - (log(a + 2*b*tan(c/2 + (
d*x)/2) + a*tan(c/2 + (d*x)/2)^2)*(a^4 + 15*b^4 - 12*a^2*b^2))/(a^7*d)
```

3.198 $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.198.1 Optimal result

Integrand size = 21, antiderivative size = 474

$$\begin{aligned}\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx &= \frac{8a^4b^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} \\ &+ \frac{12a^2b^2(a^2+b^2)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} \\ &+ \frac{a^4(2a^2+b^2)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}\,d} \\ &+ \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))^2} - \frac{3a\cos(c+dx)}{4(a+b)^4d(1-\sin(c+dx))} \\ &+ \frac{\cos(c+dx)}{12(a+b)^3d(1-\sin(c+dx))} - \frac{\cos(c+dx)}{12(a-b)^3d(1+\sin(c+dx))^2} \\ &+ \frac{3a\cos(c+dx)}{4(a-b)^4d(1+\sin(c+dx))} - \frac{\cos(c+dx)}{12(a-b)^3d(1+\sin(c+dx))} \\ &+ \frac{a^4b\cos(c+dx)}{2\left(a^2-b^2\right)^3d(a+b\sin(c+dx))^2} \\ &+ \frac{3a^5b\cos(c+dx)}{2\left(a^2-b^2\right)^4d(a+b\sin(c+dx))} \\ &+ \frac{4a^3b^3\cos(c+dx)}{\left(a^2-b^2\right)^4d(a+b\sin(c+dx))}\end{aligned}$$

output
```
8*a^4*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(9/2)
/d+12*a^2*b^2*(a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(
a^2-b^2)^(9/2)/d+a^4*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)
^(1/2))/(a^2-b^2)^(9/2)/d+1/12*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))^2-3/4*a
*cos(d*x+c)/(a+b)^4/d/(1-sin(d*x+c))+1/12*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+
c))-1/12*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))^2+3/4*a*cos(d*x+c)/(a-b)^4/d/
(1+sin(d*x+c))-1/12*cos(d*x+c)/(a-b)^3/d/(1+sin(d*x+c))+1/2*a^4*b*cos(d*x+
c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))^2+3/2*a^5*b*cos(d*x+c)/(a^2-b^2)^4/d/(a+
b*sin(d*x+c))+4*a^3*b^3*cos(d*x+c)/(a^2-b^2)^4/d/(a+b*sin(d*x+c))
```

3.198.2 Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.74

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{96a^2(2a^4+21a^2b^2+12b^4)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{9/2}} - \frac{\sec^3(c+dx)\left(-264a^6b-358a^4b^3+8a^2b^5-16b^7-8\left(44a^6b+55a^4b^3+8a^2b^5-2b^7\right)\cos(2(c\right.}{}}{}$$

input
```
Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

output
```
((96*a^2*(2*a^4 + 21*a^2*b^2 + 12*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/(a^2 - b^2)^(9/2) - (Sec[c + d*x]^3*(-264*a^6*b - 358*a^4*b
^3 + 8*a^2*b^5 - 16*b^7 - 8*(44*a^6*b + 55*a^4*b^3 + 8*a^2*b^5 - 2*b^7)*Co
s[2*(c + d*x)] - 2*(28*a^6*b + 89*a^4*b^3 - 12*a^2*b^5)*Cos[4*(c + d*x)] +
 22*a^5*b^2*Sin[c + d*x] - 264*a^3*b^4*Sin[c + d*x] + 32*a*b^6*Sin[c + d*x
] + 32*a^7*Sin[3*(c + d*x)] - 91*a^5*b^2*Sin[3*(c + d*x)] - 244*a^3*b^4*Si
n[3*(c + d*x)] - 12*a*b^6*Sin[3*(c + d*x)] - 17*a^5*b^2*Sin[5*(c + d*x)] -
 76*a^3*b^4*Sin[5*(c + d*x)] - 12*a*b^6*Sin[5*(c + d*x)]))/((a^2 - b^2)^4*
(a + b*Sin[c + d*x])^2))/(96*d)
```

3.198.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^4}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3210}$$

$$\int \left(\frac{6a^2b^2(a^2+b^2)}{(a^2-b^2)^4(a+b\sin(c+dx))}+\frac{a^4}{(a^2-b^2)^2(a+b\sin(c+dx))^3}+\frac{4a^3b^2}{(a^2-b^2)^3(a+b\sin(c+dx))^2}+\frac{}{4(a+b)^4(}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{12a^2b^2(a^2+b^2)\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{9/2}}+\frac{3a^5b\cos(c+dx)}{2d(a^2-b^2)^4(a+b\sin(c+dx))}+\frac{a^4(2a^2+b^2)\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{9/2}}+\frac{8a^4b^2\arctan\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{9/2}}+\frac{a^4b\cos(c+dx)}{2d(a^2-b^2)^3(a+b\sin(c+dx))^2}+\frac{4a^3b^3\cos(c+dx)}{d(a^2-b^2)^4(a+b\sin(c+dx))}-\frac{3a\cos(c+dx)}{4d(a+b)^4(1-\sin(c+dx))}+\frac{3a\cos(c+dx)}{4d(a-b)^4(\sin(c+dx)+1)}+\frac{\cos(c+dx)}{12d(a+b)^3(1-\sin(c+dx))}-\frac{\cos(c+dx)}{12d(a-b)^3(\sin(c+dx)+1)}+\frac{\cos(c+dx)}{12d(a+b)^3(1-\sin(c+dx))^2}-\frac{\cos(c+dx)}{12d(a-b)^3(\sin(c+dx)+1)^2}$$

input
```
Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

output
```
(8*a^4*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^
(9/2)*d) + (12*a^2*b^2*(a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]])/((a^2 - b^2)^(9/2)*d) + (a^4*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c
 + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(9/2)*d) + Cos[c + d*x]/(12*(a
+ b)^3*d*(1 - Sin[c + d*x])^2) - (3*a*Cos[c + d*x])/(4*(a + b)^4*d*(1 - Si
n[c + d*x])) + Cos[c + d*x]/(12*(a + b)^3*d*(1 - Sin[c + d*x])) - Cos[c +
d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])^2) + (3*a*Cos[c + d*x])/(4*(a - b)
^4*d*(1 + Sin[c + d*x])) - Cos[c + d*x]/(12*(a - b)^3*d*(1 + Sin[c + d*x])
) + (a^4*b*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])^2) + (3*a
^5*b*Cos[c + d*x])/(2*(a^2 - b^2)^4*d*(a + b*Sin[c + d*x])) + (4*a^3*b^3*C
os[c + d*x])/((a^2 - b^2)^4*d*(a + b*Sin[c + d*x]))
```

3.198.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3210
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

3.198.4 Maple [A] (verified)

Time = 2.98 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.75

method	result
derivativedivides	$-\frac{1}{3(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a+b}{2(a+b)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3(a-b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{2(a-b)^3(}$
default	$-\frac{1}{3(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{1}{2(a+b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-2a+b}{2(a+b)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{3(a-b)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}+\frac{}{2(a-b)^3(}$
risch	Expression too large to display

input
```
int(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/3/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^3/(tan(1/2*d*x+1/2*c)
-1)^2-1/2/(a+b)^4*(-2*a+b)/(tan(1/2*d*x+1/2*c)-1)-1/3/(a-b)^3/(tan(1/2*d*x
+1/2*c)+1)^3+1/2/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)^2-1/2/(a-b)^4*(-2*a-b)/(ta
n(1/2*d*x+1/2*c)+1)+2*a^2/(a-b)^4/(a+b)^4*((1/2*b^2*a*(5*a^2+6*b^2)*tan(1/
2*d*x+1/2*c)^3+1/2*b*(4*a^4+15*a^2*b^2+14*b^4)*tan(1/2*d*x+1/2*c)^2+11/2*b
^2*a*(a^2+2*b^2)*tan(1/2*d*x+1/2*c)+2*b*a^4+7/2*b^3*a^2)/(a*tan(1/2*d*x+1/
2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(2*a^4+21*a^2*b^2+12*b^4)/(a^2-b^2)
^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2))))
```

3.198.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 1249, normalized size of antiderivative = 2.64

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/12*(4*a^8*b - 16*a^6*b^3 + 24*a^4*b^5 - 16*a^2*b^7 + 4*b^9 - 2*(28*a^8*
b + 61*a^6*b^3 - 101*a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(8*a^8*b - 2
5*a^6*b^3 + 27*a^4*b^5 - 11*a^2*b^7 + b^9)*cos(d*x + c)^2 - 3*((2*a^6*b^2
+ 21*a^4*b^4 + 12*a^2*b^6)*cos(d*x + c)^5 - 2*(2*a^7*b + 21*a^5*b^3 + 12*a
^3*b^5)*cos(d*x + c)^3*sin(d*x + c) - (2*a^8 + 23*a^6*b^2 + 33*a^4*b^4 + 1
2*a^2*b^6)*cos(d*x + c)^3)*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c
)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*
cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) -
 a^2 - b^2)) - 2*(2*a^9 - 8*a^7*b^2 + 12*a^5*b^4 - 8*a^3*b^6 + 2*a*b^8 + (
17*a^7*b^2 + 59*a^5*b^4 - 64*a^3*b^6 - 12*a*b^8)*cos(d*x + c)^4 - 2*(4*a^9
 - 9*a^7*b^2 + 3*a^5*b^4 + 5*a^3*b^6 - 3*a*b^8)*cos(d*x + c)^2)*sin(d*x +
c))/((a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*
d*cos(d*x + c)^5 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3
*b^9 - a*b^11)*d*cos(d*x + c)^3*sin(d*x + c) - (a^12 - 4*a^10*b^2 + 5*a^8*
b^4 - 5*a^4*b^8 + 4*a^2*b^10 - b^12)*d*cos(d*x + c)^3), 1/6*(2*a^8*b - 8*a
^6*b^3 + 12*a^4*b^5 - 8*a^2*b^7 + 2*b^9 - (28*a^8*b + 61*a^6*b^3 - 101*a^4
*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 2*(8*a^8*b - 25*a^6*b^3 + 27*a^4*b^5 -
 11*a^2*b^7 + b^9)*cos(d*x + c)^2 - 3*((2*a^6*b^2 + 21*a^4*b^4 + 12*a^2*b^
6)*cos(d*x + c)^5 - 2*(2*a^7*b + 21*a^5*b^3 + 12*a^3*b^5)*cos(d*x + c)^3*s
in(d*x + c) - (2*a^8 + 23*a^6*b^2 + 33*a^4*b^4 + 12*a^2*b^6)*cos(d*x + ...
```

3.198.6 Sympy [F]

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\tan^4{(c+dx)}}{(a+b\sin{(c+dx)})^3}\,dx$$

input
```
integrate(tan(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

3.198.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.198.8 Giac [A] (verification not implemented)

Time = 1.06 (sec) , antiderivative size = 632, normalized size of antiderivative = 1.33

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{3\,(2\,a^6+21\,a^4b^2+12\,a^2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\sqrt{a^2-b^2}} + \frac{3\left(5\,a^5b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^6b\tan\ldots\right.}{(a^8\ldots}}{\ldots}$$

input
```
integrate(tan(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/3*(3*(2*a^6 + 21*a^4*b^2 + 12*a^2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^8 - 4*
a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(a^2 - b^2)) + 3*(5*a^5*b^2*tan
(1/2*d*x + 1/2*c)^3 + 6*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*tan(1/2*d
*x + 1/2*c)^2 + 15*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 + 14*a^2*b^5*tan(1/2*d*x
 + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d*x + 1/2*c) + 22*a^3*b^4*tan(1/2*d*x + 1
/2*c) + 4*a^6*b + 7*a^4*b^3)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b
^8)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 2*(3*a^
5*tan(1/2*d*x + 1/2*c)^5 + 24*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 9*a*b^4*tan
(1/2*d*x + 1/2*c)^5 - 9*a^4*b*tan(1/2*d*x + 1/2*c)^4 - 24*a^2*b^3*tan(1/2*
d*x + 1/2*c)^4 - 3*b^5*tan(1/2*d*x + 1/2*c)^4 - 10*a^5*tan(1/2*d*x + 1/2*c
)^3 - 56*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2*d*x + 1/2*c)^3 +
 36*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 36*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a
^5*tan(1/2*d*x + 1/2*c) + 24*a^3*b^2*tan(1/2*d*x + 1/2*c) + 9*a*b^4*tan(1/
2*d*x + 1/2*c) - 15*a^4*b - 20*a^2*b^3 - b^5)/((a^8 - 4*a^6*b^2 + 6*a^4*b^
4 - 4*a^2*b^6 + b^8)*(tan(1/2*d*x + 1/2*c)^2 - 1)^3))/d
```

3.198.9 Mupad [B] (verification not implemented)

Time = 10.80 (sec) , antiderivative size = 1099, normalized size of antiderivative = 2.32

$$\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(2a^6b+21a^4b^3+12a^2b^5)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8} - \frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(14a^7+23a^5b^2+242a^3b^4+36ab^6)}{3(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)} - \frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(2a^5+21a^3b^2+12ab^4)}{3(a^6-3a^4b^2+3a^2b^4-b^6)}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(2a^2+12b^2)-\tan\left(\frac{c}{2}\right.\right.}$$
$$+\frac{a^2\,\mathrm{atan}\left(\frac{\frac{a^2(2a^4+21a^2b^2+12b^4)(2a^8b-8a^6b^3+12a^4b^5-8a^2b^7+2b^9)}{2(a+b)^{9/2}(a-b)^{9/2}}+\frac{a^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(2a^4+21a^2b^2+12b^4)(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}{(a+b)^{9/2}(a-b)^{9/2}}}{2a^6+21a^4b^2+12a^2b^4}\right.}{d(a+b)^{9/2}(a-b)^{9/2}}$$

input
```
int(tan(c + d*x)^4/(a + b*sin(c + d*x))^3,x)
```

output
```
((3*tan(c/2 + (d*x)/2)^8*(2*a^6*b + 12*a^2*b^5 + 21*a^4*b^3))/(a^8 + b^8 -
 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) - (2*tan(c/2 + (d*x)/2)^5*(36*a*b^6 +
14*a^7 + 242*a^3*b^4 + 23*a^5*b^2))/(3*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4
- 4*a^6*b^2)) - (4*tan(c/2 + (d*x)/2)^7*(12*a*b^4 + 2*a^5 + 21*a^3*b^2))/(
3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(30*a^4*b
 + 4*b^5 + 71*a^2*b^3))/(3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (a^2*(42
*a^4*b + 2*b^5 + 61*a^2*b^3))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^
4*b^2)) + (10*tan(c/2 + (d*x)/2)^4*(4*a^6*b + 43*a^2*b^5 + 16*a^4*b^3))/(3
*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^
3*(16*a*b^6 - 2*a^7 + 131*a^3*b^4 + 65*a^5*b^2))/(3*(a^2 - b^2)*(a^6 - b^6
 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*tan(c/2 + (d*x)/2)^6*(22*a^6*b + 12*b^7 +
153*a^2*b^5 + 233*a^4*b^3))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2)) + (a^3*tan(c/2 + (d*x)/2)^9*(2*a^4 + 12*b^4 + 21*a^2*b^2))/(a^8 + b^
8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2) - (a*tan(c/2 + (d*x)/2)*(8*b^6 - 6*
a^6 + 208*a^2*b^4 + 105*a^4*b^2))/(3*(a^2 - b^2)*(a^6 - b^6 + 3*a^2*b^4 -
3*a^4*b^2)))/(d*(tan(c/2 + (d*x)/2)^4*(2*a^2 + 12*b^2) - tan(c/2 + (d*x)/2
)^6*(2*a^2 + 12*b^2) + a^2*tan(c/2 + (d*x)/2)^10 - a^2 + tan(c/2 + (d*x)/2
)^2*(a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^8*(a^2 - 4*b^2) + 8*a*b*tan(c/2 + (
d*x)/2)^3 - 8*a*b*tan(c/2 + (d*x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 - 4*a*
b*tan(c/2 + (d*x)/2))) + (a^2*atan(((a^2*(2*a^4 + 12*b^4 + 21*a^2*b^2)*...
```

3.198. $\int \frac{\tan^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.199 $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.199.1 Optimal result

Integrand size = 21, antiderivative size = 350

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = &-\frac{4a^2b^2\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} \\ &-\frac{a^2(2a^2+b^2)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} \\ &-\frac{2b^2(3a^2+b^2)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}\,d} \\ &+\frac{\cos(c+dx)}{2(a+b)^3d(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2(a-b)^3d(1+\sin(c+dx))} \\ &-\frac{a^2b\cos(c+dx)}{2\left(a^2-b^2\right)^2d(a+b\sin(c+dx))^2} \\ &-\frac{3a^3b\cos(c+dx)}{2\left(a^2-b^2\right)^3d(a+b\sin(c+dx))} \\ &-\frac{2ab^3\cos(c+dx)}{\left(a^2-b^2\right)^3d(a+b\sin(c+dx))}\end{aligned}$$

output
```
-4*a^2*b^2*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-b^2)^(7/2
)/d-a^2*(2*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/(a^2-
b^2)^(7/2)/d-2*b^2*(3*a^2+b^2)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(
1/2))/(a^2-b^2)^(7/2)/d+1/2*cos(d*x+c)/(a+b)^3/d/(1-sin(d*x+c))-1/2*cos(d*
x+c)/(a-b)^3/d/(1+sin(d*x+c))-1/2*a^2*b*cos(d*x+c)/(a^2-b^2)^2/d/(a+b*sin(
d*x+c))^2-3/2*a^3*b*cos(d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))-2*a*b^3*cos(
d*x+c)/(a^2-b^2)^3/d/(a+b*sin(d*x+c))
```

3.199.2 Mathematica [A] (verified)

Time = 2.45 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.61

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{-\frac{2(2a^4+11a^2b^2+2b^4)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}}+\sin\left(\frac{1}{2}(c+dx)\right)\left(\frac{2}{(a+b)^3\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2}{(a-b)^3\left(\cos\left(\frac{1}{2}(c+dx\right.\right.}\right.}{2d}$$

input
```
Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

output
```
((-2*(2*a^4 + 11*a^2*b^2 + 2*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
 - b^2]])/(a^2 - b^2)^(7/2) + Sin[(c + d*x)/2]*(2/((a + b)^3*(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])) + 2/((a - b)^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]))) - (a*b*Cos[c + d*x]*(4*a^3 + 3*a*b^2 + b*(3*a^2 + 4*b^2)*Sin[c + d*
x]))/((a - b)^3*(a + b)^3*(a + b*Sin[c + d*x])^2))/(2*d)
```

3.199.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 3210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(c+dx)^2}{(a+b\sin(c+dx))^3}dx$$
$$\Big\downarrow \text{3210}$$
$$\int \left(-\frac{a^2}{(a^2-b^2)(a+b\sin(c+dx))^3}-\frac{2ab^2}{(a^2-b^2)^2(a+b\sin(c+dx))^2}+\frac{b^2(3a^2+b^2)}{(b^2-a^2)^3(a+b\sin(c+dx))}-\frac{}{2(a+b)^3}\right.$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2(2a^2+b^2)\arctan\left(\frac{a\tan(\frac{1}{2}(c+dx))+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}}-\frac{4a^2b^2\arctan\left(\frac{a\tan(\frac{1}{2}(c+dx))+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}}-\frac{2b^2(3a^2+b^2)\arctan\left(\frac{a\tan(\frac{1}{2}(c+dx))+b}{\sqrt{a^2-b^2}}\right)}{d(a^2-b^2)^{7/2}}-\frac{a^2b\cos(c+dx)}{2d(a^2-b^2)^2(a+b\sin(c+dx))^2}-\frac{2ab^3\cos(c+dx)}{d(a^2-b^2)^3(a+b\sin(c+dx))}-\frac{3a^3b\cos(c+dx)}{2d(a^2-b^2)^3(a+b\sin(c+dx))}+\frac{\cos(c+dx)}{2d(a+b)^3(1-\sin(c+dx))}-\frac{\cos(c+dx)}{2d(a-b)^3(\sin(c+dx)+1)}$$

input
```
Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

output
```
(-4*a^2*b^2*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)
^(7/2)*d) - (a^2*(2*a^2 + b^2)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/((a^2 - b^2)^(7/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTan[(b + a*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]])/((a^2 - b^2)^(7/2)*d) + Cos[c + d*x]/(2*(a + b
)^3*d*(1 - Sin[c + d*x])) - Cos[c + d*x]/(2*(a - b)^3*d*(1 + Sin[c + d*x])
) - (a^2*b*Cos[c + d*x])/(2*(a^2 - b^2)^2*d*(a + b*Sin[c + d*x])^2) - (3*a
^3*b*Cos[c + d*x])/(2*(a^2 - b^2)^3*d*(a + b*Sin[c + d*x])) - (2*a*b^3*Cos
[c + d*x])/((a^2 - b^2)^3*d*(a + b*Sin[c + d*x]))
```

3.199.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3210
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

3.199.4 Maple [A] (verified)

Time = 2.03 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.74

method	result
derivativedivides	$-\frac{2\left(\frac{\left(\frac{5}{2}a^3b^2+a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(4a^4+11a^2b^2+6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a\,b^2\left(11a^2+10b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{a^2b\left(4a^2+3b^2\right)}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\cdots}{(a-b)^3(a+b)^3}\cdots}{d}$
default	$-\frac{2\left(\frac{\left(\frac{5}{2}a^3b^2+a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(4a^4+11a^2b^2+6b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a\,b^2\left(11a^2+10b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}+\frac{a^2b\left(4a^2+3b^2\right)}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}+\cdots}{(a-b)^3(a+b)^3}\cdots}{d}$
risch	$\frac{i\left(-2ib^5e^{5i(dx+c)}-2ib^5e^{i(dx+c)}+18ia^4b\,e^{i(dx+c)}+4ib^5e^{3i(dx+c)}+2ia^2b^3e^{3i(dx+c)}+29ia^2b^3e^{i(dx+c)}+6a^5e^{4i(dx+c)}+33a\cdots\right.}{\left(1+e^{2i(dx+c)}\right)\cdots}$

input
```
int(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/(a-b)^3/(a+b)^3*(((5/2*a^3*b^2+a*b^4)*tan(1/2*d*x+1/2*c)^3+1/2*b*(
4*a^4+11*a^2*b^2+6*b^4)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(11*a^2+10*b^2)*tan
(1/2*d*x+1/2*c)+1/2*a^2*b*(4*a^2+3*b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1
/2*d*x+1/2*c)+a)^2+1/2*(2*a^4+11*a^2*b^2+2*b^4)/(a^2-b^2)^(1/2)*arctan(1/2
*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/(a-b)^3/(tan(1/2*d*x+1/2
*c)+1)-1/(a+b)^3/(tan(1/2*d*x+1/2*c)-1))
```

3.199.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 934, normalized size of antiderivative = 2.67

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

$$= \left[\frac{4\,a^6b-12\,a^4b^3+12\,a^2b^5-4\,b^7+2\,(8\,a^6b+a^4b^3-11\,a^2b^5+2\,b^7)\cos{(dx+c)}^2+\left((2\,a^4b^2+11\,a^2b^4+2\right.}{}\right.$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/4*(4*a^6*b - 12*a^4*b^3 + 12*a^2*b^5 - 4*b^7 + 2*(8*a^6*b + a^4*b^3 - 1
1*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4 + 2*b^6)*cos(
d*x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)*sin(d*x + c)
- (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)
*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a
*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*
x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(2*a^7 - 6*a^5*b^2 + 6*a^3
*b^4 - 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2)*sin(d*x +
 c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d*cos(d*x + c)^
3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*s
in(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^4 + 2*a^4*b^6 - 3*a^2*b^8 + b^10
)*d*cos(d*x + c)), 1/2*(2*a^6*b - 6*a^4*b^3 + 6*a^2*b^5 - 2*b^7 + (8*a^6*b
 + a^4*b^3 - 11*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + ((2*a^4*b^2 + 11*a^2*b^4
 + 2*b^6)*cos(d*x + c)^3 - 2*(2*a^5*b + 11*a^3*b^3 + 2*a*b^5)*cos(d*x + c)
*sin(d*x + c) - (2*a^6 + 13*a^4*b^2 + 13*a^2*b^4 + 2*b^6)*cos(d*x + c))*sq
rt(a^2 - b^2)*arctan(-(a*sin(d*x + c) + b)/(sqrt(a^2 - b^2)*cos(d*x + c)))
 - (2*a^7 - 6*a^5*b^2 + 6*a^3*b^4 - 2*a*b^6 - 5*(a^5*b^2 + a^3*b^4 - 2*a*b
^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^
2*b^8 + b^10)*d*cos(d*x + c)^3 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*
b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) - (a^10 - 3*a^8*b^2 + 2*a^6*b^...
```

3.199.6 Sympy [F]

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\tan^2{(c+dx)}}{(a+b\sin{(c+dx)})^3}\,dx$$

input
```
integrate(tan(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(tan(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.199.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.199.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.10

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\frac{(2a^4+11a^2b^2+2b^4)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{a^2-b^2}} + \frac{2\left(a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3a^2b-b^3\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)} + 5a \cdots}{\cdots}$$

input
```
integrate(tan(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-((2*a^4 + 11*a^2*b^2 + 2*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) +
arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*sqrt(a^2 - b^2)) + 2*(a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*
tan(1/2*d*x + 1/2*c) - 3*a^2*b - b^3)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(tan(1/2*d*x + 1/2*c)^2 - 1)) + (5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b
^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 11*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^2 + 6*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d
*x + 1/2*c) + 10*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b + 3*a^2*b^3)/((a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x
+ 1/2*c) + a)^2))/d
```

3.199.9 Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.79

$$\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{\frac{5\,(2\,a^4\,b+a^2\,b^3)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(2\,a^5+a^3\,b^2+12\,a\,b^4)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^4\,b+6\,a^2\,b^3+7\,b^5)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(2\,a^4\,b+11\,a^2\,b^3 \cdots)}{a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6} \cdots}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^2+4\,b^2)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^2+4\,b^2) \cdots\right)} - \frac{\mathrm{atan}\left(\frac{\frac{(2\,a^4+11\,a^2\,b^2+2\,b^4)\,(2\,a^6\,b-6\,a^4\,b^3+6\,a^2\,b^5-2\,b^7)}{2\,(a+b)^{7/2}\,(a-b)^{7/2}}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a^4+11\,a^2\,b^2+2\,b^4)\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}{(a+b)^{7/2}\,(a-b)^{7/2}}}{2\,a^4+11\,a^2\,b^2+2\,b^4}\right)\,(2\,a^4+11\,a^2\,b^2 \cdots)}{d\,(a+b)^{7/2}\,(a-b)^{7/2}}$$

input
```
int(tan(c + d*x)^2/(a + b*sin(c + d*x))^3,x)
```

output
```
((5*(2*a^4*b + a^2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (2*tan(c/2
+ (d*x)/2)^3*(12*a*b^4 + 2*a^5 + a^3*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2) + (2*tan(c/2 + (d*x)/2)^2*(2*a^4*b + 7*b^5 + 6*a^2*b^3))/(a^6 - b^6 +
 3*a^2*b^4 - 3*a^4*b^2) - (3*tan(c/2 + (d*x)/2)^4*(2*a^4*b + 2*b^5 + 11*a^
2*b^3))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) + (a*tan(c/2 + (d*x)/2)*(18*b^
4 - 2*a^4 + 29*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2) - (a*tan(c/2
+ (d*x)/2)^5*(2*a^4 + 2*b^4 + 11*a^2*b^2))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2))/(d*(a^2*tan(c/2 + (d*x)/2)^6 - a^2 - tan(c/2 + (d*x)/2)^2*(a^2 + 4*b
^2) + tan(c/2 + (d*x)/2)^4*(a^2 + 4*b^2) + 4*a*b*tan(c/2 + (d*x)/2)^5 - 4*
a*b*tan(c/2 + (d*x)/2))) - (atan((((2*a^4 + 2*b^4 + 11*a^2*b^2)*(2*a^6*b -
 2*b^7 + 6*a^2*b^5 - 6*a^4*b^3))/(2*(a + b)^(7/2)*(a - b)^(7/2)) + (a*tan(
c/2 + (d*x)/2)*(2*a^4 + 2*b^4 + 11*a^2*b^2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4
*b^2))/((a + b)^(7/2)*(a - b)^(7/2)))/(2*a^4 + 2*b^4 + 11*a^2*b^2))*(2*a^4
 + 2*b^4 + 11*a^2*b^2))/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

3.199. $\int \frac{\tan^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.200 $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3} dx$

3.200.1 Optimal result

Integrand size = 21, antiderivative size = 202

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3} dx = -\frac{(2a^4-9a^2b^2+6b^4)\arctan\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)}{a^4(a^2-b^2)^{3/2}d} + \frac{3b\text{arctanh}(\cos(c+dx))}{a^4d} - \frac{(5a^2-6b^2)\cot(c+dx)}{2a^3(a^2-b^2)d} + \frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2} + \frac{(2a^2-3b^2)\cot(c+dx)}{2a^2(a^2-b^2)d(a+b\sin(c+dx))}$$

output
```
-(2*a^4-9*a^2*b^2+6*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/
a^4/(a^2-b^2)^(3/2)/d+3*b*arctanh(cos(d*x+c))/a^4/d-1/2*(5*a^2-6*b^2)*cot(
d*x+c)/a^3/(a^2-b^2)/d+1/2*cot(d*x+c)/a/d/(a+b*sin(d*x+c))^2+1/2*(2*a^2-3*
b^2)*cot(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sin(d*x+c))
```

3.200.2 Mathematica [A] (verified)

Time = 3.76 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.97

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3} dx = \frac{-\frac{2(2a^4-9a^2b^2+6b^4)\arctan\left(\frac{b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}} - a\cot\left(\frac{1}{2}(c+dx)\right) + 6b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 6b\log\left(\sin\left(\frac{1}{2}(c+\right.\right.}{2a^4d}$$

input
```
Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

output
```
((-2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*ArcTan[(b + a*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/(a^2 - b^2)^(3/2) - a*Cot[(c + d*x)/2] + 6*b*Log[Cos[(c + d*x)/2]
] - 6*b*Log[Sin[(c + d*x)/2]] - (a^2*b*Cos[c + d*x])/(a + b*Sin[c + d*x])^
2 + (a*b*(-3*a^2 + 4*b^2)*Cos[c + d*x])/((a - b)*(a + b)*(a + b*Sin[c + d*
x])) + a*Tan[(c + d*x)/2])/(2*a^4*d)
```

3.200.3 Rubi [A] (verified)

Time = 1.51 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.23, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {3042, 3202, 3042, 3535, 3042, 3535, 3042, 3534, 25, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^2(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3202}$$

$$\int \frac{\left(1-\sin^2(c+dx)\right)\csc^2(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1-\sin(c+dx)^2}{\sin(c+dx)^2(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3535}$$

$$\frac{\int \frac{\csc^2(c+dx)\left(3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin^2(c+dx)\right)}{(a+b\sin(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{3\left(a^2-b^2\right)-2\left(a^2-b^2\right)\sin(c+dx)^2}{\sin(c+dx)^2(a+b\sin(c+dx))^2}dx}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3535

$$\frac{\frac{\int \frac{\csc^2(c+dx)\left(5a^4-11b^2a^2-b\left(a^2-b^2\right)\sin(c+dx)a+6b^4-\left(2a^2-3b^2\right)\left(a^2-b^2\right)\sin^2(c+dx)\right)}{a+b\sin(c+dx)}dx}{a(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{\int \frac{5a^4-11b^2a^2-b\left(a^2-b^2\right)\sin(c+dx)a+6b^4-\left(2a^2-3b^2\right)\left(a^2-b^2\right)\sin(c+dx)^2}{\sin(c+dx)^2(a+b\sin(c+dx))}dx}{a(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3534

$$\frac{\frac{\frac{\int -\frac{\csc(c+dx)\left(6b\left(a^2-b^2\right)^2+a\left(2a^4-5b^2a^2+3b^4\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}dx}{a}-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{-\frac{\int \frac{\csc(c+dx)\left(6b\left(a^2-b^2\right)^2+a\left(2a^4-5b^2a^2+3b^4\right)\sin(c+dx)\right)}{a+b\sin(c+dx)}dx}{a}-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{-\frac{\int \frac{6b\left(a^2-b^2\right)^2+a\left(2a^4-5b^2a^2+3b^4\right)\sin(c+dx)}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$\Big\downarrow$ 3480

$$\frac{\frac{-\frac{\frac{6b\left(a^2-b^2\right)^2\int\csc(c+dx)dx}{a}+\frac{\left(2a^6-11a^4b^2+15a^2b^4-6b^6\right)\int\frac{1}{a+b\sin(c+dx)}dx}{a}}{a}-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

3.200. $\displaystyle\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\frac{\frac{6b\left(a^2-b^2\right)^2\int\csc(c+dx)dx}{a}+\frac{\left(2a^6-11a^4b^2+15a^2b^4-6b^6\right)\int\frac{1}{a+b\sin(c+dx)}dx}{a}}{a}-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 3139$$

$$\frac{\frac{-\frac{\frac{6b\left(a^2-b^2\right)^2\int\csc(c+dx)dx}{a}+\frac{2\left(2a^6-11a^4b^2+15a^2b^4-6b^6\right)\int\frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}}{a}-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(2\cdots}{\cdots}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{\frac{-\frac{\frac{6b\left(a^2-b^2\right)^2\int\csc(c+dx)dx}{a}-\frac{4\left(2a^6-11a^4b^2+15a^2b^4-6b^6\right)\int\frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}}{a}-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+d\cdots}{ad}}{a(a^2-b^2)}\cdots}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 217$$

$$\frac{\frac{-\frac{\frac{6b\left(a^2-b^2\right)^2\int\csc(c+dx)dx}{a}+\frac{2\left(2a^6-11a^4b^2+15a^2b^4-6b^6\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}}{a}-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{\frac{(2a^2-3b^2)\cot(c+dx)}{ad(a+b\sin(c+dx))}+\frac{-\frac{\left(5a^4-11a^2b^2+6b^4\right)\cot(c+dx)}{ad}-\frac{\frac{2\left(2a^6-11a^4b^2+15a^2b^4-6b^6\right)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}-\frac{6b\left(a^2-b^2\right)^2\text{arctanh}(\cos(c+\cdots}{ad}}{a}}{a(a^2-b^2)}}{2a\left(a^2-b^2\right)}+\frac{\cot(c+dx)}{2ad(a+b\sin(c+dx))^2}$$

3.200. $\int\frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

input
```
Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x])^3,x]
```

output
```
Cot[c + d*x]/(2*a*d*(a + b*Sin[c + d*x])^2) + ((-(((2*(2*a^6 - 11*a^4*b^2
+ 15*a^2*b^4 - 6*b^6)*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^
2])])/(a*Sqrt[a^2 - b^2]*d) - (6*b*(a^2 - b^2)^2*ArcTanh[Cos[c + d*x]])/(a
*d))/a) - ((5*a^4 - 11*a^2*b^2 + 6*b^4)*Cot[c + d*x])/(a*d))/(a*(a^2 - b^2
)) + ((2*a^2 - 3*b^2)*Cot[c + d*x])/(a*d*(a + b*Sin[c + d*x])))/(2*a*(a^2
- b^2))
```

3.200.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.200. $\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.200.4 Maple [A] (verified)

Time = 4.01 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{2\left(\frac{\frac{a\,b^2\left(5a^2-6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{b\left(4a^4+3a^2b^2-10b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{a\,b^2\left(11a^2-14b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-2b^2}+\frac{a^2b(\cdots}{2a\cdots}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\cdots\right)}{a^4}\cdots}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{2\left(\frac{\frac{a\,b^2\left(5a^2-6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{b\left(4a^4+3a^2b^2-10b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2-2b^2}+\frac{a\,b^2\left(11a^2-14b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-2b^2}+\frac{a^2b(\cdots}{2a\cdots}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+a\right)^2}\cdots\right)}{a^4}\cdots}{d}$
risch	$\frac{i\left(-2ia^3b\,e^{5i(dx+c)}+3ia\,b^3e^{5i(dx+c)}+20ib\,a^3e^{3i(dx+c)}-24ib^3a\,e^{3i(dx+c)}+6a^4e^{4i(dx+c)}-3b^2e^{4i(dx+c)}a^2-6b^4e^{4i(dx+c)}-\cdots\right.}{\left(e^{2i(dx+c)}-1\right)\left(-ib\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}\cdots\right.}$

input
```
int(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/a^3*tan(1/2*d*x+1/2*c)-2/a^4*((1/2*a*b^2*(5*a^2-6*b^2)/(a^2-b^2)*
tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4+3*a^2*b^2-10*b^4)/(a^2-b^2)*tan(1/2*d*x+
1/2*c)^2+1/2*a*b^2*(11*a^2-14*b^2)/(a^2-b^2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*
(4*a^2-5*b^2)/(a^2-b^2))/(a*tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)
^2+1/2*(2*a^4-9*a^2*b^2+6*b^4)/(a^2-b^2)^(3/2)*arctan(1/2*(2*a*tan(1/2*d*x
+1/2*c)+2*b)/(a^2-b^2)^(1/2)))-1/2/a^3/tan(1/2*d*x+1/2*c)-3/a^4*b*ln(tan(1
/2*d*x+1/2*c)))
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. 2(191) = 382.

Time = 0.63 (sec) , antiderivative size = 1394, normalized size of antiderivative = 6.90

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/4*(2*(5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - 2*(8*a^6*b -
17*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^3*b^3
+ 12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 7
*a^4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*cos(d*x + c
)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*
a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x
+ c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b
^2)) - 2*(2*a^7 + a^5*b^2 - 9*a^3*b^4 + 6*a*b^6)*cos(d*x + c) + 6*(2*a^5*b
^2 - 4*a^3*b^4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2
+ (a^6*b - a^4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x +
 c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 6*(2*a^5*b^2 - 4*a^3*b^
4 + 2*a*b^6 - 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cos(d*x + c)^2 + (a^6*b - a^
4*b^3 - a^2*b^5 + b^7 - (a^4*b^3 - 2*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*
x + c))*log(-1/2*cos(d*x + c) + 1/2))/(2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d*c
os(d*x + c)^2 - 2*(a^9*b - 2*a^7*b^3 + a^5*b^5)*d + ((a^8*b^2 - 2*a^6*b^4
+ a^4*b^6)*d*cos(d*x + c)^2 - (a^10 - a^8*b^2 - a^6*b^4 + a^4*b^6)*d)*sin(
d*x + c)), -1/2*((5*a^5*b^2 - 11*a^3*b^4 + 6*a*b^6)*cos(d*x + c)^3 - (8*a^
6*b - 17*a^4*b^3 + 9*a^2*b^5)*cos(d*x + c)*sin(d*x + c) + (4*a^5*b - 18*a^
3*b^3 + 12*a*b^5 - 2*(2*a^5*b - 9*a^3*b^3 + 6*a*b^5)*cos(d*x + c)^2 + (2*a
^6 - 7*a^4*b^2 - 3*a^2*b^4 + 6*b^6 - (2*a^4*b^2 - 9*a^2*b^4 + 6*b^6)*co...
```

3.200.6 Sympy [F]

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\cot^2{(c+dx)}}{(a+b\sin{(c+dx)})^3}\,dx$$

input
```
integrate(cot(d*x+c)**2/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**2/(a + b*sin(c + d*x))**3, x)
```

3.200.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.200.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.68

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = -\frac{\frac{2\,(2\,a^4-9\,a^2b^2+6\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{(a^6-a^4b^2)\sqrt{a^2-b^2}}+\frac{2\left(5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4b\tan(\cdots}{\cdots}}{\cdots}$$

input
```
integrate(cot(d*x+c)^2/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/2*(2*(2*a^4 - 9*a^2*b^2 + 6*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(
a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/((a^6 - a^4*b^2
)*sqrt(a^2 - b^2)) + 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^4*tan(1/2
*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 3*a^2*b^3*tan(1/2*d*x +
 1/2*c)^2 - 10*b^5*tan(1/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c
) - 14*a*b^4*tan(1/2*d*x + 1/2*c) + 4*a^4*b - 5*a^2*b^3)/((a^6 - a^4*b^2)*
(a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*d*x + 1/2*c) + a)^2) + 6*b*log(abs
(tan(1/2*d*x + 1/2*c)))/a^4 - tan(1/2*d*x + 1/2*c)/a^3 - (6*b*tan(1/2*d*x
+ 1/2*c) - a)/(a^4*tan(1/2*d*x + 1/2*c)))/d
```

3.200.9 Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 1762, normalized size of antiderivative = 8.72

$$\int \frac{\cot^2(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^2/(a + b*sin(c + d*x))^3,x)
```

output
```
tan(c/2 + (d*x)/2)/(2*a^3*d) - (a^2 - (2*tan(c/2 + (d*x)/2)*(7*a*b^3 - 6*a
^3*b))/(a^2 - b^2) + (tan(c/2 + (d*x)/2)^4*(a^4 - 12*b^4 + 9*a^2*b^2))/(a^
2 - b^2) + (2*tan(c/2 + (d*x)/2)^2*(a^4 - 16*b^4 + 12*a^2*b^2))/(a^2 - b^2
) + (2*tan(c/2 + (d*x)/2)^3*(6*a^4*b - 10*b^5 + a^2*b^3))/(a*(a^2 - b^2)))
/(d*(2*a^5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^3*(4*a^5 + 8*a^3*b^2)
 + 2*a^5*tan(c/2 + (d*x)/2) + 8*a^4*b*tan(c/2 + (d*x)/2)^2 + 8*a^4*b*tan(c
/2 + (d*x)/2)^4)) - (3*b*log(tan(c/2 + (d*x)/2)))/(a^4*d) - (atan((((-(a +
 b)^3*(a - b)^3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4
- 15*a^6*b^2)/(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7
 + 60*a^4*b^5 - 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) + (((2*a^10*b - 2
*a^8*b^3)/(a^8 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a
^8*b^4 - 20*a^10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)
^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*
b^2))*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + ((-(a + b)^3*(a - b)^
3)^(1/2)*(a^4 + 3*b^4 - (9*a^2*b^2)/2)*((2*a^8 + 12*a^4*b^4 - 15*a^6*b^2)/
(a^8 - a^6*b^2) + (tan(c/2 + (d*x)/2)*(10*a^8*b - 24*a^2*b^7 + 60*a^4*b^5
- 46*a^6*b^3))/(a^9 + a^5*b^4 - 2*a^7*b^2) - (((2*a^10*b - 2*a^8*b^3)/(a^8
 - a^6*b^2) - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^6*b^6 + 22*a^8*b^4 - 20*a^
10*b^2))/(a^9 + a^5*b^4 - 2*a^7*b^2))*(-(a + b)^3*(a - b)^3)^(1/2)*(a^4 +
3*b^4 - (9*a^2*b^2)/2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*1i)/(...
```

3.201 $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.201.1 Optimal result

Integrand size = 21, antiderivative size = 289

$$\begin{aligned}\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx =& \frac{(2a^4-19a^2b^2+20b^4)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}d} \\ &- \frac{b(9a^2-20b^2)\operatorname{arctanh}(\cos(c+dx))}{2a^6d} \\ &+ \frac{(17a^2-60b^2)\cot(c+dx)}{6a^5d} \\ &- \frac{(a^2-5b^2)\cot(c+dx)\csc(c+dx)}{a^4bd} \\ &+ \frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2} \\ &- \frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2} \\ &+ \frac{(3a^2-20b^2)\cot(c+dx)\csc(c+dx)}{6a^3bd(a+b\sin(c+dx))}\end{aligned}$$

output
```
-1/2*b*(9*a^2-20*b^2)*arctanh(cos(d*x+c))/a^6/d+1/6*(17*a^2-60*b^2)*cot(d*
x+c)/a^5/d-(a^2-5*b^2)*cot(d*x+c)*csc(d*x+c)/a^4/b/d+1/6*(3*a^2-5*b^2)*cot
(d*x+c)*csc(d*x+c)/a^2/b/d/(a+b*sin(d*x+c))^2-1/3*cot(d*x+c)*csc(d*x+c)^2/
a/d/(a+b*sin(d*x+c))^2+1/6*(3*a^2-20*b^2)*cot(d*x+c)*csc(d*x+c)/a^3/b/d/(a
+b*sin(d*x+c))+(2*a^4-19*a^2*b^2+20*b^4)*arctan((b+a*tan(1/2*d*x+1/2*c))/(
a^2-b^2)^(1/2))/a^6/d/(a^2-b^2)^(1/2)
```

3.201.2 Mathematica [A] (verified)

Time = 6.32 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.59

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{(2a^4-19a^2b^2+20b^4)\arctan\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(b\cos\left(\frac{1}{2}(c+dx)\right)+a\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{a^2-b^2}}\right)}{a^6\sqrt{a^2-b^2}d}$$
$$+\frac{\left(2a^2\cos\left(\frac{1}{2}(c+dx)\right)-9b^2\cos\left(\frac{1}{2}(c+dx)\right)\right)\csc\left(\frac{1}{2}(c+dx)\right)}{3a^5d}+\frac{3b\csc^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d}$$
$$-\frac{\cot\left(\frac{1}{2}(c+dx)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)}{24a^3d}+\frac{(-9a^2b+20b^3)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d}$$
$$+\frac{(9a^2b-20b^3)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2a^6d}-\frac{3b\sec^2\left(\frac{1}{2}(c+dx)\right)}{8a^4d}$$
$$+\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(-2a^2\sin\left(\frac{1}{2}(c+dx)\right)+9b^2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5d}$$
$$+\frac{a^2b\cos(c+dx)-b^3\cos(c+dx)}{2a^4d(a+b\sin(c+dx))^2}+\frac{3a^2b\cos(c+dx)-8b^3\cos(c+dx)}{2a^5d(a+b\sin(c+dx))}$$
$$+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)}{24a^3d}$$

input
```
Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

output
```
((2*a^4 - 19*a^2*b^2 + 20*b^4)*ArcTan[(Sec[(c + d*x)/2]*(b*Cos[(c + d*x)/2
] + a*Sin[(c + d*x)/2]))/Sqrt[a^2 - b^2]])/(a^6*Sqrt[a^2 - b^2]*d) + ((2*a
^2*Cos[(c + d*x)/2] - 9*b^2*Cos[(c + d*x)/2])*Csc[(c + d*x)/2])/(3*a^5*d)
+ (3*b*Csc[(c + d*x)/2]^2)/(8*a^4*d) - (Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^
2)/(24*a^3*d) + ((-9*a^2*b + 20*b^3)*Log[Cos[(c + d*x)/2]])/(2*a^6*d) + ((
9*a^2*b - 20*b^3)*Log[Sin[(c + d*x)/2]])/(2*a^6*d) - (3*b*Sec[(c + d*x)/2]
^2)/(8*a^4*d) + (Sec[(c + d*x)/2]*(-2*a^2*Sin[(c + d*x)/2] + 9*b^2*Sin[(c
+ d*x)/2]))/(3*a^5*d) + (a^2*b*Cos[c + d*x] - b^3*Cos[c + d*x])/(2*a^4*d*(
a + b*Sin[c + d*x])^2) + (3*a^2*b*Cos[c + d*x] - 8*b^3*Cos[c + d*x])/(2*a^
5*d*(a + b*Sin[c + d*x])) + (Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(24*a^3*
d)
```

3.201.3 Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.24, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.810, Rules used = {3042, 3203, 3042, 3534, 3042, 3534, 27, 3042, 3534, 27, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan(c+dx)^4(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{ 3203}$$

$$\frac{\int \frac{\csc^3(c+dx)\left(-3\left(a^2-5b^2\right)\sin^2(c+dx)-2ab\sin(c+dx)+2\left(3a^2-10b^2\right)\right)}{(a+b\sin(c+dx))^2}dx}{6a^2b}+\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \frac{-3\left(a^2-5b^2\right)\sin(c+dx)^2-2ab\sin(c+dx)+2\left(3a^2-10b^2\right)}{\sin(c+dx)^3(a+b\sin(c+dx))^2}dx}{6a^2b}+\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{ 3534}$$

$$\frac{\frac{\int \frac{\csc^3(c+dx)\left(-2\left(3a^4-23b^2a^2+20b^4\right)\sin^2(c+dx)-5ab\left(a^2-b^2\right)\sin(c+dx)+12\left(a^4-6b^2a^2+5b^4\right)\right)}{a+b\sin(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{\left(3a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))}}{6a^2b}+\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{\int \frac{-2\left(3a^4-23b^2a^2+20b^4\right)\sin(c+dx)^2-5ab\left(a^2-b^2\right)\sin(c+dx)+12\left(a^4-6b^2a^2+5b^4\right)}{\sin(c+dx)^3(a+b\sin(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{\left(3a^2-20b^2\right)\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))}}{6a^2b}+\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{\frac{\int -\frac{2\csc^2(c+dx)\left(-10a\left(a^2-b^2\right)\sin(c+dx)b^2-6\left(a^4-6b^2a^2+5b^4\right)\sin^2(c+dx)b+\left(17a^4-77b^2a^2+60b^4\right)b\right)}{a+b\sin(c+dx)}dx}{2a}-\frac{6\left(a^4-6a^2b^2+5b^4\right)\cot(c+dx)\csc(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2-\cdots}{\cdots}}{6a^2b}$$

$$\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{\int \frac{\csc^2(c+dx)\left(-10a\left(a^2-b^2\right)\sin(c+dx)b^2-6\left(a^4-6b^2a^2+5b^4\right)\sin^2(c+dx)b+\left(17a^4-77b^2a^2+60b^4\right)b\right)}{a+b\sin(c+dx)}dx}{a}-\frac{6\left(a^4-6a^2b^2+5b^4\right)\cot(c+dx)\csc(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2-\cdots}{\cdots}}{6a^2b}$$

$$\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{-\frac{\int \frac{-10a\left(a^2-b^2\right)\sin(c+dx)b^2-6\left(a^4-6b^2a^2+5b^4\right)\sin(c+dx)^2b+\left(17a^4-77b^2a^2+60b^4\right)b}{\sin(c+dx)^2(a+b\sin(c+dx))}dx}{a}-\frac{6\left(a^4-6a^2b^2+5b^4\right)\cot(c+dx)\csc(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2-20b^2)\cot(c+\cdots}{ad(a+b\sin(\cdots}}{6a^2b}$$

$$\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 3534$$

$$\frac{\frac{-\frac{\frac{\int -\frac{3\csc(c+dx)\left(\left(9a^4-29b^2a^2+20b^4\right)b^2+2a\left(a^4-6b^2a^2+5b^4\right)\sin(c+dx)b\right)}{a+b\sin(c+dx)}dx}{a}-\frac{b\left(17a^4-77a^2b^2+60b^4\right)\cot(c+dx)}{ad}}{a}-\frac{6\left(a^4-6a^2b^2+5b^4\right)\cot(c+dx)\csc(c+dx)}{ad}}{a(a^2-b^2)}\cdots}{6a^2b}$$

$$\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{-\frac{\frac{-\frac{3\int \frac{\csc(c+dx)\left(\left(9a^4-29b^2a^2+20b^4\right)b^2+2a\left(a^4-6b^2a^2+5b^4\right)\sin(c+dx)b\right)}{a+b\sin(c+dx)}dx}{a}-\frac{b\left(17a^4-77a^2b^2+60b^4\right)\cot(c+dx)}{ad}}{a}-\frac{6\left(a^4-6a^2b^2+5b^4\right)\cot(c+dx)\csc(c+dx)}{ad}}{a(a^2-b^2)}\cdots}{6a^2b}$$

$$\frac{\left(3a^2-5b^2\right)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

3.201. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$-\frac{\frac{-\frac{-\frac{3\int \frac{(9a^4-29b^2a^2+20b^4)b^2+2a(a^4-6b^2a^2+5b^4)\sin(c+dx)b}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}}{a}-\frac{b(17a^4-77a^2b^2+60b^4)\cot(c+dx)}{ad}-\frac{6(a^4-6a^2b^2+5b^4)\cot(c+dx)\csc(c+dx)}{ad}}{a(a^2-b^2)}+\frac{(3a^2-20\cdots}{a\cdots}}{6a^2b}$$

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 3480$$

$$-\frac{\frac{-\frac{-\frac{3\left(\frac{b^2(9a^4-29a^2b^2+20b^4)\int \csc(c+dx)dx}{a}+\frac{b(2a^6-21a^4b^2+39a^2b^4-20b^6)\int \frac{1}{a+b\sin(c+dx)}dx}{a}\right)}{a}}{a}-\frac{b(17a^4-77a^2b^2+60b^4)\cot(c+dx)}{ad}-\frac{6(a^4-6a^2b^2+5b^4)\cdots}{ad\cdots}}{a(a^2-b^2)}\cdots}{6a^2b}$$

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\frac{-\frac{-\frac{3\left(\frac{b^2(9a^4-29a^2b^2+20b^4)\int \csc(c+dx)dx}{a}+\frac{b(2a^6-21a^4b^2+39a^2b^4-20b^6)\int \frac{1}{a+b\sin(c+dx)}dx}{a}\right)}{a}}{a}-\frac{b(17a^4-77a^2b^2+60b^4)\cot(c+dx)}{ad}-\frac{6(a^4-6a^2b^2+5b^4)\cdots}{ad\cdots}}{a(a^2-b^2)}\cdots}{6a^2b}$$

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 3139$$

$$-\frac{\frac{-\frac{-\frac{3\left(\frac{b^2(9a^4-29a^2b^2+20b^4)\int \csc(c+dx)dx}{a}+\frac{2b(2a^6-21a^4b^2+39a^2b^4-20b^6)\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}\right)}{a}}{a}-\frac{b(17a^4-77a^2b^2+\cdots}{a\cdots}}{a(a^2-b^2)}\cdots}{6a^2b}$$

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \; 1083$$

$$-\frac{\frac{-\frac{-\frac{3\left(\frac{b^2(9a^4-29a^2b^2+20b^4)\int \csc(c+dx)dx}{a}-\frac{4b(2a^6-21a^4b^2+39a^2b^4-20b^6)\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4(a^2-b^2)}d\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad}\right)}{a}}{a}-\frac{b(17a^4-\cdots}{\cdots}}{a(a^2-b^2)}\cdots}{6a^2b}$$

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

3.201. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$\downarrow$ 217

$$-\frac{-\frac{-\frac{3\left(\frac{b^2(9a^4-29a^2b^2+20b^4)\int\csc(c+dx)dx}{a}+\frac{2b(2a^6-21a^4b^2+39a^2b^4-20b^6)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}}\right)}{a}-\frac{b(17a^4-77a^2b^2+60b^4)\cot(c+dx)}{ad}}{a}-\frac{6(a^4-\cdots}{\cdots}}{a(a^2-b^2)}}{6a^2b}$$

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

$\downarrow$ 4257

$$\frac{(3a^2-5b^2)\cot(c+dx)\csc(c+dx)}{6a^2bd(a+b\sin(c+dx))^2}+$$

$$\frac{\frac{(3a^2-20b^2)\cot(c+dx)\csc(c+dx)}{ad(a+b\sin(c+dx))}+\frac{-\frac{6(a^4-6a^2b^2+5b^4)\cot(c+dx)\csc(c+dx)}{ad}-\frac{-\frac{b(17a^4-77a^2b^2+60b^4)\cot(c+dx)}{ad}-\frac{3\left(\frac{2b(2a^6-21a^4b^2+39a^2b^4-20b^6)\cdots}{ad\sqrt{a^2\cdots}}\cdots\right)}{\cdots}}{\cdots}}{a(a^2-b^2)}}{6a^2b}$$

$$\frac{\cot(c+dx)\csc^2(c+dx)}{3ad(a+b\sin(c+dx))^2}$$

input
```
Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x])^3,x]
```

output
```
((3*a^2 - 5*b^2)*Cot[c + d*x]*Csc[c + d*x])/(6*a^2*b*d*(a + b*Sin[c + d*x]
)^2) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*a*d*(a + b*Sin[c + d*x])^2) + ((-(
((-3*((2*b*(2*a^6 - 21*a^4*b^2 + 39*a^2*b^4 - 20*b^6)*ArcTan[(2*b + 2*a*Ta
n[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/(a*Sqrt[a^2 - b^2]*d) - (b^2*(9*a^4
- 29*a^2*b^2 + 20*b^4)*ArcTanh[Cos[c + d*x]])/(a*d)))/a - (b*(17*a^4 - 77*
a^2*b^2 + 60*b^4)*Cot[c + d*x])/(a*d))/a) - (6*(a^4 - 6*a^2*b^2 + 5*b^4)*C
ot[c + d*x]*Csc[c + d*x])/(a*d))/(a*(a^2 - b^2)) + ((3*a^2 - 20*b^2)*Cot[c
 + d*x]*Csc[c + d*x])/(a*d*(a + b*Sin[c + d*x])))/(6*a^2*b)
```

3.201. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.201.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3203
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Simp[(3*a^2 + b^2*(m - 2))*Cos[e + f*x]*((a + b*Sin[e
+ f*x])^(m + 1)/(3*a^2*b*f*(m + 1)*Sin[e + f*x]^2)), x] - Simp[1/(3*a^2*b*(
m + 1))   Int[((a + b*Sin[e + f*x])^(m + 1)/Sin[e + f*x]^3)*Simp[6*a^2 - b^
2*(m - 1)*(m - 2) + a*b*(m + 1)*Sin[e + f*x] - (3*a^2 - b^2*m*(m - 2))*Sin[
e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0] && L
tQ[m, -1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

3.201. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.201.4 Maple [A] (verified)

Time = 7.81 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2}{8a^5}+\frac{2\left(\left(\frac{5}{2}a^3b^2-5a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(4a^4-a^2b^2\cdots\right.}{\cdots}\right.}{\left(a\left(\tan\cdots\right.\right.}}{\cdots}$
default	$\frac{\frac{\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab-5a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+24\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b^2}{8a^5}+\frac{2\left(\left(\frac{5}{2}a^3b^2-5a\,b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b\left(4a^4-a^2b^2\cdots\right.}{\cdots}\right.}{\left(a\left(\tan\cdots\right.\right.}}{\cdots}$
risch	$\frac{102ia^4e^{4i(dx+c)}+240ib^4e^{2i(dx+c)}-298ib^2e^{4i(dx+c)}a^2+6a^3b\,e^{9i(dx+c)}-30a\,b^3e^{9i(dx+c)}-60ib^4+17ia^2b^2-60ib^4e^{8i(dx+c\cdots}}{\cdots}$

input
```
int(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8/a^5*(1/3*a^2*tan(1/2*d*x+1/2*c)^3-3*tan(1/2*d*x+1/2*c)^2*a*b-5*a^
2*tan(1/2*d*x+1/2*c)+24*tan(1/2*d*x+1/2*c)*b^2)+2/a^6*(((5/2*a^3*b^2-5*a*b
^4)*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-a^2*b^2-18*b^4)*tan(1/2*d*x+1/2*c)^2
+1/2*a*b^2*(11*a^2-26*b^2)*tan(1/2*d*x+1/2*c)+1/2*a^2*b*(4*a^2-9*b^2))/(a*
tan(1/2*d*x+1/2*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(2*a^4-19*a^2*b^2+20*
b^4)/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/
2)))-1/24/a^3/tan(1/2*d*x+1/2*c)^3-1/8*(-5*a^2+24*b^2)/a^5/tan(1/2*d*x+1/2
*c)+3/8/a^4*b/tan(1/2*d*x+1/2*c)^2+1/2/a^6*b*(9*a^2-20*b^2)*ln(tan(1/2*d*x
+1/2*c)))
```

3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(274) = 548$.

Time = 0.60 (sec) , antiderivative size = 2027, normalized size of antiderivative = 7.01

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/12*(2*(17*a^5*b^2 - 77*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^5 - 4*(4*a^7 +
3*a^5*b^2 - 67*a^3*b^4 + 60*a*b^6)*cos(d*x + c)^3 - 3*(4*a^5*b - 38*a^3*b^
3 + 40*a*b^5 + 2*(2*a^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^4 - 4*(2*a
^5*b - 19*a^3*b^3 + 20*a*b^5)*cos(d*x + c)^2 + (2*a^6 - 17*a^4*b^2 + a^2*b
^4 + 20*b^6 + (2*a^4*b^2 - 19*a^2*b^4 + 20*b^6)*cos(d*x + c)^4 - (2*a^6 -
15*a^4*b^2 - 18*a^2*b^4 + 40*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2
+ b^2)*log(((2*a^2 - b^2)*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2
+ 2*(a*cos(d*x + c)*sin(d*x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) + 6*(2*a^7 - 3*a^5*b^2 -
 19*a^3*b^4 + 20*a*b^6)*cos(d*x + c) - 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b
^6 + 2*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 -
 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5
 + 20*b^7 + (9*a^4*b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b -
11*a^4*b^3 - 38*a^2*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*co
s(d*x + c) + 1/2) + 3*(18*a^5*b^2 - 58*a^3*b^4 + 40*a*b^6 + 2*(9*a^5*b^2 -
 29*a^3*b^4 + 20*a*b^6)*cos(d*x + c)^4 - 4*(9*a^5*b^2 - 29*a^3*b^4 + 20*a*
b^6)*cos(d*x + c)^2 + (9*a^6*b - 20*a^4*b^3 - 9*a^2*b^5 + 20*b^7 + (9*a^4*
b^3 - 29*a^2*b^5 + 20*b^7)*cos(d*x + c)^4 - (9*a^6*b - 11*a^4*b^3 - 38*a^2
*b^5 + 40*b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)
- 2*(2*(14*a^6*b - 59*a^4*b^3 + 45*a^2*b^5)*cos(d*x + c)^3 - 3*(11*a^6*...
```

3.201.6 Sympy [F]

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$

input
```
integrate(cot(d*x+c)**4/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**4/(a + b*sin(c + d*x))**3, x)
```

3.201.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.201.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.56

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$$
$$= \frac{\frac{12\,(9\,a^2b-20\,b^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|\right)}{a^6} + \frac{24\,(2\,a^4-19\,a^2b^2+20\,b^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(a)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b}{\sqrt{a^2-b^2}}\right)\right)}{\sqrt{a^2-b^2}\,a^6} + \frac{24\left(5\,a^3b^2\tan\left(\frac{1}{2}\right.\right.}{}}{}$$

input
```
integrate(cot(d*x+c)^4/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
1/24*(12*(9*a^2*b - 20*b^3)*log(abs(tan(1/2*d*x + 1/2*c)))/a^6 + 24*(2*a^4
 - 19*a^2*b^2 + 20*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(a) + arctan(
(a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b^2)))/(sqrt(a^2 - b^2)*a^6) + 24*
(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*a^
4*b*tan(1/2*d*x + 1/2*c)^2 - a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 18*b^5*tan(1
/2*d*x + 1/2*c)^2 + 11*a^3*b^2*tan(1/2*d*x + 1/2*c) - 26*a*b^4*tan(1/2*d*x
 + 1/2*c) + 4*a^4*b - 9*a^2*b^3)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*tan(1/2*
d*x + 1/2*c) + a)^2*a^6) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 9*a^5*b*tan(1/2*d
*x + 1/2*c)^2 - 15*a^6*tan(1/2*d*x + 1/2*c) + 72*a^4*b^2*tan(1/2*d*x + 1/2
*c))/a^9 - (198*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 440*b^3*tan(1/2*d*x + 1/2*c
)^3 - 15*a^3*tan(1/2*d*x + 1/2*c)^2 + 72*a*b^2*tan(1/2*d*x + 1/2*c)^2 - 9*
a^2*b*tan(1/2*d*x + 1/2*c) + a^3)/(a^6*tan(1/2*d*x + 1/2*c)^3))/d
```

3.201.9 Mupad [B] (verification not implemented)

Time = 6.71 (sec) , antiderivative size = 1261, normalized size of antiderivative = 4.36

$$\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^4/(a + b*sin(c + d*x))^3,x)
```

output

```
tan(c/2 + (d*x)/2)^3/(24*a^3*d) - (tan(c/2 + (d*x)/2)*((3*(a^2 + 4*b^2))/(
8*a^5) + 1/(4*a^3) - (9*b^2)/(2*a^5)))/d + (tan(c/2 + (d*x)/2)^6*(5*a^4 -
80*b^4 + 16*a^2*b^2) + tan(c/2 + (d*x)/2)^4*((29*a^4)/3 - 304*b^4 + 72*a^2
*b^2) - a^4/3 + tan(c/2 + (d*x)/2)^2*((13*a^4)/3 - (40*a^2*b^2)/3) - tan(c
/2 + (d*x)/2)^3*(156*a*b^3 - (170*a^3*b)/3) - (tan(c/2 + (d*x)/2)^5*(144*b
^5 - 55*a^4*b + 104*a^2*b^3))/a + (5*a^3*b*tan(c/2 + (d*x)/2))/3)/(d*(8*a^
7*tan(c/2 + (d*x)/2)^3 + 8*a^7*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^5
*(16*a^7 + 32*a^5*b^2) + 32*a^6*b*tan(c/2 + (d*x)/2)^4 + 32*a^6*b*tan(c/2
+ (d*x)/2)^6)) + (log(tan(c/2 + (d*x)/2))*(9*a^2*b - 20*b^3))/(2*a^6*d) -
(3*b*tan(c/2 + (d*x)/2)^2)/(8*a^4*d) + (atan((((-(a + b)*(a - b))^(1/2)*(a
^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*b^4 - 28*a^8*b^2)/a^10 + (
tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a^6*b^3))/a^9 + ((-(a + b)*
(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6*a^12 - 8*a^10*b^2))/a^9)*
(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2))*1i)/(a^8 - a^6*b^2) + ((
-(a + b)*(a - b))^(1/2)*(a^4 + 10*b^4 - (19*a^2*b^2)/2)*((2*a^10 + 40*a^6*
b^4 - 28*a^8*b^2)/a^10 + (tan(c/2 + (d*x)/2)*(13*a^8*b + 80*a^4*b^5 - 76*a
^6*b^3))/a^9 - ((-(a + b)*(a - b))^(1/2)*(2*a^2*b - (tan(c/2 + (d*x)/2)*(6
*a^12 - 8*a^10*b^2))/a^9)*(a^4 + 10*b^4 - (19*a^2*b^2)/2))/(a^8 - a^6*b^2)
)*1i)/(a^8 - a^6*b^2))/((18*a^6*b - 400*b^7 + 560*a^2*b^5 - 211*a^4*b^3)/a
^10 + (2*tan(c/2 + (d*x)/2)*(4*a^6 - 200*b^6 + 230*a^2*b^4 - 58*a^4*b^2...
```

3.201. $\int \frac{\cot^4(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.202 $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.202.1 Optimal result

Integrand size = 21, antiderivative size = 492

$$\begin{aligned}\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = &-\frac{\sqrt{a^2-b^2}(2a^4-29a^2b^2+42b^4)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^8d}\\
&+\frac{b(45a^4-200a^2b^2+168b^4)\,\text{arctanh}(\cos(c+dx))}{8a^8d}\\
&-\frac{(91a^4-645a^2b^2+630b^4)\cot(c+dx)}{30a^7d}\\
&+\frac{(8a^4-79a^2b^2+84b^4)\cot(c+dx)\csc(c+dx)}{8a^6bd}\\
&-\frac{(15a^4-187a^2b^2+210b^4)\cot(c+dx)\csc^2(c+dx)}{30a^5b^2d}\\
&-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}\\
&+\frac{(5a^4-60a^2b^2+63b^4)\cot(c+dx)\csc^2(c+dx)}{60a^3b^2d(a+b\sin(c+dx))^2}\\
&+\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}\\
&+\frac{(4a^4-54a^2b^2+63b^4)\cot(c+dx)\csc^2(c+dx)}{12a^4b^2d(a+b\sin(c+dx))}\end{aligned}$$

output
```
1/8*b*(45*a^4-200*a^2*b^2+168*b^4)*arctanh(cos(d*x+c))/a^8/d-1/30*(91*a^4-
645*a^2*b^2+630*b^4)*cot(d*x+c)/a^7/d+1/8*(8*a^4-79*a^2*b^2+84*b^4)*cot(d*
x+c)*csc(d*x+c)/a^6/b/d-1/30*(15*a^4-187*a^2*b^2+210*b^4)*cot(d*x+c)*csc(d
*x+c)^2/a^5/b^2/d-1/3*cot(d*x+c)*csc(d*x+c)/b/d/(a+b*sin(d*x+c))^2+1/12*a*
cot(d*x+c)*csc(d*x+c)^2/b^2/d/(a+b*sin(d*x+c))^2+1/60*(5*a^4-60*a^2*b^2+63
*b^4)*cot(d*x+c)*csc(d*x+c)^2/a^3/b^2/d/(a+b*sin(d*x+c))^2+7/20*b*cot(d*x+
c)*csc(d*x+c)^3/a^2/d/(a+b*sin(d*x+c))^2-1/5*cot(d*x+c)*csc(d*x+c)^4/a/d/(
a+b*sin(d*x+c))^2+1/12*(4*a^4-54*a^2*b^2+63*b^4)*cot(d*x+c)*csc(d*x+c)^2/a
^4/b^2/d/(a+b*sin(d*x+c))-(2*a^4-29*a^2*b^2+42*b^4)*arctan((b+a*tan(1/2*d*
x+1/2*c))/(a^2-b^2)^(1/2))*(a^2-b^2)^(1/2)/a^8/d
```

3.202.2 Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 448, normalized size of antiderivative = 0.91

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \frac{-\frac{3840(2a^6-31a^4b^2+71a^2b^4-42b^6)\arctan\left(\frac{b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 480b(45a^4-200a^2b^2+168b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - \cdots}{\cdots}$$

input
```
Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

output
```
((-3840*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*ArcTan[(b + a*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 480*b*(45*a^4 - 200*a^2*b^2 +
 168*b^4)*Log[Cos[(c + d*x)/2]] - 480*b*(45*a^4 - 200*a^2*b^2 + 168*b^4)*L
og[Sin[(c + d*x)/2]] + (2*a*Cot[c + d*x]*Csc[c + d*x]^6*(-784*a^6 + 3256*a
^4*b^2 + 7860*a^2*b^4 - 12600*b^6 + 2*(384*a^6 - 2131*a^4*b^2 - 6315*a^2*b
^4 + 9450*b^6)*Cos[2*(c + d*x)] + (-368*a^6 + 824*a^4*b^2 + 6060*a^2*b^4 -
 7560*b^6)*Cos[4*(c + d*x)] + 182*a^4*b^2*Cos[6*(c + d*x)] - 1290*a^2*b^4*
Cos[6*(c + d*x)] + 1260*b^6*Cos[6*(c + d*x)] - 8156*a^5*b*Sin[c + d*x] + 4
2270*a^3*b^3*Sin[c + d*x] - 37800*a*b^5*Sin[c + d*x] + 3956*a^5*b*Sin[3*(c
 + d*x)] - 20715*a^3*b^3*Sin[3*(c + d*x)] + 18900*a*b^5*Sin[3*(c + d*x)] -
 608*a^5*b*Sin[5*(c + d*x)] + 3975*a^3*b^3*Sin[5*(c + d*x)] - 3780*a*b^5*S
in[5*(c + d*x)]))/(b + a*Csc[c + d*x])^2)/(3840*a^8*d)
```

3.202.3 Rubi [A] (verified)

Time = 4.20 (sec) , antiderivative size = 605, normalized size of antiderivative = 1.23, number of steps used = 26, number of rules used = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.190, Rules used = {3042, 3205, 27, 3042, 3534, 27, 3042, 3534, 27, 3042, 3534, 25, 3042, 3534, 25, 3042, 3534, 27, 3042, 3480, 3042, 3139, 1083, 217, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(c+dx)^6(a+b\sin(c+dx))^3}dx$$

$$\Big\downarrow \text{3205}$$

$$\frac{\int \frac{4\csc^4(c+dx)\left(-5\left(2a^4-20b^2a^2+21b^4\right)\sin^2(c+dx)-3ab\left(5a^2-3b^2\right)\sin(c+dx)+3\left(5a^4-44b^2a^2+42b^4\right)\right)}{(a+b\sin(c+dx))^3}dx}{240a^2b^2}+\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\csc^4(c+dx)\left(-5\left(2a^4-20b^2a^2+21b^4\right)\sin^2(c+dx)-3ab\left(5a^2-3b^2\right)\sin(c+dx)+3\left(5a^4-44b^2a^2+42b^4\right)\right)}{(a+b\sin(c+dx))^3}dx}{60a^2b^2}+\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{-5\left(2a^4-20b^2a^2+21b^4\right)\sin(c+dx)^2-3ab\left(5a^2-3b^2\right)\sin(c+dx)+3\left(5a^4-44b^2a^2+42b^4\right)}{\sin(c+dx)^4(a+b\sin(c+dx))^3}dx}{60a^2b^2}+\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\frac{\int \frac{2\csc^4(c+dx)\left(-4\left(5a^6-65b^2a^4+123b^4a^2-63b^6\right)\sin^2(c+dx)-ab\left(20a^4-41b^2a^2+21b^4\right)\sin(c+dx)+3\left(10a^6-114b^2a^4+209b^4a^2-105b^6\right)\right)}{(a+b\sin(c+dx))^2}dx}{2a(a^2-b^2)}+\frac{(5a^4-60a^2b^2+}{ad(}}{60a^2b^2}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 27

$$\frac{\frac{\int \frac{\csc^4(c+dx)\left(-4\left(5a^6-65b^2a^4+123b^4a^2-63b^6\right)\sin^2(c+dx)-ab\left(20a^4-41b^2a^2+21b^4\right)\sin(c+dx)+3\left(10a^6-114b^2a^4+209b^4a^2-105b^6\right)\right)}{(a+b\sin(c+dx))^2}dx}{a(a^2-b^2)}+\frac{(5a^4-60a^2b^2+6}{ad(a}}{60a^2b^2}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\int \frac{-4\left(5a^6-65b^2a^4+123b^4a^2-63b^6\right)\sin(c+dx)^2-ab\left(20a^4-41b^2a^2+21b^4\right)\sin(c+dx)+3\left(10a^6-114b^2a^4+209b^4a^2-105b^6\right)}{\sin(c+dx)^4(a+b\sin(c+dx))^2}dx}{a(a^2-b^2)}+\frac{(5a^4-60a^2b^2+63b^4)\cot(c+}{ad(a+b\sin(c+dx}}{60a^2b^2}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3534

$$\frac{\frac{\frac{\int \frac{3\csc^4(c+dx)\left(-5\left(4a^4-54b^2a^2+63b^4\right)\sin^2(c+dx)\left(a^2-b^2\right)^2+2\left(15a^4-187b^2a^2+210b^4\right)\left(a^2-b^2\right)^2-ab\left(10a^2-21b^2\right)\sin(c+dx)\left(a^2-b^2\right)^2\right)}{a+b\sin(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{5\left(a^2-b^2\right)\left(4a^4\right.}{}}{a(a^2-b^2)}}{60a^2b^2}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 27

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$\frac{\frac{\frac{3\int \frac{\csc^4(c+dx)\left(-5\left(4a^4-54b^2a^2+63b^4\right)\sin^2(c+dx)\left(a^2-b^2\right)^2+2\left(15a^4-187b^2a^2+210b^4\right)\left(a^2-b^2\right)^2-ab\left(10a^2-21b^2\right)\sin(c+dx)\left(a^2-b^2\right)^2\right)}{a+b\sin(c+dx)}dx}{a\left(a^2-b^2\right)}+\frac{5\left(a^2-b^2\right)\left(4a^4\cdots\right.}{\cdots}}{a(a^2-b^2)}}{60a^2b^2}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3042

$$\frac{\frac{\frac{3\int \frac{-5\left(4a^4-54b^2a^2+63b^4\right)\sin(c+dx)^2\left(a^2-b^2\right)^2+2\left(15a^4-187b^2a^2+210b^4\right)\left(a^2-b^2\right)^2-ab\left(10a^2-21b^2\right)\sin(c+dx)\left(a^2-b^2\right)^2}{\sin(c+dx)^4(a+b\sin(c+dx))}dx}{a\left(a^2-b^2\right)}+\frac{5\left(a^2-b^2\right)\left(4a^4-54a^2b^2+63b^4\cdots\right.}{ad(a+b\sin(c\cdots}}{a(a^2-b^2)}}{60a^2b^2}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3534

$$\frac{\frac{3\left(\frac{\int -\frac{\csc^3(c+dx)\left(-4b\left(15a^4-187b^2a^2+210b^4\right)\sin^2(c+dx)\left(a^2-b^2\right)^2+15b\left(8a^4-79b^2a^2+84b^4\right)\left(a^2-b^2\right)^2-ab^2\left(62a^2-105b^2\right)\sin(c+dx)\left(a^2-b^2\right)^2\right)}{a+b\sin(c+dx)}dx}{3a}-\frac{2\left(a^2-\cdots\right.}{\cdots}\right.}{a\left(a^2-b^2\right)}}{a(a^2-b^2)}\;\;60\cdots$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 25

$$\frac{\frac{3\left(-\frac{\int \frac{\csc^3(c+dx)\left(-4b\left(15a^4-187b^2a^2+210b^4\right)\sin^2(c+dx)\left(a^2-b^2\right)^2+15b\left(8a^4-79b^2a^2+84b^4\right)\left(a^2-b^2\right)^2-ab^2\left(62a^2-105b^2\right)\sin(c+dx)\left(a^2-b^2\right)^2\right)}{a+b\sin(c+dx)}dx}{3a}-\frac{2\left(a^2-\cdots\right.}{\cdots}\right.}{a\left(a^2-b^2\right)}}{a(a^2-b^2)}\;\;60\cdots$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3042

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$3\left(-\frac{\int \frac{-4b\left(15a^4-187b^2a^2+210b^4\right)\sin(c+dx)^2\left(a^2-b^2\right)^2+15b\left(8a^4-79b^2a^2+84b^4\right)\left(a^2-b^2\right)^2-ab^2\left(62a^2-105b^2\right)\sin(c+dx)\left(a^2-b^2\right)^2}{\sin(c+dx)^3(a+b\sin(c+dx))}dx}{3a}-\frac{2\left(a^2-b^2\right)^2\left(15a^4-18\right.}{}\right.$$

$$\frac{}{a\left(a^2-b^2\right)}$$

$$\frac{}{a(a^2-b^2)}$$

$$\frac{}{60a^2b^2}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3534

$$3\left(-\frac{\frac{\int -\frac{\csc^2(c+dx)\left(-a\left(311a^2-420b^2\right)\left(a^2-b^2\right)^2\sin(c+dx)b^3-15\left(a^2-b^2\right)^2\left(8a^4-79b^2a^2+84b^4\right)\sin^2(c+dx)b^2+4\left(a^2-b^2\right)^2\left(91a^4-645b^2a^2+630b^4\right)b^2\right)}{a+b\sin(c+dx)}dx}{2a}-\frac{1}{}}{3a}\right.$$

$$\frac{}{a\left(a^2-b^2\right)}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 25

$$3\left(-\frac{-\frac{\int \frac{\csc^2(c+dx)\left(-a\left(311a^2-420b^2\right)\left(a^2-b^2\right)^2\sin(c+dx)b^3-15\left(a^2-b^2\right)^2\left(8a^4-79b^2a^2+84b^4\right)\sin^2(c+dx)b^2+4\left(a^2-b^2\right)^2\left(91a^4-645b^2a^2+630b^4\right)b^2\right)}{a+b\sin(c+dx)}dx}{2a}-\frac{1}{}}{3a}\right.$$

$$\frac{}{a\left(a^2-b^2\right)}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3042

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$\frac{\frac{3\left(-\frac{-\frac{\int \frac{-a\left(311a^2-420b^2\right)\left(a^2-b^2\right)^2\sin(c+dx)b^3-15\left(a^2-b^2\right)^2\left(8a^4-79b^2a^2+84b^4\right)\sin(c+dx)^2b^2+4\left(a^2-b^2\right)^2\left(91a^4-645b^2a^2+630b^4\right)b^2}{\sin(c+dx)^2(a+b\sin(c+dx))}dx}{2a}-\frac{15b\left(8a^4-79a^2\cdots\right)}{\cdots}}{3a}\cdots\right)}{a\left(a^2-b^2\right)}\cdots}{a(a^2-\cdots)}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3534

$$\frac{3\left(-\frac{-\frac{\frac{\int -\frac{15\csc(c+dx)\left(\left(a^2-b^2\right)^2\left(45a^4-200b^2a^2+168b^4\right)b^3+a\left(a^2-b^2\right)^2\left(8a^4-79b^2a^2+84b^4\right)\sin(c+dx)b^2\right)}{a+b\sin(c+dx)}dx}{a}-\frac{4b^2\left(a^2-b^2\right)^2\left(91a^4-645a^2b^2+630b^4\right)\cot(c\cdots)}{ad}\cdots}{2a}}{3a}\cdots\right)}{a\left(a^2-b^2\right)}\cdots$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 27

$$\frac{3\left(-\frac{-\frac{-\frac{15\int \frac{\csc(c+dx)\left(\left(a^2-b^2\right)^2\left(45a^4-200b^2a^2+168b^4\right)b^3+a\left(a^2-b^2\right)^2\left(8a^4-79b^2a^2+84b^4\right)\sin(c+dx)b^2\right)}{a+b\sin(c+dx)}dx}{a}-\frac{4b^2\left(91a^4-645a^2b^2+630b^4\right)\left(a^2-b^2\right)^2\cot(c\cdots)}{ad}\cdots}{2a}}{3a}\cdots\right)}{a\left(a^2-b^2\right)}\cdots$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$\downarrow$ 3042

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$3\left(-\frac{-\frac{-\frac{15\int\frac{(a^2-b^2)^2(45a^4-200b^2a^2+168b^4)b^3+a(a^2-b^2)^2(8a^4-79b^2a^2+84b^4)\sin(c+dx)b^2}{\sin(c+dx)(a+b\sin(c+dx))}dx}{a}}{2a}-\frac{4b^2(91a^4-645a^2b^2+630b^4)(a^2-b^2)^2\cot(c+dx)}{ad}}{3a}-\frac{15b(8\cdots}{\cdots}\cdots}{a(a^2-b^2)}\cdots\right)\cdots$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\downarrow \text{3480}$$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{4b^2(a^2-b^2)^3(2a^4-29a^2b^2+42b^4)\int\frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b^3(a^2-b^2)^2(45a^4-200a^2b^2+168b^4)\int\csc(c+dx)dx}{a}\right)}{a}-\frac{4b^2(91a^4-645a^2b^2+630b^4)(a^2-\cdots}{ad}\cdots}{2a}\cdots}{3a}\cdots}{a(a^2-b^2)}\cdots\right)\cdots$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\downarrow \text{3042}$$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{4b^2(a^2-b^2)^3(2a^4-29a^2b^2+42b^4)\int\frac{1}{a+b\sin(c+dx)}dx}{a}+\frac{b^3(a^2-b^2)^2(45a^4-200a^2b^2+168b^4)\int\csc(c+dx)dx}{a}\right)}{a}-\frac{4b^2(91a^4-645a^2b^2+630b^4)(a^2-\cdots}{ad}\cdots}{2a}\cdots}{3a}\cdots}{a(a^2-b^2)}\cdots\right)\cdots$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\downarrow \text{3139}$$

3.202. $\int\frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{8b^2\left(a^2-b^2\right)^3\left(2a^4-29a^2b^2+42b^4\right)\int \frac{1}{a\tan^2\left(\frac{1}{2}(c+dx)\right)+2b\tan\left(\frac{1}{2}(c+dx)\right)+a}d\tan\left(\frac{1}{2}(c+dx)\right)}{ad}+\frac{b^3\left(a^2-b^2\right)^2\left(45a^4-200a^2b^2+168b^4\right)\int \csc(c+dx)dx}{a}\right.}{a}}{2a}}{3a}\right.$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{1083}$$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{b^3\left(a^2-b^2\right)^2\left(45a^4-200a^2b^2+168b^4\right)\int \csc(c+dx)dx}{a}-\frac{16b^2\left(a^2-b^2\right)^3\left(2a^4-29a^2b^2+42b^4\right)\int \frac{1}{-\left(2b+2a\tan\left(\frac{1}{2}(c+dx)\right)\right)^2-4\left(a^2-b^2\right)}d\left(2b+2a\tan\left(\frac{1}{2}\right.\right.}{ad}\right.}{a}}{2a}}{3a}\right.$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{217}$$

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{b^3(a^2-b^2)^2(45a^4-200a^2b^2+168b^4)\int\csc(c+dx)dx}{a}+\frac{8b^2(a^2-b^2)^{5/2}(2a^4-29a^2b^2+42b^4)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}\right)}{a}-\frac{4b^2(91a^4-645a^2b^2 \cdots}{\cdots}}{2a}}{3a}\cdots\right)$$

$$\cdots a(a^2-b^2)\cdots$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

$$\Big\downarrow \text{4257}$$

$$\frac{7b\cot(c+dx)\csc^3(c+dx)}{20a^2d(a+b\sin(c+dx))^2}+$$

$$3\left(-\frac{-\frac{-\frac{15\left(\frac{8b^2(a^2-b^2)^{5/2}(2a^4-29a^2b^2+42b^4)\arctan\left(\frac{2a\tan\left(\frac{1}{2}(c+dx)\right)+2b}{2\sqrt{a^2-b^2}}\right)}{ad}-\frac{b^3(a^2-b^2)^2(45a^4-200a^2b^2+168b^4)\text{arctanh}(\cos(c+dx))}{ad}\right)}{a}-\frac{4b^2(91a^4 \cdots}{\cdots}}{2a}}{3a}\cdots\right)$$

$$\cdots a(a^2\cdots$$

$$\frac{a\cot(c+dx)\csc^2(c+dx)}{12b^2d(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc^4(c+dx)}{5ad(a+b\sin(c+dx))^2}-\frac{\cot(c+dx)\csc(c+dx)}{3bd(a+b\sin(c+dx))^2}$$

input
```
Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x])^3,x]
```

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

output
```
-1/3*(Cot[c + d*x]*Csc[c + d*x])/(b*d*(a + b*Sin[c + d*x])^2) + (a*Cot[c +
 d*x]*Csc[c + d*x]^2)/(12*b^2*d*(a + b*Sin[c + d*x])^2) + (7*b*Cot[c + d*x
]*Csc[c + d*x]^3)/(20*a^2*d*(a + b*Sin[c + d*x])^2) - (Cot[c + d*x]*Csc[c
+ d*x]^4)/(5*a*d*(a + b*Sin[c + d*x])^2) + (((5*a^4 - 60*a^2*b^2 + 63*b^4)
*Cot[c + d*x]*Csc[c + d*x]^2)/(a*d*(a + b*Sin[c + d*x])^2) + ((3*((-2*(a^2
 - b^2)^2*(15*a^4 - 187*a^2*b^2 + 210*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(3
*a*d) - (-1/2*((-15*((8*b^2*(a^2 - b^2)^(5/2)*(2*a^4 - 29*a^2*b^2 + 42*b^4
)*ArcTan[(2*b + 2*a*Tan[(c + d*x)/2])/(2*Sqrt[a^2 - b^2])])/(a*d) - (b^3*(
a^2 - b^2)^2*(45*a^4 - 200*a^2*b^2 + 168*b^4)*ArcTanh[Cos[c + d*x]])/(a*d)
))/a - (4*b^2*(a^2 - b^2)^2*(91*a^4 - 645*a^2*b^2 + 630*b^4)*Cot[c + d*x])
/(a*d))/a - (15*b*(a^2 - b^2)^2*(8*a^4 - 79*a^2*b^2 + 84*b^4)*Cot[c + d*x]
*Csc[c + d*x])/(2*a*d))/(3*a)))/(a*(a^2 - b^2)) + (5*(a^2 - b^2)*(4*a^4 -
54*a^2*b^2 + 63*b^4)*Cot[c + d*x]*Csc[c + d*x]^2)/(a*d*(a + b*Sin[c + d*x]
)))/(a*(a^2 - b^2)))/(60*a^2*b^2)
```

3.202.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

rule 3139
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + 2*b*e*x + a
*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[a^2 - b^2, 0]
```

rule 3205
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^6,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(5*a*f*Sin[
e + f*x]^5)), x] + (Simp[Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*m*
Sin[e + f*x]^2)), x] + Simp[a*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b
^2*f*m*(m - 1)*Sin[e + f*x]^3)), x] - Simp[b*(m - 4)*Cos[e + f*x]*((a + b*S
in[e + f*x])^(m + 1)/(20*a^2*f*Sin[e + f*x]^4)), x] + Simp[1/(20*a^2*b^2*m*
(m - 1))   Int[((a + b*Sin[e + f*x])^m/Sin[e + f*x]^4)*Simp[60*a^4 - 44*a^2
*b^2*(m - 1)*m + b^4*m*(m - 1)*(m - 3)*(m - 4) + a*b*m*(20*a^2 - b^2*m*(m -
 1))*Sin[e + f*x] - (40*a^4 + b^4*m*(m - 1)*(m - 2)*(m - 4) - 20*a^2*b^2*(m
 - 1)*(2*m + 1))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f, m}, x] &
& NeQ[a^2 - b^2, 0] && NeQ[m, 1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

3.202.4 Maple [A] (verified)

Time = 14.79 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{3}+8a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40a\,b^3\left(\tan^2\left(\frac{dx}{2}\right.\right.}{32a^7}}{}$
default	$\frac{\frac{\frac{a^4\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{3b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{2}-\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^4}{3}+8a^2b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+24a^3b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-40a\,b^3\left(\tan^2\left(\frac{dx}{2}\right.\right.}{32a^7}}{}$
risch	Expression too large to display

input
```
int(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/32/a^7*(1/5*a^4*tan(1/2*d*x+1/2*c)^5-3/2*b*tan(1/2*d*x+1/2*c)^4*a^3
-7/3*tan(1/2*d*x+1/2*c)^3*a^4+8*a^2*b^2*tan(1/2*d*x+1/2*c)^3+24*a^3*b*tan(
1/2*d*x+1/2*c)^2-40*a*b^3*tan(1/2*d*x+1/2*c)^2+22*a^4*tan(1/2*d*x+1/2*c)-2
16*a^2*b^2*tan(1/2*d*x+1/2*c)+240*b^4*tan(1/2*d*x+1/2*c))-2/a^8*(((5/2*b^2
*a^5-19/2*b^4*a^3+7*b^6*a)*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^6-9*a^4*b^2-21*
a^2*b^4+26*b^6)*tan(1/2*d*x+1/2*c)^2+1/2*a*b^2*(11*a^4-49*a^2*b^2+38*b^4)*
tan(1/2*d*x+1/2*c)+1/2*a^2*b*(4*a^4-17*a^2*b^2+13*b^4))/(a*tan(1/2*d*x+1/2
*c)^2+2*b*tan(1/2*d*x+1/2*c)+a)^2+1/2*(2*a^6-31*a^4*b^2+71*a^2*b^4-42*b^6)
/(a^2-b^2)^(1/2)*arctan(1/2*(2*a*tan(1/2*d*x+1/2*c)+2*b)/(a^2-b^2)^(1/2)))
-1/160/a^3/tan(1/2*d*x+1/2*c)^5-1/96*(-7*a^2+24*b^2)/a^5/tan(1/2*d*x+1/2*c
)^3-1/32*(22*a^4-216*a^2*b^2+240*b^4)/a^7/tan(1/2*d*x+1/2*c)+3/64/a^4*b/ta
n(1/2*d*x+1/2*c)^4-1/4/a^6*b*(3*a^2-5*b^2)/tan(1/2*d*x+1/2*c)^2-1/8/a^8*b*
(45*a^4-200*a^2*b^2+168*b^4)*ln(tan(1/2*d*x+1/2*c)))
```

3.202.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1244 vs. $2(467) = 934$.

Time = 0.66 (sec) , antiderivative size = 2571, normalized size of antiderivative = 5.23

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="fricas")
```

output
```
[-1/240*(8*(91*a^5*b^2 - 645*a^3*b^4 + 630*a*b^6)*cos(d*x + c)^7 - 4*(92*a
^7 + 67*a^5*b^2 - 3450*a^3*b^4 + 3780*a*b^6)*cos(d*x + c)^5 + 40*(14*a^7 -
 37*a^5*b^2 - 303*a^3*b^4 + 378*a*b^6)*cos(d*x + c)^3 - 60*(2*(2*a^5*b - 2
9*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^6 - 4*a^5*b + 58*a^3*b^3 - 84*a*b^5 - 6
*(2*a^5*b - 29*a^3*b^3 + 42*a*b^5)*cos(d*x + c)^4 + 6*(2*a^5*b - 29*a^3*b^
3 + 42*a*b^5)*cos(d*x + c)^2 + ((2*a^4*b^2 - 29*a^2*b^4 + 42*b^6)*cos(d*x
+ c)^6 - 2*a^6 + 27*a^4*b^2 - 13*a^2*b^4 - 42*b^6 - (2*a^6 - 23*a^4*b^2 -
45*a^2*b^4 + 126*b^6)*cos(d*x + c)^4 + (4*a^6 - 52*a^4*b^2 - 3*a^2*b^4 + 1
26*b^6)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*log(((2*a^2 - b^2)*
cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2 + 2*(a*cos(d*x + c)*sin(d*
x + c) + b*cos(d*x + c))*sqrt(-a^2 + b^2))/(b^2*cos(d*x + c)^2 - 2*a*b*sin
(d*x + c) - a^2 - b^2)) - 60*(4*a^7 - 17*a^5*b^2 - 58*a^3*b^4 + 84*a*b^6)*
cos(d*x + c) + 15*(90*a^5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 -
200*a^3*b^4 + 168*a*b^6)*cos(d*x + c)^6 + 6*(45*a^5*b^2 - 200*a^3*b^4 + 16
8*a*b^6)*cos(d*x + c)^4 - 6*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b^6)*cos(d*x
 + c)^2 + (45*a^6*b - 155*a^4*b^3 - 32*a^2*b^5 + 168*b^7 - (45*a^4*b^3 - 2
00*a^2*b^5 + 168*b^7)*cos(d*x + c)^6 + (45*a^6*b - 65*a^4*b^3 - 432*a^2*b^
5 + 504*b^7)*cos(d*x + c)^4 - (90*a^6*b - 265*a^4*b^3 - 264*a^2*b^5 + 504*
b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - 15*(90*a^
5*b^2 - 400*a^3*b^4 + 336*a*b^6 - 2*(45*a^5*b^2 - 200*a^3*b^4 + 168*a*b...
```

3.202.6 Sympy [F]

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \int \frac{\cot^6{(c+dx)}}{(a+b\sin{(c+dx)})^3}\,dx$$

input
```
integrate(cot(d*x+c)**6/(a+b*sin(d*x+c))**3,x)
```

output
```
Integral(cot(c + d*x)**6/(a + b*sin(c + d*x))**3, x)
```

3.202.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

3.202.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 731, normalized size of antiderivative = 1.49

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cot(d*x+c)^6/(a+b*sin(d*x+c))^3,x, algorithm="giac")
```

output
```
-1/960*(120*(45*a^4*b - 200*a^2*b^3 + 168*b^5)*log(abs(tan(1/2*d*x + 1/2*c
)))/a^8 + 960*(2*a^6 - 31*a^4*b^2 + 71*a^2*b^4 - 42*b^6)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(a) + arctan((a*tan(1/2*d*x + 1/2*c) + b)/sqrt(a^2 - b
^2)))/(sqrt(a^2 - b^2)*a^8) + 960*(5*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 19*a
^3*b^4*tan(1/2*d*x + 1/2*c)^3 + 14*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 4*a^6*b*
tan(1/2*d*x + 1/2*c)^2 - 9*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 - 21*a^2*b^5*tan
(1/2*d*x + 1/2*c)^2 + 26*b^7*tan(1/2*d*x + 1/2*c)^2 + 11*a^5*b^2*tan(1/2*d
*x + 1/2*c) - 49*a^3*b^4*tan(1/2*d*x + 1/2*c) + 38*a*b^6*tan(1/2*d*x + 1/2
*c) + 4*a^6*b - 17*a^4*b^3 + 13*a^2*b^5)/((a*tan(1/2*d*x + 1/2*c)^2 + 2*b*
tan(1/2*d*x + 1/2*c) + a)^2*a^8) - (12330*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 5
4800*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 46032*b^5*tan(1/2*d*x + 1/2*c)^5 - 6
60*a^5*tan(1/2*d*x + 1/2*c)^4 + 6480*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 - 7200
*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 720*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1200*a^
2*b^3*tan(1/2*d*x + 1/2*c)^3 + 70*a^5*tan(1/2*d*x + 1/2*c)^2 - 240*a^3*b^2
*tan(1/2*d*x + 1/2*c)^2 + 45*a^4*b*tan(1/2*d*x + 1/2*c) - 6*a^5)/(a^8*tan(
1/2*d*x + 1/2*c)^5) - (6*a^12*tan(1/2*d*x + 1/2*c)^5 - 45*a^11*b*tan(1/2*d
*x + 1/2*c)^4 - 70*a^12*tan(1/2*d*x + 1/2*c)^3 + 240*a^10*b^2*tan(1/2*d*x
+ 1/2*c)^3 + 720*a^11*b*tan(1/2*d*x + 1/2*c)^2 - 1200*a^9*b^3*tan(1/2*d*x
+ 1/2*c)^2 + 660*a^12*tan(1/2*d*x + 1/2*c) - 6480*a^10*b^2*tan(1/2*d*x + 1
/2*c) + 7200*a^8*b^4*tan(1/2*d*x + 1/2*c))/a^15)/d
```

3.202.9 Mupad [B] (verification not implemented)

Time = 6.94 (sec) , antiderivative size = 1614, normalized size of antiderivative = 3.28

$$\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cot(c + d*x)^6/(a + b*sin(c + d*x))^3,x)
```

output
```
tan(c/2 + (d*x)/2)^5/(160*a^3*d) - (tan(c/2 + (d*x)/2)^3*((a^2 + 4*b^2)/(3
2*a^5) + 1/(24*a^3) - (3*b^2)/(8*a^5)))/d + (tan(c/2 + (d*x)/2)*(1/(8*a^3)
 - (3*(a^2 + 4*b^2))/(32*a^5) - (6*b*((6*b*((3*(a^2 + 4*b^2))/(32*a^5) + 1
/(8*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(1024*a^6) + (9*b*(
a^2 + 4*b^2))/(16*a^6)))/a + (3*(a^2 + 4*b^2)*((3*(a^2 + 4*b^2))/(32*a^5)
+ 1/(8*a^3) - (9*b^2)/(8*a^5)))/a^2 + (3*b*(384*a^2*b + 256*b^3))/(512*a^7
)))/d - (tan(c/2 + (d*x)/2)^3*((187*a^5*b)/15 - 14*a^3*b^3) + a^6/5 + tan(
c/2 + (d*x)/2)^4*((263*a^6)/15 + 112*a^2*b^4 - (358*a^4*b^2)/3) + tan(c/2
+ (d*x)/2)^5*(1216*a*b^5 + (1519*a^5*b)/6 - 1360*a^3*b^3) - tan(c/2 + (d*x
)/2)^2*((29*a^6)/15 - (14*a^4*b^2)/5) + tan(c/2 + (d*x)/2)^8*(22*a^6 + 448
*b^6 - 368*a^2*b^4 - 56*a^4*b^2) + tan(c/2 + (d*x)/2)^6*((125*a^6)/3 + 217
6*b^6 - 2112*a^2*b^4 + 112*a^4*b^2) + (8*tan(c/2 + (d*x)/2)^7*(30*a^6*b +
104*b^7 + 36*a^2*b^5 - 149*a^4*b^3))/a - (7*a^5*b*tan(c/2 + (d*x)/2))/10)/
(d*(32*a^9*tan(c/2 + (d*x)/2)^5 + 32*a^9*tan(c/2 + (d*x)/2)^9 + tan(c/2 +
(d*x)/2)^7*(64*a^9 + 128*a^7*b^2) + 128*a^8*b*tan(c/2 + (d*x)/2)^6 + 128*a
^8*b*tan(c/2 + (d*x)/2)^8)) + (tan(c/2 + (d*x)/2)^2*((3*b*((3*(a^2 + 4*b^2
))/(32*a^5) + 1/(8*a^3) - (9*b^2)/(8*a^5)))/a - (384*a^2*b + 256*b^3)/(204
8*a^6) + (9*b*(a^2 + 4*b^2))/(32*a^6)))/d - (log(tan(c/2 + (d*x)/2))*(45*a
^4*b + 168*b^5 - 200*a^2*b^3))/(8*a^8*d) - (3*b*tan(c/2 + (d*x)/2)^4)/(64*
a^4*d) - (atan((((-(a + b)*(a - b))^(1/2)*(a^4 + 21*b^4 - (29*a^2*b^2)/...
```

3.202. $\int \frac{\cot^6(c+dx)}{(a+b\sin(c+dx))^3}\,dx$

3.203 $\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx$

3.203.1 Optimal result

Integrand size = 23, antiderivative size = 271

$$\begin{aligned}\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx &= \frac{a^3\,\text{Hypergeometric2F1}\left(1,\frac{1+p}{2},\frac{3+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} \\ &+ \frac{3a^2b\cos^2(e+fx)^{\frac{1+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+p}{2},\frac{2+p}{2},\frac{4+p}{2},\sin^2(e+fx)\right)\sin(e+fx)(g\tan(e+fx))^{1+p}}{fg(2+p)} \\ &+ \frac{b^3\cos^2(e+fx)^{\frac{1+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+p}{2},\frac{4+p}{2},\frac{6+p}{2},\sin^2(e+fx)\right)\sin^3(e+fx)(g\tan(e+fx))^{1+p}}{fg(4+p)} \\ &+ \frac{3ab^2\,\text{Hypergeometric2F1}\left(2,\frac{3+p}{2},\frac{5+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{3+p}}{fg^3(3+p)}\end{aligned}$$

output
```
a^3*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(p+
1)/f/g/(p+1)+3*a^2*b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/
2*p],[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(p+1)/f/g/(2+p)+b^3
*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([2+1/2*p, 1/2+1/2*p],[3+1/2*p],sin(f
*x+e)^2)*sin(f*x+e)^3*(g*tan(f*x+e))^(p+1)/f/g/(4+p)+3*a*b^2*hypergeom([2,
 3/2+1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/g^3/(3+p)
```

3.203.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 15.59 (sec) , antiderivative size = 4791, normalized size of antiderivative = 17.68

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

output
```
(2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^3*(2 + p)*Appel
lF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*b*(6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2] - 6*a*b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + p)*(3*a^2*AppellF1[1 +
p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 4*b^2*(Appe
llF1[1 + p/2, p, 3, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] - Ap
pellF1[1 + p/2, p, 4, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]))*
Tan[(e + f*x)/2]))*(g*Tan[e + f*x])^p*(-1/8*(b^3*Sin[3*(e + f*x)]*Tan[e +
f*x]^p) - a^3*Sin[e + f*x]^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p + ((3*I)/8)*b
^3*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*b^3*Sin[2*(e + f*
x)]^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/8 - (I/8)*b^3*Sin[2*(e + f*x)]^3*Si
n[3*(e + f*x)]*Tan[e + f*x]^p + Cos[e + f*x]^3*(a^3*Cos[3*(e + f*x)]*Tan[e
 + f*x]^p - I*a^3*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^3*((
I/8)*b^3*Cos[3*(e + f*x)]*Tan[e + f*x]^p + (b^3*Sin[3*(e + f*x)]*Tan[e + f
*x]^p)/8) + Sin[e + f*x]^2*((-3*a^2*b*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/2 +
 ((3*I)/2)*a^2*b*Sin[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p) + Sin[e
 + f*x]*((-3*a*b^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4 + ((3*I)/2)*a*b^2*Si
n[2*(e + f*x)]*Sin[3*(e + f*x)]*Tan[e + f*x]^p + (3*a*b^2*Sin[2*(e + f*x)]
^2*Sin[3*(e + f*x)]*Tan[e + f*x]^p)/4) + Cos[2*(e + f*x)]^2*((-3*b^3*Si...
```

3.203.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p dx$$

$$\Big\downarrow \; 3201$$

$$\int \left(a^3(g\tan(e+fx))^p+3a^2b\sin(e+fx)(g\tan(e+fx))^p+3ab^2\sin^2(e+fx)(g\tan(e+fx))^p+b^3\sin^3(e+fx)(g\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{a^3(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,\frac{p+1}{2},\frac{p+3}{2},-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{3a^2b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{p+1}{2},\frac{p+2}{2},\frac{p+4}{2},\sin^2(e+fx)\right)}{fg(p+2)}+\frac{3ab^2(g\tan(e+fx))^{p+3}\,\text{Hypergeometric2F1}\left(2,\frac{p+3}{2},\frac{p+5}{2},-\tan^2(e+fx)\right)}{fg^3(p+3)}+\frac{b^3\sin^3(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{p+1}{2},\frac{p+4}{2},\frac{p+6}{2},\sin^2(e+fx)\right)}{fg(p+4)}$$

input
```
Int[(a + b*Sin[e + f*x])^3*(g*Tan[e + f*x])^p,x]
```

output
```
(a^3*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e
+ f*x])^(1 + p))/(f*g*(1 + p)) + (3*a^2*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyp
ergeometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*
x]*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^3*(Cos[e + f*x]^2)^((1 + p
)/2)*Hypergeometric2F1[(1 + p)/2, (4 + p)/2, (6 + p)/2, Sin[e + f*x]^2]*Si
n[e + f*x]^3*(g*Tan[e + f*x])^(1 + p))/(f*g*(4 + p)) + (3*a*b^2*Hypergeome
tric2F1[2, (3 + p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p)
)/(f*g^3*(3 + p))
```

3.203. $\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx$

3.203.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.203.4 Maple [F]

$$\int (a+b\sin(fx+e))^3 (g\tan(fx+e))^p\,dx$$

input
```
int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

output
```
int((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x)
```

3.203.5 Fricas [F]

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \int (b\sin(fx+e)+a)^3(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

output
```
integral(-(3*a*b^2*cos(f*x + e)^2 - a^3 - 3*a*b^2 + (b^3*cos(f*x + e)^2 -
3*a^2*b - b^3)*sin(f*x + e))*(g*tan(f*x + e))^p, x)
```

3.203.6 Sympy [F]

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx=\int (g\tan(e+fx))^p\,(a+b\sin(e+fx))^3\,dx$$

input
```
integrate((a+b*sin(f*x+e))**3*(g*tan(f*x+e))**p,x)
```

output
```
Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**3, x)
```

3.203.7 Maxima [F]

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx=\int (b\sin(fx+e)+a)^3(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

3.203.8 Giac [F]

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx=\int (b\sin(fx+e)+a)^3(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))^3*(g*tan(f*x+e))^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e) + a)^3*(g*tan(f*x + e))^p, x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(e+fx))^3(g\tan(e+fx))^p\,dx = \int (g\tan(e+f\,x))^p\,(a+b\,\sin(e+f\,x))^3\,dx$$

input
```
int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^3,x)
```

output
```
int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^3, x)
```

3.204 $\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx$

3.204.1 Optimal result

Integrand size = 23, antiderivative size = 186

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \frac{a^2\,\text{Hypergeometric2F1}\left(1,\frac{1+p}{2},\frac{3+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{2ab\cos^2(e+fx)^{\frac{1+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+p}{2},\frac{2+p}{2},\frac{4+p}{2},\sin^2(e+fx)\right)\sin(e+fx)(g\tan(e+fx))^{1+p}}{fg(2+p)} + \frac{b^2\,\text{Hypergeometric2F1}\left(2,\frac{3+p}{2},\frac{5+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{3+p}}{fg^3(3+p)}$$

output
```
a^2*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(p+
1)/f/g/(p+1)+2*a*b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*
p],[2+1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(p+1)/f/g/(2+p)+b^2*h
ypergeom([2, 3/2+1/2*p],[5/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(3+p)/f/
g^3/(3+p)
```

3.204.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 14.64 (sec) , antiderivative size = 2464, normalized size of antiderivative = 13.25

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \text{Result too large to show}$$

input
```
Integrate[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

output
```
(2*(Cos[e + f*x]*Sec[(e + f*x)/2]^2)^p*Tan[(e + f*x)/2]*(a^2*(2 + p)*Appel
lF1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 4*b*(b*(2 + p)*AppellF1[(1 + p)/2, p, 2, (3 + p)/2, Tan[(e + f*x)/2]^2, -
Tan[(e + f*x)/2]^2] - b*(2 + p)*AppellF1[(1 + p)/2, p, 3, (3 + p)/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + a*(1 + p)*AppellF1[1 + p/2, p, 2, 2
+ p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2]))*(g*Tan[
e + f*x])^p*(-1/4*(b^2*Cos[2*(e + f*x)]^3*Tan[e + f*x]^p) + (I/4)*b^2*Sin[
2*(e + f*x)]*Tan[e + f*x]^p + I*a^2*Sin[e + f*x]^2*Sin[2*(e + f*x)]*Tan[e
+ f*x]^p + (b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/2 - (I/4)*b^2*Sin[2*(e
+ f*x)]^3*Tan[e + f*x]^p + Cos[e + f*x]^2*(a^2*Cos[2*(e + f*x)]*Tan[e + f*
x]^p - I*a^2*Sin[2*(e + f*x)]*Tan[e + f*x]^p) + Cos[2*(e + f*x)]^2*((b^2*T
an[e + f*x]^p)/2 + a*b*Sin[e + f*x]*Tan[e + f*x]^p - (I/4)*b^2*Sin[2*(e +
f*x)]*Tan[e + f*x]^p) + Sin[e + f*x]*(I*a*b*Sin[2*(e + f*x)]*Tan[e + f*x]^
p + a*b*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p) + Cos[2*(e + f*x)]*(-1/4*(b^2*T
an[e + f*x]^p) - a*b*Sin[e + f*x]*Tan[e + f*x]^p - a^2*Sin[e + f*x]^2*Tan[
e + f*x]^p - (b^2*Sin[2*(e + f*x)]^2*Tan[e + f*x]^p)/4) + Cos[e + f*x]*((-
I)*a*b*Cos[2*(e + f*x)]^2*Tan[e + f*x]^p + a*b*Sin[2*(e + f*x)]*Tan[e + f*
x]^p + 2*a^2*Sin[e + f*x]*Sin[2*(e + f*x)]*Tan[e + f*x]^p - I*a*b*Sin[2*(e
 + f*x)]^2*Tan[e + f*x]^p + Cos[2*(e + f*x)]*(I*a*b*Tan[e + f*x]^p + (2*I)
*a^2*Sin[e + f*x]*Tan[e + f*x]^p))))/(f*(1 + p)*(2 + p)*((2*p*(Cos[e + ...
```

3.204.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^pdx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a^2(g\tan(e+fx))^p+2ab\sin(e+fx)(g\tan(e+fx))^p+b^2\sin^2(e+fx)(g\tan(e+fx))^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(g\tan(e+fx))^{p+1}\text{Hypergeometric2F1}\left(1,\frac{p+1}{2},\frac{p+3}{2},-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{2ab\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\text{Hypergeometric2F1}\left(\frac{p+1}{2},\frac{p+2}{2},\frac{p+4}{2},\sin^2(e+fx)\right)}{fg(p+2)}+\frac{b^2(g\tan(e+fx))^{p+3}\text{Hypergeometric2F1}\left(2,\frac{p+3}{2},\frac{p+5}{2},-\tan^2(e+fx)\right)}{fg^3(p+3)}$$

input
```
Int[(a + b*Sin[e + f*x])^2*(g*Tan[e + f*x])^p,x]
```

output
```
(a^2*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e
+ f*x])^(1 + p))/(f*g*(1 + p)) + (2*a*b*(Cos[e + f*x]^2)^((1 + p)/2)*Hyper
geometric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]
*(g*Tan[e + f*x])^(1 + p))/(f*g*(2 + p)) + (b^2*Hypergeometric2F1[2, (3 +
p)/2, (5 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e + f*x])^(3 + p))/(f*g^3*(3 + p)
)
```

3.204.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.204.4 Maple [F]

$$\int (a+b\sin(fx+e))^2 (g\tan(fx+e))^p\, dx$$

input
```
int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

output
```
int((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x)
```

3.204.5 Fricas [F]

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\, dx = \int (b\sin(fx+e)+a)^2(g\tan(fx+e))^p\, dx$$

input
```
integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

output
```
integral(-(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^2 - b^2)*(g*tan(f*x
 + e))^p, x)
```

3.204.6 Sympy [F]

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\, dx = \int (g\tan(e+fx))^p (a+b\sin(e+fx))^2\, dx$$

input
```
integrate((a+b*sin(f*x+e))**2*(g*tan(f*x+e))**p,x)
```

output
```
Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**2, x)
```

3.204.7 Maxima [F]

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int (b\sin{(fx+e)}+a)^2(g\tan{(fx+e)})^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

3.204.8 Giac [F]

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int (b\sin{(fx+e)}+a)^2(g\tan{(fx+e)})^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))^2*(g*tan(f*x+e))^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e) + a)^2*(g*tan(f*x + e))^p, x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(e+fx))^2(g\tan(e+fx))^p\,dx = \int (g\tan(e+f\,x))^p\,(a+b\,\sin{(e+f\,x)})^2\,dx$$

input
```
int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^2,x)
```

output
```
int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^2, x)
```

3.205 $\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx$

3.205.1 Optimal result

Integrand size = 21, antiderivative size = 129

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \frac{a\,\text{Hypergeometric2F1}\left(1,\frac{1+p}{2},\frac{3+p}{2},-\tan^2(e+fx)\right)(g\tan(e+fx))^{1+p}}{fg(1+p)} + \frac{b\cos^2(e+fx)^{\frac{1+p}{2}}\,\text{Hypergeometric2F1}\left(\frac{1+p}{2},\frac{2+p}{2},\frac{4+p}{2},\sin^2(e+fx)\right)\sin(e+fx)(g\tan(e+fx))^{1+p}}{fg(2+p)}$$

output
```
a*hypergeom([1, 1/2+1/2*p],[3/2+1/2*p],-tan(f*x+e)^2)*(g*tan(f*x+e))^(p+1)
/f/g/(p+1)+b*(cos(f*x+e)^2)^(1/2+1/2*p)*hypergeom([1+1/2*p, 1/2+1/2*p],[2+
1/2*p],sin(f*x+e)^2)*sin(f*x+e)*(g*tan(f*x+e))^(p+1)/f/g/(2+p)
```

3.205.2 Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.

Time = 5.56 (sec) , antiderivative size = 849, normalized size of antiderivative = 6.58

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \frac{}{f\left(\sec^2\left(\frac{1}{2}(e+fx)\right)\left(a(2+p)\,\text{AppellF1}\left(\frac{1+p}{2},p,1,\frac{3+p}{2},\tan^2\left(\frac{1}{2}(e+fx)\right),-\tan^2\left(\frac{1}{2}(e+fx)\right)\right)+2b(1+p\right.\right.}$$

input
```
Integrate[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

output
```
(2*(a + b*Sin[e + f*x])*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 + p)/2, p,
 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*Appe
llF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[
(e + f*x)/2])*(g*Tan[e + f*x])^p)/(f*(Sec[(e + f*x)/2]^2*(a*(2 + p)*Appell
F1[(1 + p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
2*b*(1 + p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2]*Tan[(e + f*x)/2]) - 16*p*Cos[(e + f*x)/2]*Csc[e + f*x]^3*Sec[e
 + f*x]*Sin[(e + f*x)/2]^5*(a*(2 + p)*AppellF1[(1 + p)/2, p, 1, (3 + p)/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 + p)*AppellF1[1 + p/2,
p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Tan[(e + f*x)/2])
+ 2*p*Csc[e + f*x]*Sec[e + f*x]*Tan[(e + f*x)/2]*(a*(2 + p)*AppellF1[(1 +
p)/2, p, 1, (3 + p)/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*b*(1 +
 p)*AppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]
^2]*Tan[(e + f*x)/2]) + 2*(1 + p)*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2]*(b*A
ppellF1[1 + p/2, p, 2, 2 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 (a*(2 + p)*(-AppellF1[(3 + p)/2, p, 2, (5 + p)/2, Tan[(e + f*x)/2]^2, -Ta
n[(e + f*x)/2]^2] + p*AppellF1[(3 + p)/2, 1 + p, 1, (5 + p)/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2])/(3 + p) + (2*b*(2 + p)*(-
2*AppellF1[2 + p/2, p, 3, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2
] + p*AppellF1[2 + p/2, 1 + p, 2, 3 + p/2, Tan[(e + f*x)/2]^2, -Tan[(e ...
```

3.205.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3201, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^pdx$$

$$\Big\downarrow \text{3201}$$

$$\int \left(a(g\tan(e+fx))^p+b\sin(e+fx)(g\tan(e+fx))^p\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(1,\frac{p+1}{2},\frac{p+3}{2},-\tan^2(e+fx)\right)}{fg(p+1)}+\frac{b\sin(e+fx)\cos^2(e+fx)^{\frac{p+1}{2}}(g\tan(e+fx))^{p+1}\,\text{Hypergeometric2F1}\left(\frac{p+1}{2},\frac{p+2}{2},\frac{p+4}{2},\sin^2(e+fx)\right)}{fg(p+2)}$$

input
```
Int[(a + b*Sin[e + f*x])*(g*Tan[e + f*x])^p,x]
```

output
```
(a*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, -Tan[e + f*x]^2]*(g*Tan[e +
f*x])^(1 + p))/(f*g*(1 + p)) + (b*(Cos[e + f*x]^2)^((1 + p)/2)*Hypergeomet
ric2F1[(1 + p)/2, (2 + p)/2, (4 + p)/2, Sin[e + f*x]^2]*Sin[e + f*x]*(g*Ta
n[e + f*x])^(1 + p))/(f*g*(2 + p))
```

3.205.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3201
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

3.205.4 Maple [F]

$$\int (a+b\sin(fx+e))\,(g\tan(fx+e))^p\,dx$$

input
```
int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

output
```
int((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x)
```

3.205.5 Fricas [F]

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \int (b\sin(fx+e)+a)(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

3.205.6 Sympy [F]

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \int (g\tan(e+fx))^p\,(a+b\sin(e+fx))\,dx$$

input
```
integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))**p,x)
```

output
```
Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x)), x)
```

3.205.7 Maxima [F]

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \int (b\sin(fx+e)+a)(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

3.205.8 Giac [F]

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \int (b\sin(fx+e)+a)(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))*(g*tan(f*x+e))^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e) + a)*(g*tan(f*x + e))^p, x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\sin(e+fx))(g\tan(e+fx))^p\,dx = \int (g\tan(e+fx))^p\,(a+b\sin(e+fx))\,dx$$

input
```
int((g*tan(e + f*x))^p*(a + b*sin(e + f*x)),x)
```

output
```
int((g*tan(e + f*x))^p*(a + b*sin(e + f*x)), x)
```

3.206 $\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$

3.206.1 Optimal result

Integrand size = 23, antiderivative size = 284

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx = \frac{ag\left(1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)^{\frac{1}{2}(-1+p)}\text{Hypergeometric2F1}\left(\frac{1-p}{2},\frac{1-p}{2},\frac{3-p}{2},\frac{\cos^2(e+fx)-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}}{1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}}\right)\sin^2(e+fx)^{\frac{1-p}{2}}(g}{(a^2-b^2)\,f(-1+p)} + \frac{b\,\text{AppellF1}\left(\frac{1-p}{2},-\frac{p}{2},1,\frac{3-p}{2},\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-p/2}(g\tan(e+fx))^p}{(-a^2+b^2)\,f(-1+p)}$$

output
```
a*g*(1-b^2*cos(f*x+e)^2/(-a^2+b^2))^(-1/2+1/2*p)*Hypergeometric2F1(1/2-1/2
*p,1/2-1/2*p,3/2-1/2*p,(cos(f*x+e)^2-b^2*cos(f*x+e)^2/(-a^2+b^2))/(1-b^2*c
os(f*x+e)^2/(-a^2+b^2)))*(sin(f*x+e)^2)^(1/2-1/2*p)*(g*tan(f*x+e))^(-1+p)/
(a^2-b^2)/f/(-1+p)+b*AppellF1(1/2-1/2*p,-1/2*p,1,3/2-1/2*p,cos(f*x+e)^2,b^
2*cos(f*x+e)^2/(-a^2+b^2))*cos(f*x+e)*(g*tan(f*x+e))^p/(-a^2+b^2)/f/(-1+p)
/((sin(f*x+e)^2)^(1/2*p))
```

3.206.2 Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 858 vs. 2(284) = 568.

Time = 14.05 (sec) , antiderivative size = 858, normalized size of antiderivative = 3.02

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$$

$$= \frac{}{a^2bf(1+p)(2+p)(a+b\sin(e+fx))\left(\frac{\sec^2(e+fx)\tan^p(e+fx)\left((a^2-b^2)(1+p)\operatorname{AppellF1}\left(\frac{2+p}{2},-\frac{1}{2},1,\frac{4+p}{2},-\tan^2(e+fx),\left(-1+\right.\right.\right.}{}\right.}$$

input
```
Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]
```

output
```
(Tan[e + f*x]^(1 + p)*(g*Tan[e + f*x])^p*((a^2 - b^2)*(1 + p)*AppellF1[(2
+ p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2
]*Tan[e + f*x] + a*(b*(2 + p)*Hypergeometric2F1[1, (1 + p)/2, (3 + p)/2, (
(-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + p)*Hypergeometric2F1[1/2, 1 + p
/2, 2 + p/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*f*(1 + p)*(2 + p)*(a
+ b*Sin[e + f*x])*((Sec[e + f*x]^2*Tan[e + f*x]^p*((a^2 - b^2)*(1 + p)*App
ellF1[(2 + p)/2, -1/2, 1, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e
 + f*x]^2]*Tan[e + f*x] + a*(b*(2 + p)*Hypergeometric2F1[1, (1 + p)/2, (3
+ p)/2, ((-a^2 + b^2)*Tan[e + f*x]^2)/a^2] - a*(1 + p)*Hypergeometric2F1[1
/2, 1 + p/2, 2 + p/2, -Tan[e + f*x]^2]*Tan[e + f*x])))/(a^2*b*(2 + p)) + (
Tan[e + f*x]^(1 + p)*((a^2 - b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 1, (4
+ p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Sec[e + f*x]^2 + (
a^2 - b^2)*(1 + p)*Tan[e + f*x]*((2*(-1 + b^2/a^2)*(2 + p)*AppellF1[1 + (2
 + p)/2, -1/2, 2, 1 + (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f
*x]^2]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + p) + ((2 + p)*AppellF1[1 + (2 + p
)/2, 1/2, 1, 1 + (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2
]*Sec[e + f*x]^2*Tan[e + f*x])/(4 + p)) + a*(-(a*(1 + p)*Hypergeometric2F1
[1/2, 1 + p/2, 2 + p/2, -Tan[e + f*x]^2]*Sec[e + f*x]^2) - 2*a*(1 + p/2)*(
1 + p)*Sec[e + f*x]^2*(-Hypergeometric2F1[1/2, 1 + p/2, 2 + p/2, -Tan[e +
f*x]^2] + 1/Sqrt[1 + Tan[e + f*x]^2]) + b*(1 + p)*(2 + p)*Csc[e + f*x]*...
```

3.206.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}dx$$
$$\Big\downarrow \ 3211$$
$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}dx$$

input
```
Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x]),x]
```

output
```
$Aborted
```

3.206.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3211
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*
x])^p, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.206.4 Maple [F]

$$\int \frac{(g\tan(fx+e))^p}{a+b\sin(fx+e)}dx$$

input
```
int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)
```

output
```
int((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x)
```

3.206.5 Fricas [F]

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx = \int \frac{(g\tan(fx+e))^p}{b\sin(fx+e)+a}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="fricas")
```

output
```
integral((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

3.206.6 Sympy [F]

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx = \int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx$$

input
```
integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e)),x)
```

output
```
Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x)), x)
```

3.206.7 Maxima [F]

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx = \int \frac{(g\tan(fx+e))^p}{b\sin(fx+e)+a}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

3.206.8 Giac [F]

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx = \int \frac{(g\tan\left(fx+e\right))^p}{b\sin\left(fx+e\right)+a}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e)),x, algorithm="giac")
```

output
```
integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a), x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g\tan(e+fx))^p}{a+b\sin(e+fx)}\,dx = \int \frac{(g\tan(e+f\,x))^p}{a+b\,\sin\left(e+f\,x\right)}\,dx$$

input
```
int((g*tan(e + f*x))^p/(a + b*sin(e + f*x)),x)
```

output
```
int((g*tan(e + f*x))^p/(a + b*sin(e + f*x)), x)
```

3.207 $\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$

3.207.1 Optimal result

Integrand size = 23, antiderivative size = 737

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$

$$= \frac{a^2\cos(e+fx)\left(1-\cos^2(e+fx)\right)^{\frac{1}{2}(-1+q)}\left(1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)^{-2+\frac{3-q}{2}+\frac{1}{2}(-1+q)}\left(\left(2(a^2-b^2)+b^2(1+q)\cos^2(e\right.\right.}{}$$

$$-\frac{a^2\cos(e+fx)\left(1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)^{\frac{1}{2}(-1+q)}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{1-q}{2},\frac{3-q}{2},\frac{\cos^2(e+fx)-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}}{1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}}\right)\sin(e}{(a^2-b^2)^2 f(-1+q)}$$

$$+\frac{b^2\cos(e+fx)\left(1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)^{\frac{1}{2}(-1+q)}\text{Hypergeometric2F1}\left(\frac{1-q}{2},\frac{1-q}{2},\frac{3-q}{2},\frac{\cos^2(e+fx)-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}}{1-\frac{b^2\cos^2(e+fx)}{-a^2+b^2}}\right)\sin(e}{(a^2-b^2)^2 f(-1+q)}$$

$$-\frac{2ab\,\text{AppellF1}\left(\frac{1-q}{2},-\frac{q}{2},2,\frac{3-q}{2},\cos^2(e+fx),\frac{b^2\cos^2(e+fx)}{-a^2+b^2}\right)\cos(e+fx)\sin^2(e+fx)^{-q/2}(g\tan(e+fx))^q}{(a^2-b^2)^2 f(-1+q)}$$

output
```
1/2*a^2*cos(f*x+e)*(1-cos(f*x+e)^2)^(-1/2+1/2*q)/(1-b^2*cos(f*x+e)^2/(-a^2
+b^2))*((2*a^2-2*b^2+b^2*(1+q)*cos(f*x+e)^2)*HurwitzLerchPhi(a^2*cos(f*x+e
)^2/(a^2-b^2)/(-1+cos(f*x+e)^2),1,1/2-1/2*q)-b^2*(-1+q)*cos(f*x+e)^2*Hurwi
tzLerchPhi(a^2*cos(f*x+e)^2/(a^2-b^2)/(-1+cos(f*x+e)^2),1,3/2-1/2*q))*sin(
f*x+e)*(sin(f*x+e)^2)^(-1/2-1/2*q)*(g*tan(f*x+e))^q/(a^2-b^2)^2/(-a^2+b^2)
/f-a^2*cos(f*x+e)*(1-b^2*cos(f*x+e)^2/(-a^2+b^2))^(-1/2+1/2*q)*Hypergeomet
ric2F1(1/2-1/2*q,1/2-1/2*q,3/2-1/2*q,(cos(f*x+e)^2-b^2*cos(f*x+e)^2/(-a^2+
b^2))/(1-b^2*cos(f*x+e)^2/(-a^2+b^2)))*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2-1/2
*q)*(g*tan(f*x+e))^q/(a^2-b^2)^2/f/(-1+q)+b^2*cos(f*x+e)*(1-b^2*cos(f*x+e)
^2/(-a^2+b^2))^(-1/2+1/2*q)*Hypergeometric2F1(1/2-1/2*q,1/2-1/2*q,3/2-1/2*
q,(cos(f*x+e)^2-b^2*cos(f*x+e)^2/(-a^2+b^2))/(1-b^2*cos(f*x+e)^2/(-a^2+b^2
)))*sin(f*x+e)*(sin(f*x+e)^2)^(-1/2-1/2*q)*(g*tan(f*x+e))^q/(a^2-b^2)^2/f/
(-1+q)-2*a*b*AppellF1(1/2-1/2*q,-1/2*q,2,3/2-1/2*q,cos(f*x+e)^2,b^2*cos(f*
x+e)^2/(-a^2+b^2))*cos(f*x+e)*(g*tan(f*x+e))^q/(a^2-b^2)^2/f/(-1+q)/((sin(
f*x+e)^2)^(1/2*q))
```

3.207.2 Mathematica [A] (warning: unable to verify)

Time = 8.18 (sec) , antiderivative size = 695, normalized size of antiderivative = 0.94

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$

$$= \frac{}{f(1+p)(a+b\sin(e+fx))^2\left(a(2+p)\left((a^2+b^2)\,\text{Hypergeometric2F1}\left(1,\frac{1+p}{2},\frac{3+p}{2},\left(-1+\frac{b^2}{a^2}\right)\tan^2(e+f\right.\right.\right.}$$

input
```
Integrate[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]
```

output
```
(Cos[e + f*x]*Sin[e + f*x]*(g*Tan[e + f*x])^p*(a*(2 + p)*((a^2 + b^2)*Hype
rgeometric2F1[1, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*
b^2*Hypergeometric2F1[2, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]
^2]) + 2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -T
an[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e + f*x]))/(f*(1 + p)*(a
 + b*Sin[e + f*x])^2*(a*(2 + p)*((a^2 + b^2)*Hypergeometric2F1[1, (1 + p)/
2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] - 2*b^2*Hypergeometric2F1[2,
(1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2]) + 2*b*(-a^2 + b^2)*A
ppellF1[(2 + p)/2, -1/2, 2, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan
[e + f*x]^2]*Tan[e + f*x] + 2*b*(-a^2 + b^2)*(1 + p)*AppellF1[(2 + p)/2, -
1/2, 2, (4 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2]*Tan[e +
 f*x] + (2*b*(-a^2 + b^2)*(2 + p)*((-4 + (4*b^2)/a^2)*AppellF1[(4 + p)/2,
-1/2, 3, (6 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + Appe
llF1[(4 + p)/2, 1/2, 2, (6 + p)/2, -Tan[e + f*x]^2, (-1 + b^2/a^2)*Tan[e +
 f*x]^2])*Tan[e + f*x]^3)/(4 + p) + a*(2 + p)*(-2*b^2*(-Hypergeometric2F1[
2, (1 + p)/2, (3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + (1 - b^2/a^
2)*Tan[e + f*x]^2)^(-2)) + (a^2 + b^2)*(-Hypergeometric2F1[1, (1 + p)/2, (
3 + p)/2, (-1 + b^2/a^2)*Tan[e + f*x]^2] + (1 + (1 - b^2/a^2)*Tan[e + f*x]
^2)^(-1)))))
```

3.207.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}dx$$
$$\Big\downarrow \text{ 3211}$$
$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}dx$$

input
```
Int[(g*Tan[e + f*x])^p/(a + b*Sin[e + f*x])^2,x]
```

output
```
$Aborted
```

3.207.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3211
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*
x])^p, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.207.4 Maple [F]

$$\int \frac{(g\tan(fx+e))^p}{(a+b\sin(fx+e))^2}dx$$

input
```
int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)
```

output
```
int((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x)
```

3.207.5 Fricas [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(g\tan(fx+e))^p}{(b\sin(fx+e)+a)^2}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(-(g*tan(f*x + e))^p/(b^2*cos(f*x + e)^2 - 2*a*b*sin(f*x + e) - a^
2 - b^2), x)
```

3.207.6 Sympy [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(g\tan{(e+fx)})^p}{(a+b\sin{(e+fx)})^2}\,dx$$

input
```
integrate((g*tan(f*x+e))**p/(a+b*sin(f*x+e))**2,x)
```

output
```
Integral((g*tan(e + f*x))**p/(a + b*sin(e + f*x))**2, x)
```

3.207.7 Maxima [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(g\tan{(fx+e)})^p}{(b\sin{(fx+e)}+a)^2}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)
```

3.207.8 Giac [F]

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(g\tan{(fx+e)})^p}{(b\sin{(fx+e)}+a)^2}\,dx$$

input
```
integrate((g*tan(f*x+e))^p/(a+b*sin(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((g*tan(f*x + e))^p/(b*sin(f*x + e) + a)^2, x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g\tan(e+fx))^p}{(a+b\sin(e+fx))^2}\,dx = \int \frac{(g\tan(e+f\,x))^p}{(a+b\,\sin{(e+f\,x)})^2}\,dx$$

input
```
int((g*tan(e + f*x))^p/(a + b*sin(e + f*x))^2,x)
```

output
```
int((g*tan(e + f*x))^p/(a + b*sin(e + f*x))^2, x)
```

3.208 $\int (a+b\sin(e+fx))^m (g\tan(e+fx))^p \, dx$

3.208.1 Optimal result

Integrand size = 23, antiderivative size = 23

$$\int (a+b\sin(e+fx))^m (g\tan(e+fx))^p \, dx = \text{Int}((a+b\sin(e+fx))^m (g\tan(e+fx))^p, x)$$

output
```
Unintegrable((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

3.208.2 Mathematica [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin(e+fx))^m (g\tan(e+fx))^p \, dx = \int (a+b\sin(e+fx))^m (g\tan(e+fx))^p \, dx$$

input
```
Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

output
```
Integrate[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p, x]
```

3.208.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {3042, 3211}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (g\tan(e+fx))^p(a+b\sin(e+fx))^m\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (g\tan(e+fx))^p(a+b\sin(e+fx))^m dx$$

$$\Big\downarrow \text{ 3211}$$

$$\int (g\tan(e+fx))^p(a+b\sin(e+fx))^m dx$$

input
```
Int[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*x])^p,x]
```

output
```
$Aborted
```

3.208.3.1 Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3211
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Unintegrable[(a + b*Sin[e + f*x])^m*(g*Tan[e + f*
x])^p, x] /; FreeQ[{a, b, e, f, g, m, p}, x]
```

3.208.4 Maple [N/A] (verified)

Not integrable

Time = 0.72 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int (a+b\sin (fx+e))^{m} (g\tan (fx+e))^{p} dx$$

input
```
int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

output
```
int((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x)
```

3.208.5 Fricas [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (b\sin (fx+e)+a)^{m}(g\tan (fx+e))^{p}\,dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="fricas")
```

output
```
integral((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

3.208.6 Sympy [N/A]

Not integrable

Time = 78.50 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx=\int (g\tan (e+fx))^{p} (a+b\sin (e+fx))^{m}\,dx$$

input
```
integrate((a+b*sin(f*x+e))**m*(g*tan(f*x+e))**p,x)
```

output
```
Integral((g*tan(e + f*x))**p*(a + b*sin(e + f*x))**m, x)
```

3.208.7 Maxima [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx = \int (b\sin(fx+e)+a)^m(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="maxima")
```

output
```
integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

3.208.8 Giac [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx = \int (b\sin(fx+e)+a)^m(g\tan(fx+e))^p\,dx$$

input
```
integrate((a+b*sin(f*x+e))^m*(g*tan(f*x+e))^p,x, algorithm="giac")
```

output
```
integrate((b*sin(f*x + e) + a)^m*(g*tan(f*x + e))^p, x)
```

3.208.9 Mupad [N/A]

Not integrable

Time = 6.96 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int (a+b\sin(e+fx))^m(g\tan(e+fx))^p\,dx = \int (g\tan(e+f\,x))^p\,(a+b\,\sin(e+f\,x))^m\,dx$$

input
```
int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^m,x)
```

output
```
int((g*tan(e + f*x))^p*(a + b*sin(e + f*x))^m, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
      return 1
    elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
        return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
          return 1
        else:
          return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
        return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
      m1 = expnType(expn.args[0])
      m2 = expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
      return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
      m1 = max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
      m1 = max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
      m1 = max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
      m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
      return max(7,m1)
    elif str(expn).find("Integral") != -1:
      m1 = max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
        else:
            return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```